dl-assignment-10

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1 Deep Learning — Assignment 10

Assignment for week 10 of the 2023 Deep Learning course (NWI-IMC070) of the Radboud University.

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Group: 31

Instructions: * Fill in your names and the name of your group. * Answer the questions and complete the code where necessary. * Keep your answers brief, one or two sentences is usually enough. * Re-run the whole notebook before you submit your work. * Save the notebook as a PDF and submit that in Brightspace together with the .ipynb notebook file. * The easiest way to make a PDF of your notebook is via File > Print Preview and then use your browser's print option to print to PDF.

1.1 Objectives

In this assignment you will 1. Build a variational autoencoder 2. Extend the model to a conditional $V\!AE$

1.2 Required software

As before you will need these libraries: * torch and torchvision for PyTorch, * d21, the library that comes with Dive into deep learning book.

All libraries can be installed with pip install.

```
from torchvision import datasets, transforms
```

1.3 10.1 MNIST dataset (no points)

In this assignment we will use the MNIST digit dataset. This dataset consists of 28×28 binary images and has 60000 training examples divided over 10 classes.

(a) Run the code below to load the MNIST dataset.

1.4 10.2 Variational Autoencoder (VAE) (3 points)

We will implement a Variational Autoencoder. This model consists of two networks: an encoder and a decoder. The encoder produces a distribution in the latent space, represented as the parameters of a normal distribution. The decoder takes the latent space representation and produces an output in the data space.

(a) Complete the implementation below. (2 points)

```
[3]: class VAE(nn.Module):
         def __init__(self, latent_size=2):
             super(VAE, self).__init__()
             self.latent_size = latent_size
             # Components of the encoder network
             self.encoder_part1 = nn.Sequential(
                 nn.Conv2d(1, 32, kernel_size=3, padding=1, stride=2), nn.ReLU(),
                 nn.Conv2d(32, 64, kernel_size=3, padding=1, stride=2), nn.ReLU(),
                 nn.Flatten(),
                 nn.Linear(7 * 7 * 64, 16), nn.ReLU()
             self.encoder_mean = nn.Linear(16, latent_size)
             self.encoder_logvar = nn.Linear(16, latent_size)
             # Components of the decoder
             self.decoder_part1_z = nn.Linear(latent_size, 7 * 7 * 64)
             self.decoder_part2 = nn.Sequential(
                 nn.ReLU(),
                 nn.ConvTranspose2d(64, 32, kernel_size=3, padding=1,_
      →output_padding=1, stride=2),
```

```
nn.ReLU(),
           nn.ConvTranspose2d(32, 1, kernel_size=3, padding=1,__
→output_padding=1, stride=2),
           # TODO: Choose an appropriate activation function for the final,
\hookrightarrow layer.
          nn.Sigmoid(),
       )
  def encode(self, x):
      h = self.encoder_part1(x)
      return self.encoder_mean(h), self.encoder_logvar(h)
  def sample_latent(self, mean_z, logvar_z):
      eps = torch.randn_like(mean_z)
      std_z = torch.exp(0.5 * logvar_z)
       # TODO: turn the sample from N(0,1) into a sample from N(0,1)
      eps *= std z
      eps += mean_z
      return eps
  def decode(self, z):
      h = self.decoder_part1_z(z)
      h = torch.reshape(h, (-1, 64, 7, 7)) # Unflatten
      return self.decoder_part2(h)
  def forward(self, x):
      mean z, logvar z = self.encode(x)
      z = self.sample latent(mean z, logvar z)
      return self.decode(z), mean_z, logvar_z
```

The decoder produces two outputs that together give the parameters of a normal distribution: mean and logvar, so μ and log(σ^2). The latter might seem strange, but there is a good reason for doing it this way.

(b) What can go wrong if the encoder network directly outputs mean and standard deviation (,)? (1 point)

Because it's more numerically stable.

1.5 10.3 Loss function (2 points)

The loss for a variational autoencoder consists of two parts: 1. The reconstruction loss, which is the log likelihood of the data, $L_{\rm R} = \log P(x \mid z)$. 2. The Kulback-Leibler divergence from the encoder output to the target distribution, $L_{\rm KL} = KL(Q(z)||P(z)|)$.

In our case the data is binary, so we can use binary cross entropy for the reconstruction loss.

The derivation of the KL loss term can be found in appendix B of the VAE paper; Kingma and Welling. Auto-Encoding Variational Bayes. ICLR, 2014. Be careful: * the paper defines $-D_{KL}$, not D_{KL} * the sum is only over the latent space. In our code this corresponds to axis=1. Use the mean over the samples in the batch (axis=0).

(a) Implement the KL loss term below. (2 points)

```
[5]: def reconstruction_loss(recon_x, x):
         # The reconstruction loss is binary cross entropy
         # Note: we normalize the loss wrt. the batch size (len(x)), but not the
      ⇒size of the image
         return F.binary_cross_entropy(recon_x, x, reduction='sum') / len(x)
     def kl_loss(mean_z, logvar_z):
         # The KL divergence between a standard normal distribution and
         # a normal distribution with given mean and log-variance.
         # TODO: your code here
         mean_z = torch.mean(mean_z, dim=0, dtype=float)
         logvar_z = torch.mean(logvar_z, dim=0, dtype=float)
         std_z = torch.exp(0.5 * logvar_z)
         return torch.sum(torch.log(std_z) + (1 + mean_z ** 2) / (2 * std_z ** 2) -_{\sqcup}
      →0.5)
     def loss_function(recon_x, x, mean_z, logvar_z):
         1_recon = reconstruction_loss(recon_x, x)
         1_kl = kl_loss(mean_z, logvar_z)
         return l_recon + l_kl, l_recon, l_kl
```

```
[6]: # Here are some unit tests for the loss function
assert kl_loss(torch.tensor([[0]]), torch.tensor([[0]])) == 0, \
    'KL loss should be 0 for =0, =1'
assert kl_loss(torch.tensor([[0]]), torch.tensor([[-1]])) > 0, \
    'KL loss should be > 0 for =0, <1'
assert kl_loss(torch.tensor([[0]]), torch.tensor([[1]])) > 0, \
    'KL loss should be > 0 for =0, >1'
assert kl_loss(torch.tensor([[1]]), torch.tensor([[0]])) > 0, \
    'KL loss should be > 0 for !=0, =1'
```

1.6 10.4 Training our VAE (3 points)

(a) Complete the training loop below (2 points)

```
[7]: def train(model, num_epochs=10, lr=1e-3):
         optimizer = torch.optim.Adam(model.parameters(), lr=lr)
         animator = d21.Animator(xlabel='epoch', xlim=[1, num_epochs], figsize=(10, u
      ⇒5),
                                 legend=['train loss', 'train recon. loss', 'train_
      ⊸KL loss'.
                                          'test loss', 'test recon. loss', 'test KL,,
      ⇔loss'])
         for epoch in range(num_epochs):
             metric = d21.Accumulator(3)
             model.train()
             for x, y in train_iter:
                 x = x.to(device)
                 optimizer.zero_grad()
                 recon_x, mean_z, logvar_z = model.forward(x)
                 loss, loss_recon, loss_kl = loss_function(recon_x, x, mean_z,_
      →logvar_z)
                 # TODO: compute the outputs and loss
                 # TODO: backpropagate and apply optimizer
                 loss.backward()
                 optimizer.step()
                 # Track our progress
                 metric.add(loss_recon.detach(), loss_kl.detach(), x.shape[0])
             # Compute test loss
             test_loss, test_loss_recon, test_loss_kl = test(model)
             # Plot
             train_loss_recon = metric[0] / metric[2]
             train_loss_kl = metric[1] / metric[2]
             train_loss = train_loss_recon + train_loss_kl
             animator.add(epoch + 1,
                          (train_loss, train_loss_recon, train_loss_kl,
                           test_loss, test_loss_recon, test_loss_kl))
         print(f'training loss {train_loss:.3f}, test loss {test_loss:.3f}')
```

```
print(f'training reconstruction loss {train_loss_recon:.3f}, test_
 ⇔reconstruction loss {test_loss_recon:.3f}')
   print(f'training KL loss {train_loss_kl:.3f}, test KL loss {test_loss_kl:.

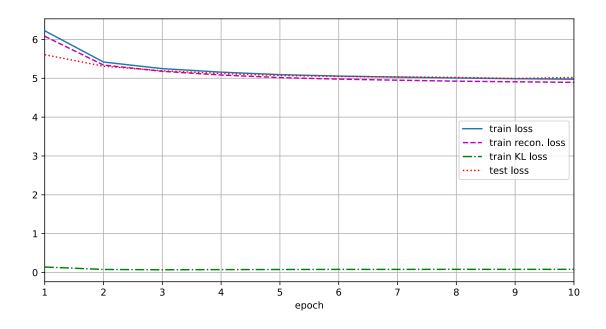
¬3f}')
def test(model):
   model.eval()
   metric = d21.Accumulator(3)
   with torch.no_grad():
        for i, (x, y) in enumerate(test_iter):
            x = x.to(device)
            recon_x, mean_z, logvar_z = model.forward(x)
            loss, loss_recon, loss_kl = loss_function(recon_x, x, mean_z,_
 →logvar_z)
            metric.add(loss_recon, loss_kl, x.shape[0])
   test_loss_recon = metric[0] / metric[2]
   test_loss_kl = metric[1] / metric[2]
   return test_loss_recon + test_loss_kl, test_loss_recon, test_loss_kl
```

(b) Train the model. (no points)

Hint: the training and test loss should both be around 5.

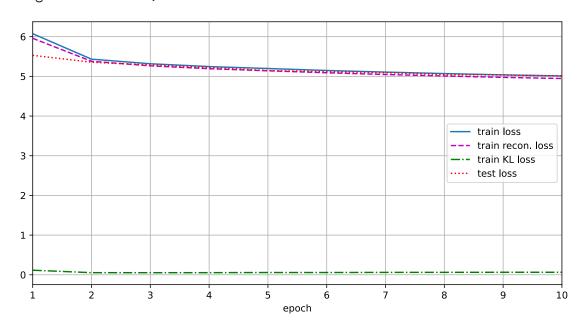
```
[8]: torch.manual_seed(12345)
  device = d2l.try_gpu()
  model = VAE().to(device)
  train(model)
```

training loss 4.976, test loss 5.021 training reconstruction loss 4.895, test reconstruction loss 4.926 training KL loss 0.081, test KL loss 0.096



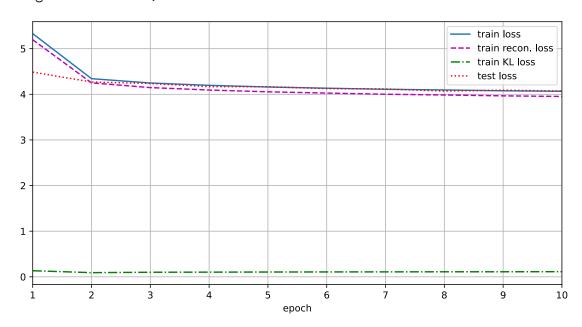
(c) If you increase the number of latent dimensions, how does that affect the reconstruction loss and the KL loss terms? (1 point)

training loss 5.010, test loss 5.011 training reconstruction loss 4.946, test reconstruction loss 4.940 training KL loss 0.064, test KL loss 0.071



```
[10]: model_5 = VAE(latent_size=5).to(device)
train(model_5)
```

training loss 4.066, test loss 4.073 training reconstruction loss 3.953, test reconstruction loss 3.963 training KL loss 0.113, test KL loss 0.110



latent_	training sizeloss	test loss	training reconstruction loss	test reconstruction loss	training KL loss	test KL loss
2	5.008	5.023	4.932	4.940	0.076	0.083
5	4.506	4.508	4.414	4.420	0.092	0.088
10	4.492	4.492	4.403	4.409	0.089	0.083

Increasing the latent size makes the reconstruction loss to go down slightly, and the KL loss does not significantly change.

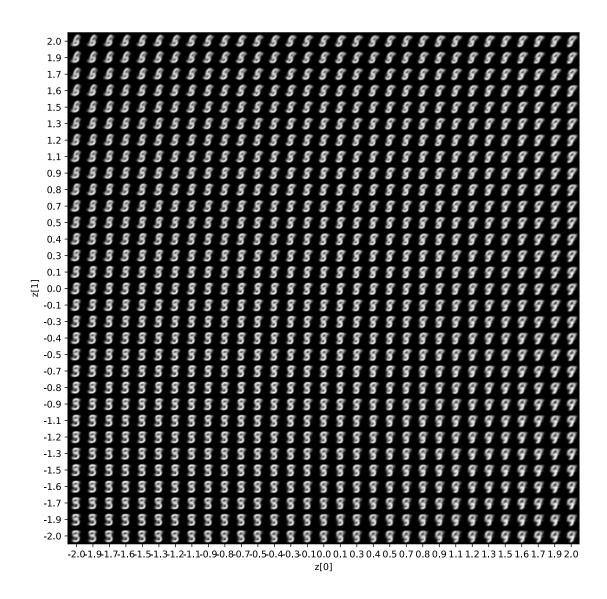
1.7 10.5 Visualizing the latent space (8 points)

We can use the function below to visualize the 2D latent space, by running the decoder on z values sampled at regular intervals.

(a) Complete the code below and run it to plot the latent space. (1 point)

```
[11]: def plot_latent(model):
    # display a n*n 2D manifold of digits
```

```
n = 31
   digit_size = 28
    scale = 2.0
   figsize = 10
   figure = np.zeros((digit_size * n, digit_size * n))
    # linearly spaced coordinates corresponding to the 2D plot
   # of digit classes in the latent space
   grid_x = np.linspace(-scale, scale, n)
   grid_y = np.linspace(-scale, scale, n)[::-1]
   for i, yi in enumerate(grid_y):
        for j, xi in enumerate(grid_x):
            # TODO: run the decoder on z = [xi, yi].
            x_decoded = model.decode(z=torch.Tensor([xi, yi]).to(device))
            figure[
            i * digit_size: (i + 1) * digit_size,
            j * digit_size: (j + 1) * digit_size,
            ] = x_decoded.detach().cpu().numpy()
   plt.figure(figsize=(figsize, figsize))
   start_range = digit_size // 2
   end_range = n * digit_size + start_range
   pixel_range = np.arange(start_range, end_range, digit_size)
   sample_range_x = np.round(grid_x, 1)
    sample_range_y = np.round(grid_y, 1)
   plt.xticks(pixel_range, sample_range_x)
   plt.yticks(pixel_range, sample_range_y)
   plt.xlabel("z[0]")
   plt.ylabel("z[1]")
   plt.imshow(figure, cmap="Greys_r")
   plt.show()
plot_latent(model)
```



(b) Would it be possible to classify digits based on this latent representation? Explain your answer. (1 point)

Yes, as a human, we can clearly identify most numbers, while some are ambiguous between two numbers.

(c) If you retrain the model, would you expect the latent space to look exactly the same. If not, what differences can you expect? (1 point)

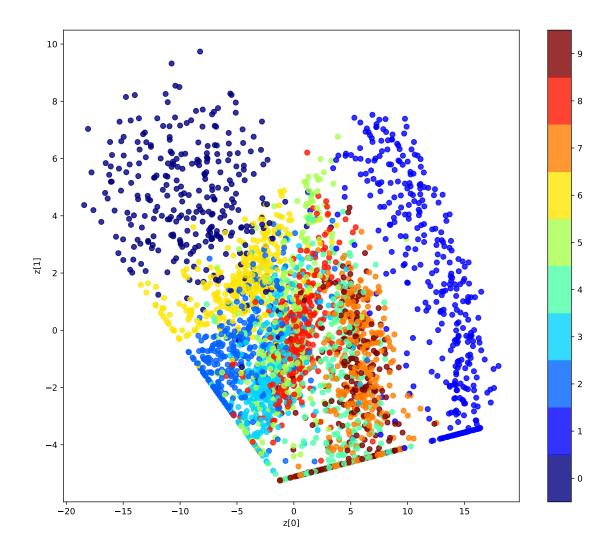
No, the latent space would change, because it is randomly initialized and therefore may use different representations for the same digits.

Another way of visualizing the latent space is by making a scatter plot of the training data in the latent space.

(d) Complete and run the code below to make a scatterplot of the training data. (1

point)

```
[12]: def scatterplot_latent(model):
          # display a 2D plot of the digit classes in the latent space
          zs, ys = [], []
          for x, y in itertools.islice(train_iter, 100):
              z_mean, _ = model.encode(x.to(device))
              zs.append(z_mean.detach().cpu())
              ys.append(y)
          zs = torch.cat(zs).numpy()
          ys = torch.cat(ys).numpy()
          cmap = plt.get_cmap('jet', 10)
          plt.figure(figsize=(12, 10))
          plt.scatter(zs[:, 0], zs[:, 1], c=ys, cmap=cmap, alpha=0.8, vmin=-0.5,
       \hookrightarrowvmax=9.5)
          plt.colorbar(ticks=np.arange(0, 10))
          plt.xlabel("z[0]")
          plt.ylabel("z[1]")
          plt.show()
      scatterplot_latent(model)
```



(e) Compare this figure to the one from plot_latent. How are the plots related? (1 point)

Note: Don't just answer "both visualize the latent space"

When looking at the same z[0], z[1] coordinates, we can clearly see that the model generates numbers that correspond to the numbers shown in the scatter plot.

(f) Compared to the figure from plot_latent, what information about the VAE is shown in this figure but not in the previous one? (1 point)

The second plot shows which part of the space is used most, i.e., the density in the latent space, and where the encoder is placing the different numbers. We can also see, how much space is between the individual data points, which is an indication of how clearly they are distinguished.

(g) What distribution should we expect the points in the latent space to follow, based on the KL divergence term in the loss function? (1 point)

We can expect some kind of normal distribution, as the KL divergence term punished based on the

normal distribution.

(h) Look at the distribution of the data in the latent space. Does the plot match the answer to the previous question? If not, why? (1 point)

Yes, we can see that the different numbers are distributed in an ellipse, with more values in the middle than on the edge.

1.8 10.6 Conditional Variational Autoencoder (10 points)

An extension of variational autoencoders uses labels to *condition* the encoder and decoder models. In this *conditional VAE*, the decoder becomes P(x|z,y) and the encoder Q(z|x,y). In practice, this means that the label y is given as an extra input to the both the encoder and the decoder.

For details see the paper Semi-Supervised Learning with Deep Generative Models; Kingma, Rezende, Mohamed, Welling; 2014.

To use the labels in the decoder, we can concatenate the label with the latent vector. Or equivalently, we can use separate weights for z and y in the first layer, so that layer computes $W_z \cdot z + W_y \cdot y + b$.

Similarly for the encoder, except there we will still use a convolutional layer for x, combined with a fully connected layer for y.

(a) Complete the implementation of the conditional VAE below. (3 points)

```
[13]: class ConditionalVAE(nn.Module):
          def __init__(self, latent_size=2, num_classes=10):
              super(ConditionalVAE, self).__init__()
              self.latent_size = latent_size
              # Components of the encoder network
              # TODO: split the first layer from the previous encoder network into au
       ⇔separate variable,
                      and add a layer to use with the y input
              self.encoder_part1_x = nn.Conv2d(1, 32, kernel_size=3, padding=1,_
       ⇔stride=2)
              self.encoder_part1_y = nn.Linear(num_classes, 32 * 14 * 14)
              self.encoder_part2 = nn.Sequential(
                  nn.ReLU(),
                  nn.Conv2d(32, 64, kernel_size=3, padding=1, stride=2), nn.ReLU(),
                  nn.Flatten(),
                  nn.Linear(7 * 7 * 64, 16), nn.ReLU()
              )
              self.encoder mean = nn.Linear(16, latent size)
              self.encoder_logvar = nn.Linear(16, latent_size)
              # Components of the decoder network
              self.decoder_part1_z = nn.Linear(latent_size, 7 * 7 * 64)
              # TODO: add layer to use with the y input
              self.decoder_part1_y = nn.Linear(num_classes, 7 * 7 * 64)
```

```
self.decoder_part2 = nn.Sequential(
          nn.ReLU(),
          nn.ConvTranspose2d(64, 32, kernel_size=3, padding=1,__
→output_padding=1, stride=2),
          nn.ReLU(),
          nn.ConvTranspose2d(32, 1, kernel size=3, padding=1,
→output_padding=1, stride=2),
          # TODO: see VAE
          nn.Sigmoid()
      )
  def encode(self, x, y):
      y = nn.functional.one_hot(y, num_classes).float()
      h = self.encoder_part1_x(x) + self.encoder_part1_y(y).reshape(-1, 32,__
⇒14, 14)
      h = self.encoder_part2(h)
      return self.encoder_mean(h), self.encoder_logvar(h)
  def sample_latent(self, mean_z, logvar_z):
      eps = torch.randn_like(mean_z)
      std_z = torch.exp(0.5 * logvar_z)
      # TODO: see VAE
      eps *= std z
      eps += mean_z
      return eps
  def decode(self, z, y):
      # TODO: use a first layer that combines z and y
      y = nn.functional.one_hot(y, num_classes).float()
      h_z = self.decoder_part1_z(z)
      h_y = self.decoder_part1_y(y)
      h = h_y + h_z
      h = torch.reshape(h, (-1, 64, 7, 7))
      return self.decoder_part2(h)
  def forward(self, x, y):
      mean_z, logvar_z = self.encode(x, y)
      z = self.sample latent(mean z, logvar z)
      return self.decode(z, y), mean_z, logvar_z
```

(b) Copy the training code from section 10.4, and modify it for a conditional VAE. (1 point)

Hint: To train the conditional VAE we need to use one-hot encoding of the labels. You can use the following code for that:

```
y = F.one_hot(y,10).float().to(device)
```

```
[14]: def train_cvae(model, num_epochs=10, lr=1e-3):
          optimizer = torch.optim.Adam(model.parameters(), lr=lr)
          animator = d21.Animator(xlabel='epoch', xlim=[1, num_epochs], figsize=(10, u
       ⇒5),
                                  legend=['train loss', 'train recon. loss', 'train⊔
       'test loss', 'test recon. loss', 'test KL
       →loss'])
          for epoch in range(num_epochs):
              metric = d21.Accumulator(3)
              model.train()
              for x, y in train_iter:
                  x = x.to(device)
                  y = y.to(device)
                  optimizer.zero_grad()
                  recon_x, mean_z, logvar_z = model.forward(x, y)
                  loss, loss_recon, loss_kl = loss_function(recon_x, x, mean_z,_
       →logvar_z)
                  # TODO: compute the outputs and loss
                  # TODO: backpropagate and apply optimizer
                  loss.backward()
                  optimizer.step()
                  # Track our progress
                  metric.add(loss_recon.detach(), loss_kl.detach(), x.shape[0])
              # Compute test loss
              test_loss, test_loss_recon, test_loss_kl = test_cvae(model)
              # Plot
              train_loss_recon = metric[0] / metric[2]
              train_loss_kl = metric[1] / metric[2]
              train_loss = train_loss_recon + train_loss_kl
              animator.add(epoch + 1,
                           (train_loss, train_loss_recon, train_loss_kl,
                            test_loss, test_loss_recon, test_loss_kl))
          print(f'training loss {train_loss:.3f}, test loss {test_loss:.3f}')
          print(f'training reconstruction loss {train_loss_recon:.3f}, test_

¬reconstruction loss {test_loss_recon:.3f}')
          print(f'training KL loss {train_loss_kl:.3f}, test KL loss {test_loss_kl:.

3f}')

      # TODO: your code here
      def test cvae(model):
          model.eval()
          metric = d21.Accumulator(3)
          with torch.no_grad():
              for i, (x, y) in enumerate(test_iter):
                  x = x.to(device)
```

```
y = y.to(device)
    recon_x, mean_z, logvar_z = model.forward(x, y)
    loss, loss_recon, loss_kl = loss_function(recon_x, x, mean_z, logvar_z)

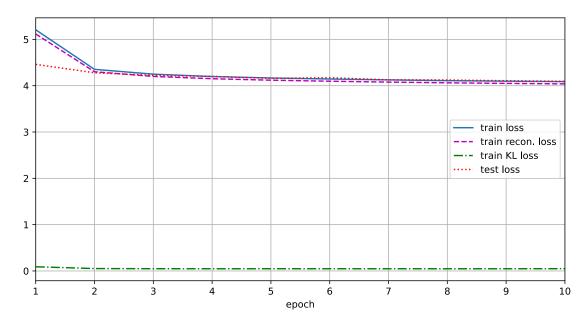
    metric.add(loss_recon, loss_kl, x.shape[0])
    test_loss_recon = metric[0] / metric[2]
    test_loss_kl = metric[1] / metric[2]
    return test_loss_recon + test_loss_kl, test_loss_recon, test_loss_kl

# TODO: your code here
```

(c) Train a conditional VAE

```
[15]: cvae_model = ConditionalVAE().to(device)
train_cvae(cvae_model)
```

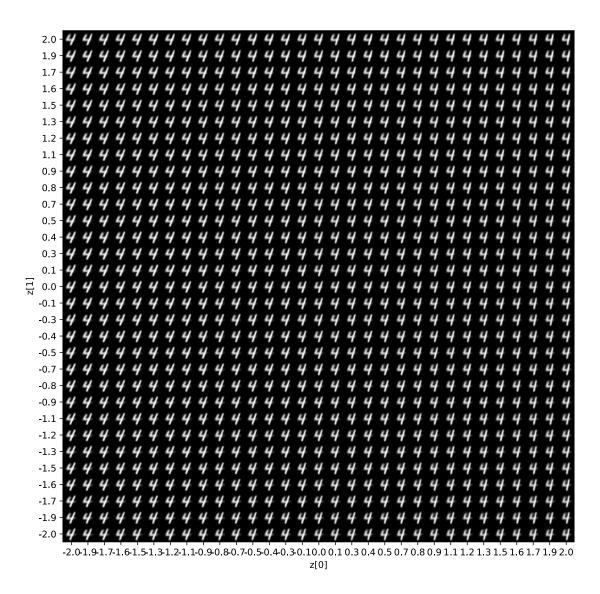
training loss 4.088, test loss 4.093 training reconstruction loss 4.039, test reconstruction loss 4.046 training KL loss 0.049, test KL loss 0.047

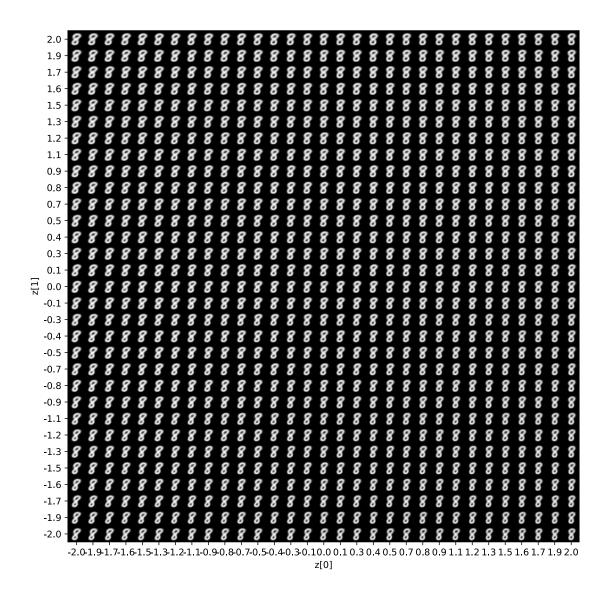


(d) Adapt the plot_latent function from section 10.5 for conditional VAEs, and use your function to visualize the latent space for the classes 4 and 8. (1 point)

```
[16]: def plot_latent_cvae(model, ys):
    # display a n*n 2D manifold of digits
    n = 31
    digit_size = 28
    scale = 2.0
```

```
figsize = 10
   figure = np.zeros((digit_size * n, digit_size * n))
    # linearly spaced coordinates corresponding to the 2D plot
    # of digit classes in the latent space
   grid_x = np.linspace(-scale, scale, n)
   grid_y = np.linspace(-scale, scale, n)[::-1]
   for y in ys:
       for i, yi in enumerate(grid_y):
            for j, xi in enumerate(grid_x):
                # TODO: run the decoder on z = [xi, yi].
                x_decoded = model.decode(torch.Tensor([xi, yi]).to(device),__
 ⇔torch.tensor([y]).to(device))
                figure[
                i * digit_size: (i + 1) * digit_size,
                j * digit_size: (j + 1) * digit_size,
                ] = x_decoded.detach().cpu().numpy()
       plt.figure(figsize=(figsize, figsize))
       start_range = digit_size // 2
        end range = n * digit size + start range
       pixel_range = np.arange(start_range, end_range, digit_size)
       sample_range_x = np.round(grid_x, 1)
       sample_range_y = np.round(grid_y, 1)
       plt.xticks(pixel_range, sample_range_x)
       plt.yticks(pixel_range, sample_range_y)
       plt.xlabel("z[0]")
       plt.ylabel("z[1]")
       plt.imshow(figure, cmap="Greys_r")
       plt.show()
plot_latent_cvae(cvae_model, [4, 8])
```





(e) What do the latent dimensions represent? Is this the same for all labels? (1 point)

The latent dimensions seem to represent the small variations on how exactly the numbers are written. The latent dimensions seem to represent the same variation in style even for different labels, which is probably caused by sharing big parts of the network.

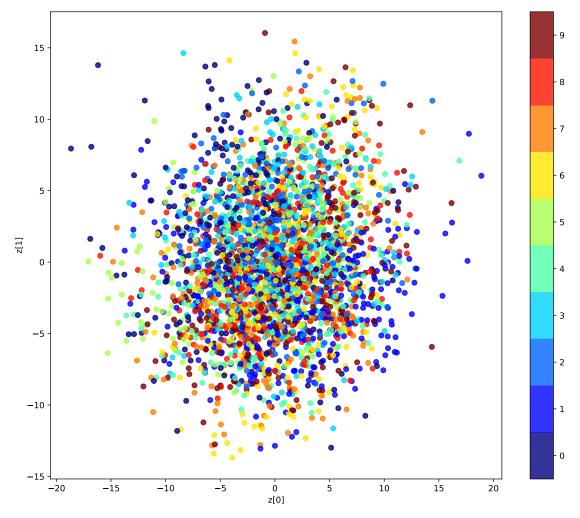
(f) Adapt scatterplot_latent to show the distribution in the latent space. (1 point)

```
[17]: def scatterplot_latent_cvae(cvae_model):
    # display a 2D plot of the digit classes in the latent space
    zs, ys = [], []
    for x, y in itertools.islice(train_iter, 100):
        z_mean, _ = cvae_model.encode(x.to(device), y.to(device))
        zs.append(z_mean.detach().cpu())
        ys.append(y)
```

```
zs = torch.cat(zs).numpy()
ys = torch.cat(ys).numpy()

cmap = plt.get_cmap('jet', 10)
plt.figure(figsize=(12, 10))
plt.scatter(zs[:, 0], zs[:, 1], c=ys, cmap=cmap, alpha=0.8, vmin=-0.5,u

ovmax=9.5)
plt.colorbar(ticks=np.arange(0, 10))
plt.xlabel("z[0]")
plt.ylabel("z[1]")
plt.ylabel("z[1]")
plt.show()
```



(g) How is this distribution in the latent space different from the distribution of the

VAE? Compare to your answer to that for question 10.5 g and h. What is the cause of these differences? (1 point)

The latent space is no longer divided into clusters per digit. This is because it's not needed anymore, because of the digit in known to the encoder and decoder. Therefore, they do not have to distinguish them anymore themselves.

(h) Would it be possible to classify digits based on the latent representation of the conditional VAE? Explain your answer. (1 point)

No, because the representation of the different digits in the latent space is not seperable anymore.

(i) Describe how you could use a conditional VAE to change the label or content of an image, while keeping the style as similar as possible. (1 point)

You would encode a digit with the style that you want and the current label and use its latent representation with a different, i.e., the target label, to decode it to a different digit with the same style.

1.9 10.7 Discussion (2 points)

(a) Is the conditional VAE a strict improvement over the normal VAE in all cases? (1 point)

No, because they are designed for different use cases. The VAE can be used to detect a number, while the other needs to know the label in advance, for example.

(b) Compare the latent representation vector z in the VAE with the input for the generator in a GAN. They are both small vectors, and they are both often called z. In what way are they the same, and in what way are the different? (1 point)

In both cases z represents the input to the generator, but in the GAN case, it's just random noise, while in the VAE case it is encoded from a real sample. This means that VAEs can keep the semantics while GANs cannot.

1.10 The end

Well done! Please double check the instructions at the top before you submit your results.

This assignment has 28 points. Version 7536f1e / 2023-11-15