Neural language models with syntax

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Contents

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ABSTRACT

In this thesis I investigate the question: *What are effective ways of incorporating syntactic structure into neural language models?*

In this thesis I:

- study a class of neural language models that merges generative transition-based parsing with recurrent neural networks in order to model sentences together with their latent syntactic structure;
- propose a new globally trained chart-based parser as an alternative proposal distribution used in the approximate marginalization;
- propose effective methods for semisupervised learning, making the syntactic structure a latent variable;
- perform targeted syntactic evaluation and compare the model's performance with that of alternative models that are based on multitask learning.

I find that:

- ...
- ...

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Introduction

This thesis investigates the question: *What are effective ways of incorporating syntactic structure into neural language models?*

We study a class of neural language models that explicitly model the hierarchical syntactic structure in addition to the sequence of words (**Buys+2015:neural-gen-dep**; **Buys+2018**; Dyer *et al.*, 2016). These models merges generative transition-based parsing with recurrent neural networks in order to model sentences together with their latent syntactic structure. The syntactic structure that decorates the words can be latent, and marginalized over, or can be given explicitly, for example as the prediction of an external parser. Although these are fundamentally joint model, they can be evaluated as regular language models (modeling only words) by (approximate) marginalization of the syntactic structure. In the case of the RNNG (Dyer *et al.*, 2016), exact marginalization is intractable due to the parametrization of the statistical model, but importance sampling provides an effective approximate method. An externally trained discriminative parser is used to obtain proposal samples. Other models provide exact marginalization, but this typically comes at the cost of a less expressive parametrization, for example one in which the features cannot be structure-dependent

(**Buys+2018**).

In this thesis I study the RNNG (Dyer *et al.*, 2016) and investigate:

The approximate marginalization I propose an alternative proposal distribution and investigate the impact.

- I propose a new discriminative chart-based neural parser that is trained with a global, Conditional Random Field (CRF), objective. The parser is an adaptation of the minimal neural parser proposed in **Stern+2017:Minimal** which is trained with a margin-based objective.

- This contrast with the typical choice for a transition-based parser as proposal (a discrminatively trained RNNG) (Dyer *et al.*, 2016).

- We posit that a globally trained model is a better proposal distribution than a locally trained transition based model: a global model has ready access to competing analyses that can be structurally dissimilar but close in probability, whereas we hypothesize that a locally trained model is prone to produce locally corrupted structures that are nearby in transition-space.

- In a transition based parser more diverse samples can be obtained by flattening the transition distributions. This causes the model to be less confident in its predictions. A downside is that this approach causes the model to explore parts of the probability space which it has not encountered during training.

- The above is a general challenge for greedy transition based models that can be answered to by training with dynamic oracles (**Goldberg+2013:dynamic**), also called 'exploration' ((**Ballesteros+2016 Stern+2017:Minimal**). These approaches can be considered instances of imitation learning (**Vlachos**; **Eisner+2012:imitation**).

- We do not consider these directions in this thesic. Dynamic oracles can produce substantial improvements in constituency parsing performance, but they must be custom designed for each transition system (**Klein+2018:reinforce**).

Semi-supervised training by including unlabeled data To make joint models competitive language models they need to make use of the vast amounts of unlabeled data that exists.

- A major drawback of these syntactic language models is that they require annotated data to be trained, and preciously little of such data exists.
- We extend the training to the unsupervised domain by optimizing a variational lower bound on the marginal probabilities that jointly optimizes the parameters of proposal model ('posterior' in this framework) with the joint model.
- We obtain gradients for this objective using the score function estimator (Fu 2006), also known as REINFORCE (Williams 1992), which is widely used in the field of deep reinforcement learning, and we introduce an effective baseline based on argmax decoding (Rennie et al. 2017), which significantly reduces the variance in this optimization procedure.
- Our CRF parser particularly excels in the role of posterior thanks the independence assumptions that allow for efficient exact computation of key quantities: the entropy term in the lower bound can be computed exactly using Inside-Outside algorithm, removing one source of variance from the gradient estimation, and the argmax decoding can be performed exactly thanks to Viterbi, making the argmax baseline even more effective.

Alternative, simpler, models There are alternatives to the methods that this thesis investigates.

- Multitask learning of a neural language model with a syntactic side objective is a competitive and robust alternative method to infuse neural language models with syntactic knowledge.
- Training the syntactic model on data that mixes gold trees with predicted 'silver' trees for unlabeled data is a competitive and robust alternative to fully principled semi-supervised learning.

- We propose a simple multitask neural language model that predicts labeled spans from the RNN hidden states, using a feature function identical identical to that used in the CRF parser. A similar strategy has recently proposed in work on semantic parsing and is called a 'syntactic scaffold' (**Swayamdipta+2018:scaffold**).
- We consider these alternatives in order to quantify significance of the latent structure, and the semisupervised training on the other hand, as measured by some external performance metric.

Targeted syntactic evaluation TBA

2

Background

In this chapter I give the nescessary background

2.1 Syntax

- Some generic stuff on syntax and constituency in natural language
- Reference (**Carnie2010:constituent**; **Everaert+2015:structures**) or something?

2.2 Parsing

- Treebanks, in particular the Penn Treebank. Treebank preprocessing. CFGs, CNF, spans. Reference figure 2.1.
- The two conceptions of a tree: as a set of *labeled spans* or as a set of *anchored rules*.
- A labeled span is a triple (ℓ, i, j) of a syntactic label ℓ together the left and right endpoints i, j that the label spans.

Labeled spans	Anchored rules
(S, 0, 10)	(S → SBAR @, 0, 3, 10)
(SBAR, 0, 3)	(SBAR → WHNP S+VP, 0, 1, 3)
(VP, 1, 3)	(S+VP → @ NP, 1, 2, 3)
⋮	⋮
(NP, 7, 9)	(NP → @ @, 7, 8, 9)

Table 2.1: Two conceptions of the tree in 2.1d.

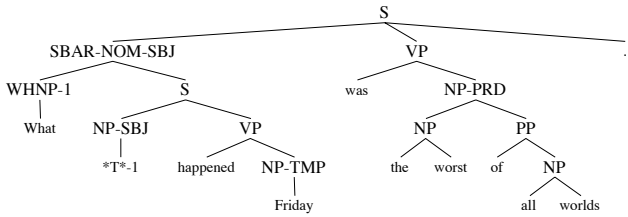
- An *anchored rule* is a four-tuple (r, i, k, j) containing the CNF rule r with the span endpoints i, j and split-point k of the left and right child (TODO: unary rules)
- For the difference, consider the following two representations of the tree in figure 2.1d given in table 2.1.
- Algorithms for parsing: global chart based, local transition based
- Dynamic programming inference versus search heuristics.
- Modelling types: generative, discriminative, log-linear, count-based, feature-based, neural network features.

2.3 Neural networks

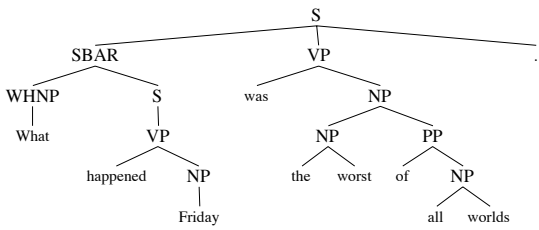
Introduce all the neural networks.

- Feedfoward, RNN, LSTM etc.
- We consider these as abstractions denoting certain parametrized functions. A Feedforward network is a function that a vector $\mathbf{x}$ produces an output vector

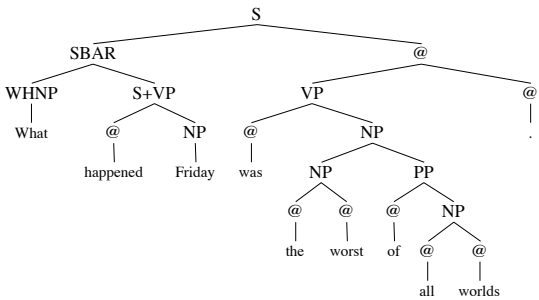
$$\textsc{Feedforward}_\theta(\mathbf{x}) = \mathbf{y}.$$



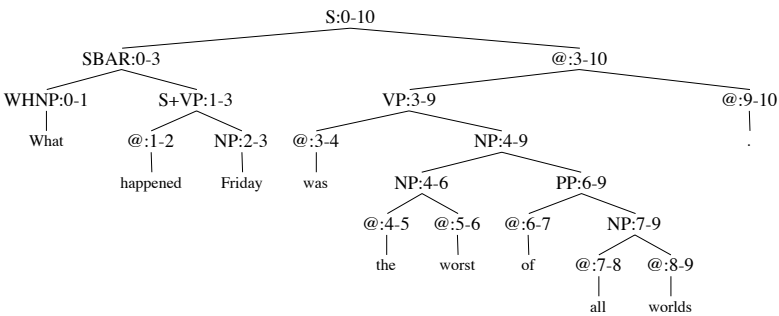
(a) Original Penn Treebank tree.



(b) Function tags and traces removed.



(c) Converted to normal form.



(d) In normal form with spans.

Figure 2.1: Converting a treebank tree (withouth part-of-speech tags).

An RNN is function parametrized by θ that takes a sequence of vectors $\mathbf{x} = \mathbf{x}_1, \ldots, \mathbf{x}_n and produces a series of output vectors \text{RNN}_\theta(\mathbf{x}) = \mathbf{y}_1, \ldots, \mathbf{y}_n$.

An LSTM is a particular way to construct the RNN function.

Maybe, maybe write down the equations for the Feedforward and the LSTM.

(Minibatch) SGD optimization.

2.4 Language models

- Briefly mention some typical approaches for langugage modelling: count based n-gram with smoothing (**Goodman+1996**; **Kneser+1995**), neural n-gram (**Bengio+2003**) and recurrent neural network (**Mikolov+2010**). Also mention some (early) syntactic approaches: count-based (**Chelba2000**; **Klein:2012:treelets**), neural (**Emami+2005**), and top-down parsing related (**Roark2001**).

- Explain the metric perplexity.

- Briefly mentions some typical datasets and some benchmarks (dataset, perplexity, number of parameters, training time).

- Mention some downsides of the perplexity metric: conflating different sources of succes in next-word prediction (simple collocations, semantics, syntax).

- Note that there exists some alternatives to perplexity: adversarial evaluation (**Smith2012:adversarial**), subject-verb agreement (**Linzen+2016:LSTM-syntax**) and grammatical acceptability judgments (**Linzen+2018:targeted**).

2.5 Multitask learning

- Give formal description of multitask learning.

- In our case of language modelling with syntactic side-objective, the learning objective is to maximize

$$\mathcal{L}(\theta, \lambda, \eta) = \sum_{(\mathbf{x},\mathbf{y})\in\mathcal{D}} \log p_{\theta,\lambda}(\mathbf{x}) + \log q_{\theta,\eta}(\mathbf{y}|\mathbf{x})$$

  with respect to the parameters θ, λ and η, where p is our model of interest, optimized for the main objective, and q is our model optimized for the side objective, which we discard after optimization.

- The key point of multitask learning is that the the two models p and q share the set of parameters θ. This means that θ will be optimized to fit both objectives well.

- The parameters in λ and η, in turn, are optimized to the each objective separately.

- The proportion and the nature of the parameters that belong to θ is a choice of the modeller and the objective.

- Name some recent examples of multitask learning in NLP: **Zhang+2016:mult** **Goldberg+2016:multitask**; **Swayamdipta+2018:scaffold**

3

Recurrent Neural Network Grammars

4

Conditional Random Field parser

In this chapter I introduce an alternative parser to act as proposal model in the approximate marginalization. The parser is a neural Conditional Random Field (CRF) parser that parser combines exact dynamic programming of chart-based parsing with the rich nonlinear featurization of neural networks. The parser is an adaptation of the chart-based parser introduced in **Stern+2017:Minimal** where it is trained with a margin-based objective.

- The chart-based approach allows efficient exact inference. The neural network is used exclusively to learn good representation from which to predict local scores; the global structured interactions are

4.1 Model

In this section I describe the probabilistic model of the parser.

1. Introduce probabilistic model, reference CRF background in apendix
2. Desribe how this is an adaptation from **Stern+2017:Minimal** to probabilistic training.

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4.1.1 Motivation

- Key point to make: this model regards a constituency tree as a collection of *labeled spans* over a sentence. Earlier models, both log-linear and neural, regard a constituency tree as a collection of *anchored rules* over a sentence (**Finkel+2008**; **Klein+2015**).
- A model over spanned rules puts more expressiveness in the state space of the dynamic program, since rules The model in **Stern+2017:Minimal** instead puts the expressiveness in the input space by using rich neural feature representations and a very unconstrained output space.
- Earlier approaches went evern further by enriching enriched meanse to lexicalize the rules **Collins2003** break the grammar's independence assumptions by annotating the rule with parent and sibling labels (**Klein+2003**).
-

4.1.2 Features

In this section I describe how the scores are computed from the bidirectional LSTM features.

- Give formal expression for 'LSTM minus features' with Feedforward scoring function

4.2 Inference

Due to the parametrization, the model allows efficient inference. In this section we describe efficient solutions to three related problems:

- Find the best parse $\mathbf{y}^* = \arg\max_{\mathbf{y}} p(\mathbf{y}|\mathbf{x})$
- Compute the normalizer $Z(\mathbf{x}) = \sum_{\mathbf{y}} \prod_{a=1}^{A} \Psi(\mathbf{x}, \mathbf{y}_a)$, where $F = \{\Psi_a\}_{a=1}^{A}$ is the set of factors in the graph.

- Compute the entropy conditioned on $\mathbf{x}$, $H(\mathbf{y}|\mathbf{x})$.

All three problems can be solved with a different instance of the same two algorithm: the inside algorithm and the outside algorithm.

4.2.1 Inside recursion

NOTE: This is a complete draft section, just for me to get the correct derivation for the implementation. In the actual section I will follow the semiring formulation, and connect it with the belief-propagation/message-passing/sum-product algorithm.

In this derivation we follow Michael Collins notes on the Inside-Outside Algorithm.[1]

Let a sentence be $x_1, \ldots, x_n$, where each x_i is a word. We are given a CFG (N, Σ, R, S) in Chomsky normal form. Let ψ be a function that maps any rule production $r \in R$ of the form $\langle A \to B\ C, i, k, j\rangle$ or $\langle A, i, i+1\rangle$ to a value $\psi(r) \geq 0$. Let a tree T be a set of such rules r with the only constraint that these rules make up a tree.

Following the minimal span parser we define ψ as

$$\log \psi(A \to B\ C, i, k, j) \triangleq s_{label}(i, j, A) + s_{span}(i, j) \tag{4.1}$$

$$\text{and} \tag{4.2}$$

$$\log \psi(A, i, i+1) \triangleq s_{label}(i, i+1, A) + s_{span}(i, i+1), \tag{4.3}$$

and thus the potential of a tree as

$$\log \Psi(T) = \sum_{r \in T} \log \psi(r) \tag{4.4}$$

$$= \sum_{\langle A,i,j\rangle \in T} s_{label}(i, j, A) + s_{span}(i, j), \tag{4.5}$$

$$\tag{4.6}$$

Note that the potential function as defined in 4.1 disregards most of the information in a binary rule. In particular we see that B, C and k, the labels and split-point of the children, are discarded.

[1] http://www.cs.columbia.edu/~mcollins/io.pdf

Now note that equation corresponds exactly to the second formula in section 3 of the minimal span-based parser paper

$$s_{tree}(T) = \sum_{(\ell(i,j))\in T} [s_{label}(i,j,\ell) + s_{span}(i,j)]. \tag{4.7}$$

which is how I derived that 4.1 is the correct formula for the rule score.

We obtain our CRF objective when we normalize this score globally

$$P(T) = \frac{\prod_{r\in T}\psi(r)}{\sum_{T'\in\mathcal{T}}\prod_{r'\in T'}\psi(r')} \tag{4.8}$$

$$\tag{4.9}$$

or equivalently

$$\log P(T) = \sum_{r\in T}\log\psi(r) - \log\sum_{T\in\mathcal{T}}\prod_{r\in T}\psi(r) \tag{4.10}$$

$$\tag{4.11}$$

From the aforementioned notes we get the following general result for the inside value α. For all $A \in N$, for all $0 \le i < n$

$$\alpha(A,i,i+1) = \psi(A,i,i+1) \tag{4.12}$$

and for all (i,j) such that $1 \le i < j \le n$:

$$\alpha(A,i,j) = \sum_{A\to BC}\sum_{k=i+1}^{j-1}\psi(A\to B\ C,i,k,j)\cdot\alpha(B,i,k)\cdot\alpha(C,k,j) \tag{4.13}$$

Note that we are considering a CFG in which the rule set is complete, i.e.

$$\langle A\to B\ C\rangle \in R \text{ for each } (A,B,C)\in N^3, \tag{4.14}$$

and recall that the labels B and C do not appear in the scoring functions in 4.1. These facts will allow us to simplify the expression in formula

4.13 as

$$\alpha(A,i,j) = \sum_{B\in N}\sum_{C\in N}\sum_{k=i+1}^{j-1} \tilde{s}_{label}(i,j,A)\cdot\tilde{s}_{span}(i,j)\alpha(B,i,k)\cdot\alpha(C,k,j) \tag{4.15a}$$

$$= \tilde{s}_{label}(i,j,A)\cdot\tilde{s}_{span}(i,j)\sum_{k=i+1}^{j-1}\sum_{B\in N}\alpha(B,i,k)\cdot\sum_{C\in N}\alpha(C,k,j) \tag{4.15b}$$

$$= \tilde{s}_{label}(i,j,A)\cdot\tilde{s}_{span}(i,j)\sum_{k=i+1}^{j-1} S(i,k)\cdot S(k,j) \tag{4.15c}$$

where we've introduced a number of notational abbreviations

$$\tilde{s}_{label}(i,j,A) = \exp(s_{label}(i,j,A)) \tag{4.16}$$

$$\tilde{s}_{span}(i,j) = \exp(s_{span}(i,j)) \tag{4.17}$$

$$S(i,j) = \sum_{A\in N}\alpha(A,i,j) \tag{4.18}$$

Note that this is the exact same formula as **??**.

From equation 4.15c we can deduce that we in fact do even need to store the values $\alpha(i,j,A)$ but that it suffices to only store the marginalized values $S(i,j)$. In this case, the recursion simplifies even further:

$$S(i,j) = \sum_{A\in N}\alpha(A,i,j) \tag{4.19a}$$

$$= \sum_{A\in N}\tilde{s}_{label}(i,j,A)\cdot\tilde{s}_{span}(i,j)\sum_{k=i+1}^{j-1} S(i,k)\cdot S(k,j) \tag{4.19b}$$

$$= \left[\sum_{A\in N}\tilde{s}_{label}(i,j,A)\cdot\tilde{s}_{span}(i,j)\right]\left[\sum_{k=i+1}^{j-1} S(i,k)\cdot S(k,j)\right] \tag{4.19c}$$

where we put explicit brackets to emphasize that independence of the subproblems of labeling and splitting. We can now recognize this as the

'inside' equivalent of the expression from the paper[2]

$$s_{best}(i,j) = \max_{\ell}[s_{label}(i,j,\ell)] + \max_{k}[s_{split}(i,k,j)]. \tag{4.20}$$

The recursions are the same; the semirings are different. The viterbi recursion given above is in the ViterbiSemiring, which uses the max operator as $\oplus$; the inside recursion given in 4.15c has standard addition $(+)$ instead.

4.2.2 Outside recursion

$$\begin{aligned}
\beta(A,i,j) &= \sum_{B\to CA\in R}\sum_{k=1}^{i-1}\psi(B\to CA,k,i-1,j)\cdot\alpha(C,k,i-1)\cdot\beta(B,k,j) \\
&\quad + \sum_{B\to AC\in R}\sum_{k=j+1}^{n}\psi(B\to A,C,i,j,k)\cdot\alpha(C,j+1,k)\cdot\beta(B,i,k) \\
&= \sum_{B\in N}\sum_{C\in N}\sum_{k=1}^{i-1}\psi(B,k,j)\cdot\alpha(C,k,i-1)\cdot\beta(B,k,j) \\
&\quad + \sum_{B\in N}\sum_{C\in N}\sum_{k=j+1}^{n}\psi(B,i,k)\cdot\alpha(C,j+1,k)\cdot\beta(B,i,k) \\
&= \sum_{k=1}^{i-1}\left[\sum_{B\in N}\psi(B,k,j)\cdot\beta(B,k,j)\right]\cdot\left[\sum_{C\in N}\alpha(C,k,i-1)\right] \\
&\quad + \sum_{k=j+1}^{n}\left[\sum_{B\in N}\psi(B,i,k)\cdot\beta(B,i,k)\right]\cdot\left[\sum_{C\in N}\alpha(C,j+1,k)\right] \\
&= \sum_{k=1}^{i-1}S'(k,j)\cdot S(k,i-1) + \sum_{k=j+1}^{n}S'(i,k)\cdot S(j+1,k)
\end{aligned}$$

[2] I believe there is actually an error in this equation: it should read $s_{label}(i,j,\ell) + s_{span}(i,j)$ instead of just $s_{label}(i,j,\ell)$. This is implied by the score for a single node, which is given by equation 4.1, taken directly from the paper.

where

$$S(i,j) = \sum_{A \in N} \alpha(A,i,j)$$
$$S'(i,j) = \sum_{A \in N} \psi(A,i,j)\beta(A,i,j)$$

4.3 Related work

1. Conditional Random Fields (**Sutton+2012:CRF**)
2. Conditional Random Field parsing with linear (**Finkel+2008**) and nonlinear features (**Klein+2015:neural-crf**)

5

Semisupervised learning

Throughout these note we write x for a sentence, y for a (latent) constituency tree, and $\mathcal{Y}(x)$ for the *yield* of x, that is, all trees that can be assigned to x. Furthermore, let $\mathbb{L}$ be a set of pairs (x, y) of sentences with gold trees, and let $\mathbb{U}$ be a set of unlabeled sentences x.

We define the following semi-supervised objective

$$\mathcal{J} \triangleq \mathcal{J}_{\mathcal{S}} + \alpha \mathcal{J}_{\mathcal{U}}, \tag{5.1}$$

were $\mathcal{J}_{\mathcal{S}}$ is the supervised objective optimized over $\mathbb{L}$ and $\mathcal{J}_{\mathcal{U}}$ the unsupervised objective optimized over $\mathbb{U}$. We introduce $\alpha \in \mathbb{R}$ as an arbitrary scalar controlling the contribution of the unsupervised objective.

5.1 Supervised objective

Let $p_\theta(x, y)$ be parametrized by a Generative RNNG (Dyer *et al.*, 2016). Then our supervised objective is

$$\mathcal{J}_{\mathcal{S}} \triangleq \sum_{(x,y)\in\mathbb{L}} \log p_\theta(x, y) \tag{5.2}$$

$$\tag{5.3}$$

This objective is optimized as usual using stochastic gradient estimates:

$$\nabla_\theta \mathcal{J}_\mathcal{S} \approx \frac{|\mathbb{L}|}{n} \sum_{i=1}^{n} \nabla_\theta \log p_\theta(x^{(i)}, y^{(i)}), \tag{5.4}$$

$$\tag{5.5}$$

where $\{(x^{(i)}, y^{(i)})\}_{i=1}^n$ is a mini-batch sampled uniformly from $\mathbb{L}$. To compute $\nabla_\theta \log p_\theta(x^{(i)}, y^{(i)})$ we rely on automatic differentiation (**Baydin+2015:AD**).

5.2 Unsupervised objective

Consider the following objective to be maximized:

$$\mathcal{J}_\mathcal{U} \triangleq \sum_{x \in \mathbb{U}} \log p(x) \tag{5.6a}$$

$$= \sum_{x \in \mathbb{U}} \log \sum_{y \in \mathcal{Y}(x)} p_\theta(x, y) \tag{5.6b}$$

This is a language modelling objective, in which we treat y as latent, and $p_\theta(x, y)$ is a generative RNNG. A consequence the independence assumptions of the RNNG (or better: lack thereof) is that the sum over trees y is no longer tractable. To optimize this objective we must fall back on approximate methods.

Variational approximation We optimize the objective using variational inference (Blei *et al.*, 2016). We introduce a posterior $q_\lambda(y|x)$ parametrised by λ and use Jensen's inequality to derive a variational lower bound:

$$\log p(x) = \log \sum_{y \in \mathcal{Y}(x)} q_\lambda(y|x) \frac{p_\theta(x, y)}{q_\lambda(y|x)} \tag{5.7a}$$

$$= \log \mathbb{E}_{q_\lambda}\left[\frac{p_\theta(x, y)}{q_\lambda(y|x)}\right] \tag{5.7b}$$

$$\geq \mathbb{E}_{q_\lambda}\left[\log \frac{p_\theta(x, y)}{q_\lambda(y|x)}\right] \tag{5.7c}$$

$$= \mathbb{E}_{q_\lambda}\left[\log p_\theta(x, y) - \log q_\lambda(y|x)\right] \triangleq \mathcal{L}(\theta, \lambda) \tag{5.7d}$$

The only requirement for q_λ is that

$$p(x, y) > 0 \Rightarrow q(y|x) > 0 \qquad \text{for all } x \text{ and } y \in \mathcal{Y}(x). \tag{5.8}$$

Any discriminatively trained parser fulfills this requirement.

The lower bound $\mathcal{L}(\theta, \lambda)$ will be optimized by gradient optimization, which means we will need to take the gradients $\nabla_\theta \mathcal{L}(\theta, \lambda)$ and $\nabla_\lambda \mathcal{L}(\theta, \lambda)$.

Gradients of joint parameters The first gradient is easy and permits a straightforward Monte-Carlo estimate:

$$\begin{aligned}
\nabla_\theta \mathcal{L}(\theta, \lambda) &= \nabla_\theta \mathbb{E}_{q_\lambda}\big[\log p_\theta(x, y) - \log q_\lambda(y|x)\big] && \text{(5.9a)}\\
&= \mathbb{E}_{q_\lambda}\big[\nabla_\theta \log p_\theta(x, y) - \nabla_\theta \log q_\lambda(y|x)\big] && \text{(5.9b)}\\
&= \mathbb{E}_{q_\lambda}\big[\nabla_\theta \log p_\theta(x, y)\big] && \text{(5.9c)}\\
&\approx \frac{1}{n}\sum_{i=1}^{n} \nabla_\theta \log p_\theta(x, y^{(i)}) && \text{(5.9d)}
\end{aligned}$$

where $y_i \sim q_\lambda(y|x)$ for $i = 1, \ldots, n$. We can move the gradient inside the expectation (second equality) because q does not depend on θ.

Gradients of posterior parameters The second gradient is harder and requires us to rewrite the objective:

$$\begin{aligned}
\nabla_\lambda \mathcal{L}(\theta, \lambda) &= \nabla_\lambda \mathbb{E}_{q_\lambda}\left[\log p_\theta(x, y) - \log q_\lambda(y|x)\right] && \text{(5.10a)}\\
&= \nabla_\lambda \sum_y \left\{q_\lambda(y|x) \log p_\theta(x, y) - q_\lambda(y|x) \log q_\lambda(y|x)\right\} && \text{(5.10b)}\\
&= \sum_y \big\{\nabla_\lambda q_\lambda(y|x) \log p_\theta(x, y) && \text{(5.10c)}\\
&\qquad - \nabla_\lambda q_\lambda(y|x) \log q_\lambda(y|x) && \text{(5.10d)}\\
&\qquad - q_\lambda(y|x) \nabla_\lambda \log q_\lambda(y|x)\big\} && \text{(5.10e)}\\
&= \sum_y \left\{\nabla_\lambda q_\lambda(y|x) \log p_\theta(x, y) - \nabla_\lambda q_\lambda(y|x) \log q_\lambda(y|x)\right\} && \text{(5.10f)}\\
&= \sum_y \left\{(\log p_\theta(x, y) - \log q_\lambda(y|x)) \nabla_\lambda q_\lambda(y|x)\right\} && \text{(5.10g)}\\
&= \sum_y \left\{(\log p_\theta(x, y) - \log q_\lambda(y|x)) q_\lambda(y|x) \nabla_\lambda \log q_\lambda(y|x)\right\} && \text{(5.10h)}\\
&= \mathbb{E}_{q_\lambda}\left[(\log p_\theta(x, y) - \log q_\lambda(y|x)) \nabla_\lambda \log q_\lambda(y|x)\right\} && \text{(5.10i)}\\
&= \mathbb{E}_{q_\lambda}\left[l(x, y) \nabla_\lambda \log q_\lambda(y|x)\right] && \text{(5.10j)}
\end{aligned}$$

Where we've defined a learning signal $l(x, y) \triangleq \log p_\theta(x, y) - \log q_\lambda(y|x)$.

In this derivation we used the identity

$$\nabla_\lambda \log q_\lambda(y|x) = \frac{\nabla_\lambda q_\lambda(y|x)}{q_\lambda(y|x)} \tag{5.11}$$

or equivalently

$$\nabla_\lambda q_\lambda(y|x) = q_\lambda(y|x) \nabla_\lambda \log q_\lambda(y|x) \tag{5.12}$$

and the fact that

$$\sum_y q_\lambda(y|x)\nabla_\lambda \log q_\lambda(y|x) = \sum_y q_\lambda(y|x)\frac{\nabla_\lambda q_\lambda(y|x)}{q_\lambda(y|x)} \tag{5.13a}$$

$$= \sum_y \nabla_\lambda q_\lambda(y|x) \tag{5.13b}$$

$$= \nabla_\lambda \sum_y q_\lambda(y|x) \tag{5.13c}$$

$$= \nabla_\lambda 1 \tag{5.13d}$$

$$= 0. \tag{5.13e}$$

$$\tag{5.13f}$$

This rewritten objective permits a straightforward MC estimate:

$$\mathbb{E}_{q_\lambda}\left[l(x,y)\nabla_\lambda \log q_\lambda(y|x)\right] \approx \frac{1}{n}\sum_{i=1}^{n} l(x,y^{(i)})\nabla_\lambda \log q_\lambda(x|y^{(i)}) \tag{5.14}$$

where $y_i \sim q_\lambda(y|x)$ for $i = 1,\ldots,n$.

This estimator has been derived (in slightly different forms) in among others (Williams, 1992), (Paisley *et al.*, 2012), (Mnih and Gregor, 2014), (Ranganath *et al.*, 2014), and (**Miao+16:LLVAE**) and is known as the REINFORCE estimator (Williams, 1992), or score function estimator (after the score function $\nabla_\theta \log p_\theta(x)$) (Fu, 2006).

5.3 Optimization

We use automatic differentiation (**Baydin+2015:AD**) to obtain all our gradient estimates.

To get the gradients in formula 5.14 we rewrite it in the form of a surrogate objective (Schulman *et al.*, 2015)

$$L(\lambda) = \frac{1}{n}\sum_{i=1}^{n} \log q_\lambda(x|y^{(i)})\text{BLOCKGRAD}(l(x,y^{(i)})) \tag{5.15}$$

where BLOCKGRAD is function that 'detaches' a parametrized function from the computation graph effectively turning it into a scalar. That is, loosely speaking

$$\text{BLOCKGRAD}(f_\theta(x)) = f(x) \tag{5.16}$$

such that

$$\nabla_\theta \text{BLOCKGRAD}(f_\theta(x)) = \nabla_\theta f(x) \tag{5.17}$$

$$= 0 \tag{5.18}$$

Then differentiation of L gives us the unbiased estimator

$$\nabla_\lambda L(\lambda) = \frac{1}{n}\sum_{i=1}^{n} l(x, y^{(i)}) \nabla_\lambda \log q_\lambda(x|y^{(i)}) \tag{5.19}$$

5.4 Variance reduction

We have derived an estimator for the gradient of the posterior parameters in the unsupervised objective. This estimator is unbiased, but is known to have high variance, often too much to be useful (Paisley *et al.*, 2012). Two effective methods to counter this are control variates and baselines (Ross, 2006).

Variance of estimator First, let's analyze the variance of our estimator. Note that our expectation is of the general form

$$\mu \triangleq \mathbb{E}[f(X)] \tag{5.20}$$

and that we estimate this quantity by generating n independent samples $X_1, \ldots, X_n \sim P(X)$ and computing

$$\hat{\mu} \triangleq \frac{1}{n}\sum_{i=1}^{n} f(X_i). \tag{5.21}$$

This is an unbiased estimator for μ with error

$$\text{MSE} = \mathbb{E}[(\mu - \hat{\mu})^2] = \text{Var}[\hat{\mu}] = \frac{\text{Var}[\hat{\mu}]}{n}, \tag{5.22}$$

which means that the error is of the order

$$\mu - \hat{\mu} \sim \sqrt{\frac{\text{Var}[\hat{\mu}]}{n}} \tag{5.23}$$

and reducing it linearly requires a quadratic number of samples.

In our particular case, the function f is

$$f_{X=x}(Y) \triangleq l(X, Y) \nabla_\lambda \log q_\lambda(Y|X = x) \tag{5.24}$$

where we have made explicit that y is the random variable, and x is given.

Control variates Consider a function $\phi(X)$ with known expectation

$$\mu_\phi \triangleq \mathbb{E}[\phi(X)] \tag{5.25}$$

Then we can define a new function $\hat{f}$ such that

$$\hat{f}(X) \triangleq f(X) - \phi(X) + \mu_\phi. \tag{5.26}$$

This function is also an estimator for μ, since

$$\mathbb{E}[\hat{f}(X)] = \mathbb{E}[f(X)] - \mu_\phi + \mu_\phi \tag{5.27}$$

$$= \mathbb{E}[f(X)], \tag{5.28}$$

and a computation shows that the variance of the new function is

$$\begin{aligned}
\mathrm{Var}[\hat{f}(X)] &= \mathbb{E}[(f(X) - \phi(X) + \mu_\phi) - \mu)^2] \\
&= \mathbb{E}[(f(X) - \phi(X) + \mu_\phi)^2] - 2\mathbb{E}[(f(X) - \phi(X) + \mu_\phi)\mu] + \mathbb{E}[\mu^2] \\
&= \mathbb{E}[(f(X) - \phi(X) + \mu_\phi)^2] - 2\mathbb{E}[(f(X) - \phi(X) + \mu_\phi)]\mu + \mu^2 \\
&= \mathbb{E}[(f(X) - \phi(X) + \mu_\phi)^2] - 2\mu^2 + \mu^2 \\
&= \mathbb{E}[f(X)^2 + \phi(X)^2 + \mu_\phi^2 - 2f(X)\phi(X) + 2f(X)\mu_\phi - 2\phi(X)\mu_\phi] - \mu^2 \\
&= \mathbb{E}[f(X)^2] - \mathbb{E}[f(X)]^2 \\
&\quad - 2(\mathbb{E}[f(X)\phi(X)] - \mathbb{E}[f(X)]\mathbb{E}[\phi(X)]) \\
&\quad + \mathbb{E}[\phi(X)^2] - \mathbb{E}[\phi(X)]^2 \\
&= \mathrm{Var}[f(X)] - 2\ \mathrm{Cov}[f(X), \phi(X)] + \mathrm{Var}[\phi(X)]
\end{aligned}$$

This means we can get a reduction in variance whenever

$$\mathrm{Cov}[f(X), \phi(X)] > \frac{1}{2}\ \mathrm{Var}[\phi(X)]. \tag{5.29}$$

The function ϕ is called a *control variate*—it allows us to control the variance of f.

From the equality above we can see that this will be the case whenever $f(X)$ and $\phi(X)$ are strongly correlated. Our choice of control variate will be made with the that in mind. Furthermore, $\mathbb{E}[\phi(X)]$ must be known. What is an optimal control variate? Typically a control variate of the form $a\phi$ is chosen with fixed, and a is optimized to maximize the correlation. This brings us to the generic formulation of a control variate:

$$\hat{f}(X) \triangleq f(X) - a(\phi(X) - \mathbb{E}[\phi(X)]) \tag{5.30}$$

with variance

$$\text{Var}[\hat{f}(X)] = \text{Var}[f(X)] - 2a \text{ Cov}[f(X), \phi(X)] + a^2 \text{ Var}[\phi(X)] \quad (5.31)$$
$$(5.32)$$

We take a derivative of this with respect to a

$$\frac{\partial}{\partial a}\text{Var}[\hat{f}(X)] = -2 \text{ Cov}[f(X), \phi(X)] + 2a \text{ Var}[\phi(X)] \quad (5.33)$$

Setting this to zero and solving for a we obtain the optimal choice for a

$$a = \frac{\text{Cov}[f(X), \phi(X)]}{\text{Var}[\phi(X)]}. \quad (5.34)$$

Plugging in this solution into the expression for $\text{Var}[\hat{f}(X)]$ and dividing by $\text{Var}[f(X)]$ we get

$$\frac{\text{Var}[\hat{f}(X)]}{\text{Var}[f(X)]} = 1 - \frac{\text{Cov}[f(X), \phi(X)]}{\text{Var}[f(X)] \text{ Var}[\phi(X)]} \quad (5.35)$$
$$= 1 - \text{corr}^2[f(X), \phi(X)], \quad (5.36)$$

which shows that given this choice of a the reduction in variance is directly determined by the correlation between $f(X)$ and $\phi(X)$.

Bringing this all together, we let our new estimator be

$$\mathbb{E}[f(X)] = \mathbb{E}[\hat{f}(X)] \approx \frac{1}{n}\sum_{i=1}^{n}[f(X_i) - a\phi(X_i)] - \mu_\phi \quad (5.37)$$

Example (Ross, 2006) Suppose we want to use simulation to determine

$$\mathbb{E}[f(X)] = \mathbb{E}[e^X] = \int_0^1 e^x dx = e - 1 \quad (5.38)$$

with $X \sim \mathcal{U}(0, 1)$. A natural control variate to use in this case is the random variable X itself: $\phi(X) \triangleq X$. We thus define the new estimator

$$\hat{f}(X) = f(X) - \phi(X) + \mathbb{E}[\phi(X)] \quad (5.39)$$
$$= e^X - X + \frac{1}{2}. \quad (5.40)$$

To compute the decrease in variance with this new estimator, we first note that

$$\begin{aligned}
\mathrm{Cov}(e^X, X) &= \mathbb{E}[Xe^X] - \mathbb{E}[X]\mathbb{E}[e^X] \\
&= \int_0^1 xe^x dx - \frac{e-1}{2} \\
&= 1 - \frac{e-1}{2} \approx 0.14086 \\
\mathrm{Var}[e^X] &= \mathbb{E}[e^{2X}] - (\mathbb{E}[e^X])^2 \\
&= \int_0^1 e^{2x} dx - (1-e^x)^2 \\
&= \frac{e^2-1}{2} - (1-e^x)^2 \approx 0.2420 \\
\mathrm{Var}[X] &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \\
&= \int_0^1 x^2 dx - \frac{1}{4} \\
&= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}.
\end{aligned}$$

When we choose a as in formula 5.34 we can use formula 5.35 to compute that

$$\frac{\mathrm{Var}[\hat{f}(X)]}{\mathrm{Var}[f(X)]} = 1 - \frac{(0.14086)^2}{\frac{0.2420}{12}} \tag{5.41}$$

$$\approx 0.0161. \tag{5.42}$$

This is a reduction of 98.4 percent! A simulation illustrates what this looks like in practice with ... samples:

6

Multitask learning

I doubt whether I should make this a separate chapter.

- I just want to describe my cool span-based multitask model, and describe the earlier CCG multitask model.
- Maybe I will make it a subsection of the Syntactic evaluation?
- Or maybe make it a really short, three page chapter?

7

Syntactic evaluation

In this section I describe the syntactic evaluation that I perform.

8

Conclusion

Here is a narrative summary of what I have shown in this thesis.

8.1 Main contributions

The main n contributions of thesis are:

- **Global training of a chart based neural parser.** Here I describe what that entails.
- **Semisupervised training of RNNGs.** Here I describe what that entails.
- **Effective baselines for the score functio estimator.** Here I describe what that entails.

8.2 Future work

We have identified possibilities for future work:

- **Something...** Here I describe what that entails.

Acknowledgements

Appendices

A

Figures

In this appendix I will put figures, for cases where there are just too many. For example:

- The barplots of the syntactic evaluation
- The training losses for the various models
- The valuation perplexity and f-score during training.

B

Data

In this section I describe in detail the data used in the experiments:

- Data preprocessing for the PTB and unlabeled data.
- Vocabulary and UNKing for all models.

C

Data

In this appendix I describe CRF's for factor graphs, together with the message passing algorithms from which forward-backward and inside-outside are derived.

References

Blei, D. M., A. Kucukelbir, and J. D. McAuliffe (2016). "Variational Inference: A Review for Statisticians". *ArXiv e-prints*. Jan.

Dyer, C., A. Kuncoro, M. Ballesteros, and N. A. Smith (2016). "Recurrent Neural Network Grammars". *CoRR*. abs/1602.07776. arXiv: 1602.07776. URL: http://arxiv.org/abs/1602.07776.

Fu, M. C. (2006). "Gradient Estimation". In: *Handbooks in Operations Research and Management Science (Volume 13)*. Ed. by B. L. N. Edited by Shane G. Henderson. Elsevier. Chap. 19. 575–616.

Mnih, A. and K. Gregor (2014). "Neural Variational Inference and Learning in Belief Networks". In: *ICML*.

Paisley, J., D. Blei, and M. Jordan (2012). "Variational Bayesian Inference with Stochastic Search". In: *Proceedings of the 29th International Conference on Machine Learning (ICML-12)*. Ed. by J. Langford and J. Pineau. *ICML '12*. New York, NY, USA: Omnipress. 1367–1374.

Ranganath, R., S. Gerrish, and D. Blei (2014). "Black Box Variational Inference". In: *Proceedings of the Seventeenth International Conference on Artificial Intelligence and Statistics*. Ed. by S. Kaski and J. Corander. Vol. 33. *Proceedings of Machine Learning Research*. Reykjavik, Iceland: PMLR. 814–822. URL: http://proceedings.mlr.press/v33/ranganath14.html.

Ross, S. M. (2006). *Simulation, Fourth Edition*. Orlando, FL, USA: Academic Press, Inc.

Schulman, J., N. Heess, T. Weber, and P. Abbeel (2015). "Gradient Estimation Using Stochastic Computation Graphs". In: *Advances in Neural Information Processing Systems 28*. Ed. by C. Cortes, N. D. Lawrence, D. D. Lee, M. Sugiyama, and R. Garnett. Curran Associates, Inc. 3528–3536.

Williams, R. J. (1992). "Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning". *Mach. Learn.* 8(3-4): 229–256.