# 11. Light & Colour

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# Agenda

- Physics of Light.
- Interactions between light and the environment
- Integrating radiance
- Types of light
- Color theory
- Color spaces

# What's the Difference?

- Light is a physical phenomenon
  - Photons of varying wavelength
  - A spectral distribution
- Colour is a perceptual phenomenon
  - How the human eye interprets it
  - Based on the rods and cones in the eye
- What you learned in school is simplified

# Photons

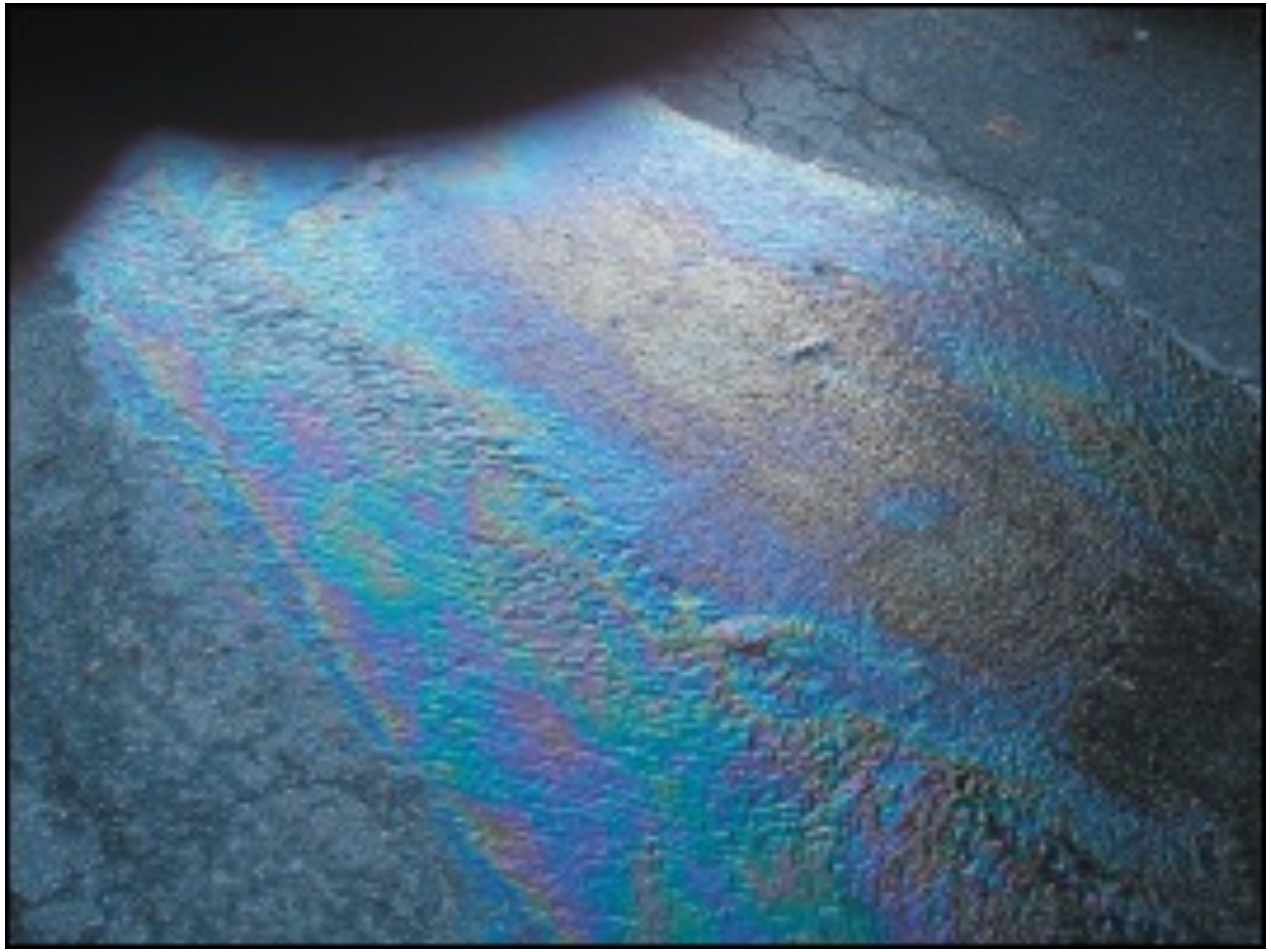
- Light is a particle – a photon
  - But also a (sine) wave with a frequency
- Energy of a photon is related to frequency:
  - $E = hf = \frac{hc}{\lambda}$
  - $h \approx 6.626 \times 10^{-34} \frac{kg\, m^2}{s}$ (Planck's constant)
  - $f = \frac{c}{\lambda}$ is the frequency
  - $c \approx 2.99 \times 10^{8} \frac{m}{s}$ is speed of light in a vacuum

# Units for Light

- Energy ($J$) depends on wavelength
- Power ($W$) is energy/unit time
- Irradiance ($W/m^2$) is power/unit area
  - Also known as radiosity
- Radiance ($W/m^2 sr$) is irradiance/unit angle
  - Conserved along a ray
  - Defers inverse-square law to surface

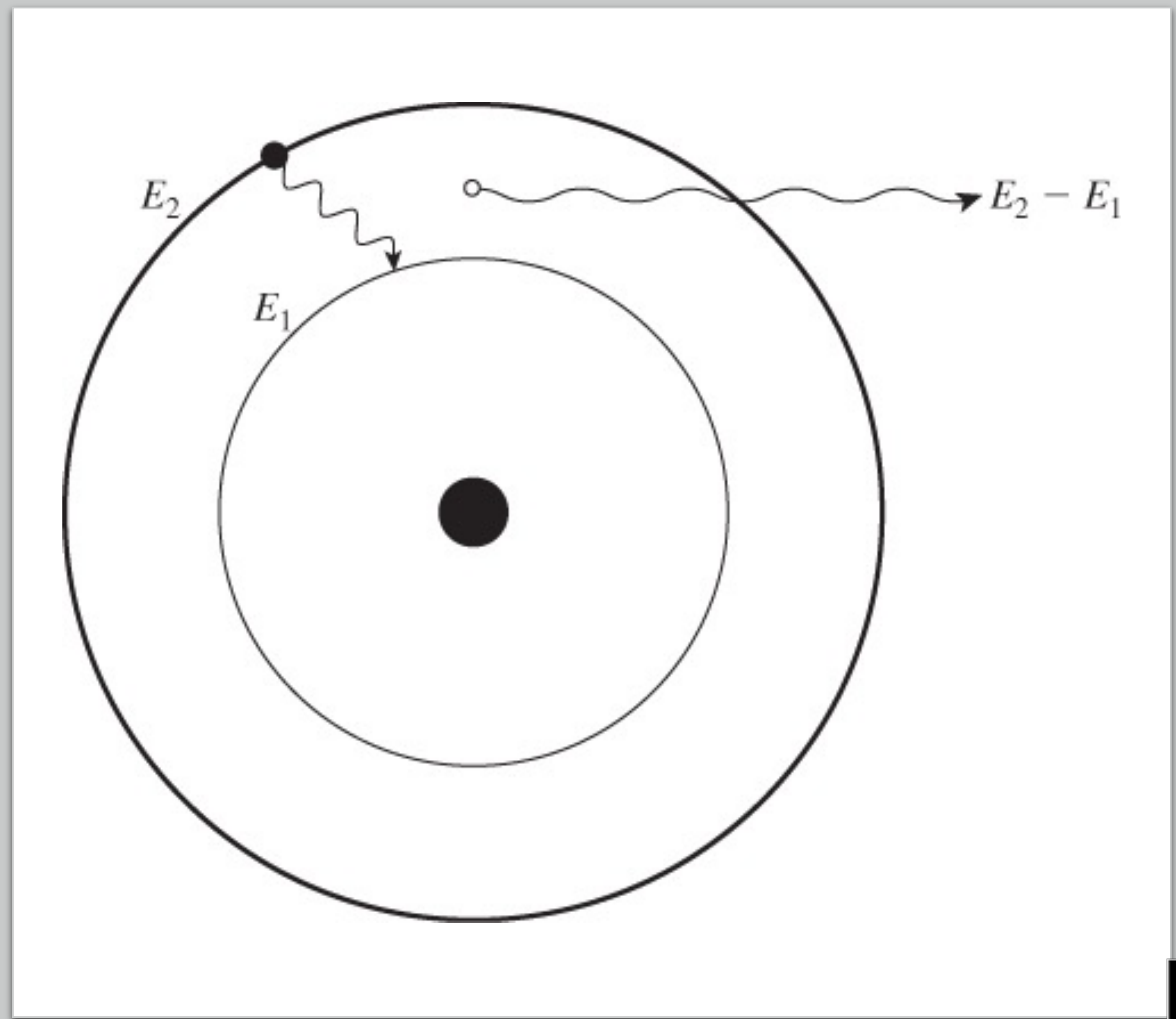
# Why This Matters

- Thin films generate rainbows of colour
- Many other phenomena also affected



# Atomic Orbitals

- Electrons orbit atoms
- Fixed radii called orbitals
  - Related to electron energy
- Electrons capture photons
  - Jump to higher orbitals
- Or they release photons
  - Drop to lower orbitals



# Photon Frequencies

- Single atom – very strict orbitals
  - Photons have specific frequencies
- Blocks of material have roaming electrons
  - So they have bands of frequencies
  - But photons still restricted to these bands

# Electron Effects

- Electrons may be tightly or loosely bound
  - Tight: photon released at same frequency
  - Loose: photon released at lower frequency
    - Spare energy absorbed as heat
  - Reflective material ⇔ conductors
  - Transparent material ⇔ insulators

# Fluorescence

- Energy absorbed, transformed to heat
- Electron releases lower-energy photon
- Lower energy => lower frequency
- Lower frequency => different colour
- UV light can drop to visible light
    - Fluorescence

# Time Delays

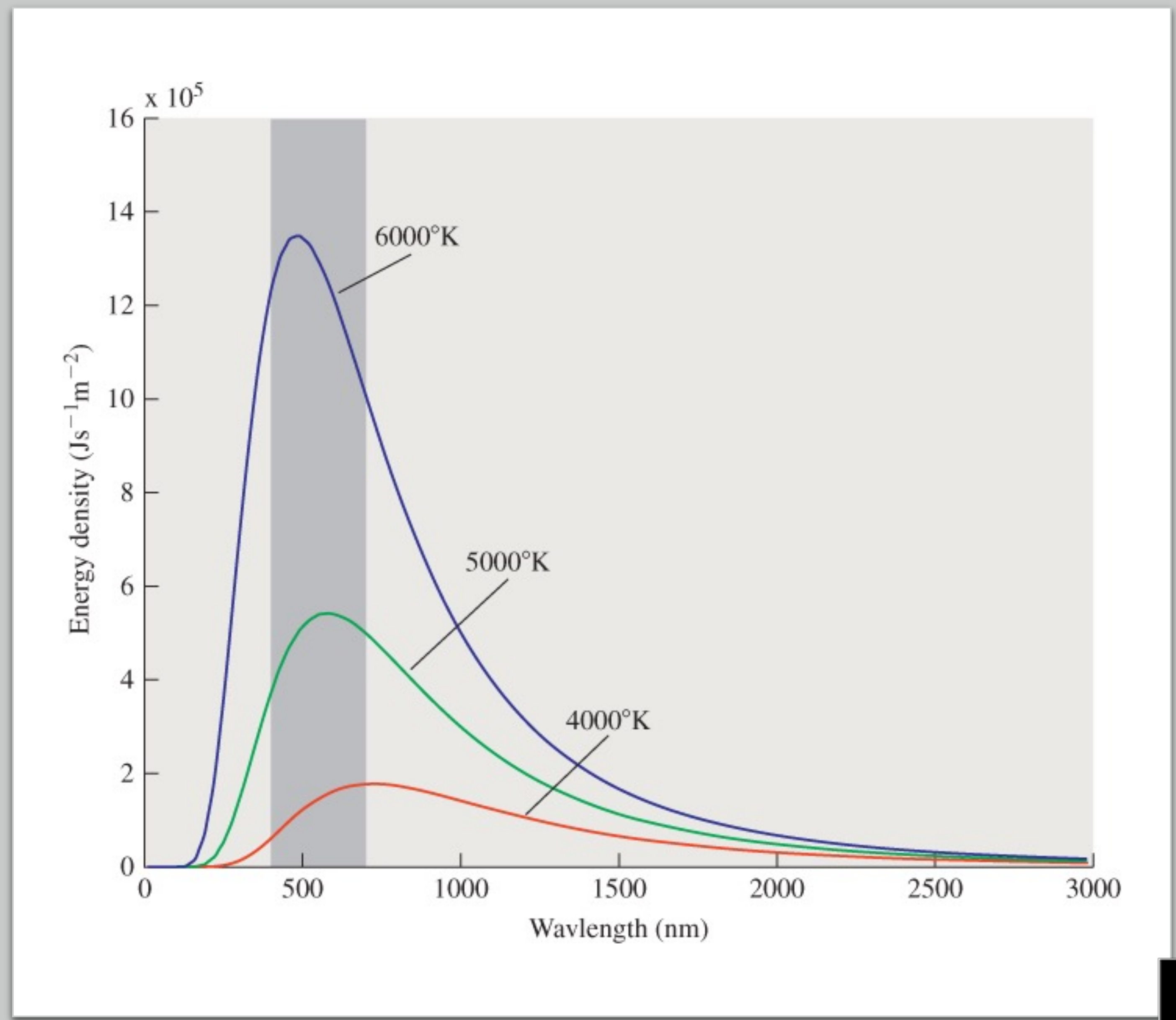
- Photon re-radiation is not immediate
- There is a time lag
- Small lag + re-emission of identical photon
  - Means the effective speed of light is slower
  - Different materials => different speeds
  - Known as virtual transition
- Large lag + energy loss
  - Phosphorescence

# Carbon

- Carbon has loosely bound electrons
  - Can absorb photons of many frequencies
  - Store energy in vibrations (heat)
  - Dark materials warm up
- Reverse this to generate photons
  - Heat up soot so atoms vibrate
  - They start kicking out photons

# Black-body Radiation

- Heat up a black body
- Radiates *lots* of photons
- Radiation depends on temperature
- Stefan-Boltzmann law
  - $Power = \sigma T^4$
  - $\sigma = 5.67\times 10^{-8}\frac{W}{m^2K^4}$



# Wave Nature of Light

- Photons *also* behave like waves
  - wave length – distance between peaks
  - wave velocity – speed of peaks

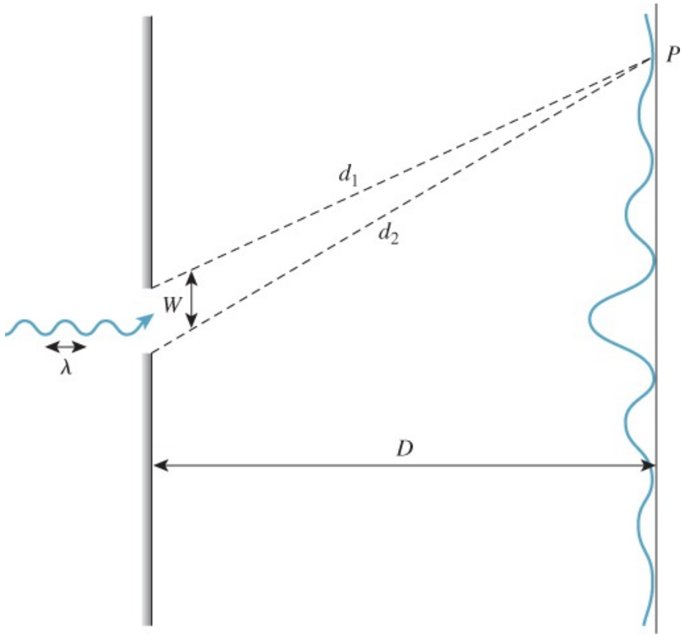
# Planar Waves

- Generalisation of sine waves to space
- Light wave has an associated electric field
- So, a wave along x-direction is:

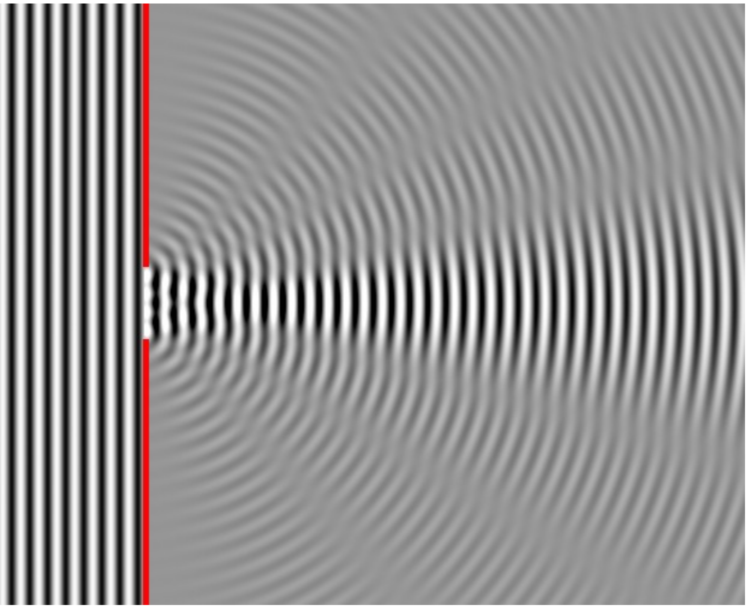
$$
\begin{aligned}
E_y(x,0,0,t) &= A_y sin\left(2\pi\frac{x}{\lambda} - 2\pi\frac{c}{\lambda}t + \Delta_y\right) \\
E_z(x,0,0,t) &= A_z sin\left(2\pi\frac{x}{\lambda} - 2\pi\frac{c}{\lambda}t + \Delta_z\right)
\end{aligned}
$$

- All others are just rotations of this

# Diffraction




At *one* distance D



At *all* distances

- Waves passing through a slit *diffract*
  - depending on width of slit

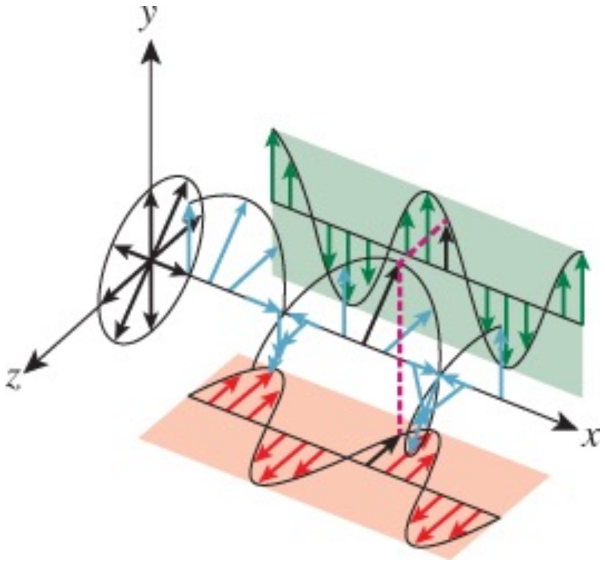
# Polarisation

- Change the origin in x or t
  - $\Delta y, \Delta z$ will change
  - But their difference will not
  - This is the basis of polarisation

$$
\begin{aligned}
E_x(x,0,0,t) &= 0 \\
E_y(x,0,0,t) &= A_y \sin\left(2\pi\frac{x}{\lambda} - 2\pi\frac{c}{\lambda}t + \Delta_y\right) \\
E_z(x,0,0,t) &= A_z \sin\left(2\pi\frac{x}{\lambda} - 2\pi\frac{c}{\lambda}t + \Delta_z\right)
\end{aligned}
$$

# Circular Polarisation

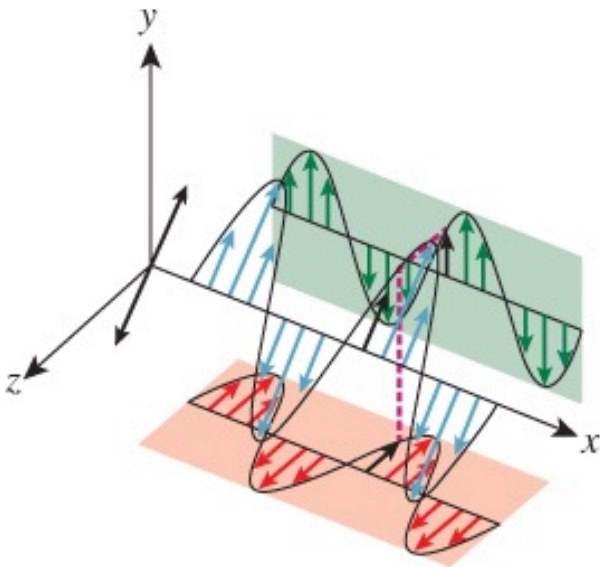
- y,z are out of phase
- Represents a helical light wave
- Projection sideways gives sine, cosine
- Projection forward gives a circle




$$
\begin{aligned}
\text{Let } t &= 0 \\
A_y &= A_z \\
\Delta_y &= 0 \\
\Delta_z &= \pi/2 \\
\text{Then} & \\
E_x(x,0,0,t) &= 0 \\
E_y(x,0,0,t) &= A_y \sin\left(2\pi\frac{x}{\lambda}\right) \\
E_z(x,0,0,t) &= A_z \cos\left(2\pi\frac{x}{\lambda}\right)
\end{aligned}
$$

# Linear Polarisation

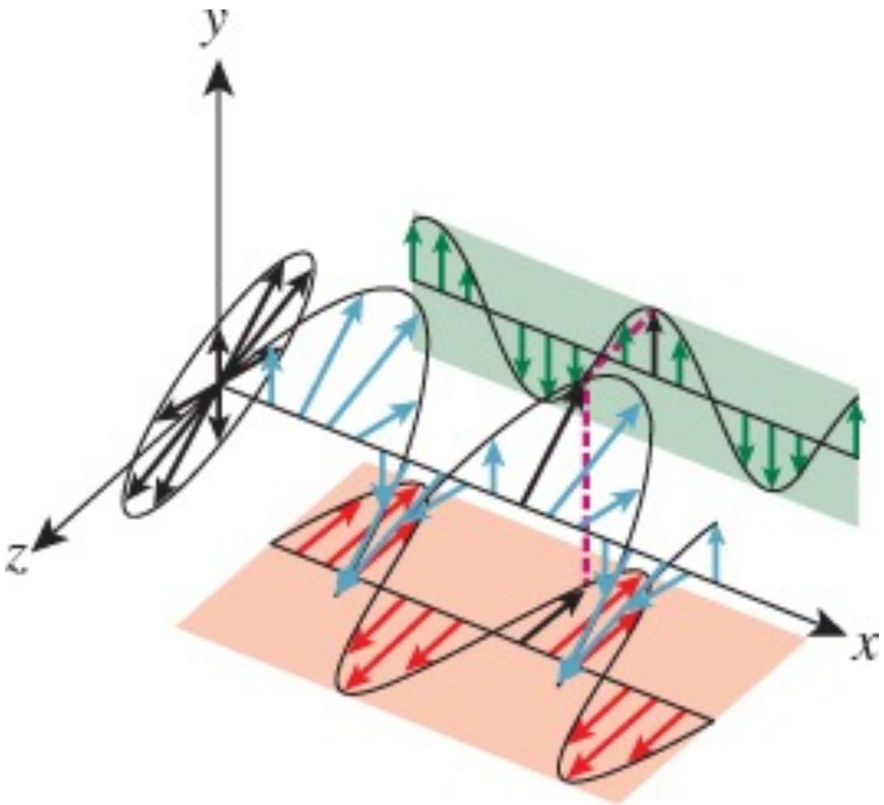
- y, z are in phase
- Light wave is planar in nature
- Projects to a straight line
    - And can have any axis $(0 \quad A_y \quad A_z)^T$




$$\begin{aligned}
\text{Let } t &= 0 \\
A_y &= A_z \\
\Delta_y &= 0 \\
\Delta_z &= 0 \\
\text{Then} & \\
E_x(x,0,0,t) &= 0 \\
E_y(x,0,0,t) &= A_y \sin\left(2\pi \frac{x}{\lambda}\right) \\
E_z(x,0,0,t) &= A_z \sin\left(2\pi \frac{x}{\lambda}\right)
\end{aligned}$$

# Elliptical Polarisation

- Any other case
  - a combination of circular and linear



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# Polarising Material

- Some material is polarising
  - Transparent to one polarisation
  - Opaque to opposite polarisation
- Commonly used for driving glasses
- Also used in LCD screens
- And in monitor glass
- More in a few minutes ...

# Refractive Index

- Recall that light speed depends on material
- The refractive index is the ratio with $c$
  - Vacuum: $s = 1.0000$
  - Air: $s = 1.0003$
  - Water: $s = 1.33$
  - Diamond: $s = 2.42$
- Actually depends on wavelength



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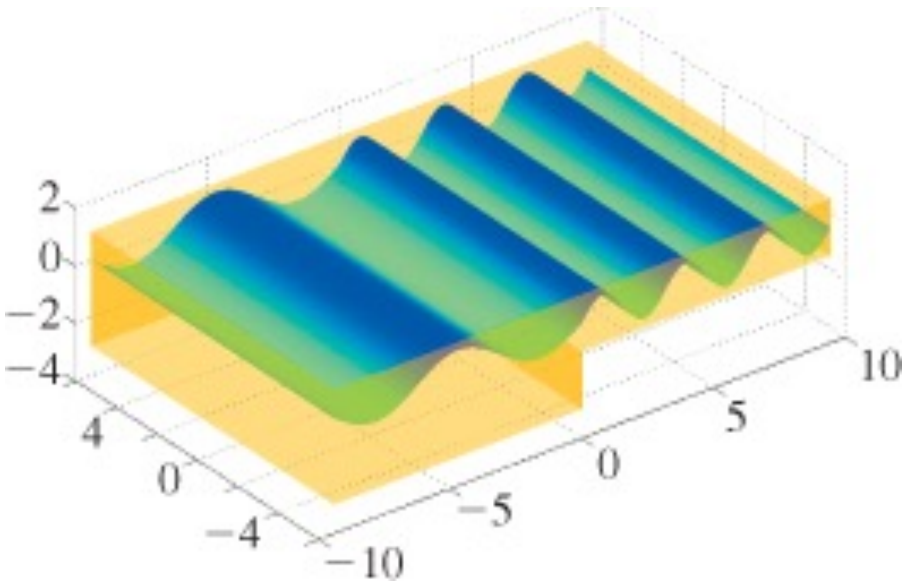
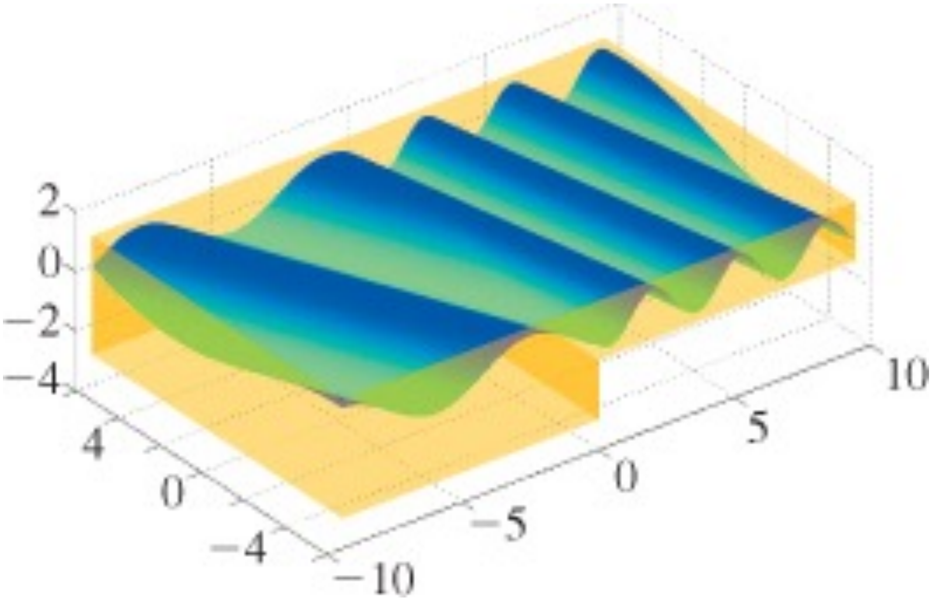
# Wave Retardation




Uniform




Slowed on One Side

- As waves slow down, they bunch up
- Frequency is constant, wavelength changes
- Slowing one side changes the direction



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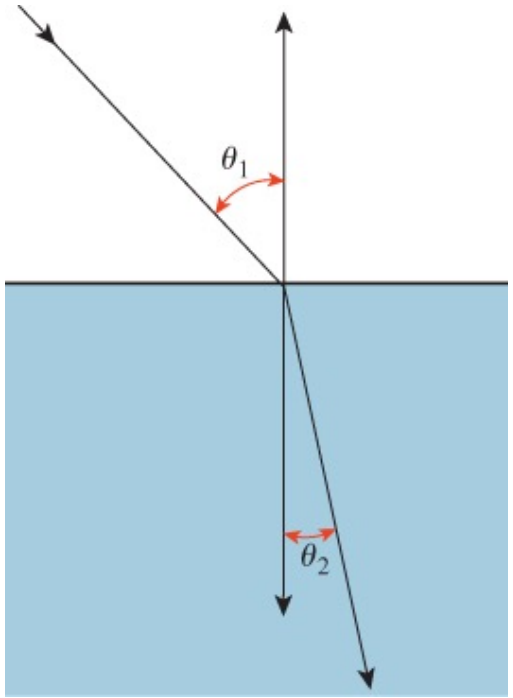
# Snell's Law

- Waves retard based on angle
- So this rotates the wave
  - Based on indices of refraction



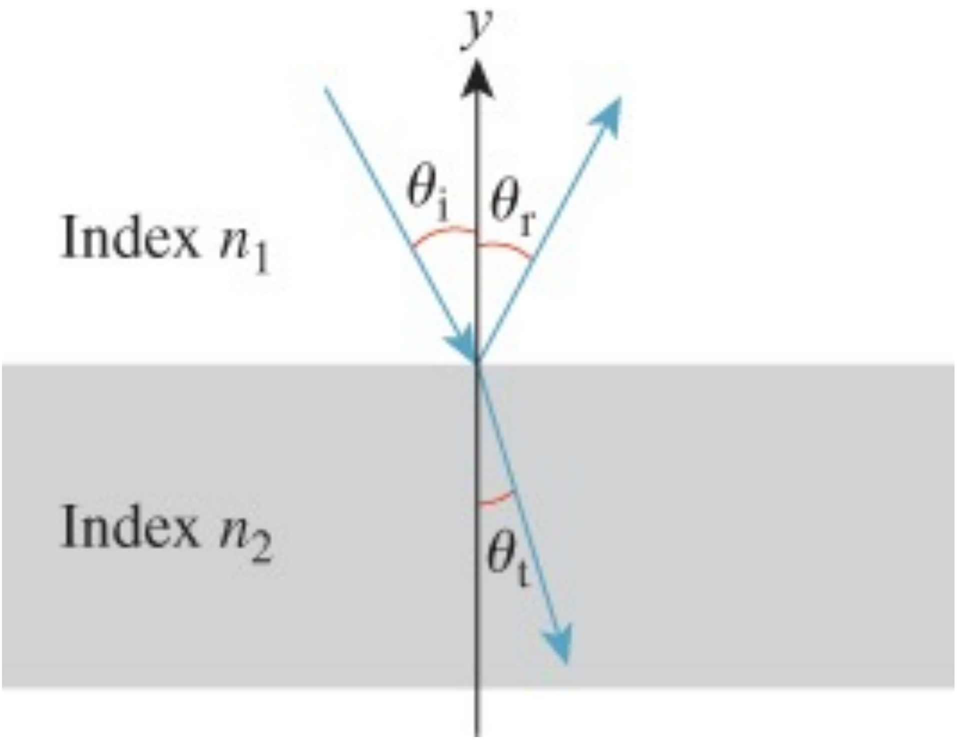

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

# Reflection & Refraction

- Not all light *can* refract
- So energy must go somewhere
  - Reflection
  - Refraction
  - Absorption
- Governed by Fresnel equations

# Polarised Reflection

- Light reflected gets polarised
  - p-polarisation:
    - parallel to surface
    - perpendicular to page
  - s-polarisation:
    - perpendicular to light
    - in plane of page



# Total Internal Reflection

- If angle is too steep, light *cannot* transmit
- All light reflects internally
- Basis of lenses, &c.



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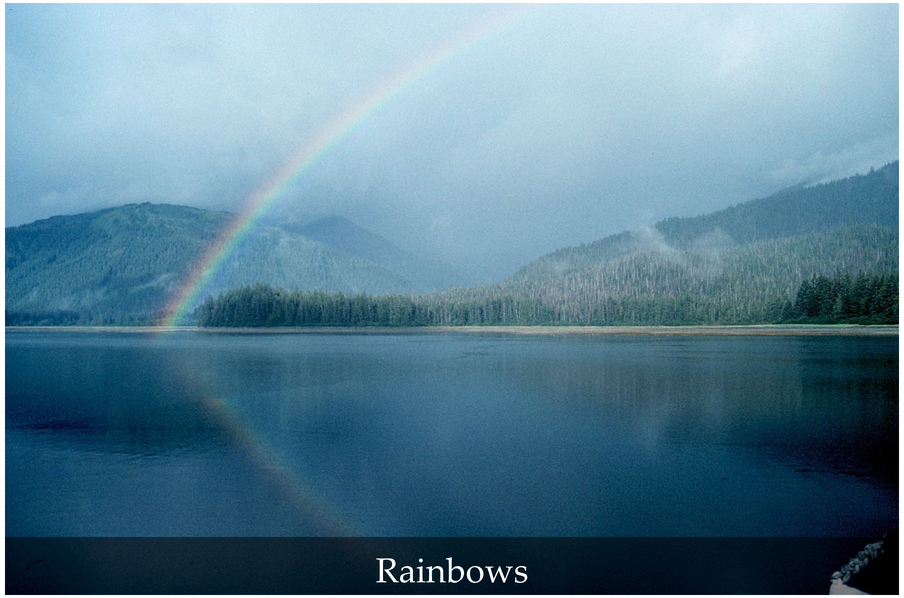
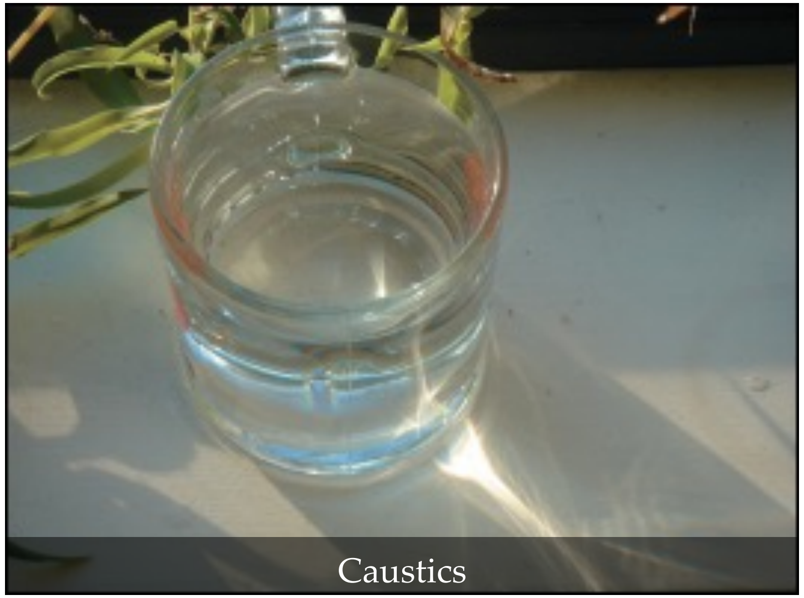
# Refractive Effects


Rainbows


CD-ROMs


Sundogs


Caustics

# Conductors

- Insulators tend to refract light
- Conductors tend to absorb then release
- Rate of release affects reflected light
  - Can be expressed as complex number
    - Real part = index of refraction
    - Imaginary part = coefficient of extinction $\kappa$

# Simpler Approach

- Treat refraction and extinction separately
- Fresnel reflectance then becomes

$$R_s = \frac{(n_2^2 + \kappa^2)\cos^2\theta_i - 2n_2\cos\theta_i + 1}{(n_2^2 + \kappa^2)\cos^2\theta_i + 2n_2\cos\theta_i + 1} \text{ and} \qquad (26.14)$$

$$R_p = \frac{(n_2^2 + \kappa^2) - 2n_2\cos\theta_i + \cos^2\theta_i}{(n_2^2 + \kappa^2) + 2n_2\cos\theta_i + \cos^2\theta_i}, \qquad (26.15)$$

- But this doesn't cover everything
  - Phenomena such as birefringence (refraction that depends on polarization)

## Without Polarisation

- Graphics usually assumes no polarisation
- Makes computations much easier
- Simplifies Fresnel equations to:

$$L(P, \omega_r) = R_F L(P, -\omega_i) \text{ and} \tag{26.18}$$

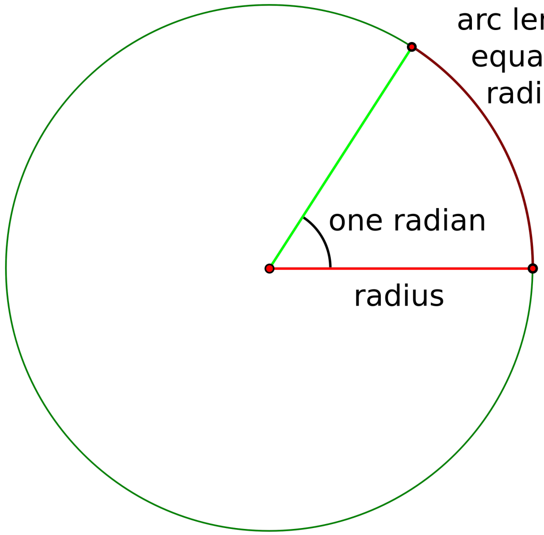
$$L(P, \omega_t) = (1 - R_F)\frac{n_2^2}{n_1^2} L(P, -\omega_i). \tag{26.19}$$

# Moving to Calculus

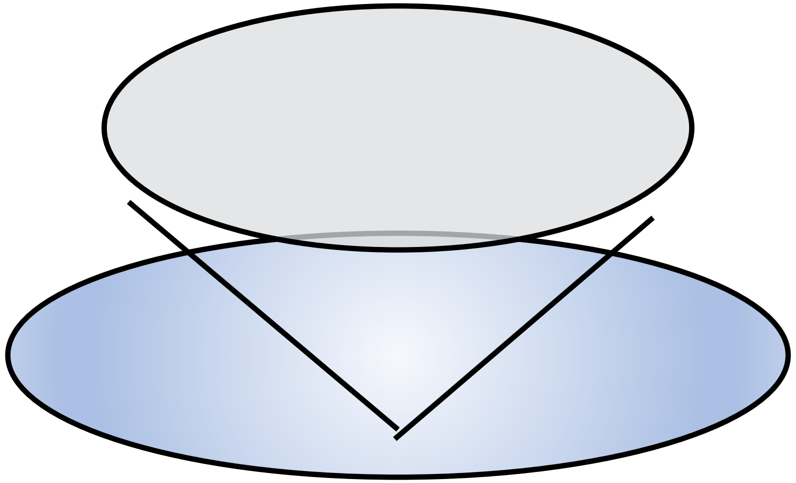
- All of this is per photon
- But there are many photons
  - At least $10^{23}$ in practice
- So we assume a continuous function
  - called a density function
- Provided that the integral comes out right

# Solid Angles

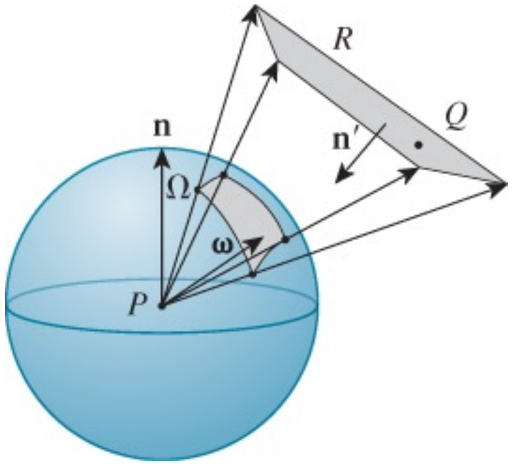
- A radian subtends a unit arc




- A steradian subtends a unit area



# Change of Variables




$$A = \int_{\omega_i \in \Omega} g(\omega_i)\ \omega_i \cdot \mathbf{n}\ d\omega \qquad (26.30)$$

becomes

$$A = \int_{Q \in R} g(N(Q))\ N(Q) \cdot \mathbf{n}\ |JN(Q)|\ dQ, \qquad (26.31)$$

becomes

$$A = \int_{s=0}^{w} \int_{t=0}^{h} g(\omega(s,t))\ \frac{|\omega(s,t) \cdot \mathbf{n}|\ |\omega(s,t) \cdot \mathbf{n}'|}{\|Q(s,t) - P\|^2}\ dt\ ds. \qquad (26.37)$$

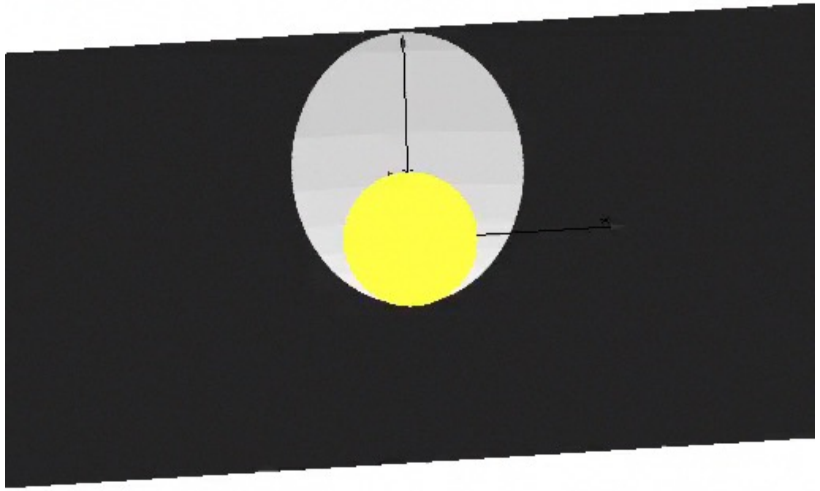
- We often have rectangular patches
- We want to integrate in steradians
- This involves a change of variables

# Integrating Radiance

- Energy is an integral of radiance
  - Over wavelengths from $\lambda_0$ to $\lambda_1$
  - Over all incoming directions $\omega$
  - Over a given pixel/patch $[x_0, x_1]\times[y_0, y_1]$
  - And over a time interval $[t_0, t_1]$

$$energy \approx \int_{t_0}^{t_1} \int_{x_0}^{x_1} \int_{y_0}^{y_1} \int_{\omega\in\Omega} \int_{\lambda_0}^{\lambda_1} L(t, (x, y, 0), -\omega, \lambda)\, d\lambda\, d\omega\, dy\, dx\, dt. \quad (26.39)$$

# But . . .



$$energy \approx \int_{t_0}^{t_1} \int_{x_0}^{x_1} \int_{y_0}^{y_1} \int_{\omega \in \Omega} \int_{\lambda_0}^{\lambda_1} L(t, (x, y, 0), -\omega, \lambda)\, d\lambda\, d\omega\, dy\, dx\, dt. \quad (26.39)$$

becomes

$$energy = \int_{t_0}^{t_1} \int_{x_0}^{x_1} \int_{y_0}^{y_1} \int_{\omega \in \Omega} \int_{\lambda_0}^{\lambda_1} L(t, (x, y, 0), -\omega, \lambda)\, \omega \cdot \mathbf{e}_3\, d\lambda\, d\omega\, dy\, dx\, dt. \quad (26.40)$$

becomes

$$energy = \int_{t_0}^{t_1} \int_{\lambda_0}^{\lambda_1} \int_{P \in R} \int_{\omega \in \Omega} L(t, P, -\omega, \lambda)\, |\omega \cdot \mathbf{n}|\, d\omega\, dP\, d\lambda\, dt, \quad (26.41)$$

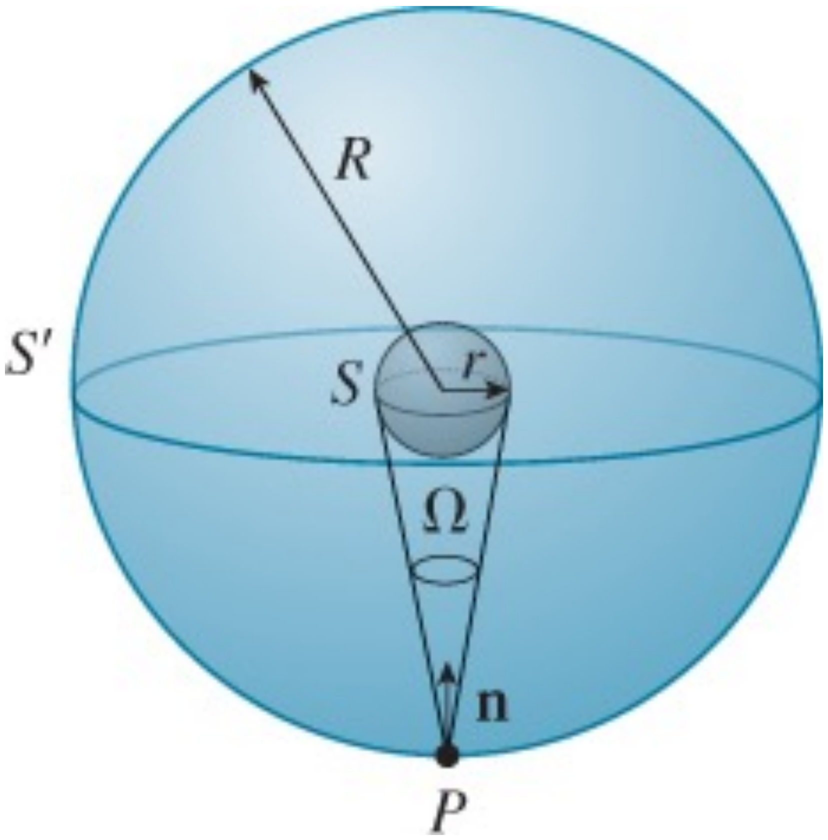
- The amount of light depends on the angle $\omega$
  - Dependence is as with diffuse Phong
  - On the dot product with the angle

# A Lambertian Emitter

- As P gets further from sphere S
  - Angle $\omega \cdot \vec{n} \to 1$ and

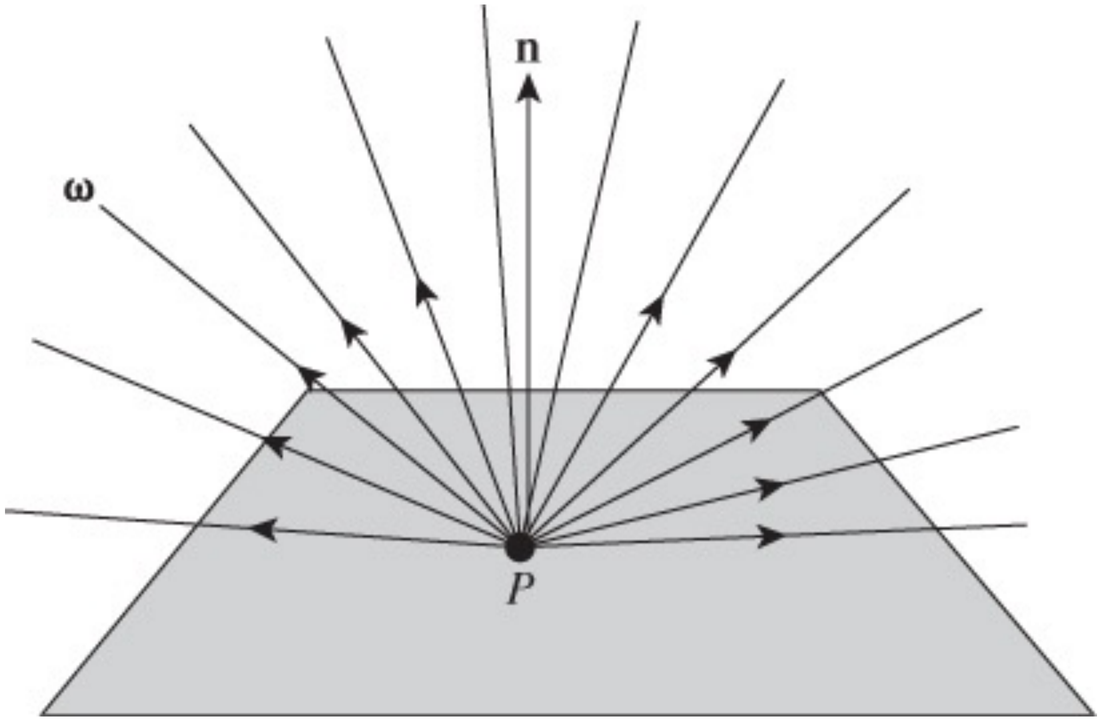
$$D = L \int 1 d\omega = L\, m(\Omega)$$

  - Substitute $m(\Omega) = \frac{\pi r^2}{R^2}$ and get $L = \frac{\Phi}{4\pi(\pi r^2)}$



# Irradiance

- Similarly, we can simplify the irradiance
- By substituting assumptions




$$E(t, P, \mathbf{n}, \lambda) = \int_{\{\omega\,:\,\omega_i \cdot \mathbf{n} \geq 0\}} L(t, P, -\omega_i, \lambda)\ \omega_i \cdot \mathbf{n}\ d\omega_i. \tag{26.64}$$

# Types of Light Source

- Sunlight (aka white light)
  - Black-body radiation at c. 6000°C
- Incandescent Bulb (Tungsten)
  - Black-body radiation at c. 2500°C
- Fluorescent Lamp (Mercury)
  - Stimulated emission with phosphorescence
- LED
  - Direct stimulated emission

# Spectral Distribution

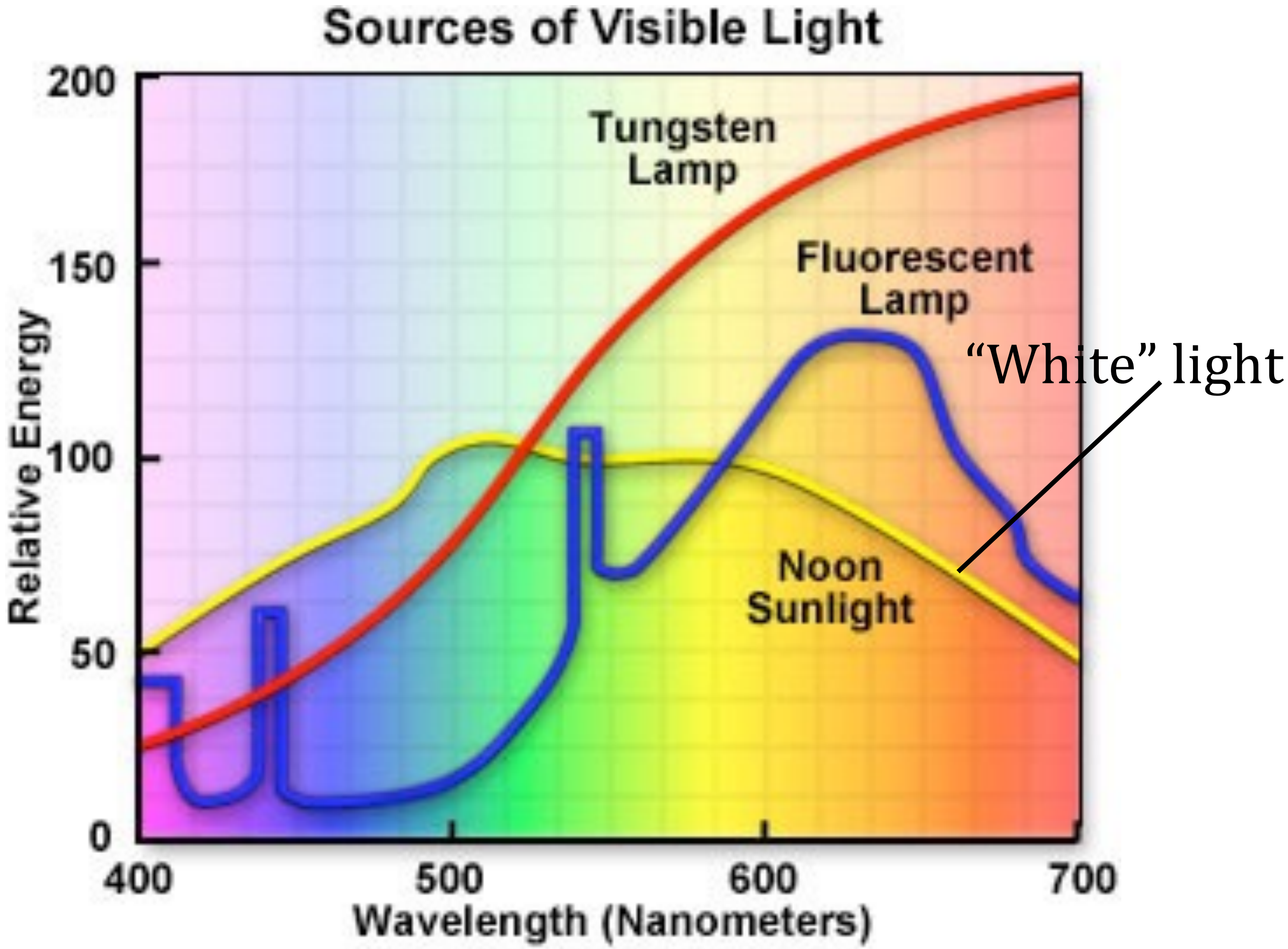
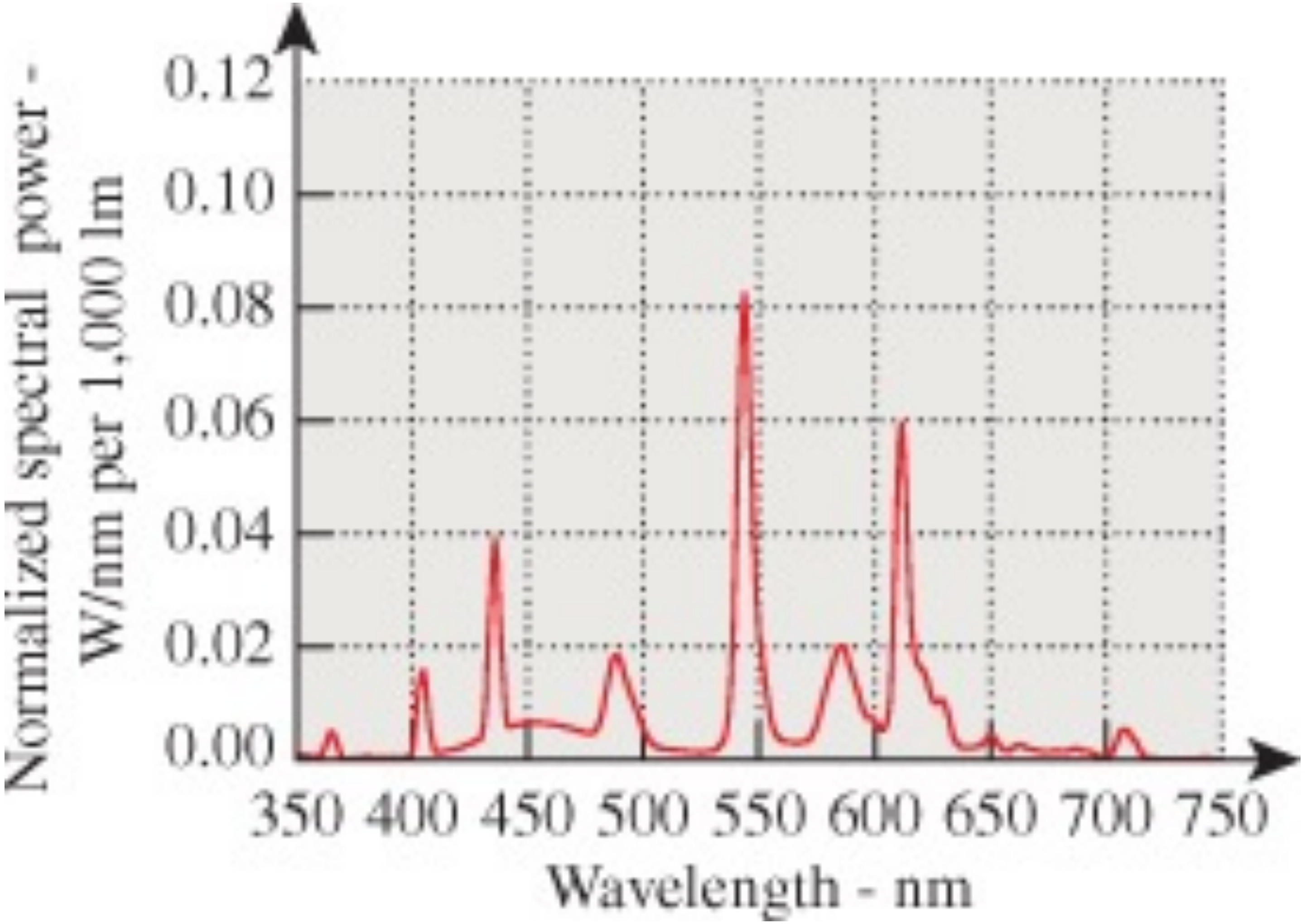


Figure 2

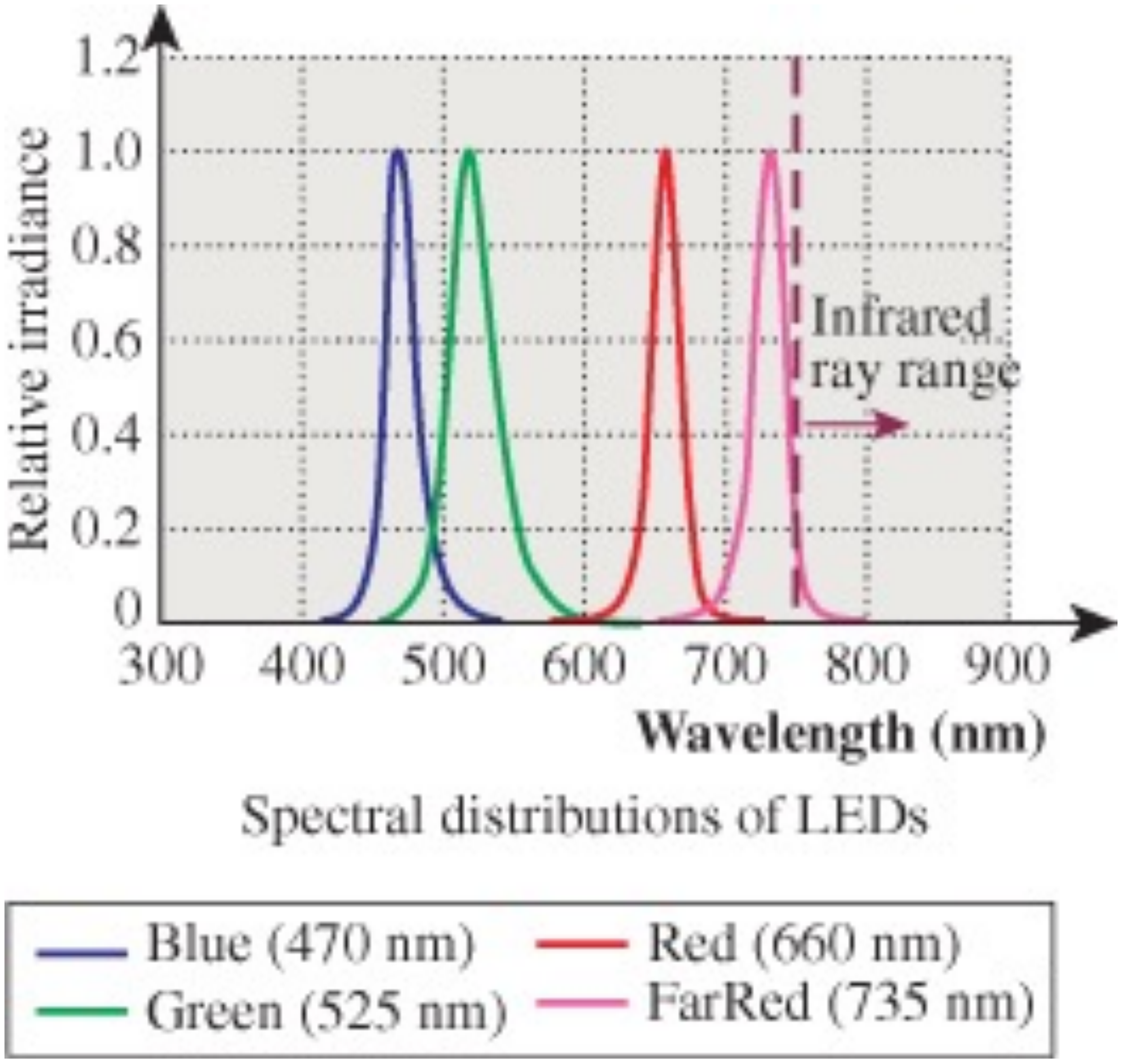
# A Fluorescent Lamp

- Note the spikes (impulses)
- Phosphors chosen so that *result* looks white
  - i.e. perceptually close to sunlight



# LED Lights

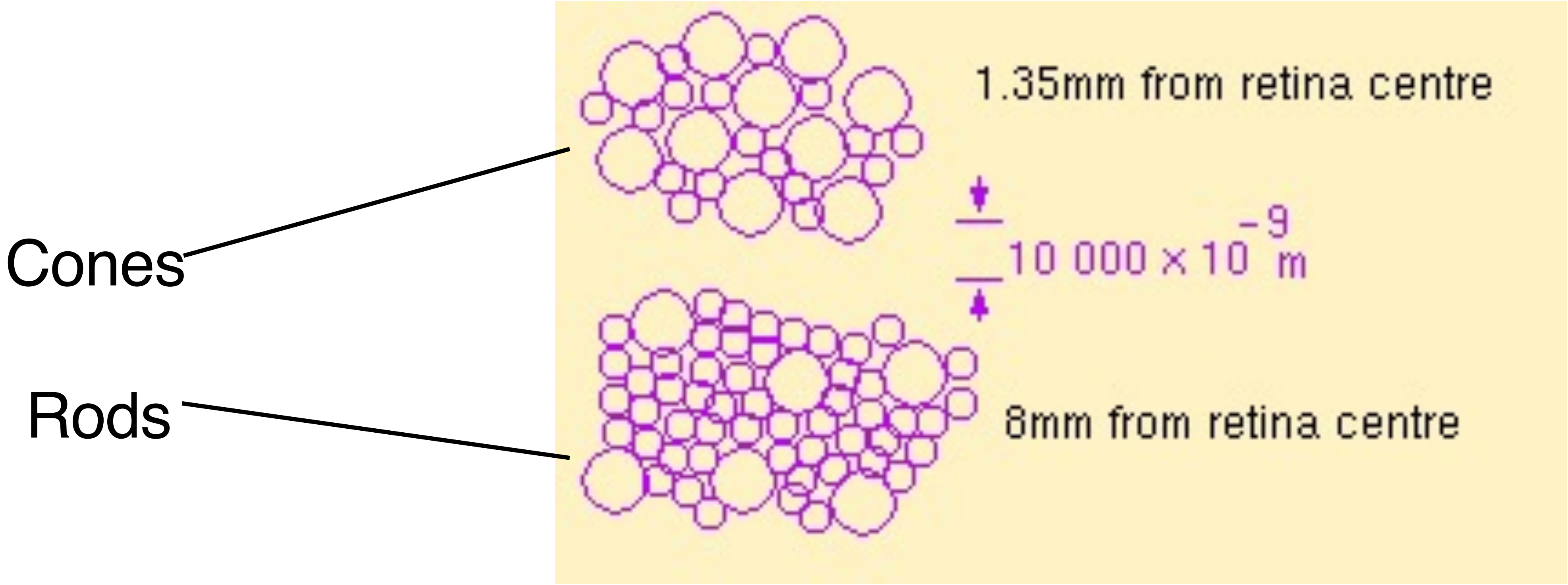
- LEDs are light-emitting diodes
- Close to *mono-spectral*
  - i.e. output only one wavelength



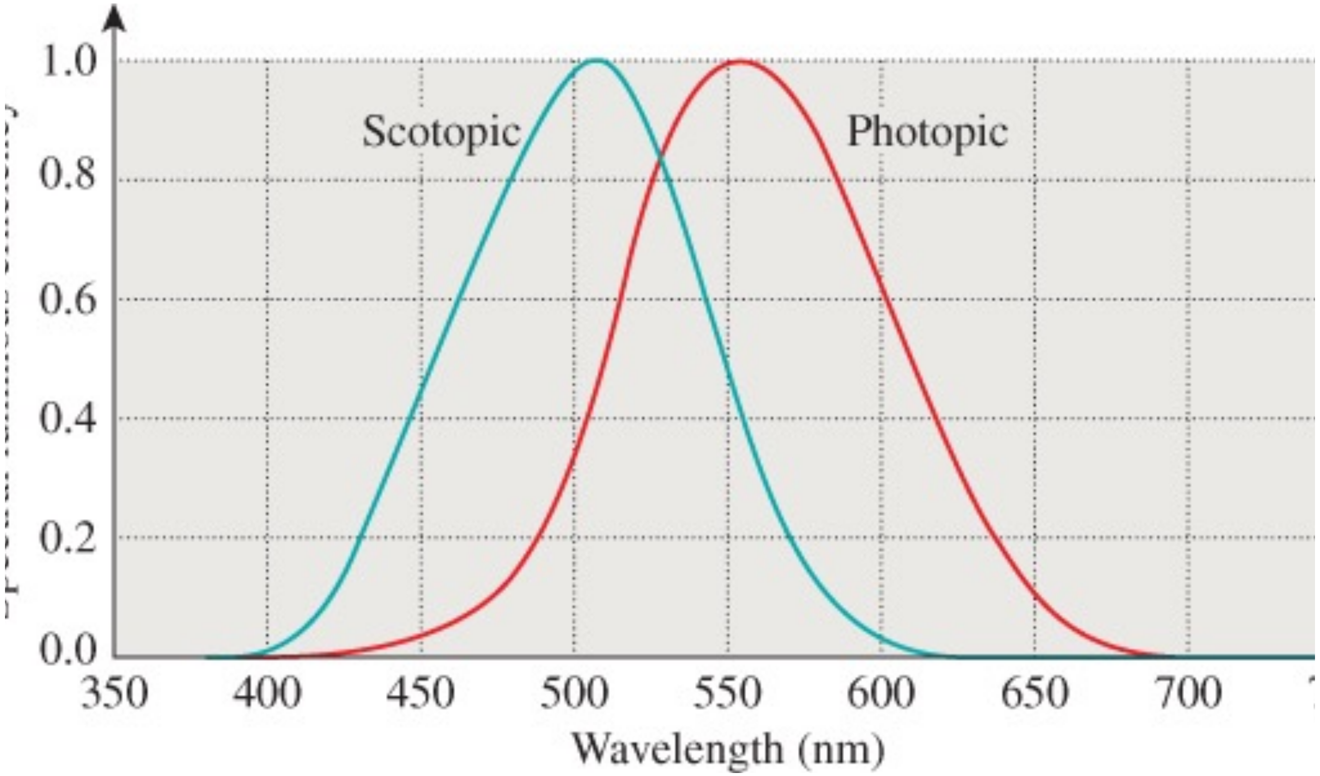

Spectral distributions of LEDs

# The Retina

- A patchwork of light-sensitive cells
  - Rods: low light conditions (B/W)
  - Cones: ordinary conditions (colour)



# Photopic & Scotopic Vision




- Photopic vision uses the cones
- Scotopic (night) vision uses the rods
- And their efficiency is different
    - Scotopic vision has trouble with red



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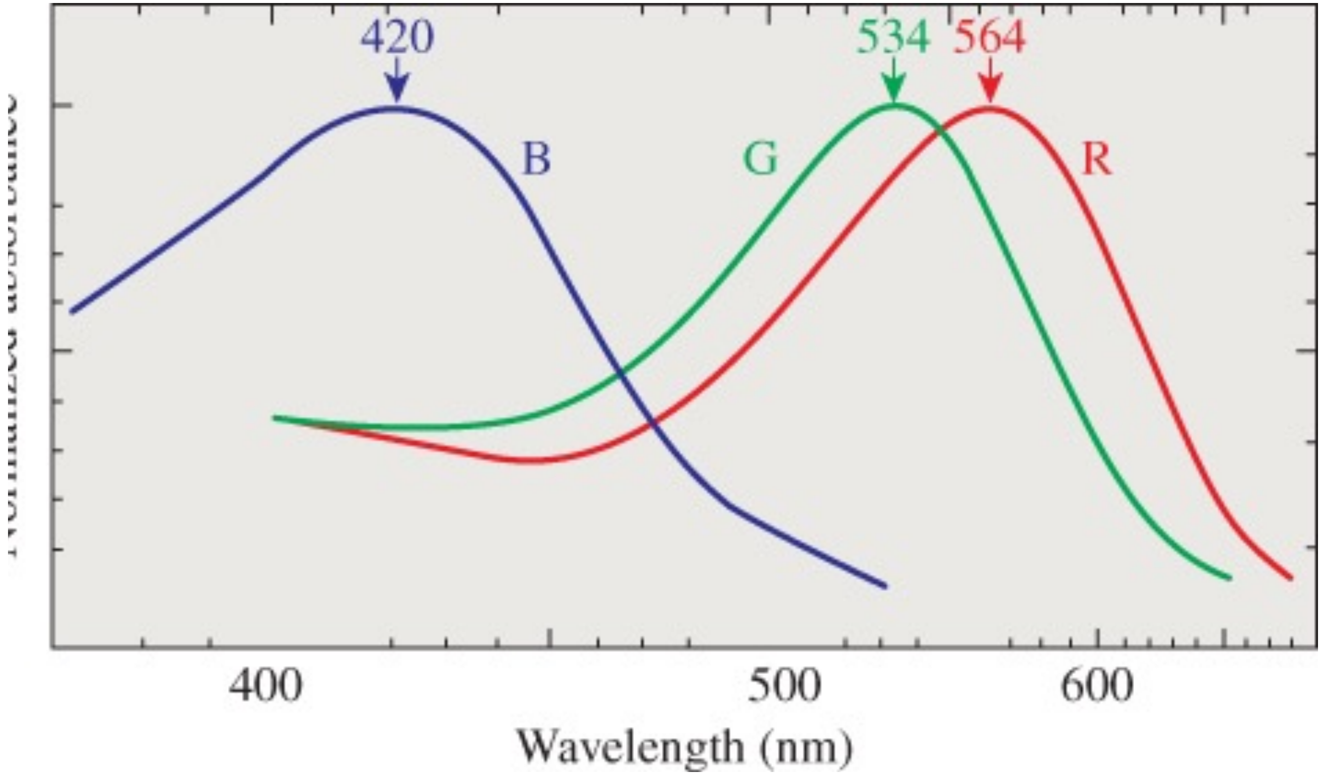
# Cones & Colour




- Three types of cones
- Respond to different wavelengths
    - Integrate over a *range* of wavelengths



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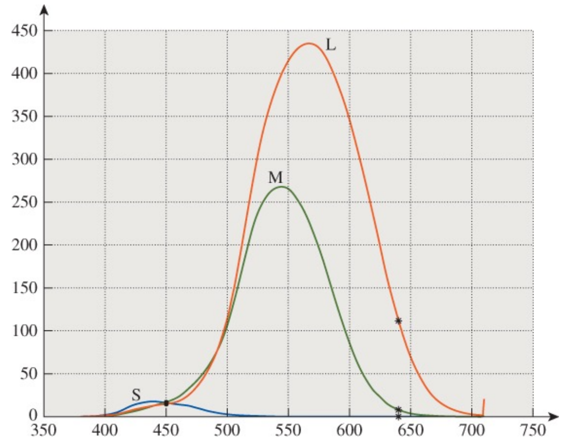
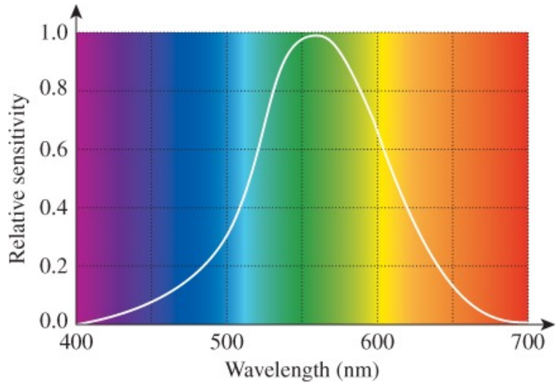
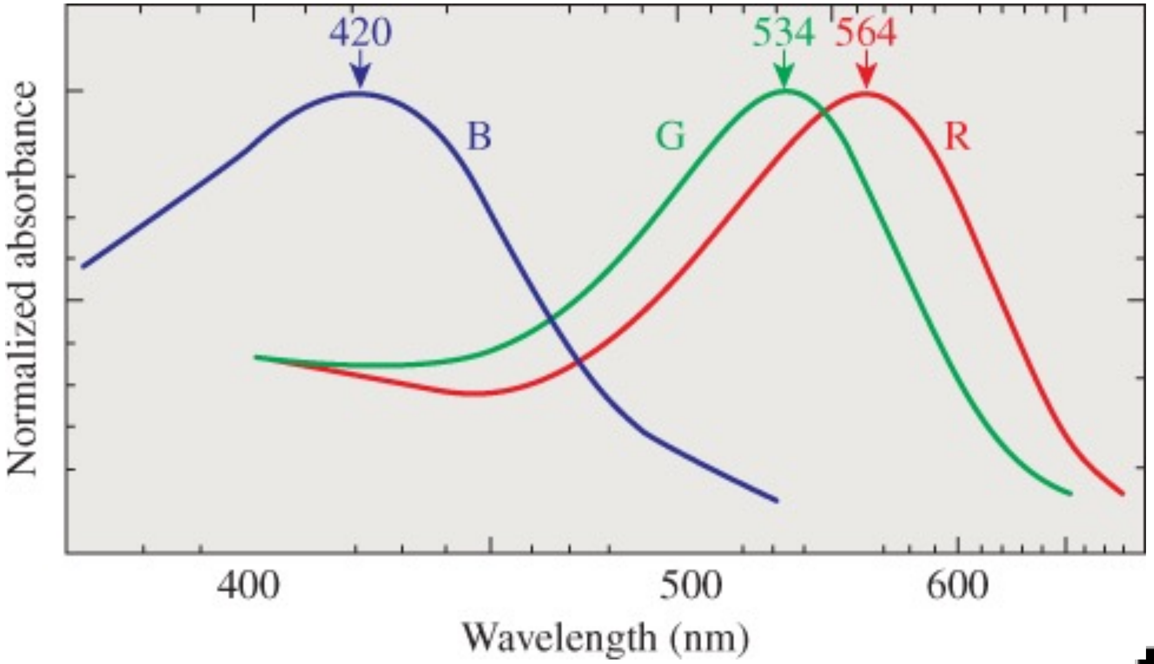
# Luminous Efficiency

- The eye works best with greenish light
- Other wavelengths fall off in response
- So blue cones are overall weak



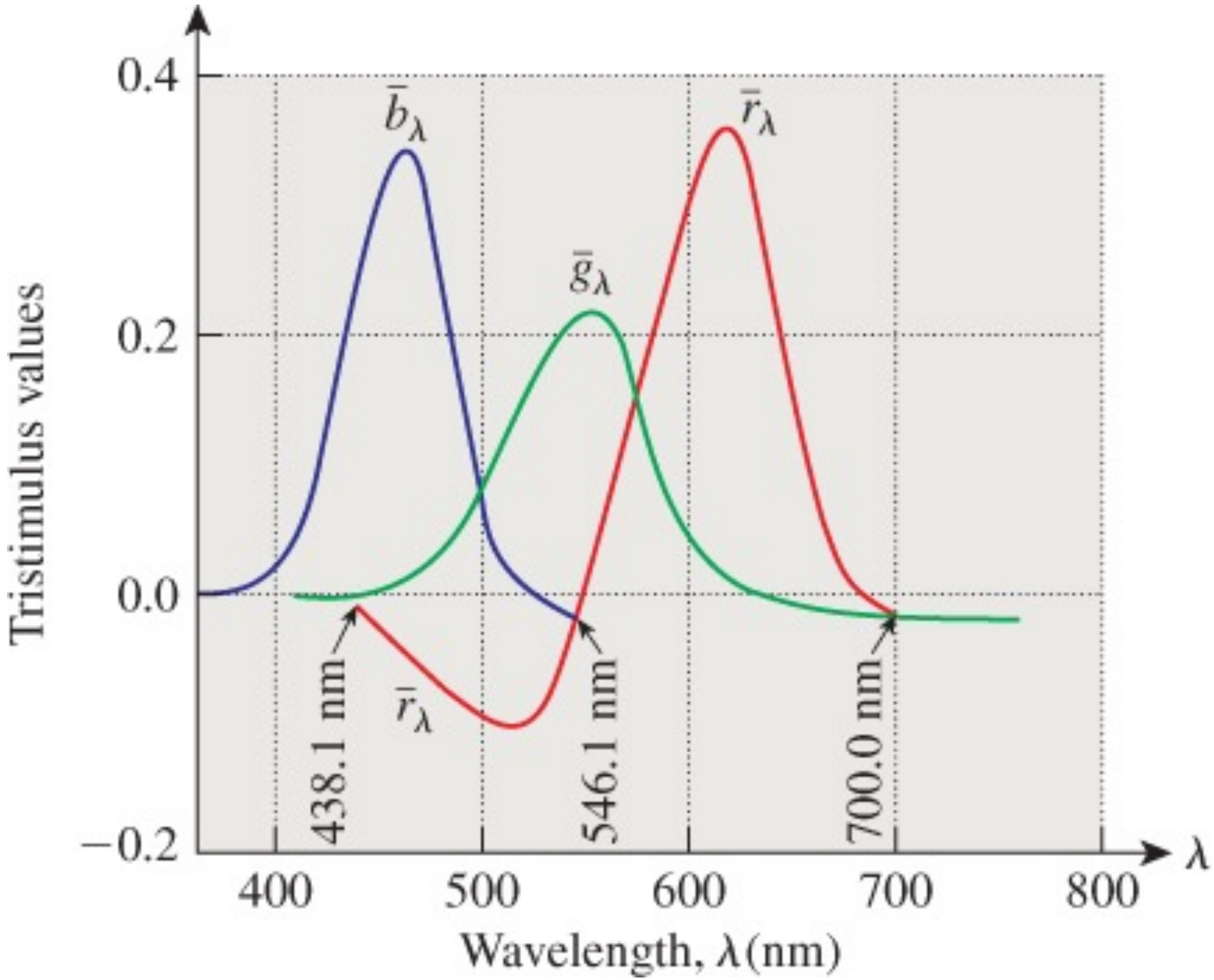



# Luminance

- Perceived brightness
- Integrate total energy over all wavelengths
  - $\int I(\lambda)e(\lambda)d\lambda$
- Notice we've now included human response
  - So this is based on perceptual studies
  - Averages over a population

# Tri-stimulus Theory

- Mix R, G & B to get any colour desired
- Human can't tell the difference
- But you need *negative* red light to do it



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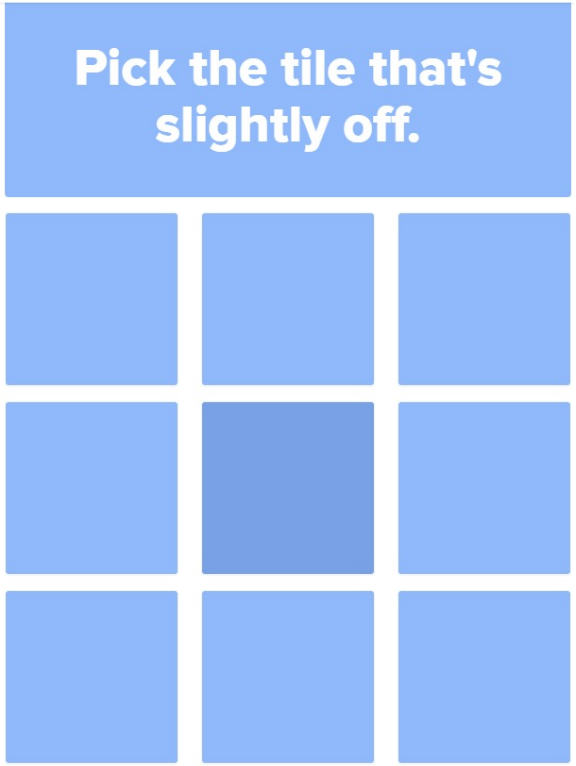
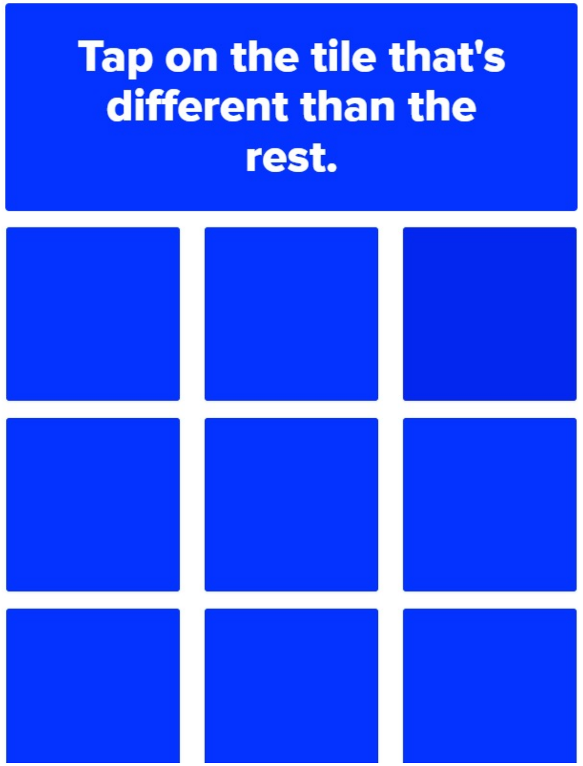
# Adaptation

- The eyes use slow chemical reactions
- But they adapt to the light level
- So their response is (mostly) logarithmic
- But too much light washes an image out
    - Even in the areas where it's dark

# Just Noticeable Difference

- Humans can just spot a 1% difference
  - at least in the range that displays use
- So we need about 100 logarithmic levels
- And 256 values is enough most of the time
- Hence our 8-bit RGB values
- But they’re not uniform
  - So we use gamma correction



# Radiance to Pixel Value

- OpenGL/PPM use a simple model:
  - 0-255 / 0.0 – 1.0 in each colour channel
- Human vision isn't like this
  - We see a wide range of intensity
    - Closer to 0.0 – 1,000,000
  - But it's logarithmic in nature
  - And we have to adjust for it

# Gamma encoding

- Assume we have
  - radiance *r*
  - min & max *possible* r
  - Δr = (max – min) / 1.0
- Then we compute

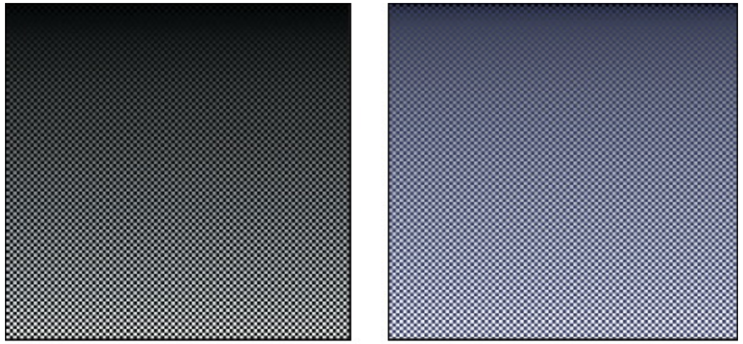
$$r' = \left[(r * \Delta r)^{1/\gamma}\right] * 255.0$$

- With γ = 2.2 (to match the human eye)

# Gamma Examples

Linear brightness Linear radiance

$\gamma = 2.0$



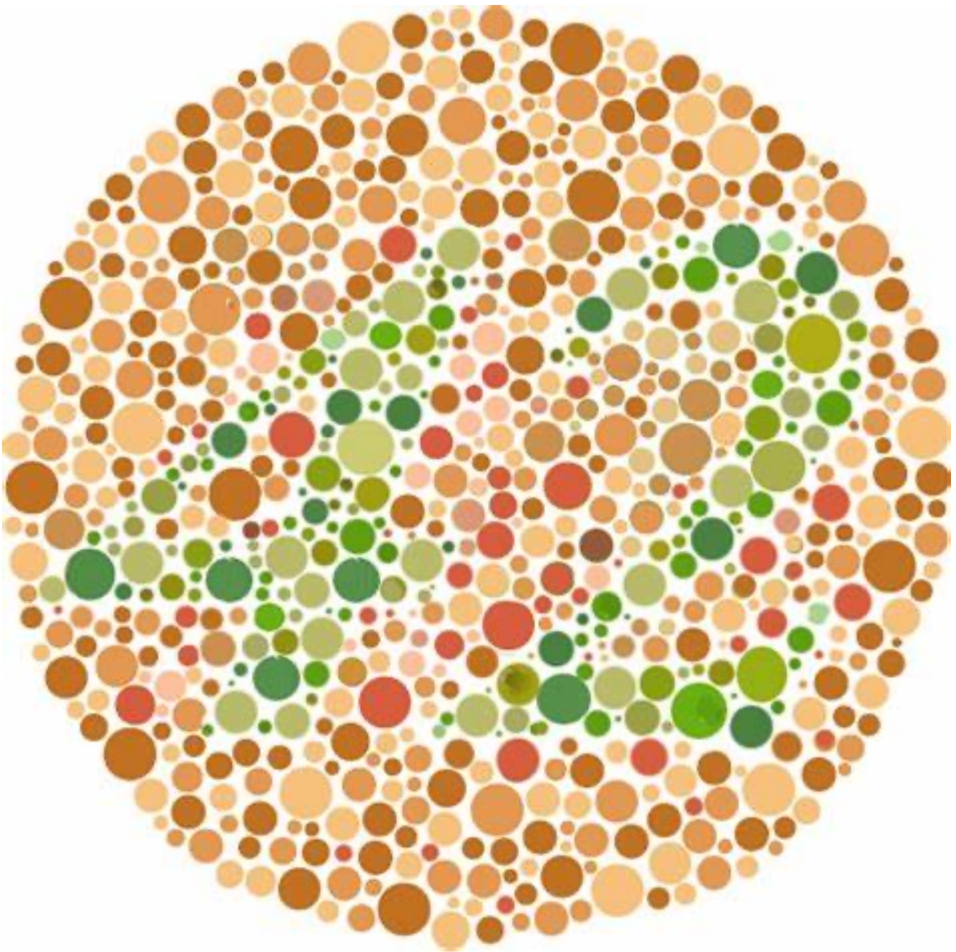
(a) (b)

$\gamma = 1.0$ $\gamma = 2.0$

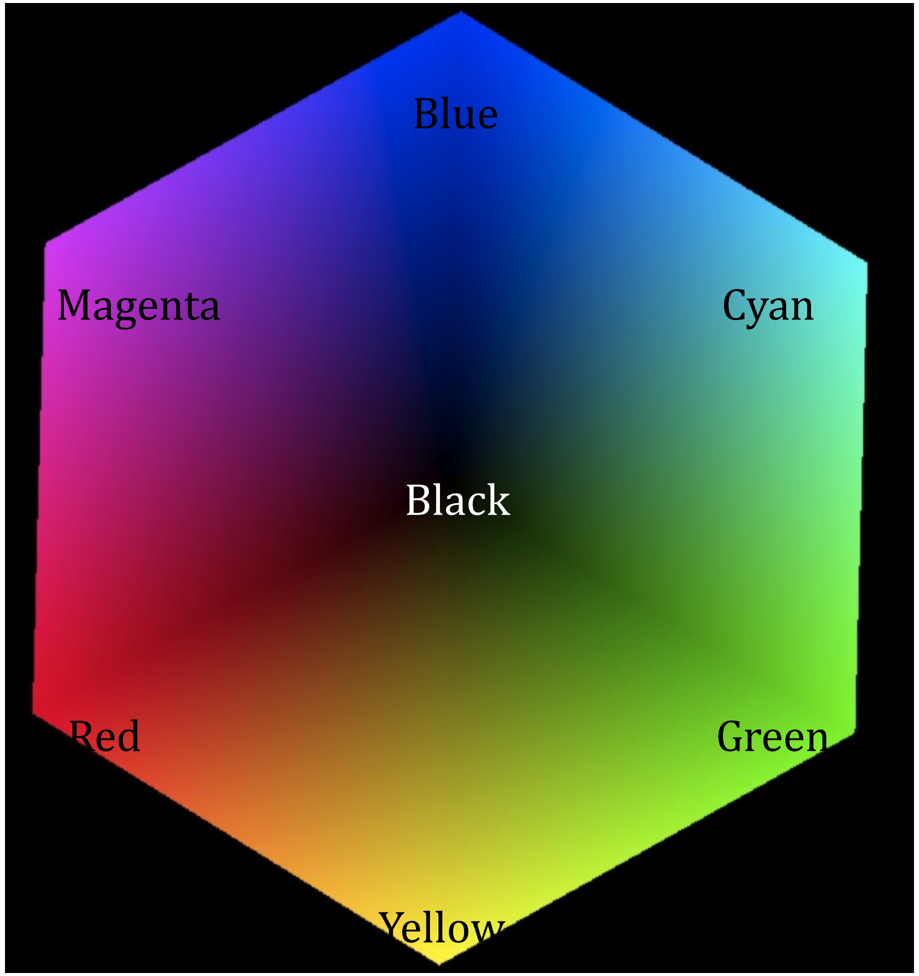
- (subject to monitor gamma, &c.)
- Compute in float
- Use gamma to convert to integers

# Colour Blindness

- About 8-10% of men, 1% of women
- Most common is red-green
  - only one cone for both red & green
- Yellow-blue also occurs
- And some people only have one type of cone
- Remember to design for colour blindness
  - NEVER rely only on colour

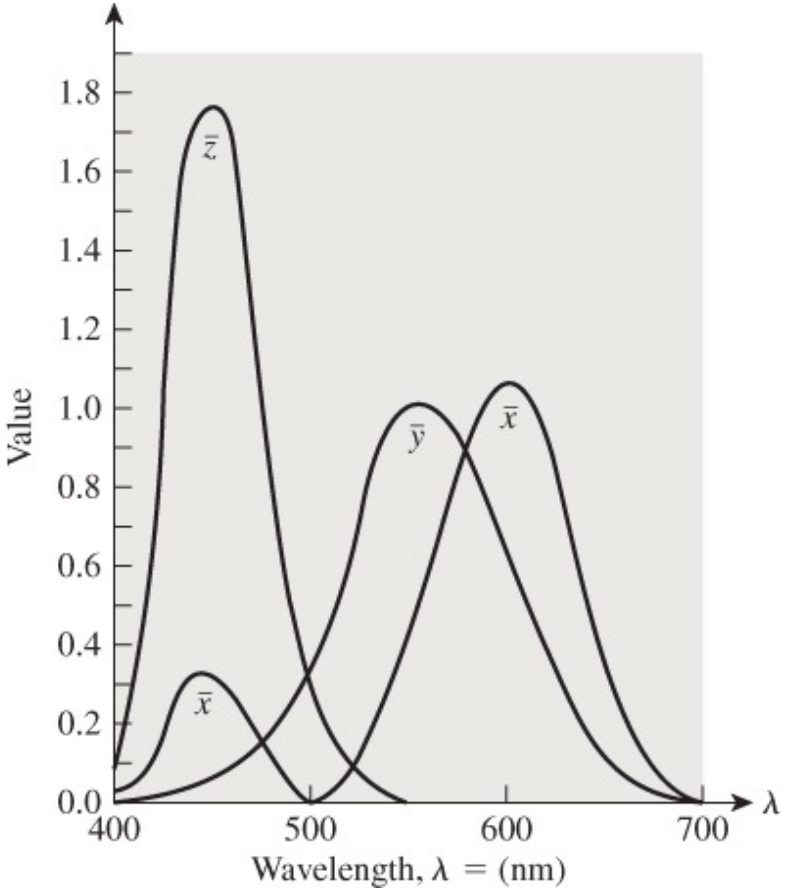


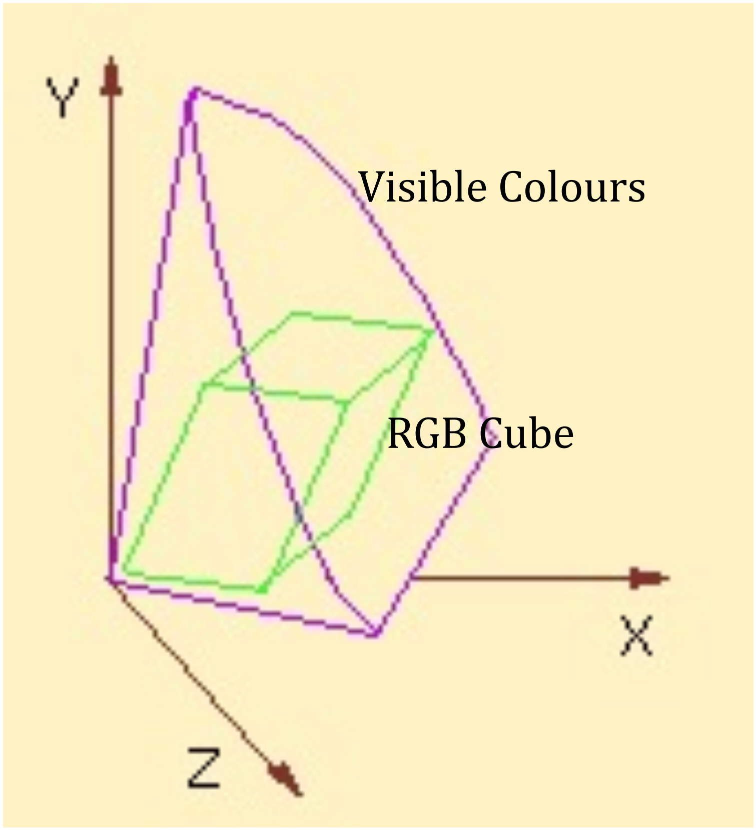
# RGB Colour Cube



# XYZ Colour Space

- By the CIE (Commission Internationale d'Eclairage)
- Every visible colour uses positive coords



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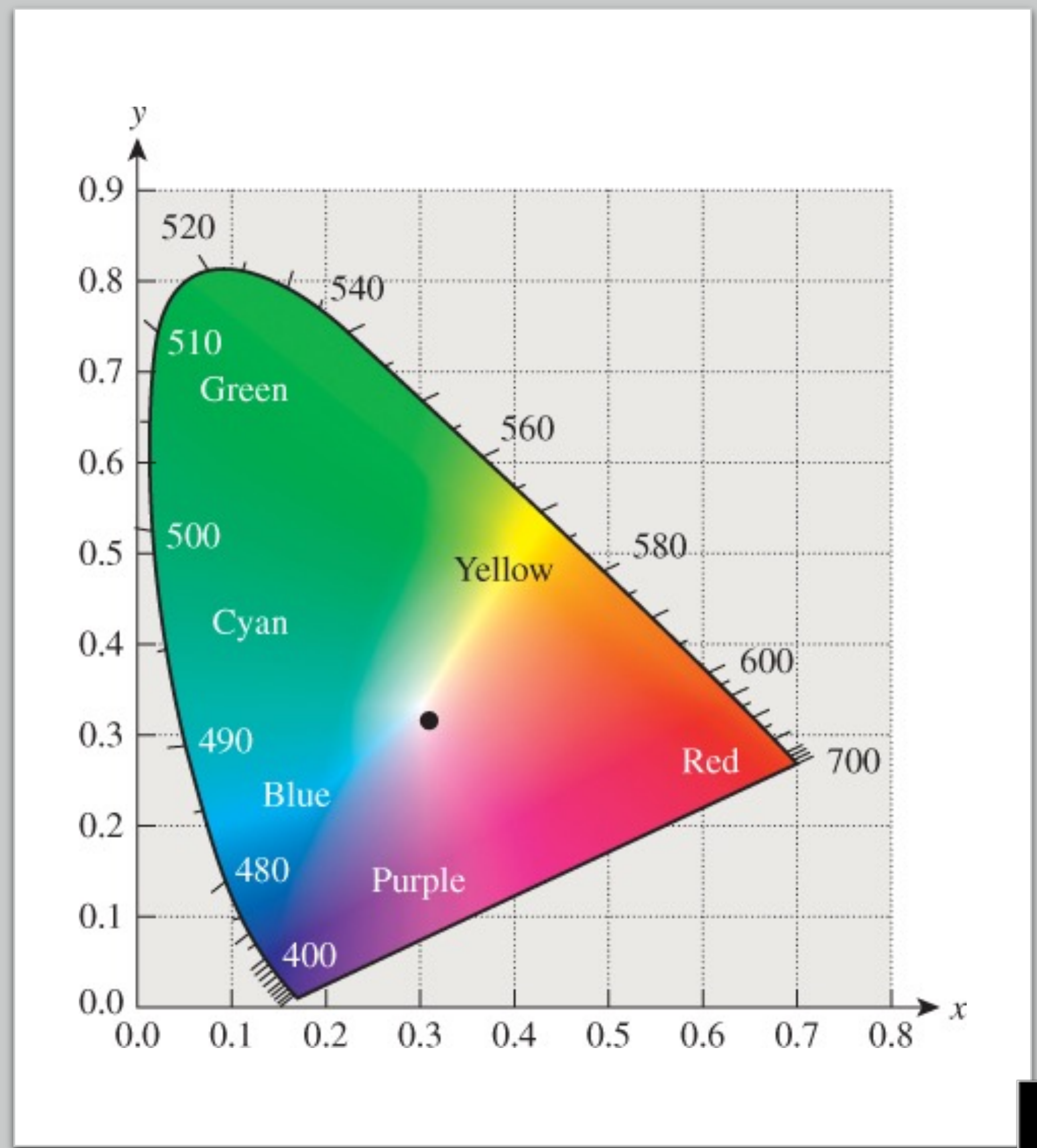
# RGB to XYZ

- Just a matrix transformation
- Converts between different colour systems

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2.36460 & -0.51515 & 0.00520 \\ -0.89653 & 1.42640 & -0.01441 \\ -0.46807 & 0.08875 & 1.00921 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

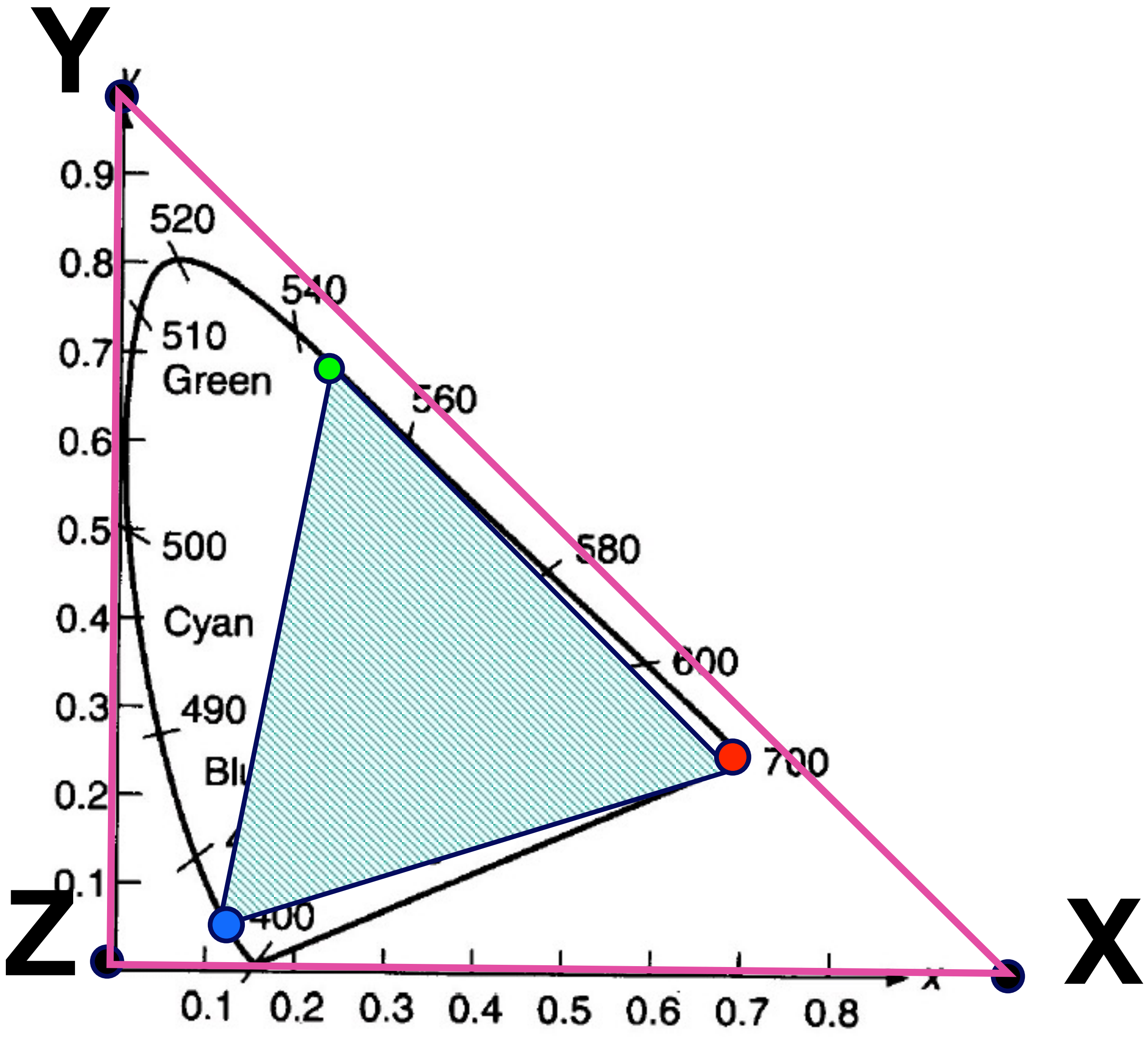
# CIE Chromaticity Diagram

- Project colour space to plane X + Y + Z = 1
- Pure colours appear along the edge



# sRGB Gamut

- Set of colours possible with standard RGB (sRGB)
- Convex hull of positions on the diagram
- Each device has a gamut
- Used to calibrate colour perception
- Still a major problem

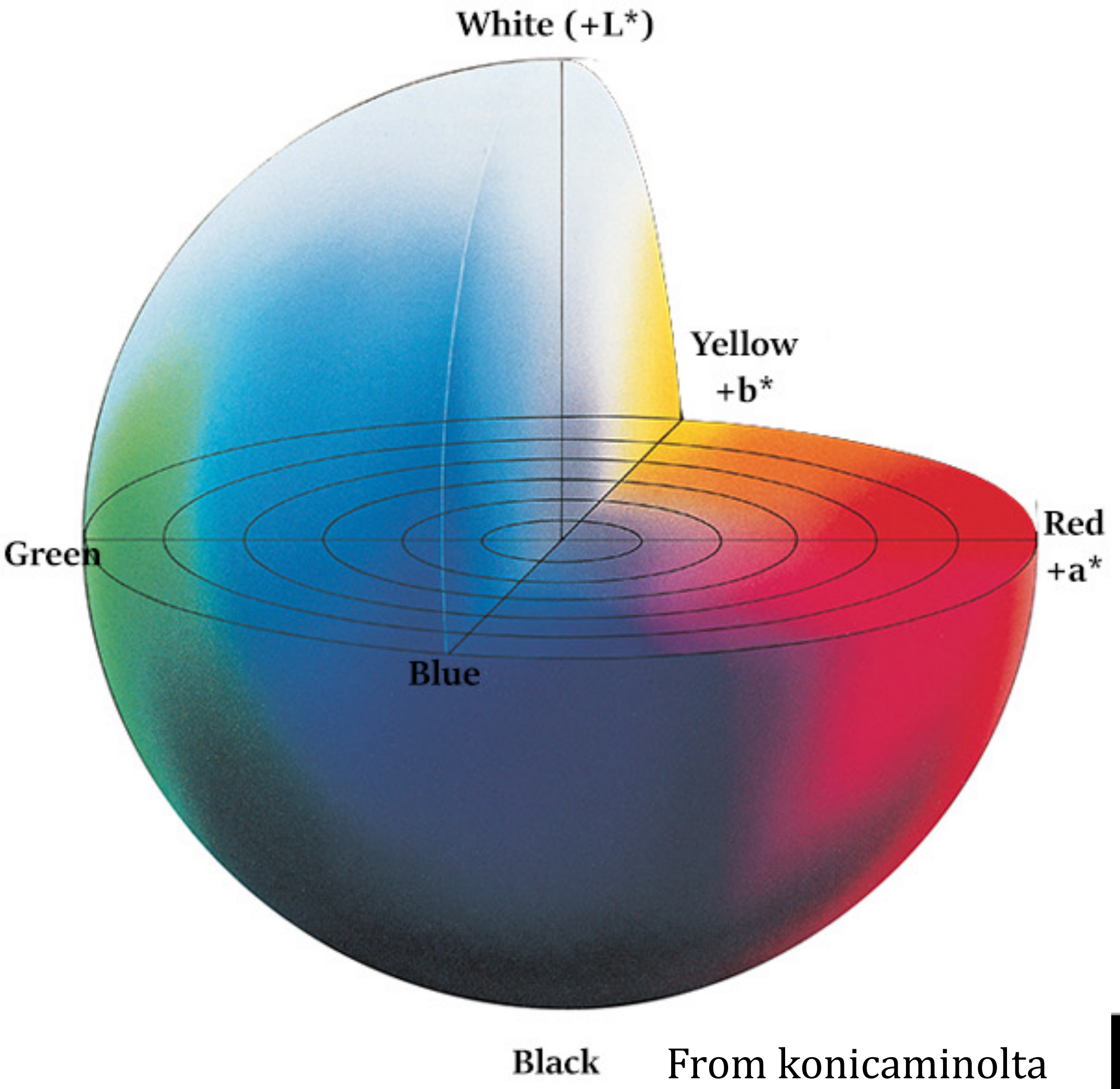


# YUV Colour Space

- Early TV was a monochrome signal
  - Measured total intensity Y of light
  - Axis from black to white
- Colour TV is backward compatible
  - Y component is still transmitted
  - Components U,V were added

# CIELAB Color space

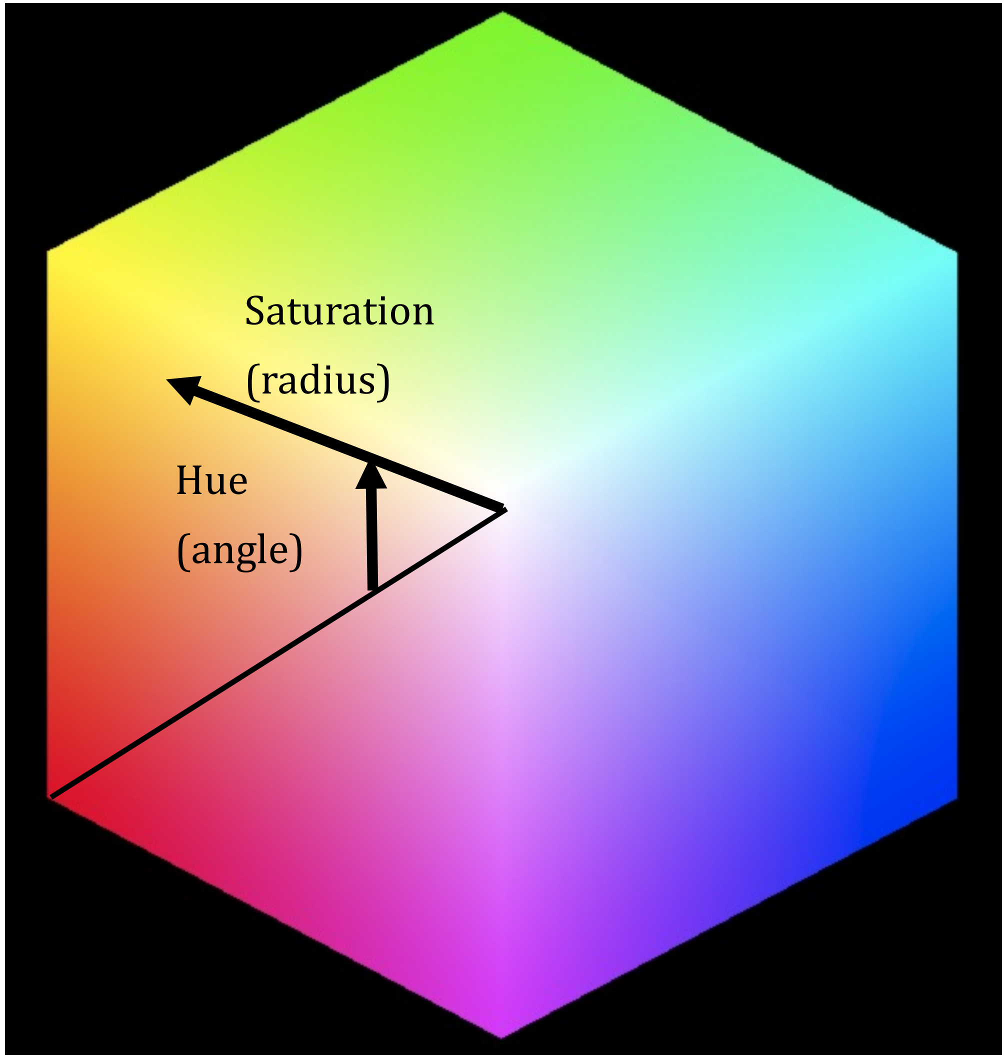
- Perceptually uniform space:
  - Separates light and chromacity (similarly to YUV)
  - But in this case not focused on transmission, but perception.
- A change of X in one of the axis should match how much change is perceived.
- Computing in 8 bits will break things. Use floats




From konicaminolta

# HSV Colour Space

- The *colour wheel*
  - H - *hue*
  - S - *saturation*
  - V - *value*
    - (or B - *brightness)*



# Choosing Colours

- How do we get “yellow”?
  - start with the rainbow: R O Y G B I V
  - mix R & G to get Y
- Or look at the colour wheel
- Or just play with RGB until we get it right



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# Additive Colour

- These models assume light is added
  - Colour is the sum of components
  - Suitable for combining light sources
- But this isn't the only way of doing it
  - Go back to the physics again

# Colour Absorption

- Objects are coloured by pigments
  - that absorb certain colours of light
- E.g. chlorophyll in leaves
  - absorbs almost all red / blue light
  - but reflects green light
  - so output light is green



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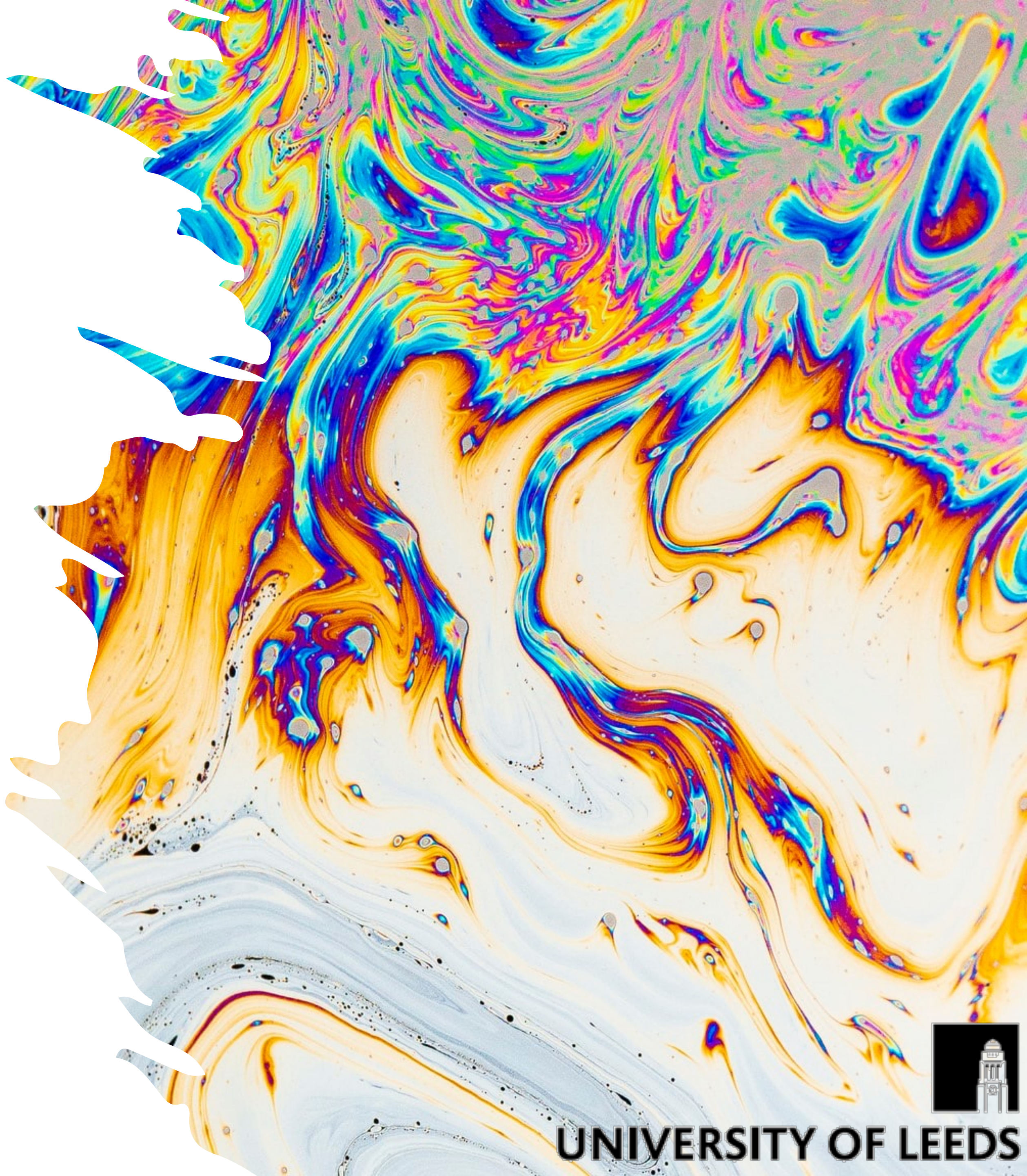
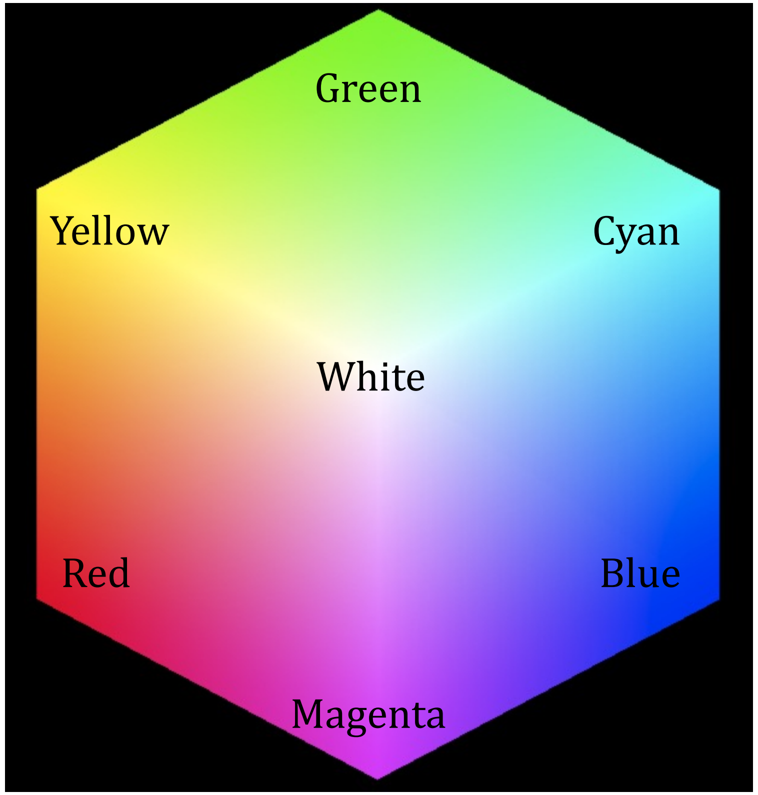
# Subtractive Colour

- For pigments, we subtract colour
  - Blue + Yellow = Green
    - blue reflects some green as well
    - so does yellow
    - green is only colour reflected by both
- We learned this colour model in school



# CMYK model

- Printers use CMYK
- Cyan removes red
- Magenta removes green
- Yellow removes blue
- Black removes everything
- Question: Can you run out of color ink just printing black and white?



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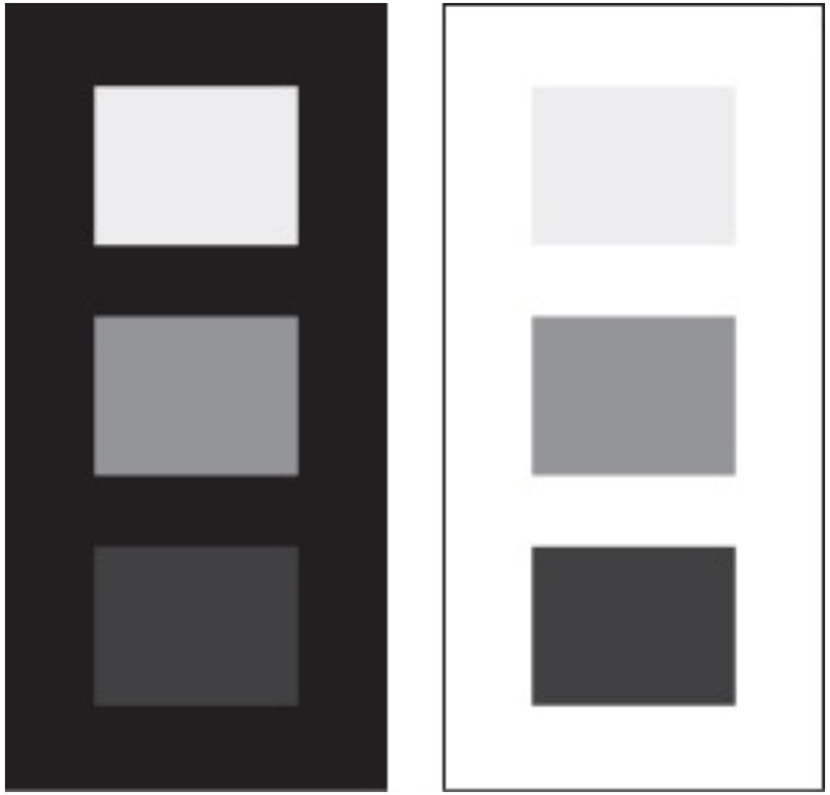
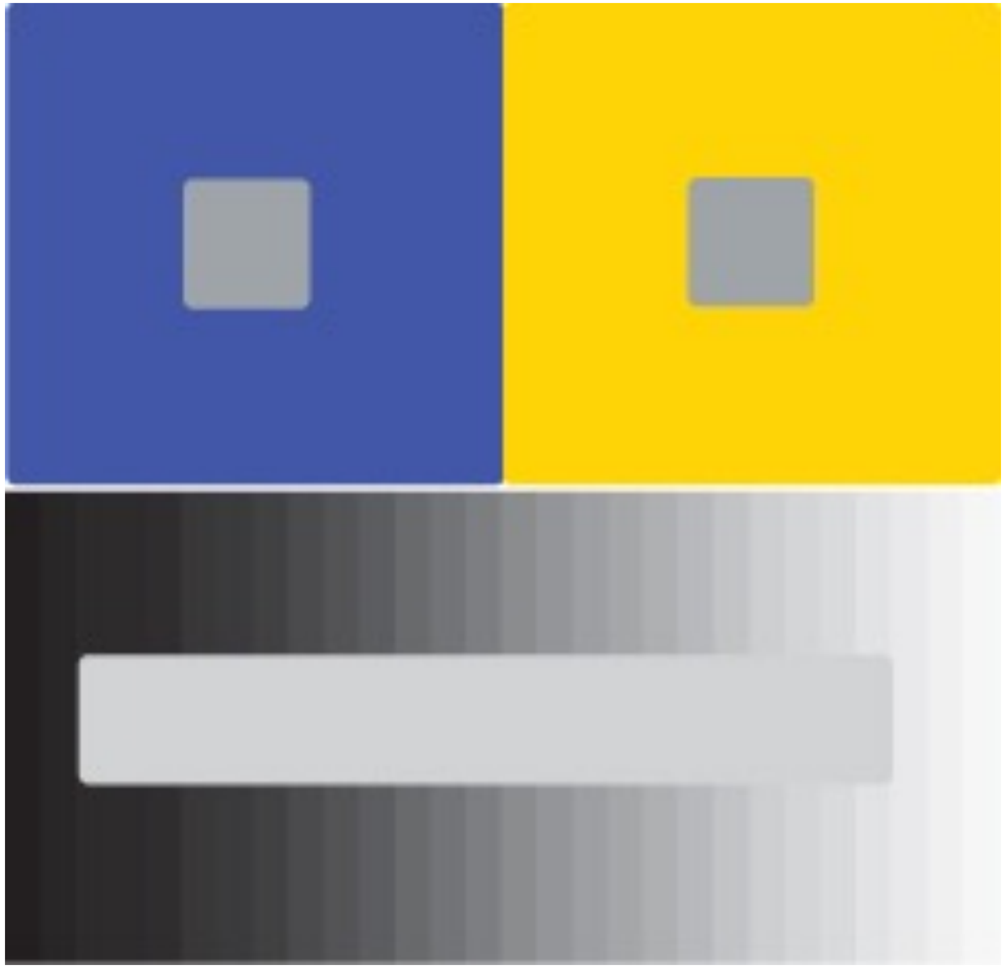
# Tone Perception

- Cones don't act separately
- Retina combines signals from cones
- Perception is affected by context
- Which set of squares is lighter?



# Simultaneous Contrast

- Tone perception also occurs in colour
- Which greys are the same here?



# Colour Interpolation

- Interpolating nearby colours works well
- Interpolating distant colours usually fails
- So films are starting to add wavelengths
  - Known as spectral rendering
- Will we end up with ROYGBIV devices?
  - Time will tell
- In the meantime, assume RGB

# Images by

- S3: Braxton Apana
- S4 S10: Sigmund
- S21: Alexandra Tran
- S22: Robbie Dawn
- S25: FLY:D
- S26: ThomasCarlPion
- S29: Etienne Girardet
- S49: Victor Freitas
- S62: Sarah Arista
- S65: Tanalee Youngblood
- S66: Daniel Olah

S50: buzzfeed.com