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Report

Lab No. 2

“Multicriteria optimization”

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1. Problem Selection

The problem of optimizing sustainable tourism in Mexico is especially relevant given the economic and social importance of tourism in the country. Mexico is recognized worldwide for its rich cultural, historical, and natural diversity, which makes it one of the most important tourist destinations in the world. Tourism plays a fundamental role in the Mexican economy, contributing significantly to the national GDP and generating employment in various regions of the country, especially in rural and coastal areas.

However, tourism development can also have negative impacts, such as environmental degradation, congestion at tourist sites, exploitation of natural and cultural resources, and socioeconomic inequality. Therefore, it is crucial to find a balance between tourism-driven economic growth and the conservation of the environment and local communities.

The problem of optimizing sustainable tourism in Mexico involves the efficient management of multiple variables that impact tourism planning and development in the country. These variables include:

- **Number of tourists allowed:** Represents the maximum number of tourists that can visit a tourist destination in each period of time.
- **Investment in tourism infrastructure:** Refers to resources allocated to the construction and improvement of tourism facilities, such as hotels, airports, roads, and tourist attractions.
- **Investment in environmental protection programs:** Represents funds allocated to the conservation and preservation of natural resources and the environment in tourism areas.
- **Expenditures on tourism promotion:** Refers to the budget dedicated to marketing and promotional activities to attract tourists to the country.
- **Investment in community development:** Represents resources allocated to the socioeconomic development of local communities that depend on tourism, including education, training, and quality of life improvement programs.
- **Maximum allowable length of stay per visit:** The maximum length of time a tourist can stay in a tourist destination in a single visit.

To ensure the viability and sustainability of tourism planning, various restrictions are established:

- **Maximum tourist limit:** To avoid overloading tourist destinations and preserve their carrying capacity, a maximum limit on the number of tourists allowed is established.

- **Total available budget:** The financial resources available for tourism planning are limited, so a total budget that cannot be exceeded is established.
- **Minimum percentage of the budget allocated to environmental protection:** Given the ecological value of many tourist destinations, a minimum percentage of the total budget is required to be allocated to environmental protection and conservation.
- **Permitted length of stay:** A limit is set on the maximum length of stay for tourists to avoid congestion at destinations and to allow for adequate visitor turnover.

All these variables and constraints must be considered in the optimization process to find the best tourism development strategy that maximizes economic and social benefits, while minimizing environmental impact and ensuring a positive experience for visitors and local communities.

The sustainable tourism optimization problem seeks to address these challenges by pursuing tourism planning that maximizes the economic benefits of tourism while minimizing its negative impact on the environment and promoting equitable social and economic development of local communities. This is not only important to ensure the long-term sustainability of the tourism industry in Mexico, but also to preserve the country's rich cultural and natural heritage for future generations.

2. Mathematical Model

Model Variables.

- x_1= Number of allowed tourists.
- x_2= Investment in tourism infrastructure (in millions of pesos).
- x_3= Investment in environmental protection programs (in millions of pesos).
- x_4= Expenditure on tourism promotion (in millions of pesos).
- x_5= Investment in community development (in millions of pesos).
- x_6= Maximum allowed duration of stay per visit (in days).

Objective Functions.

1. Maximize tourism income: $f_1(x) = x_1\ (100 - 0.1\ (x_6))$
2. Minimize environmental impact: $f_2(x) = -x_3$
3. Maximize tourist satisfaction: $f_3(x) = \frac{80\ (x_1)}{(x_3)(x_6)}$
4. Maximize social benefits for local communities: $f_4(x) = x_5$
5. Minimize congestion at tourist sites: $f_5(x) = -(\frac{x_1}{1000} + 0.5(x_6))$

Constraints.

$x_1 \leq 100{,}000$ (Maximum limit of tourists).

$x_2 + x_3 + x_4 + x_5 \leq 500$ (Total available budget in millions of pesos).

$x_3 \geq 0.1\,(x_2 + x_4 + x_5)$ (Minimum 10% of the budget in environmental protection).

$5 \leq x_6 \leq 30$ (Allowed duration of stay).

$x_i \geq 0$ (Non-negativity).

3. Resolution

Method 1: Linear Scalarization (Exact)

Explanation:

- In this method, we approach the multicriteria optimization problem by combining all the objective functions into a single scalarized function using specific weights for each objective.
- In the context of sustainable tourism in Mexico, this involves assigning weights to each objective, such as tourism revenue, environmental impact, tourist satisfaction, social benefits, and congestion at tourism sites. For example, a higher weight could be assigned to tourism revenue, reflecting its greater importance in the Mexican context, while a lower weight could be assigned to environmental impact.
- The linear scalarization algorithm then searches for the best solution within the constraints by maximizing the composite objective function. This solution represents a tourism plan that achieves an optimal balance between the different objectives considered, such as maximizing tourism revenues while minimizing environmental impact and promoting social benefits.

Implementation:

- In the code, this method is implemented in the linear_scalarization function. It takes as input the weights of the objective functions, the bounds on the variables, and the number of samples to generate.
- The function then iterates through multiple random samples within the given bounds and selects the one that maximizes the composite objective function. The best solution found, and its objective function value are then returned.

Method 2: Custom Genetic Algorithm (Approximate)

Explanation:

- In this method, we tackle the problem using a heuristic approach inspired by natural selection and evolution. In the context of sustainable tourism in Mexico, this algorithm simulates the process of evolving solutions to find those that are optimal or near optimal.
- This is achieved by generating an initial population of random solutions representing potential tourism plans. These solutions are then evaluated using the composite objective function, which considers all objectives of the problem. Through iterations of selection, reproduction, and mutation, the algorithm aims to improve the quality of solutions in each generation.
- The best solution found after a specific number of generations represents a tourism plan that may not be globally optimal but is a good approximation given the nature of the problem and available resources.

Implementation:

- In the code, this method is implemented in the genetic_algorithm function. It takes as input the composite objective function, population size, number of generations, variable bounds, and mutation rate.
- The function then iteratively evolves a population of solutions using crossover and mutation operations to improve their fitness with respect to the composite objective function. The best solution found is returned at the end.

Test Generation:

To generate tests for the implemented algorithms, we need to define variables and constraints that represent realistic scenarios in the context of sustainable tourism in Mexico. Here's how it was implemented in the code:

- **Variables:** We defined six variables representing aspects of tourism planning, such as the number of allowed tourists, investments in infrastructure and environmental protection, promotion expenses, investment in community development, and maximum duration of stay.
- **Constraints:** We defined five constraints representing limitations or requirements in the tourism planning process, such as maximum tourist limits, total budgetary constraints, minimum allocation of budget for environmental protection, and maximum duration of stay.

These variables and constraints were then used as input parameters in the implemented algorithms to test their performance in generating optimal or near-optimal tourism plans while adhering to the defined constraints.

4. Analysis of results

Results from Linear Scalarization Optimization

Best Solution Found:

$$[9.92765102 \times 10^4, 225.87044, 77.5593932, 132.481225, 33.1164287, 24.7861223]$$

This solution suggests:

- Allowing approximately 99,276 tourists.
- Investing approximately 226 million pesos in tourism infrastructure.
- Investing approximately 78 million pesos in environmental protection programs.
- Spending approximately 132 million pesos on tourism promotion.
- Investing approximately 33 million pesos in community development.
- Setting a maximum stay duration of approximately 25 days.
- Objective function value: 1,937,111.66 This value indicates the composite evaluation of the objectives under the assigned weights, suggesting a high level of achievement of the desired objectives under the weighting scheme used.

Results from the Genetic Algorithm

Best Solution Found:

$$[8.45877144 \times 10^4, 17.7800152, 92.6274832, 16.8540884, 52.3553764, 12.4462674]$$

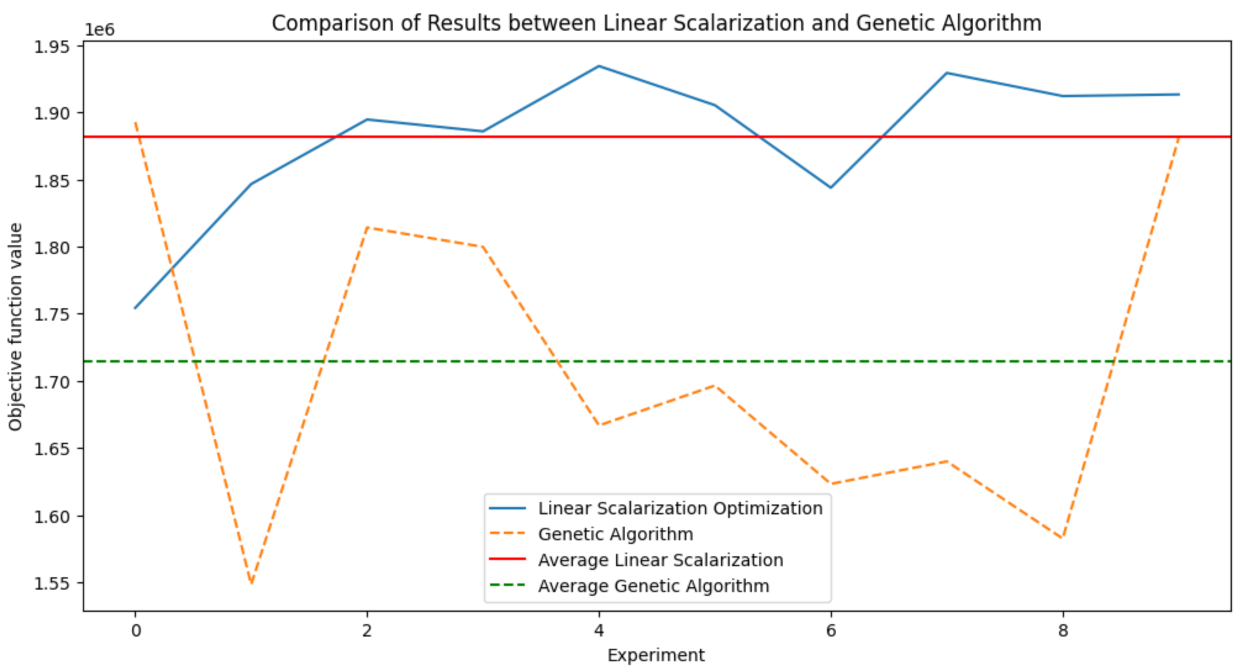
This solution suggests:

- Allowing approximately 84,588 tourists.
- Investing approximately 18 million pesos in tourism infrastructure.
- Investing approximately 93 million pesos in environmental protection programs.
- Spending approximately 17 million pesos on tourism promotion.
- Investing approximately 52 million pesos in community development.
- Setting a maximum stay duration of approximately 12 days.
- Objective function value: 1,671,845.99 This value, being lower than that obtained with Linear Scalarization, suggests that under the same weightings, the solution from the Genetic Algorithm achieves a lower level of fulfillment of the combined objectives.

Comparative Analysis

- **Efficiency in Specific Objectives:** The solution from Linear Scalarization appears to be more efficient in maximizing the number of tourists and the budgets for infrastructure and promotional tourism. In contrast, the Genetic Algorithm prioritizes environmental protection and community development, allocating more resources to these areas.
- **Adaptability:** The Genetic Algorithm, by exploring the solution space more dynamically, offers solutions that may be more adaptable to changes in weightings or in the constraints of the problem.
- **Policy Preferences:** Depending on the policy priorities of tourism, one solution might be preferred over another. For example, if environmental sustainability is a priority, the solution from the Genetic Algorithm could be more attractive.

Experiments on tests



1. **Graph Lines:**
   - The **blue solid line** represents the values from the Linear Scalarization Optimization for each of the ten experiments.
   - The **orange dashed line** represents the values from the Genetic Algorithm for the same experiments.

2. **Horizontal Lines:**
    - The **red solid line** indicates the average objective function value for the Linear Scalarization across all experiments.
    - The **green dashed line** shows the average objective function value for the Genetic Algorithm.

Observations from the Graph

- The Linear Scalarization consistently produces higher objective function values across all experiments compared to the Genetic Algorithm, indicating it may be more effective in maximizing the defined objective function under the given constraints and weights.
- The Genetic Algorithm's results show more volatility and generally lower values, suggesting that while it may explore a broader solution space and potentially find more diverse solutions, it struggles to consistently reach as high an objective function value as Linear Scalarization in this context.
- The average performance (indicated by the horizontal lines) further emphasizes the superior average effectiveness of Linear Scalarization in this particular set of experiments.

Detailed Results

- **Average Objective Function Value with Linear Scalarization Optimization:** 1,881,970.85
- **Average Objective Function Value with Genetic Algorithm:** 1,714,441.14

Interpretation of Results

- **Linear Scalarization**:
    - This method has proven to be more consistent and effective, yielding higher average results.
    - The higher average objective function value suggests that Linear Scalarization might be more suited for scenarios where a high degree of optimization is required on specific weighted criteria.

- **Genetic Algorithm**:
    - Despite its lower average, the Genetic Algorithm provides valuable insights due to its explorative approach, which could potentially uncover unique or non-intuitive solutions that linear approaches might miss.
    - Its variability could be useful in cases where the search landscape is rugged or has many local optima, offering a higher chance of escaping local optima albeit at the cost of consistency.

Considerations for Use

- **Choice of Algorithm**: The choice between these algorithms should consider the specific needs and constraints of the problem. If reliability and performance on specific metrics are paramount, Linear Scalarization appears favorable. If the goal is to explore a variety of solutions in a complex landscape, or if the model's constraints and weights are subject to change, a Genetic Algorithm might provide more flexibility.
- **Adjustment of Parameters**: Both algorithms could potentially benefit from a fine-tuning of parameters — for Linear Scalarization, this could mean adjusting weights and for the Genetic Algorithm, experimenting with mutation rates, crossover strategies, and population dynamics might yield improvements in stability and performance.

5. Justify theoretically the sorting of functions

Linear Scalarization

Theoretical Justification:

1. **Weighting of Objectives:** Linear Scalarization transforms a multi-criteria optimization problem into a single-criterion problem by linearly summing the objective functions, each weighted by a coefficient that reflects its relative importance. This weighting allows specific strategic decisions to be reflected in the final solution.
2. **Ordering by Importance:** The order in which functions are considered can be relevant if truncation strategies are implemented or if the optimization process stops early based on certain conditions. Although in pure linear summation all functions contribute simultaneously, in practice, adjusting weights based on

clear priorities can direct the search towards more promising regions of the solution space under certain criteria.

3. **Direct Impact on the Solution:** Weights in Linear Scalarization directly determine the impact of each objective function on the final solution. A higher weighting means that the corresponding objective will have a greater impact on the direction and nature of the optimal solution.

Genetic Algorithm

Theoretical Justification:

1. **Diversity of Solutions:** In genetic algorithms, although functions are not "ordered" as in Linear Scalarization, the way solutions are evaluated and interpreted can vary. For example, a fitness function that is a composite of various objectives needs to consider how each objective contributes to the total fitness value.
2. **Selection and Survival:** The evolutionary theory underlying genetic algorithms implies that solutions that best fit (according to the fitness function) are more likely to "survive" and reproduce. If the fitness function is a weighted sum of different objectives, then indirectly it is prioritizing those objectives with greater weights, like ordering in terms of their influence on the survival of solutions.
3. **Exploration vs. Exploitation:** In the context of optimization using genetic algorithms, the selection of functions and how they are weighted within the fitness function influences the balance between exploration (discovering new areas of the search space) and exploitation (refining solutions in known areas). Prioritizing certain functions can direct this balance towards exploring areas that maximize more critical specific objectives.

6. Conclusion

The implementation of advanced optimization methods to address sustainable tourism planning in Mexico has unveiled significant aspects about how Linear Scalarization and Genetic Algorithms can be utilized to tackle complex challenges in a crucial economic and social context. Through detailed analysis of these methods, we've gained valuable insights that clarify their applicability and effectiveness in real-world scenarios.

Linear Scalarization has proven to be extremely effective in producing consistent, high-quality solutions that satisfy multiple optimized criteria simultaneously. This method is particularly suitable for situations where the goals and constraints of the problem are well-defined and stable. The ability to assign specific weights to different objectives allows planners to precisely align solutions with strategic priorities, which is essential in tourism planning, where diverse interests—such as economic revenue, environmental conservation, and social benefits—must be carefully balanced.

On the other hand, the Genetic Algorithm stands out for its flexibility and ability to explore a broad solution space, uncovering potential configurations that might be overlooked by more rigid methods. Although this approach may not always achieve the same level of optimization as Linear Scalarization, its value lies in its adaptability to changing conditions and its potential to innovate in tourism planning. This method is particularly valuable in contexts where decision criteria are dynamic or where innovation is a key goal.

From a comparative perspective, although Linear Scalarization provides more predictable and potentially more optimized results on average, the Genetic Algorithm offers the advantage of generating diverse solutions that may better adapt to unexpected changes or explore new opportunities that might go unnoticed in a more conventional approach. The choice between these methods should be guided by the specific needs of the problem and the environment in which they are implemented. For stable contexts with clear requirements, Linear Scalarization is ideal, while in scenarios that require adaptability and responsiveness to new information, the Genetic Algorithm could be preferable.

In conclusion, employing these optimization methods in the planning of sustainable tourism not only reflects a commitment to innovation and continuous improvement in Mexico's tourism sector but also underscores the need for solutions that effectively balance economic benefits, environmental sustainability, and social development. Both methods, with their respective strengths and weaknesses, offer valuable tools for designing tourism strategies that are long-term viable, sustainable, and beneficial for all stakeholders, thus ensuring the preservation of Mexico's valuable cultural and natural resources for future generations.

7. Code

https://colab.research.google.com/drive/16SXrHBJYxOuogILa55lebUc3MpWuJWCq?usp=sharing