FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
ITMO UNIVERSITY

Report
on the practical task No. 4
“Algorithms for unconstrained nonlinear optimization. Stochastic and metaheuristic algorithms”

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St. Petersburg
2023

Goal

1.Use simulated annealing, differential evolution, particle swarm optimization (3 in 4) as well as LMA methods and Newton methods to solve minimization problems:

$$D(a,b,c,d) = \sum_{k=0}^{1000} (F(x_k, a, b, c, d) - y_k)^2.$$

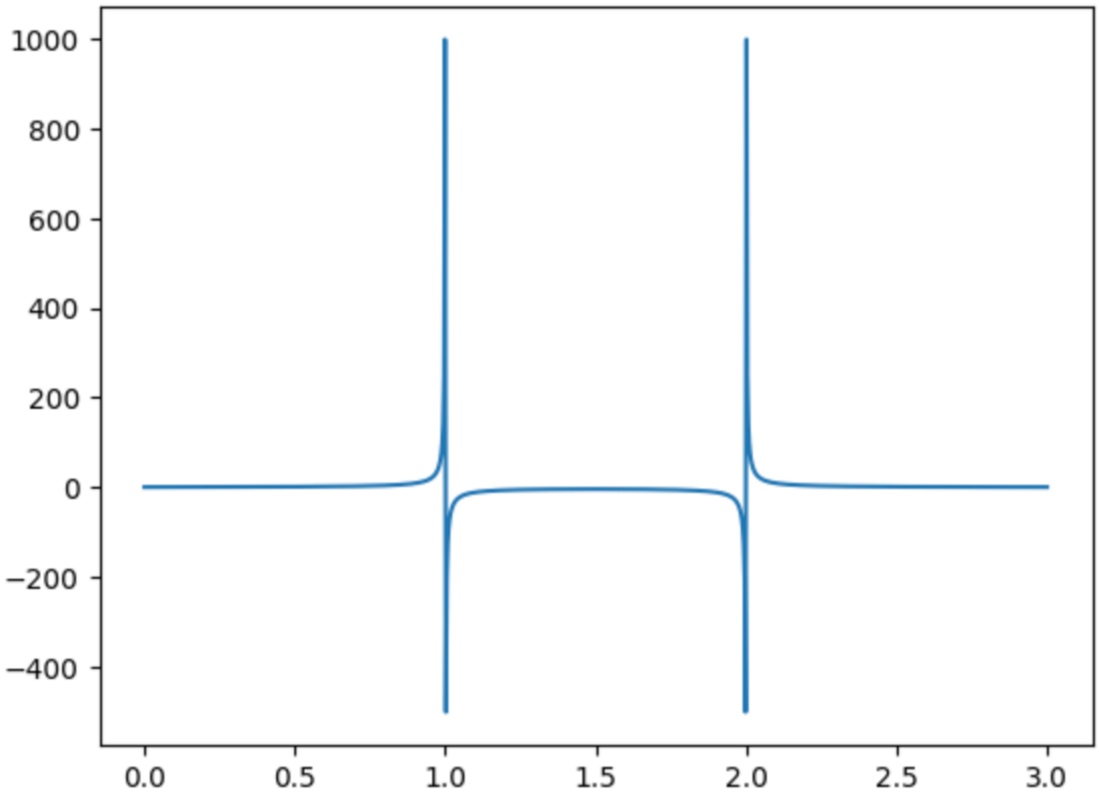
2. Solve the corresponding Travelling Salesman Problem with simulated annealing method in the transport connection of 50 cities and visualization.

Formulation of the problem

Q1.

1.Noisy random

X_data	*np.range(0, 3.003, 0.003)*
Y_data	$y_k = \begin{cases} -100 + \delta_k, & f(x_k) < -100, \\ f(x_k) + \delta_k, & -100 \le f(x_k) \le 100, \\ 100 + \delta_k, & f(x_k) > 100, \end{cases}$ $f(x) = \dfrac{1}{x^2 - 3x + 2},$



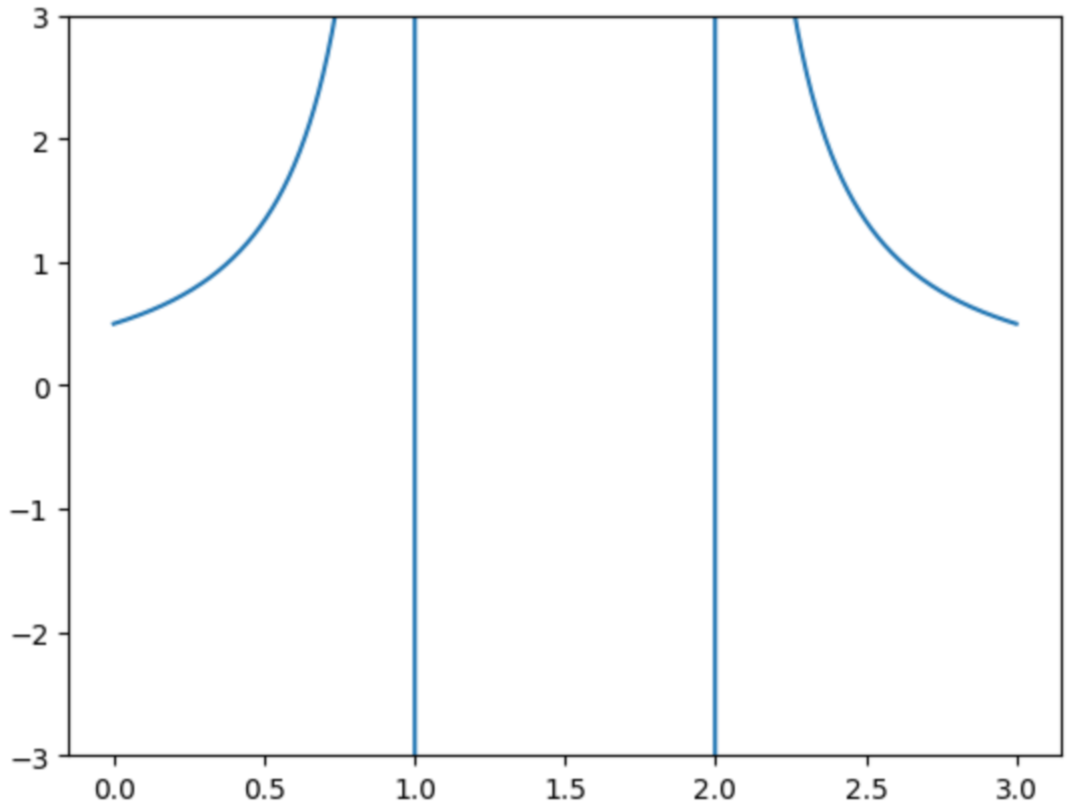
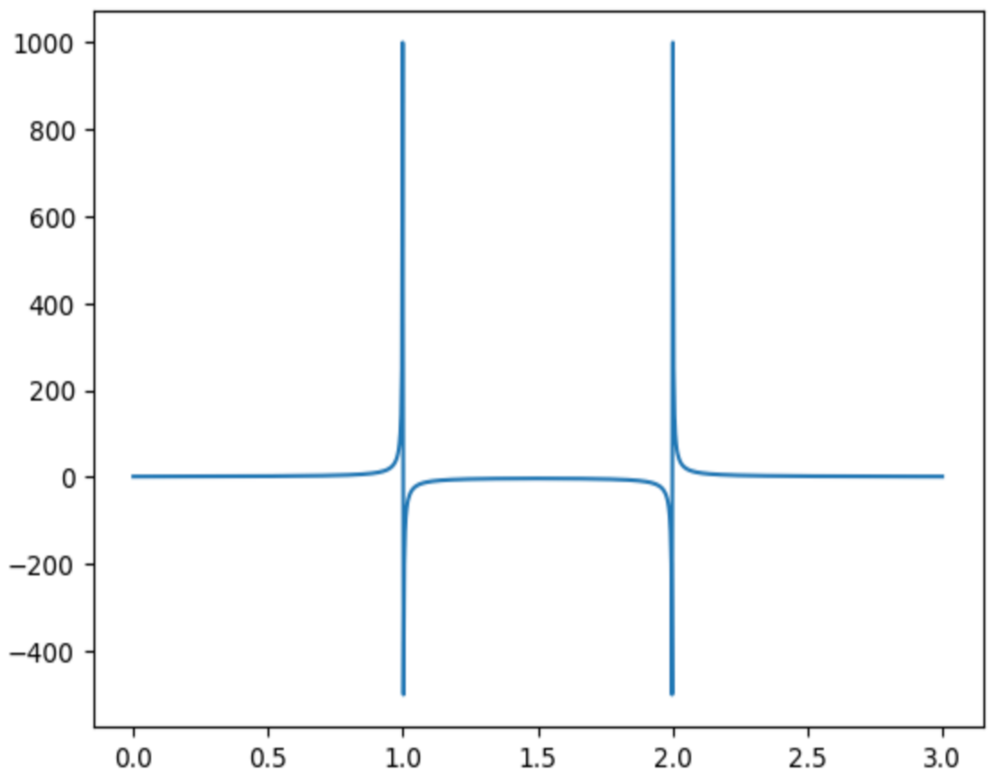


Figure-1 the plot of f(x) (δ=0)



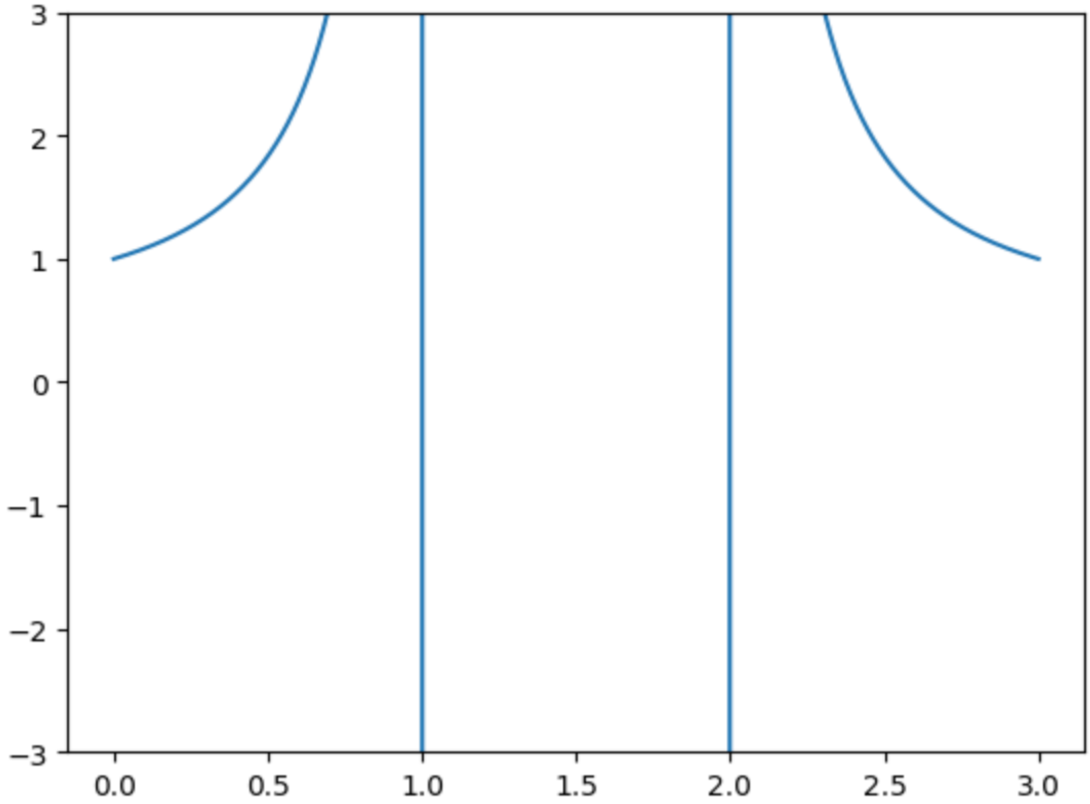


Figure-2 the plot of f(x) (δ=0.5)

Features:1. the value range of x is (0,3), f(x) equal to 1/(x**2+3*x+2), as well as other parts of the piecewise function determine the upper and lower limits of f(x)

2.For f(x) the variables are only δ.

Approximated function:

$$F(x, a, b, c, d) = \frac{ax + b}{x^2 + cx + d}$$

When x is known, there are still 4 unknown quantities: a, b, c, d. Therefore, the optimal solution of this function can only be the least squares method.

Q2.

For Task 2, we had to select at least 15 cities in the world that have land transport connections with each other. We selected 50 cities, which we obtained the coordinates from the web page attached to the assignment instructions.

In the following, we will show the selected cities on a map:

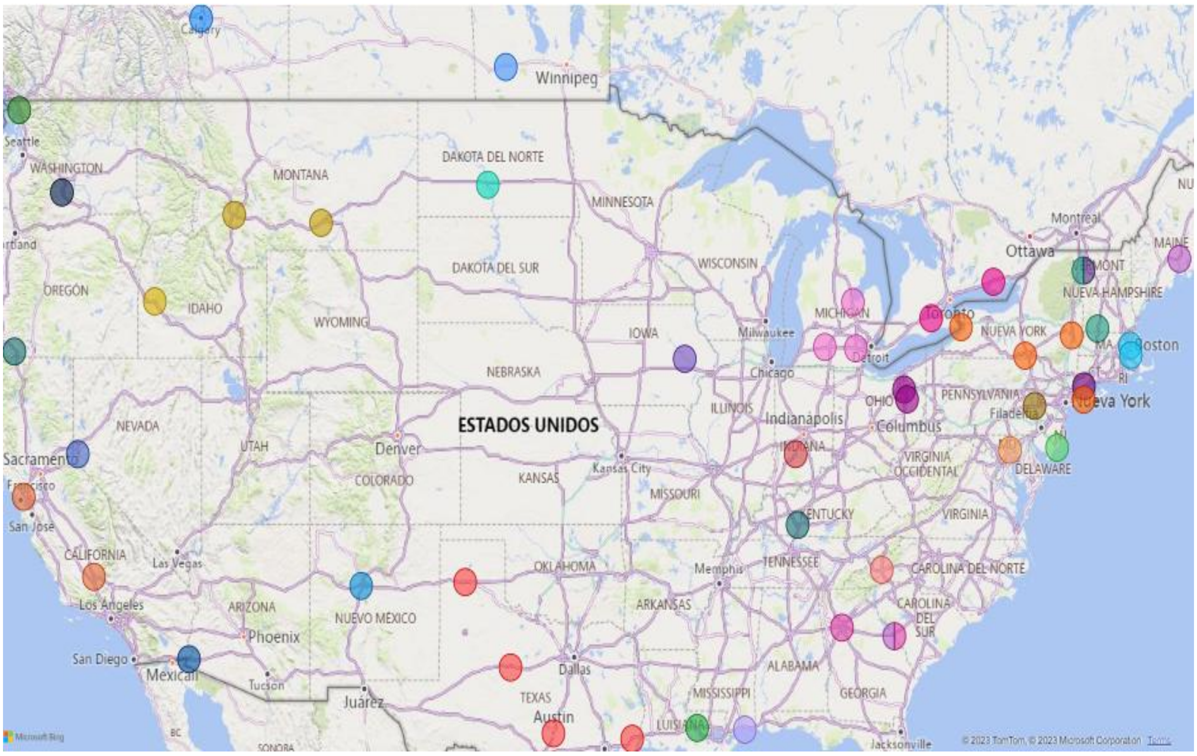


Figure-3 Map of the 50 selected cities

As we can see, there are some cities that are very close and others that are very far away, so we will make use of the Simulated Annealing method to solve the corresponding traveling salesman problem and find the shortest route, passing through each city only once and returning to the city of origin.

Brief theoretical part

1. Monto Carlo(stochastic) and meta-heuristic method
   Monto Carlo method that finds the optimization solution by the random sampling is one of numerical calculation method.
   Key: According to weak law of large numbers, the outcome of random sampling is equal to mathematical expectation of this problem which integral is approximate to optimization solution of problem.
2. Simulated annealing
   Simulated annealing is a stochastic method of simulating the annealing process. It defines the temperature function Tk+1=αTk. Starting from the initial solution, a point a` is selected from k nearest neighbors according to the nearest neighbor function to compare with it. If a` is a better solution than ak, replace it, otherwise replace it with exp (-(ak-a`)/Tk) and update Tk.Stop Tk=0 in the end.
3. Differential evolution
   The evolutionary algorithms are processing which update candidate solutions to get optimization solution.
   Select agents x and agents a,b,c different from x.if ri∈（0，1<p,yi=ai+w(bi-ci),else if :yi=xi. Then compare it with the optimal solution and choose whether to retain it.
4. Particle swarm optimization
   In particle swarm optimization, each particle which have certainly velocity remove with group optimal solution, particle optimal solution as well as speed inertia.

Results

1. Task 1：

1.1 Nelder-Mead method

```
Optimization terminated successfully.
         Current function value: 137111.171137
         Iterations: 240
         Function evaluations: 432
1 [-1.18360252  1.18419057 -2.00085937  1.00088268]
```

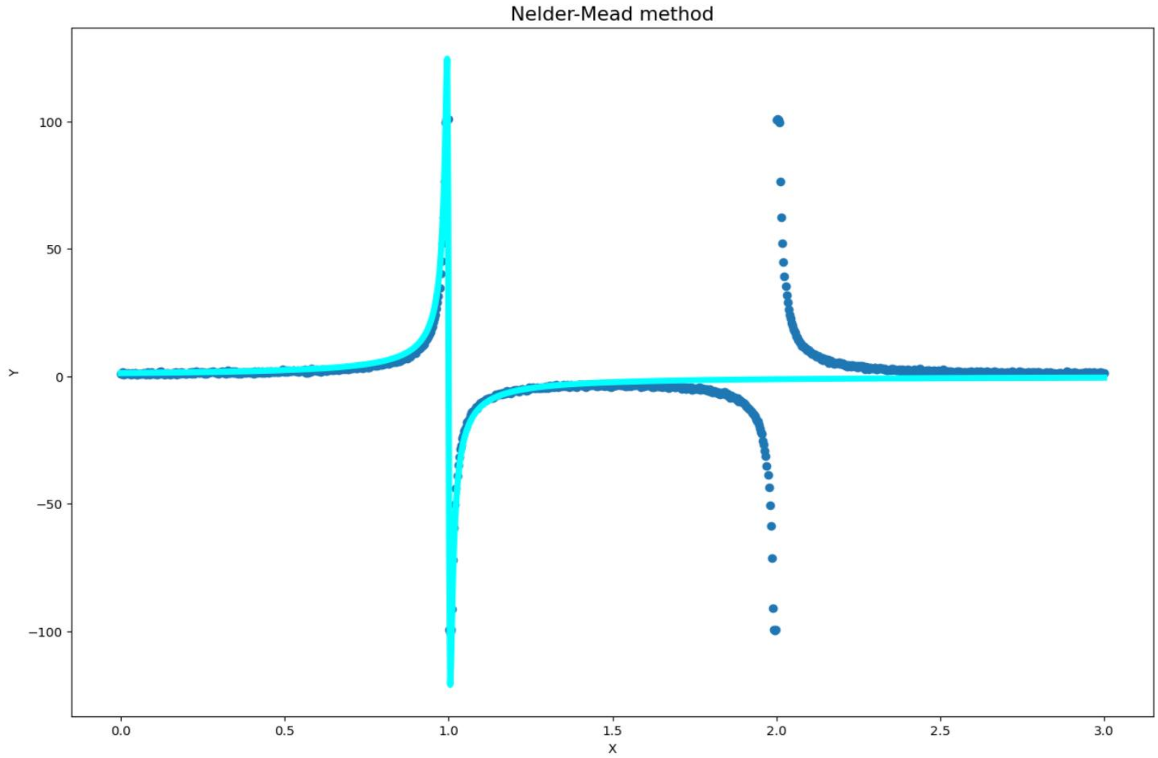


Figure-4 Nelder-Mead method

1.2 LMA method

```
         Current function value: 137111.171137
         Iterations: 240
         Function evaluations: 432
[-1.18360252  1.18419057 -2.00085937  1.00088268]
```

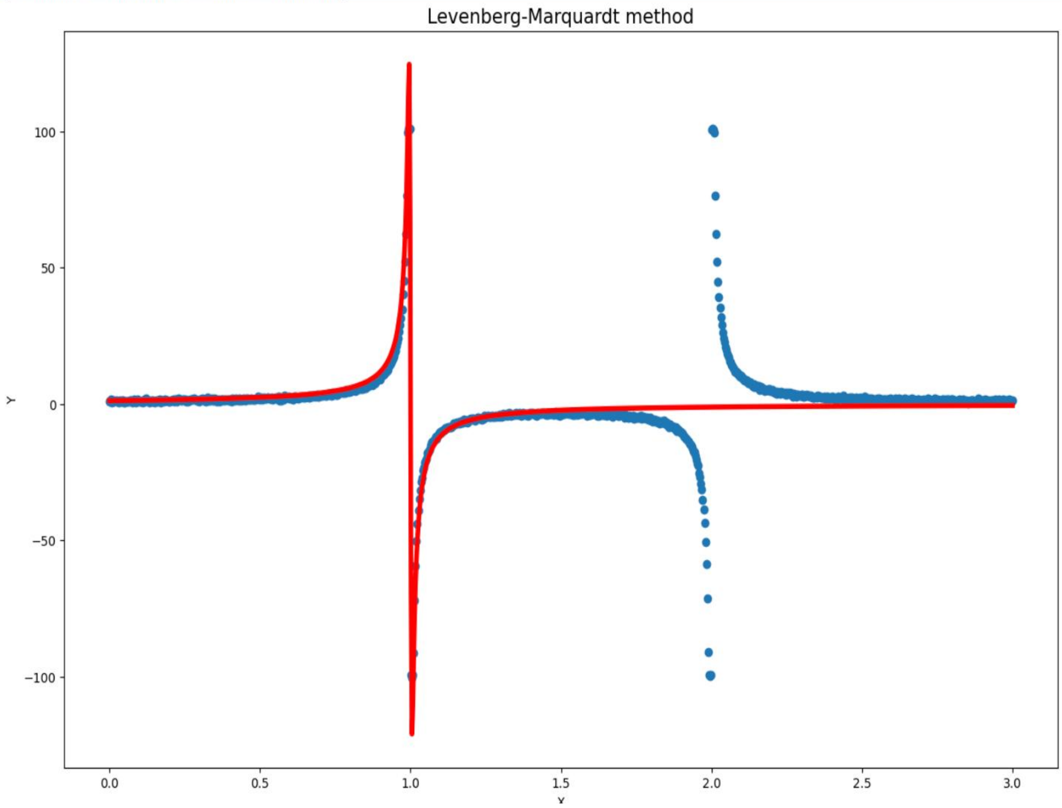


Figure-5 LMA method

1.3 Simulated Annealing method

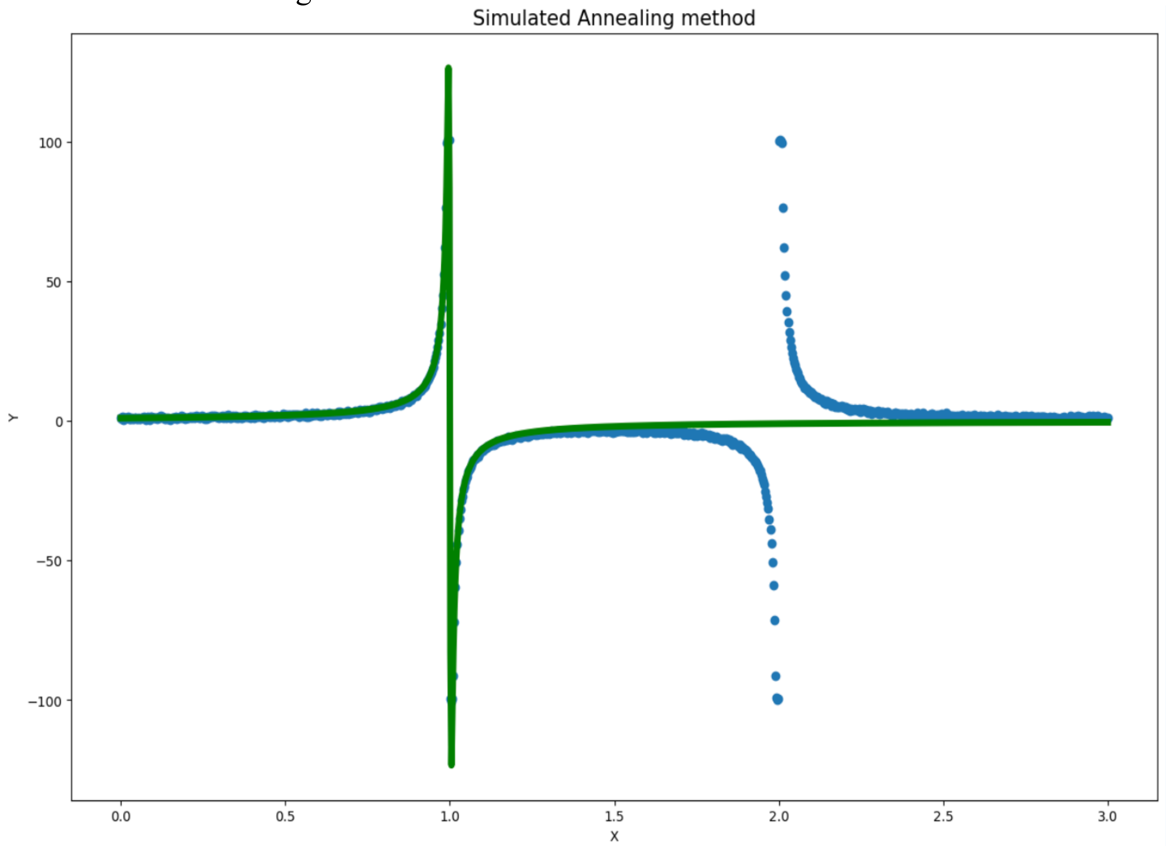


Figure-6 Simulated Annealing method

1.4 Particle swarm method

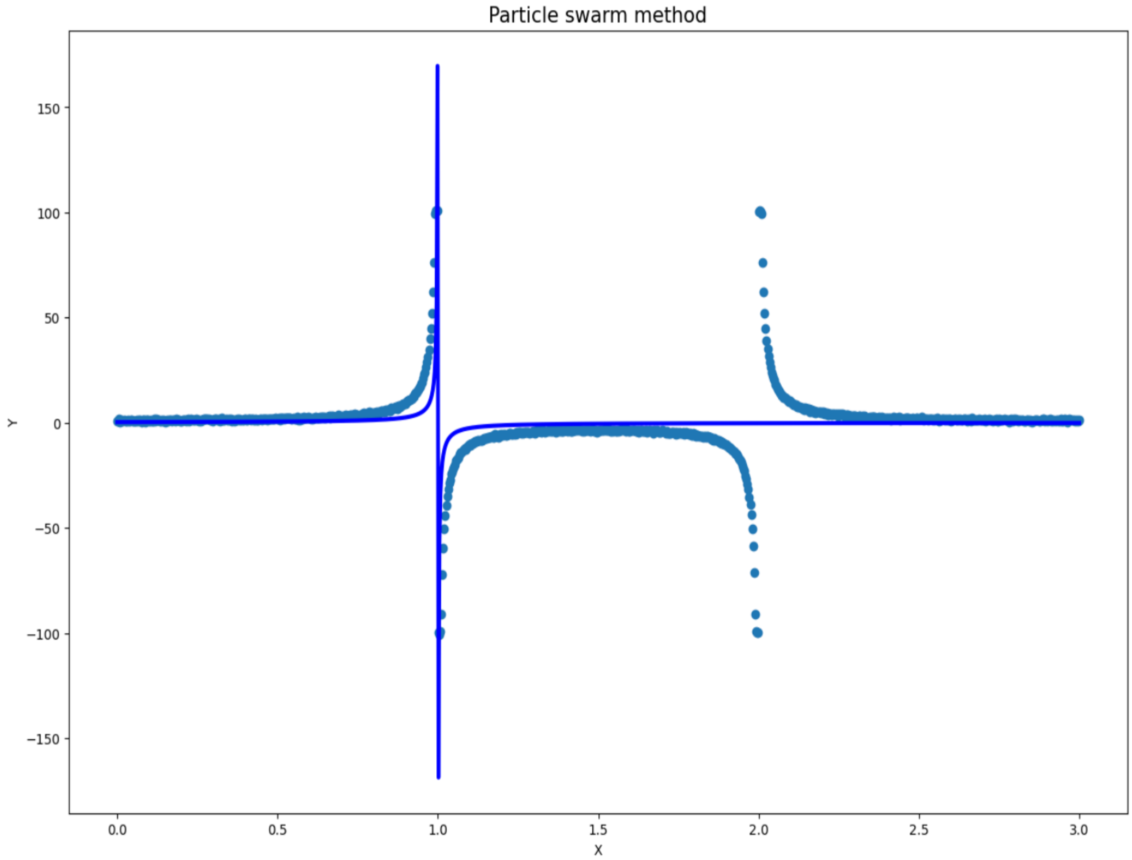


Figure-7 Particle swarm method

1.5 Differential Evolution method

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[-1.01298171  1.01349053 -2.0009197   1.00093614]
```

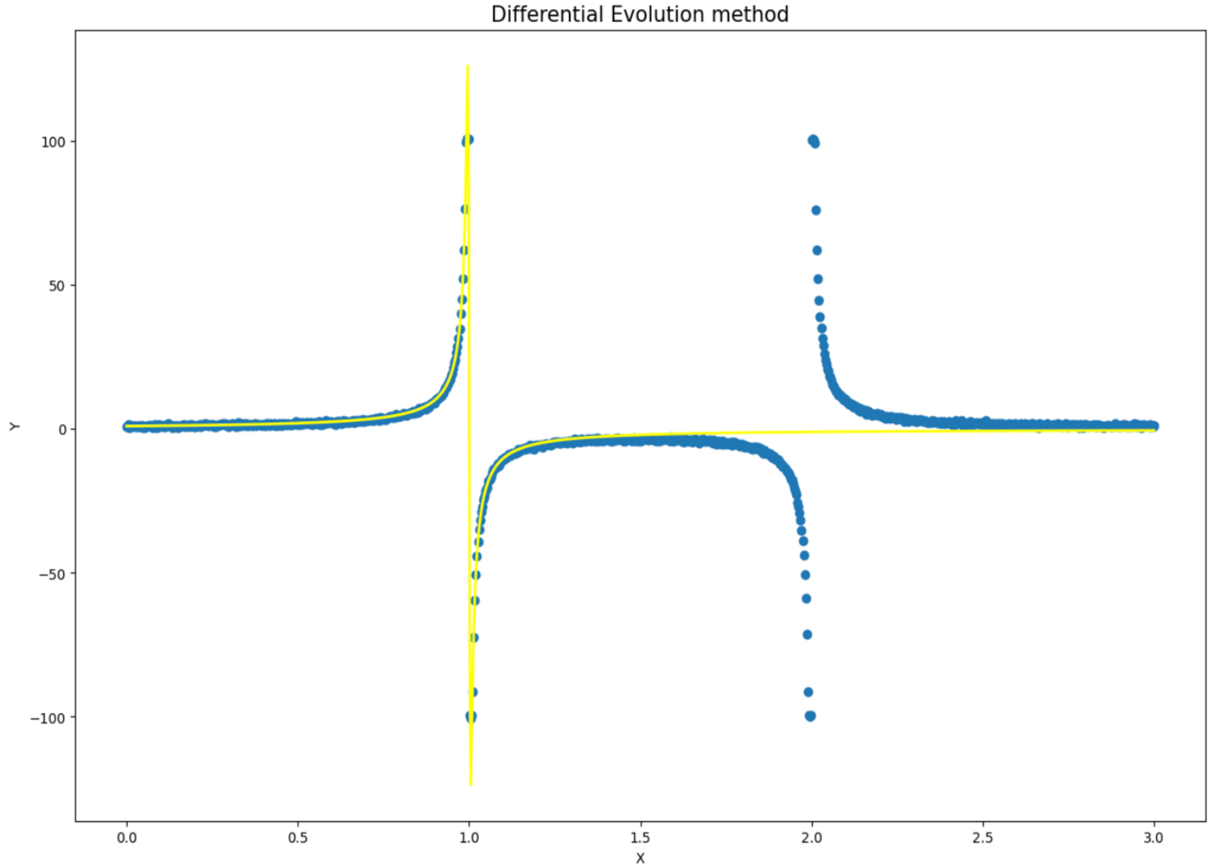

Figure-7 Differential Evolution method

1.6 Comparison

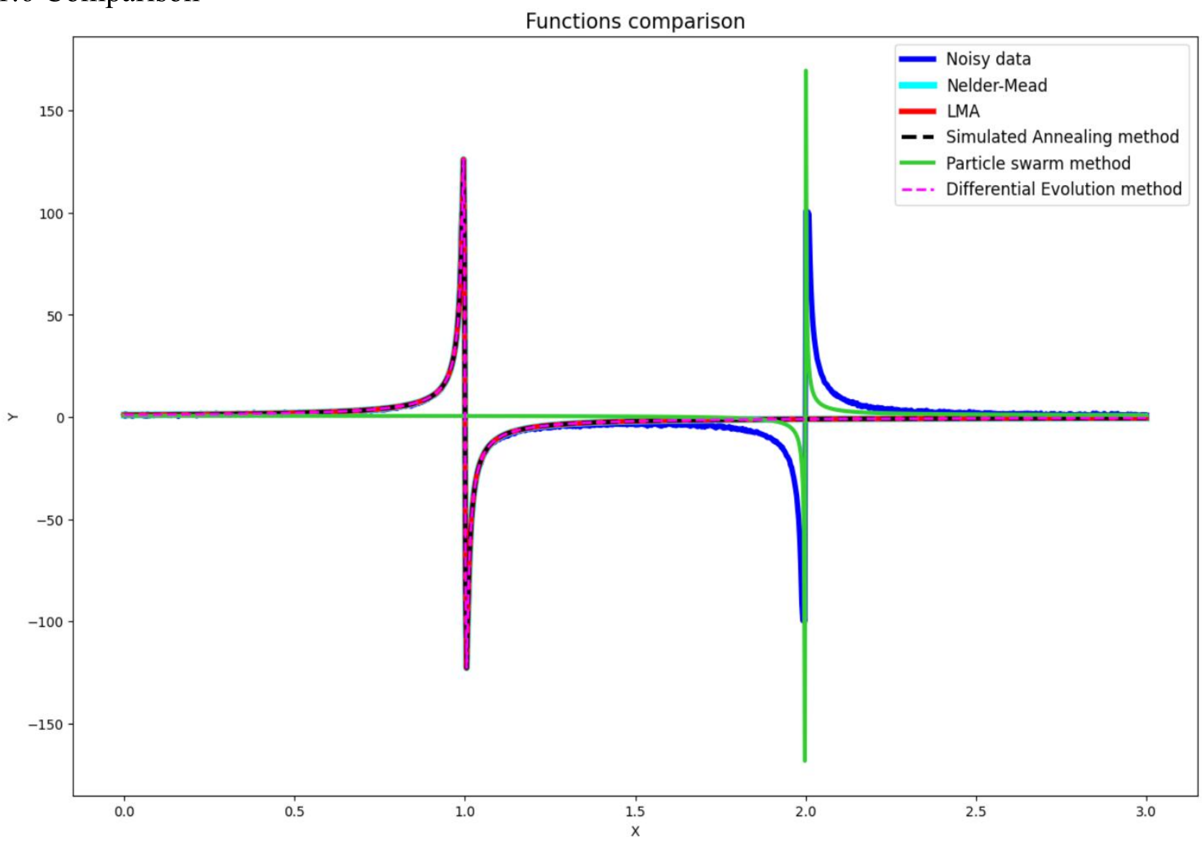

Figure-8 Comparison

Discussion:

This time, 5 methods are used for discussion: Nelder-Mead LMA simulated annealing method, particle swarm method, and differential evolution method. The value of x is probably between -3 and 3, more accurately between -2 and 2.

Regarding the missing part of the image, we need to discuss it. It is worth noting that when the particle swarm method is used for calculation, when the upper and lower limits are large enough, the function will become the other half of the symmetry of the existing image.

Task 2

As already mentioned, consisted of choosing at least 15 cities in the world that have land transport connections between them and calculating the matrix of distances between them and applying the Simulated Annealing method to find the shortest route that a salesperson must travel to visit all the selected cities and return to the point of origin. We selected 50 US and Canadian cities. The coordinates were obtained from the web page that was written in the task instructions or you can see the csv file with the data, which is in the same GitHub repository found in the appendix section.

Simulated annealing was used to solve the traveling salesman problem on a data set of 50 points in a plane, i.e., the 50 cities. First, an initial random trajectory was chosen.

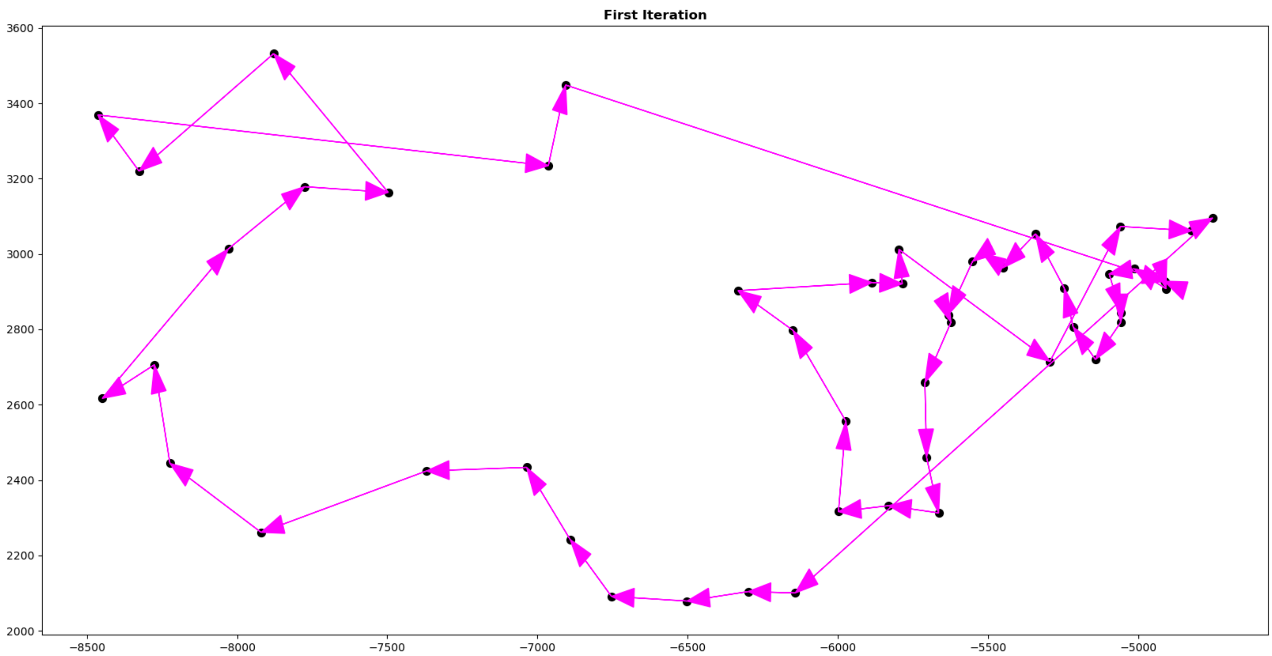

Figure 9 – First Iteration in Travelling Salesman Problem.

Then, the route was optimized so that in each iteration the cities will reconnect with a certain probability that depends on how optimal the new solution is. The resulting path is shown below:

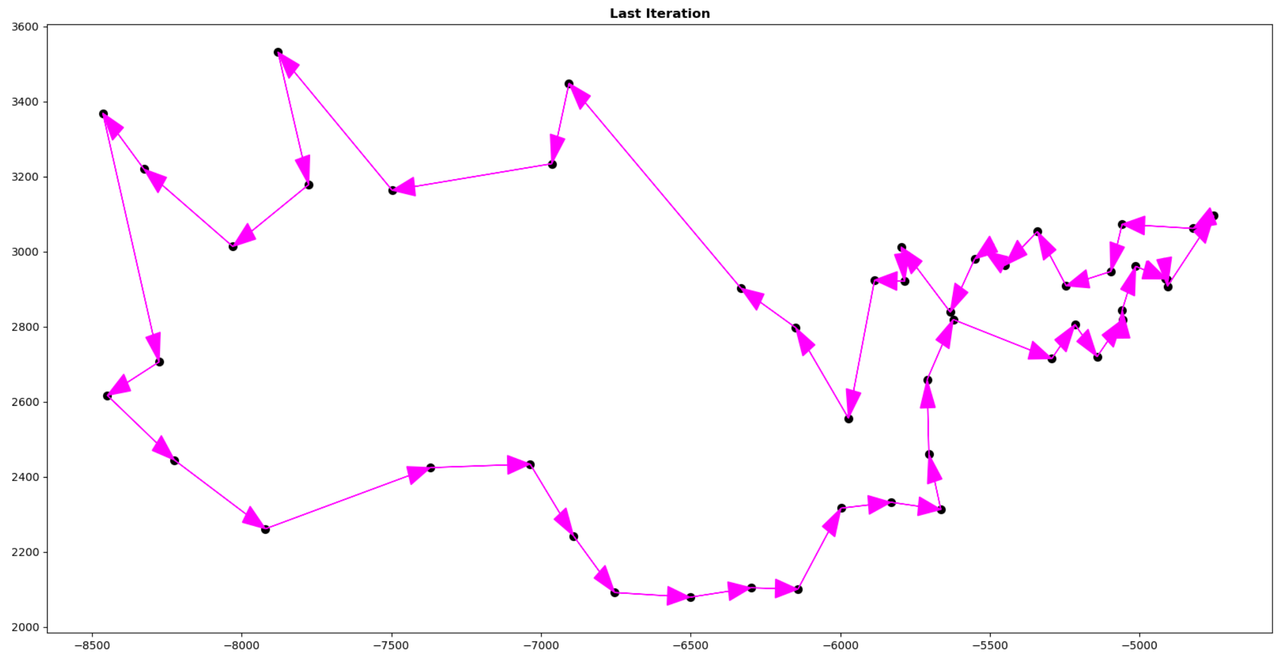

Figure 10 – Last Iteration in Travelling Salesman Problem

We can see in the graph above that the algorithm found the shortest distance between the cities and returned to the same starting point.

First Iteration	Last Iteration
15,637.053	11,679.349

Figure 11 – First Iteration VS Last Iteration (Distance)

The initial path had a length of 15,637.053 units, while the optimized path is 11,679.349 units. Optimizing the path was a great improvement, and the path looks quite optimal.

Conclusions

1. About question 1, It is worth noting that part of the function image is missing during the fitting process. For the LMA and SIMPLEX methods, this may belong to local optimization, but for the remaining algorithms they all belong to global optimization, so this is not true, so the specific situation still requires explore

2.Our conclusions for the Travelling Salesman Problem (TSP) exercise are that it is a difficult problem to solve, since its optimal solution would require exponentially increasing time as a function of the number of cities.

Despite its complexity, the TSP is very important for many reasons. For instance, it has practical applications in various areas, such as in vehicle route planning, task assignment for inspectors on a production line, circuit design for electrical systems, robotic exploration, etc.

In conclusion, although the TSP is a difficult problem to solve, its importance lies in its practical relevance and its ability to improve our understanding of computational complexity in general.