AE3212-II

Flight Dynamics Assignment

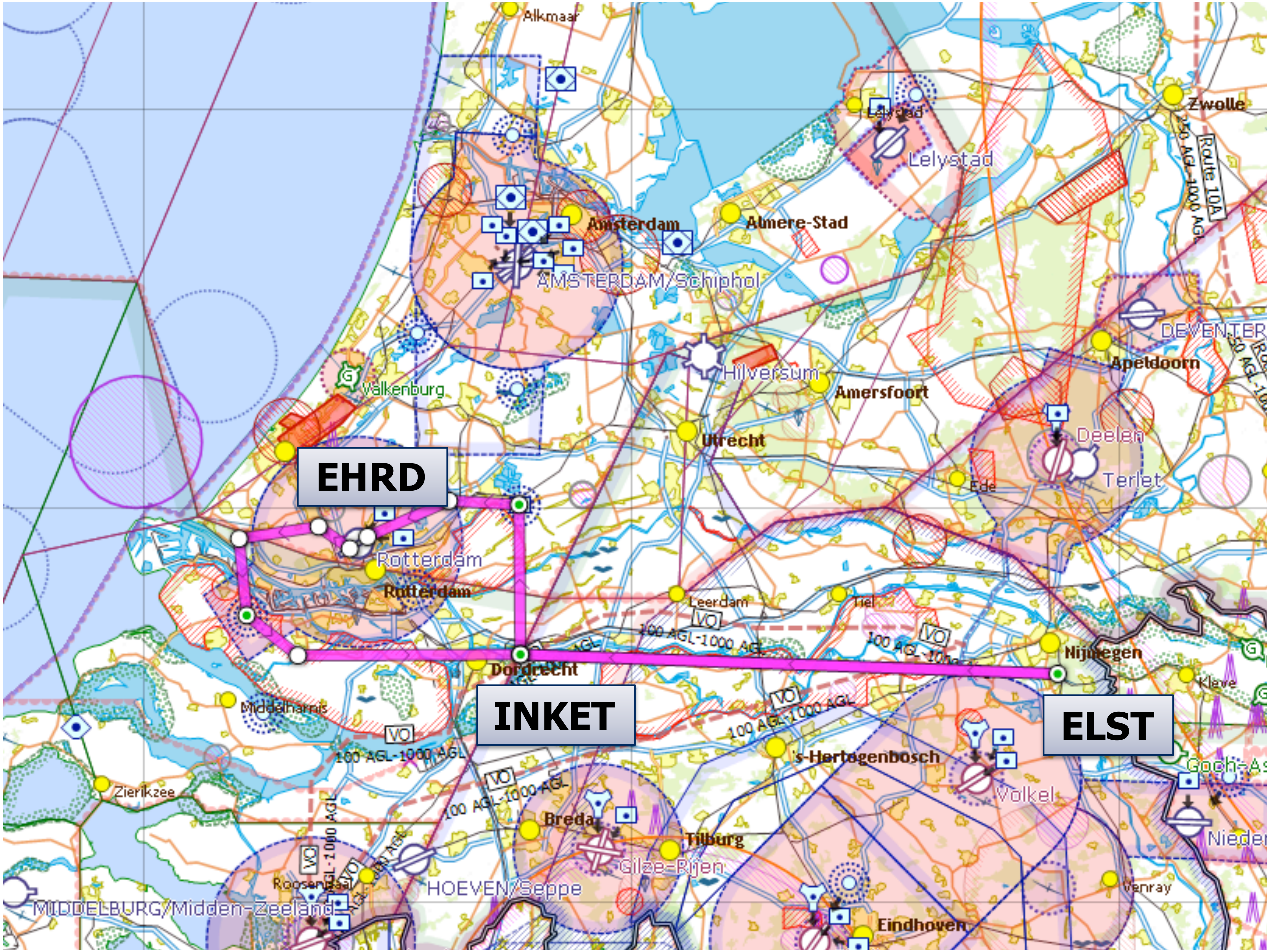
Course schedules

	AE3212-I	*Flight Test*	*AE3212-II*
Week 3.1	*Lectures*		*Structures assignment*
Week 3.2	*Lectures*		
Week 3.3	*Lectures*		
Week 3.4	*Lectures*	*Test flights*	*Flight Dynamics assignment*
Week 3.5	*Lectures*	*Test flights*	
Week 3.6	*Lectures*	*Test flights*	
Week 3.7	*Lectures*		

Check Brightspace!

Admission to the flight test

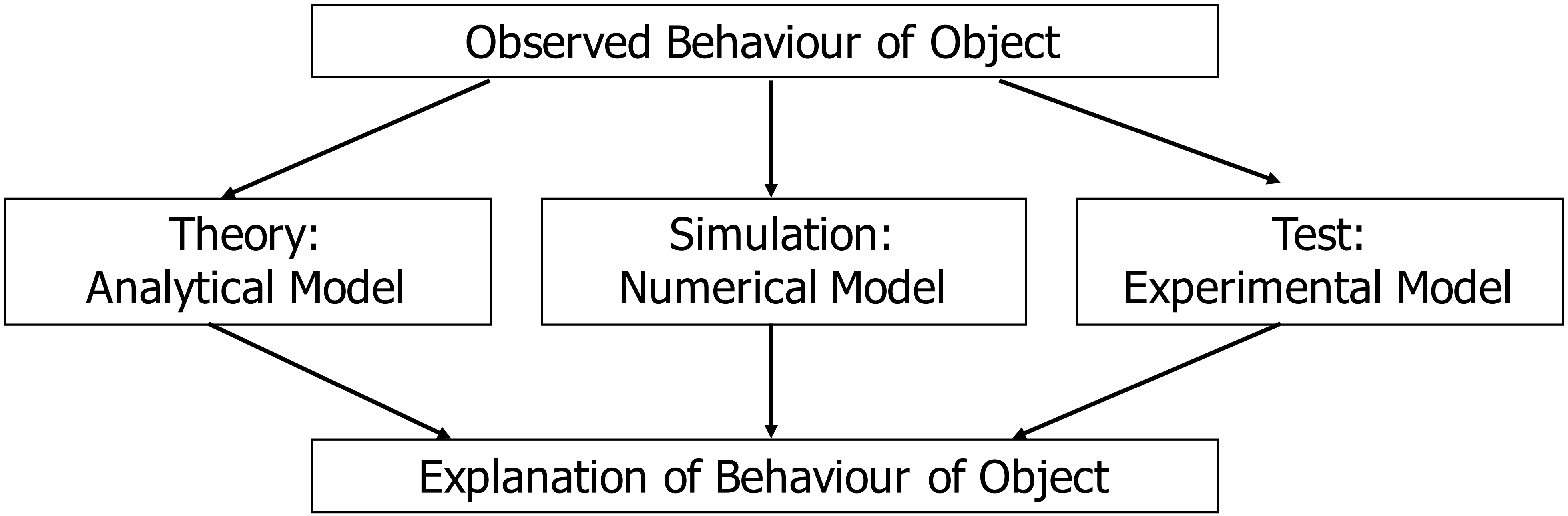
- Good progress in SVV Structures Assignment
- Currently taking AE3212-I FD
- Not flown flight test before
- Sign the attendance list!



EHRD
INKET
ELST
Alkmaar
Lelystad
Zwolle
Route 10A
Amsterdam
AMSTERDAM/Schiphol
Almere-Stad
DEVENTER
Apeldoorn
Hilversum
Amersfoort
Valkenburg
Utrecht
Deelen
Terlet
Ede
Rotterdam
Leerdam
Tiel
Nijmegen
Kleve
Dordrecht
100 AGL-1000 AGL
VO
Middelharnis
's-Hertogenbosch
Volkel
Zierikzee
Breda
Tilburg
Gilze-Rijen
HOEVEN/Seppe
Roosendaal
MIDDELBURG/Midden-Zeeland
Eindhoven
Venray

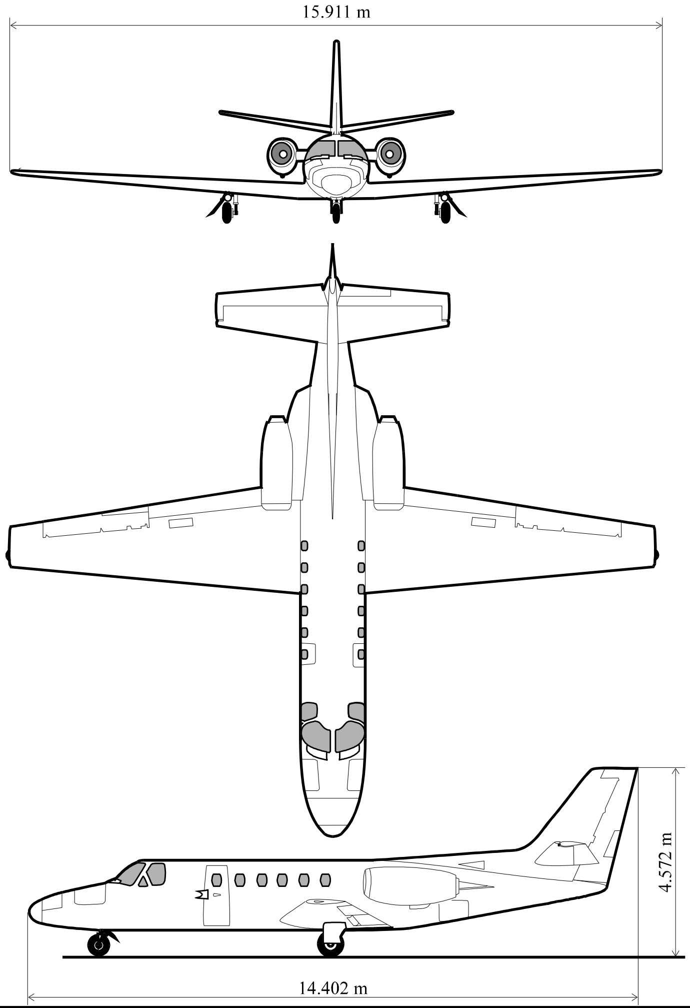
Simulation, Verification and Validation

Theoretical analysis, computer simulation and measuring or testing are used to evaluate, verify and validate observed performance or failure of aerospace vehicles and phenomena



AE3212-II Flight Dynamics Assignment

- New aircraft design
- Limited data available
- Predict dynamic behavior
- Adjust design to improve behavior



Model development

$C_{m\alpha} = 1.435$

$G_m = 7\,dB$

$ROC = 4500\,ft/\min$

PREDICT

$C_{m\alpha} = \cancel{1.435}\quad 1.327$

$G_m = \cancel{7\,dB}\quad 5$

$ROC = \cancel{4500}\,ft/\min\quad 4250$

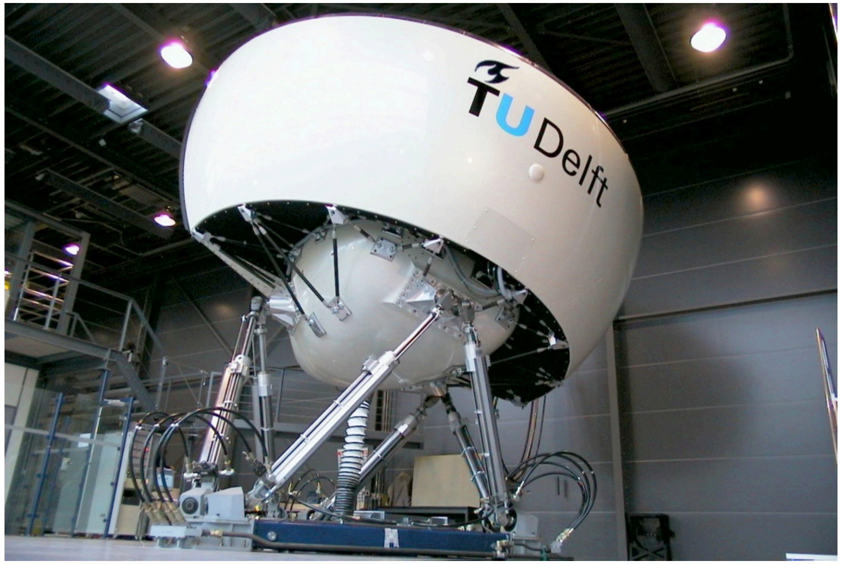
VALIDATE

Model validation test method

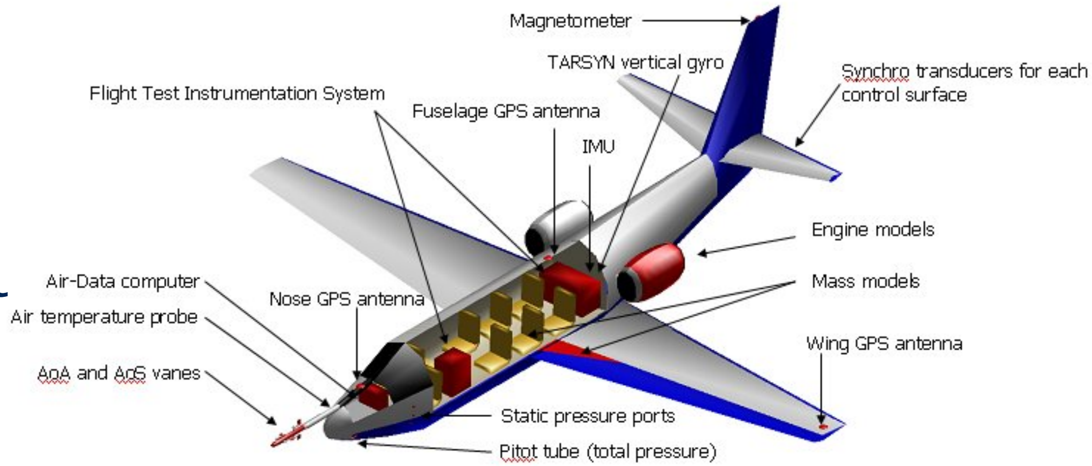
- Build a mathematical model
- Predict performance and behavior of the aircraft
- Flight test the model
- Adjust the mathematical model

Flight Simulation purposes

- Fly before you build
- Testing dangerous conditions
- Training simulators
- Future modifications



Flight Test Data



<u>Stationary measurements series 1</u>

- To determine a number of aerodynamic coefficients

<u>Stationary measurements series 2</u>

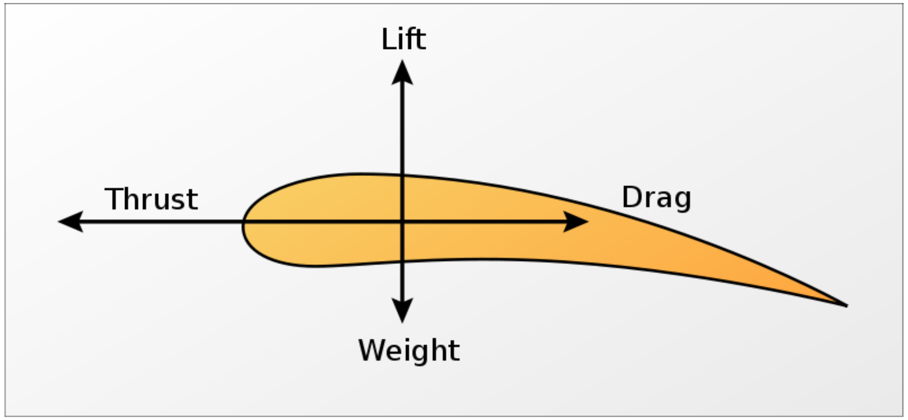
- To determine a number of stability derivatives
- To determine longitudinal stability

<u>Dynamic measurements</u>

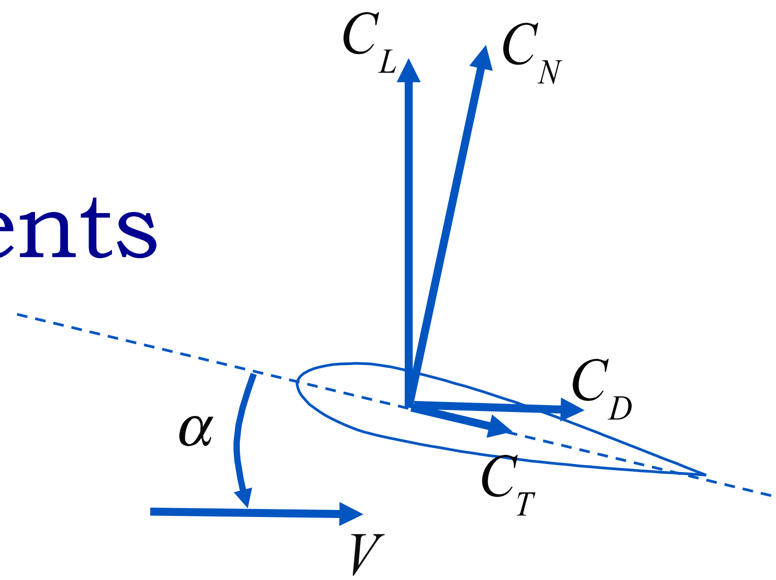
- To determine Eigenmotion characteristics
- To validate simulation results

First measurement series

- Steady, horizontal, symmetrical flight
- Constant altitude, varying thrust
- Data for speedrange $V_S \rightarrow V_{MO}$
- One configuration
  - Gear up, flaps up



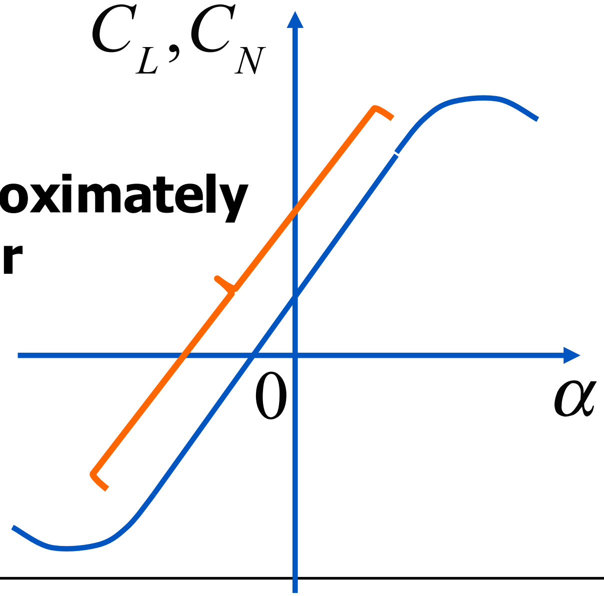
Aerodynamic coefficients



Steady, horizontal flight

Vertical equilibrium:

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = C_{L_\alpha}(\alpha - \alpha_0) \approx C_{N_\alpha}(\alpha - \alpha_0)$$

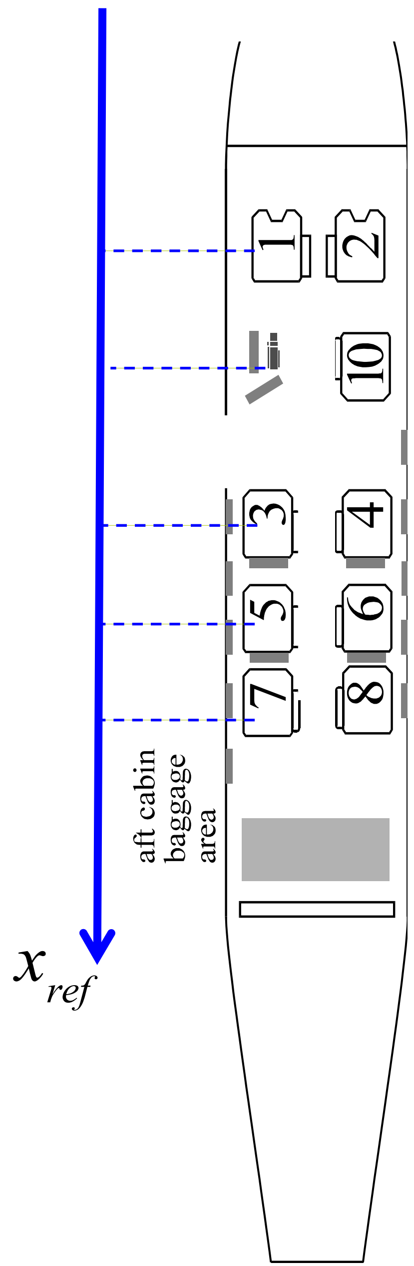


→ **We can measure H, V_{ias} and α, $W_{fuel\ used}$, *T (temperature)*.....**

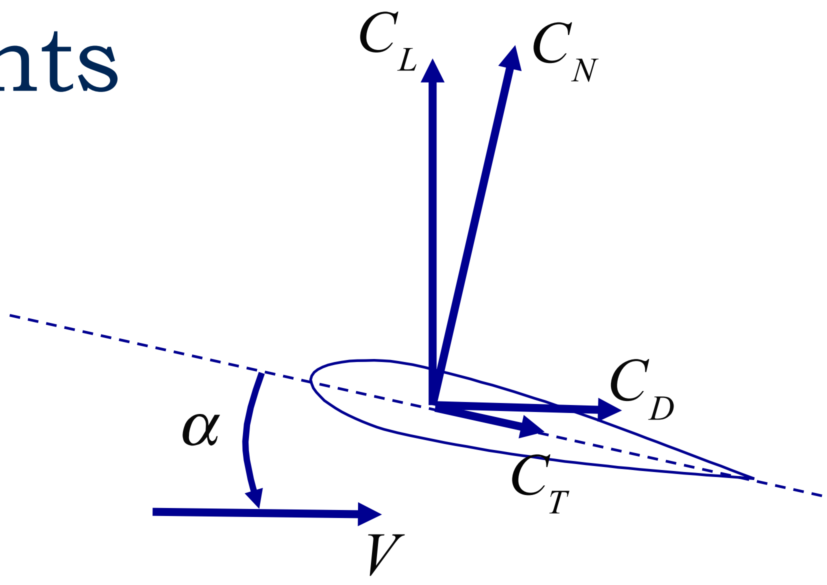
→ ***and determine ρ, V_{tas} and W***

payload computations			
crew and pax	*[inches]*	*mass [lbs]*	*moment [llsinches]*
seat 1	131		
seat 2	131		
seat 3	214		
seat 4	214		
seat 5	251		
seat 6	251		
seat 7	288		
seat 8	288		
seat 10	170		
baggage			
nose	74		
aft cabin	321		
	338		
payload			

mass and balance computations		
item	*mass [lbs]*	*moment [lbsnches]*
basic empty mass		
$x_{cg_{datum}}$ at BEM = _____		
payload		
zero fuel mass		
$x_{cg_{datum}}$ at ZFM = _____		
fuel load		
ramp mass		
$x_{cg_{datum}}$ at RM = _____		



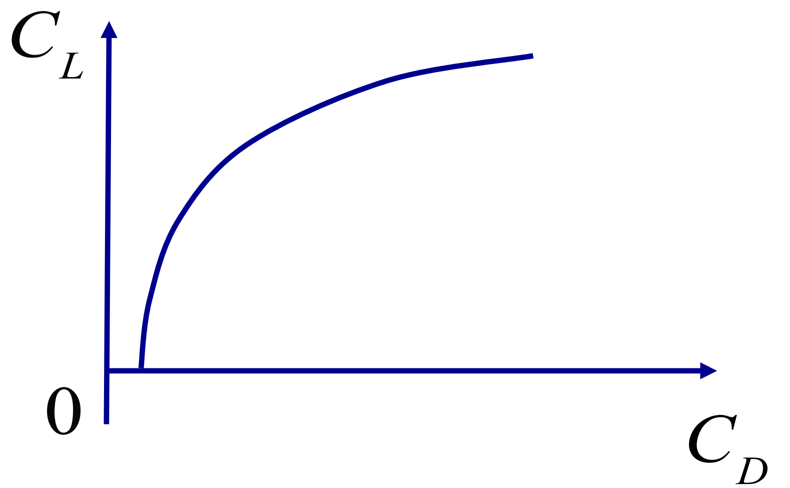
Aerodynamic coefficients



Steady, horizontal flight:

Horizontal equilibrium:

$$C_D = \frac{T}{\frac{1}{2}\rho V^2 S} = C_{D_0} + \frac{C_L{}^2}{\pi A e}$$

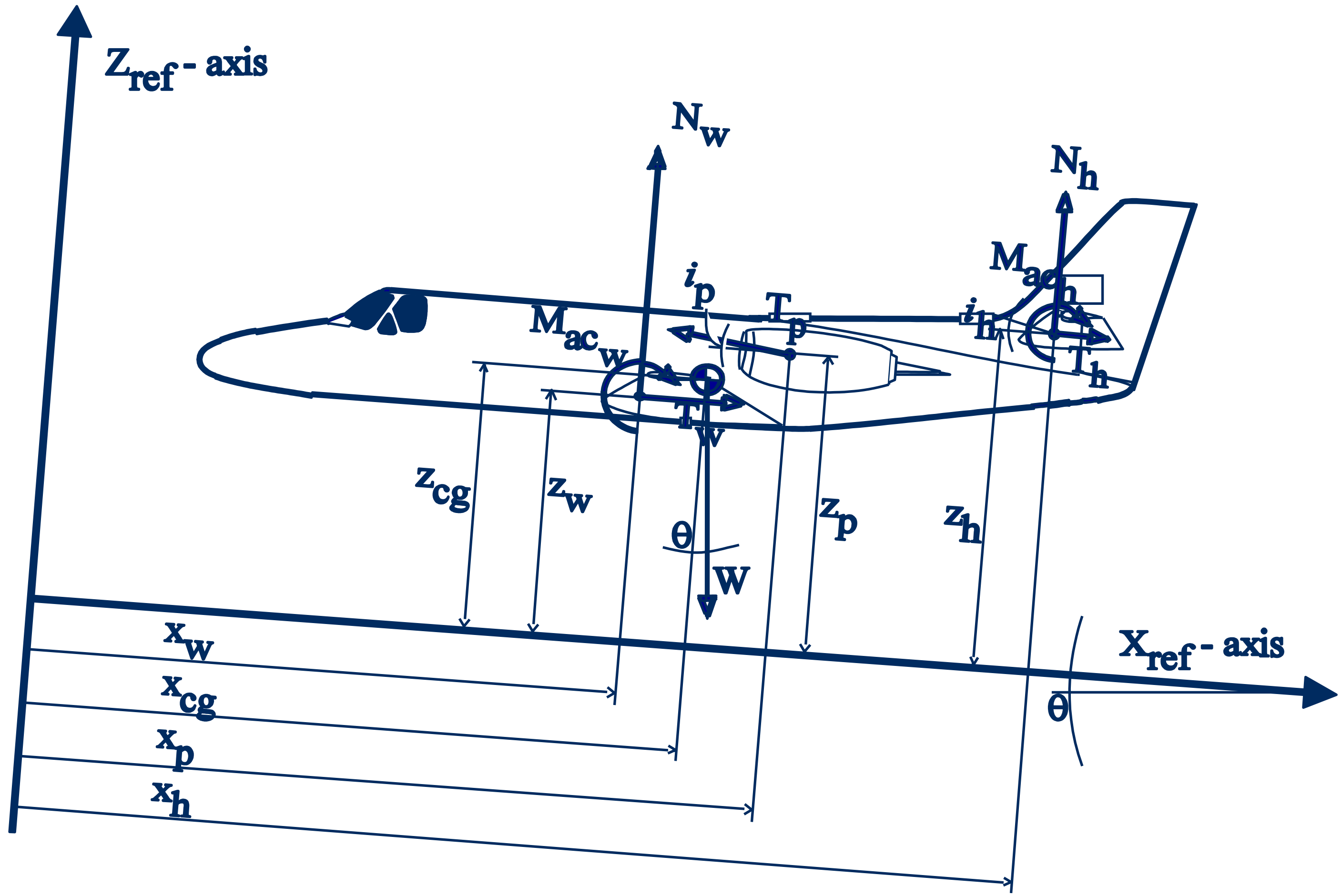


→ T, V, C_{D_0} and e can be determined from flight test data

Second measurement series

- Quasi-steady, horizontal flight
- Constant thrust, varying altitude
- Small speedrange around V_{TR}
- One configuration
  - Gear up, flaps up

Forces and moments in symmetric flight



$$M = C_m \tfrac{1}{2} \rho V^2 S \bar{c}$$

Moment equilibrium

$$C_m = C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_\delta}\delta_e + C_{m_{\delta_f}}\delta_f + C_{m_{T_c}}T_c + C_{m_{\lg}}\Big|_{\lg\ \text{down}} = 0$$

Static stability for: $$C_{m_\alpha} = \frac{dC_m}{d\alpha} < 0$$

Normal pitch control: $$C_{m_\delta} = \frac{dC_m}{d\delta_e} < 0$$

Elevator trim curve $\delta_e - \alpha$

Equilibrium, landing gear up, flaps up

$$C_m = C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_\delta}\delta_e + C_{m_{\delta_f}}\delta_f + C_{m_{T_c}}T_c + C_{m_{lg}}\Big|_{\text{lg down}} = 0$$

Rewrite:

$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_{Tc}}T_c\right\} = f(\alpha)$$

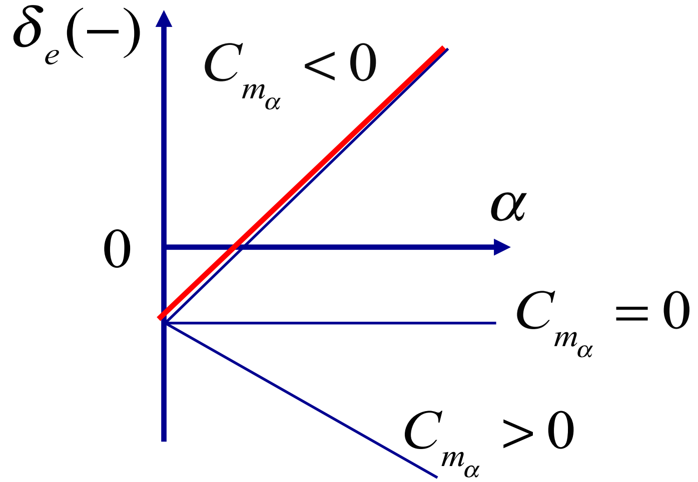
Slope:

$$\frac{d\delta_e}{d\alpha} = -\frac{1}{C_{m_\delta}}C_{m_\alpha}$$

$\ominus$

$$\frac{d\boldsymbol{\delta_e}}{d\boldsymbol{\alpha}} < \mathbf{0}$$

Static stability



Elevator trim curve $\delta_e - \alpha$

Equilibrium, landing gear up, flaps up

$$C_m = C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_\delta}\delta_e + C_{m_{T_c}} T_c = 0$$

Rewrite:

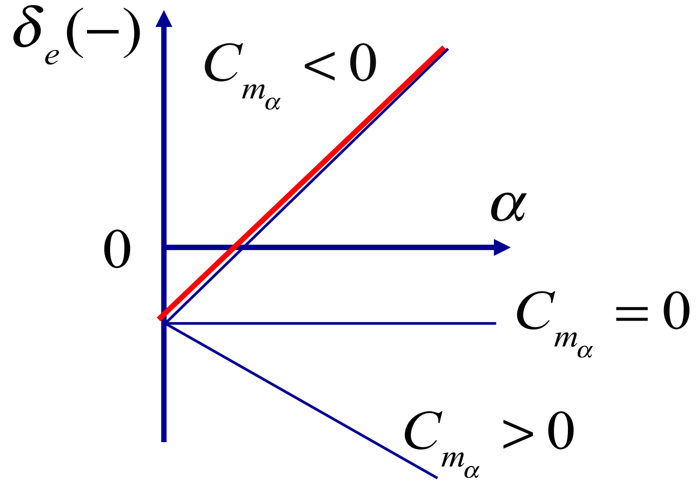
$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_{Tc}} T_c\right\} = f(\alpha)$$

Slope:

$$\frac{d\delta_e}{d\alpha} = -\frac{1}{\underset{\ominus}{C_{m_\delta}}} C_{m_\alpha}$$

$$\frac{d\boldsymbol{\delta_e}}{d\boldsymbol{\alpha}} < \mathbf{0}$$

Static stability



Test condition

Equilibrium, landing gear up, flaps up, constant thrust

$$C_m = C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_\delta}\delta_e + C_{m_{\delta_{cf}}}\delta_{cf} + C_{m_{T_c}} T_c + C_{m_{\mathrm{lg}}}\Big|_{\mathrm{lg\ down}} = 0$$

Rewrite:

$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_{Tc}} T_c\right\} = f(\alpha)$$

Using Force Equilibrium W=N:

$$C_N \approx C_{N_\alpha}\left(\alpha - \alpha_0\right) \approx \frac{W}{\frac{1}{2}\rho V^2 S} \Rightarrow \left(\alpha - \alpha_0\right) = \frac{1}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S}$$

Elevator trim curve $\delta_e - V$

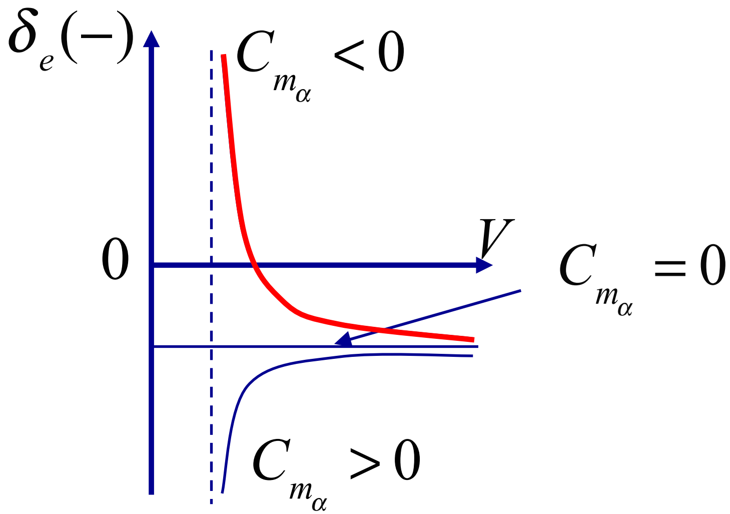
$$\delta_{e_{\mathrm{eq}}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + \frac{C_{m_\alpha}}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S} + C_{m_{T_c}} T_c\right\} = f(V)$$

C_{m_δ}: $\ominus$ $\quad$ C_{N_α}: $\oplus$

Slope:

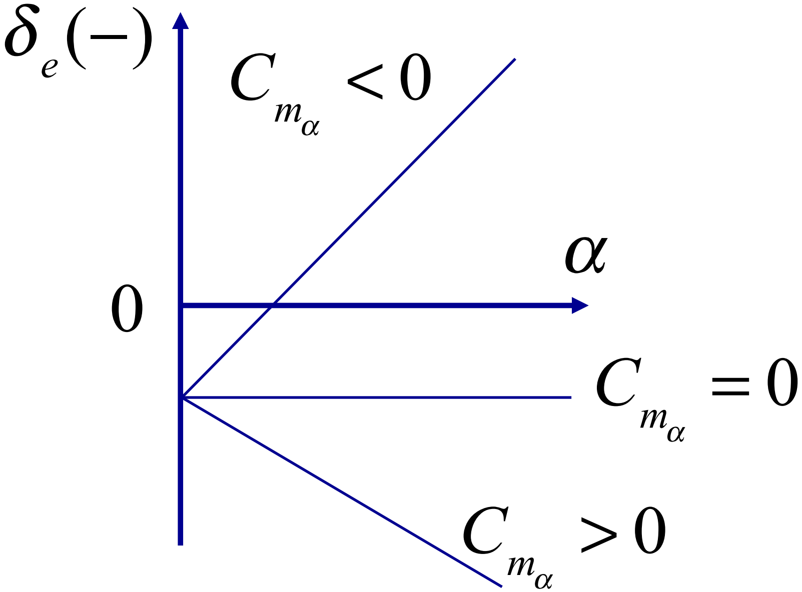
$$\frac{d\delta_e}{dV} = \frac{4W}{\rho V^3 S}\frac{1}{C_{m_\delta}}\frac{C_{m_\alpha}}{C_{N_\alpha}}$$

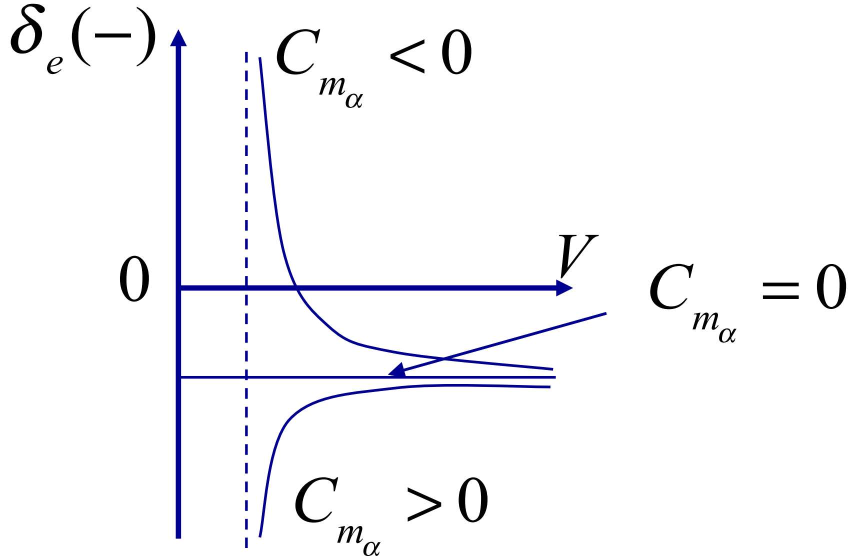
C_{m_δ}: $\ominus$ $\quad$ C_{N_α}: $\oplus$



Static stability ($C_{m_\alpha} < 0$ **) for:** $\dfrac{d\delta_{e_{\mathrm{eq}}}}{dV} > 0$

Elevator trim curve





Static stability for: $\dfrac{d\delta_{e_{eq}}}{d\alpha} < 0$ $\qquad$ $\dfrac{d\delta_{e_{eq}}}{dV} > 0$

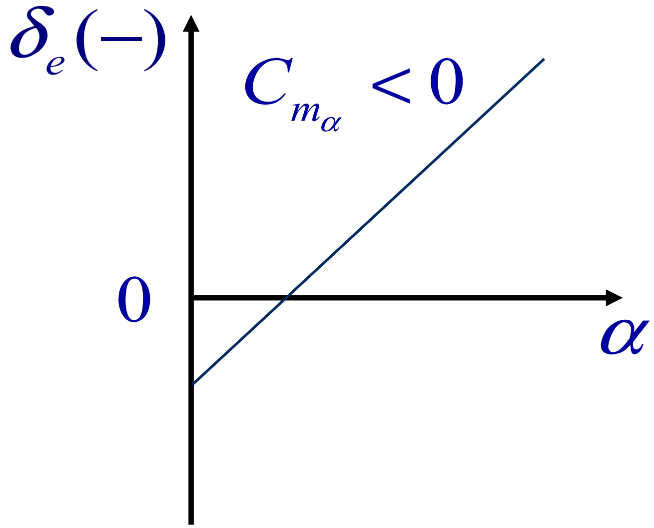
→ We can measure *V*, α and δ_e

Elevator Trim Curve Measurements

Quasi-steady, horizontal flight

Moment equilibrium:

Slope: $$\frac{d\delta_e}{d\alpha} = -\frac{1}{C_{m_\delta}} C_{m_\alpha}$$

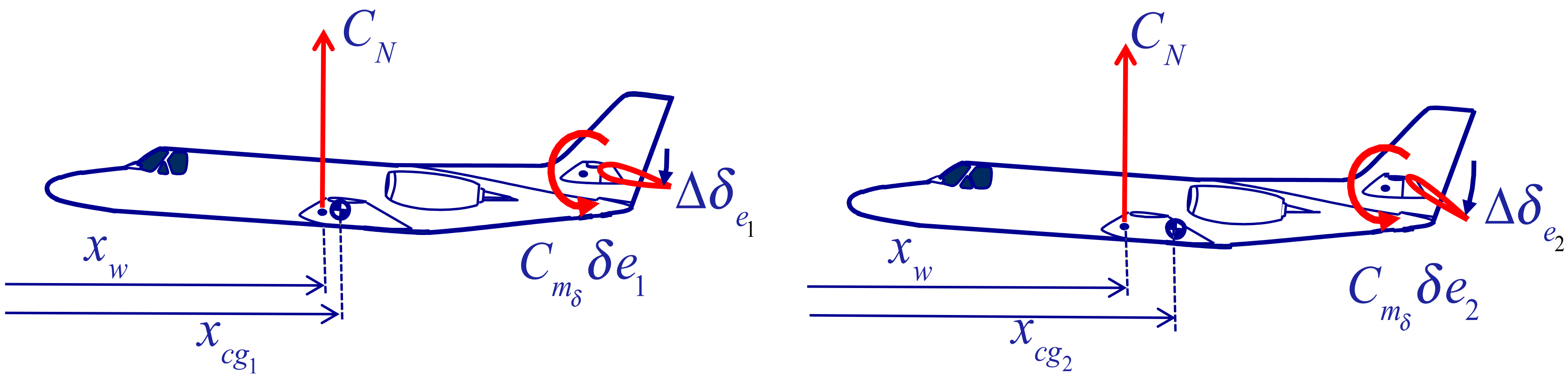


Stable if: $$\frac{dC_m}{d\alpha} = C_{m_\alpha} < 0$$

Long. stability: $$C_{m_\alpha} = -\frac{d\delta_e}{d\alpha} C_{m_\delta}$$

Determining elevator effectiveness C_{m_δ}

$$C_m = C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_\delta}\delta_e + C_{m_{\delta_f}}\delta_f + C_{m_{T_c}}T_c + C_{m_{\mathrm{lg}}}\Big|_{\mathrm{lg\ down}} = 0$$

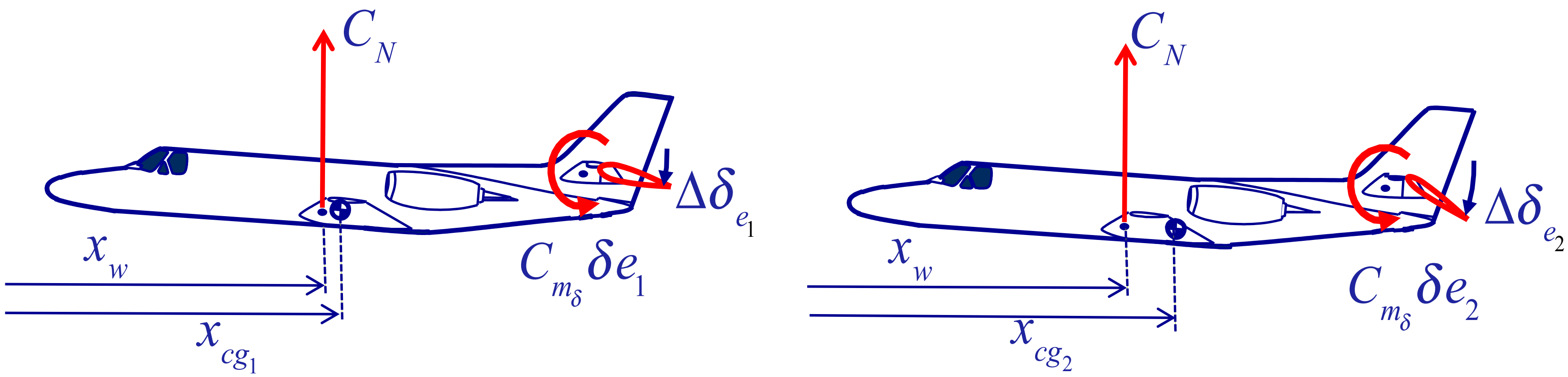


$$\left.\begin{aligned} \Delta C_m &= C_{m_\delta} \cdot \Delta\delta_e \\ \Delta C_m &= C_N \frac{x_{cg2} - x_{cg1}}{\bar{c}} \end{aligned}\right\} \quad C_{m_\delta} = -\frac{1}{\Delta\delta_e} C_N \frac{\Delta x_{cg}}{\bar{c}}$$

with $C_N = \dfrac{W}{\frac{1}{2}\rho V^2 S}$

Determining elevator effectiveness C_{m_δ}

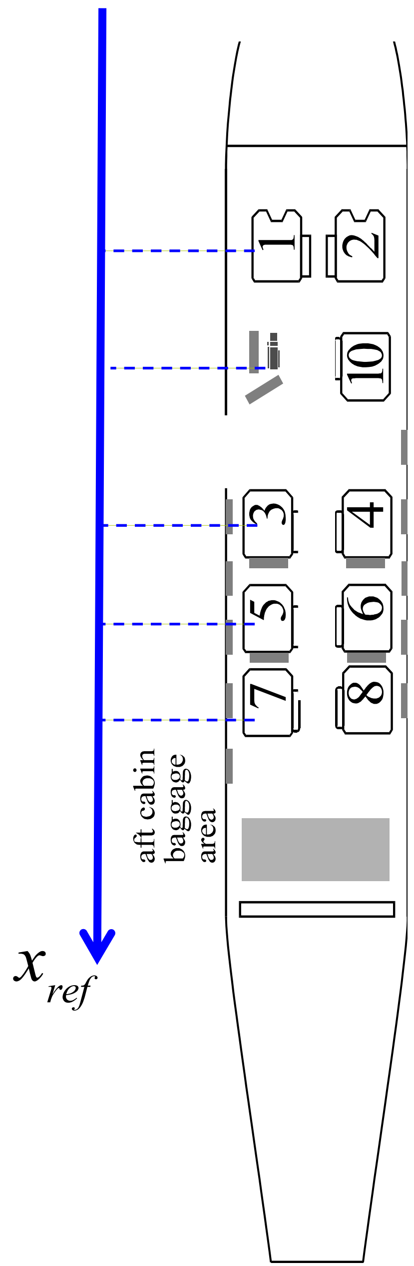
$$C_m = C_m\big|_{\delta_e=0} + C_{m_\delta}\delta_e$$



$$\left.\begin{aligned}\Delta C_m &= C_{m_\delta}\cdot\Delta\delta_e \\ \Delta C_m &= C_N\frac{x_{cg2}-x_{cg1}}{\bar{c}}\end{aligned}\right\} \quad C_{m_\delta} = -\frac{1}{\Delta\delta_e}C_N\frac{\Delta x_{cg}}{\bar{c}} \quad \text{with } C_N = \frac{W}{\frac{1}{2}\rho V^2 S}$$

payload computations			
crew and pax	*[inches]*	*mass [lbs]*	*moment [llsinches]*
seat 1	131		
seat 2	131		
seat 3	214		
seat 4	214		
seat 5	251		
seat 6	251		
seat 7	288		
seat 8	288		
seat 10	170		
baggage			
nose	74		
aft cabin	321		
	338		
payload			

mass and balance computations		
item	*mass [lbs]*	*moment [lbsnches]*
basic empty mass $x_{cg_{datum}}$ at BEM = _____ **payload**		
zero fuel mass $x_{cg_{datum}}$ at ZFM = _____ **fuel load**		
ramp mass $x_{cg_{datum}}$ at RM = _____		



Data reduction

<u>Test conditions</u>

- Uncontrollable variables — air temperature, density
- Adjustable variables — mass, cg
- Controllable variables — altitude, airspeed, angle of attack

In order to compare results, data must be adjusted with respect to standard conditions

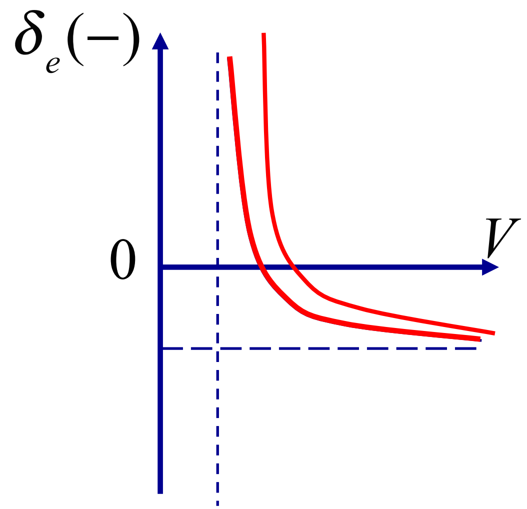
Reduced Airspeed

- $V_i \approx V_c$ when instrument and position errors are discarded
- By converting V_c to the equivalent airspeed V_e, the atmospheric variables are reduced to ISA values.
- V_e is the airspeed that gives the same dynamic pressure at Sea Level ISA, as the true airspeed V_t at altitude

$$\frac{1}{2}\rho_0 V_e^2 = \frac{1}{2}\rho V_t^2$$

Reduced elevator trim curve $\delta_e^* - \tilde{V}_e$

$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + \frac{C_{m_\alpha}}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S} + C_{m_{T_c}} T_c\right\}$$



Data points for V = 120 m/s:

	δ_e
m = 6200 kg	-0.4
m = 4742 kg	-0.6

Reduced (or corrected) EAS:

$$\tilde{V}_e = V_e\sqrt{\frac{W_s}{W}}$$

Building the mathematical model

- Based on flight dynamics theory
- Use parameters derived from flight test data
- Test the model
- Predict dynamic behavior

State Space Representation

General Form:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

x	**- state vector**	**A**	**- state matrix**
y	**- output vector**	**B**	**- input matrix**
u	**- input vector**	**C**	**- output matrix**
		D	**- direct matrix**

State Space Representation

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- **very compact notation**
- **we can apply linear algebra**
- **computers can easily work with matrices**

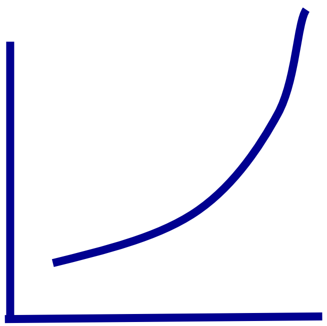
Solution of the equations

$$\dot{x} = Ax$$

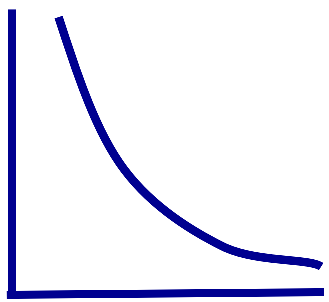
General solution:

$$\bar{x}(t) = ce^{\lambda t}\bar{v}$$

System behavior (λ is real):



$$\lambda > 0$$



$$\lambda < 0$$

Eigenvalues and stability

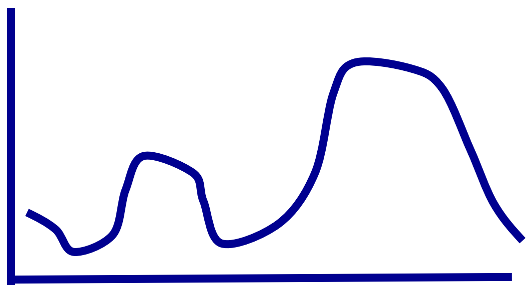
$$\overline{x}(t) = ce^{\lambda t}\overline{v}$$

Complex eigenvalues:

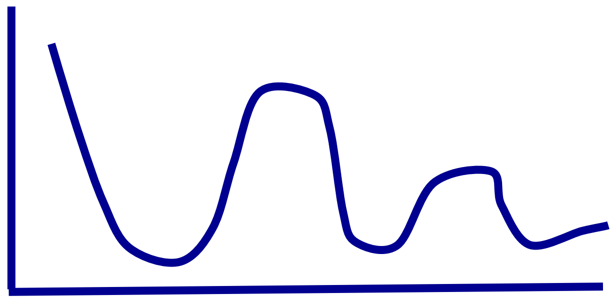
$$\lambda = a + ib$$

$$\Rightarrow \overline{x}(t) = ce^{(a+ib)t}\overline{v} = ce^{at}(\cos b + i\sin b)\overline{v}$$

System behavior real part:



$$a > 0$$

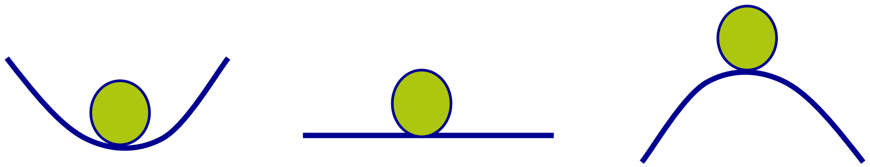


$$a < 0$$

Eigenvalues and stability

$$Ax = \lambda x$$

Positive real part	undamped
Negative real part	damped
Imaginary part	oscillatory



Longitudinal EOM

Linearized, homogenous, deviation equations for symmetric motions:

$$\begin{bmatrix} C_{X_u} - 2\mu_c D_c & C_{X_\alpha} & C_{Z_0} & 0 \\ C_{Z_u} & C_{Z_\alpha} + \left(C_{Z_{\dot{\alpha}}} - 2\mu_c\right) D_c & -C_{X_0} & C_{Z_q} + 2\mu_c \\ 0 & 0 & -D_c & 1 \\ C_{m_u} & C_{m_\alpha} + C_{m_{\dot{\alpha}}} D_c & 0 & C_{m_q} - 2\mu_c K_Y^2 D_c \end{bmatrix} \begin{bmatrix} \hat{u} \\ \alpha \\ \theta \\ \frac{q\bar{c}}{V} \end{bmatrix} = \underline{0}$$

- Eigenvalues describe stability of short period and phugoid

Lateral EOM

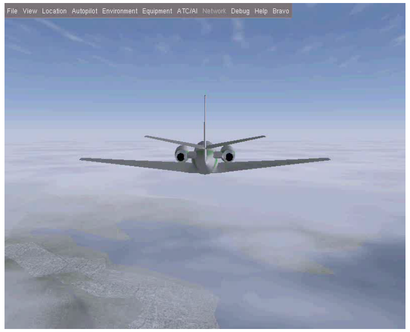
$$\begin{bmatrix} C_{Y_\beta} + \left(C_{Y_{\dot\beta}} - 2\mu_b\right) D_b & C_L & C_{Y_p} & C_{Y_r} - 4\mu_b \\ 0 & -\frac{1}{2}D_b & 1 & 0 \\ C_{\ell_\beta} & 0 & C_{\ell_p} - 4\mu_b K_X^2 D_b & C_{\ell_r} + 4\mu_b K_{XZ} D_b \\ C_{n_\beta} + C_{n_{\dot\beta}} D_b & 0 & C_{n_p} + 4\mu_b K_{XZ} D_b & C_{n_r} - 4\mu_b K_Z^2 D_b \end{bmatrix} \begin{bmatrix} \beta \\ \varphi \\ \frac{pb}{2V} \\ \frac{rb}{2V} \end{bmatrix} = \underline{0}$$

- Eigenvalues describe stability of dutch roll, aperiodic roll and spiral
- Derivation will be treated in coming weeks

Eigenmotions

Symmetric

- **Short period**
- **Phugoid**

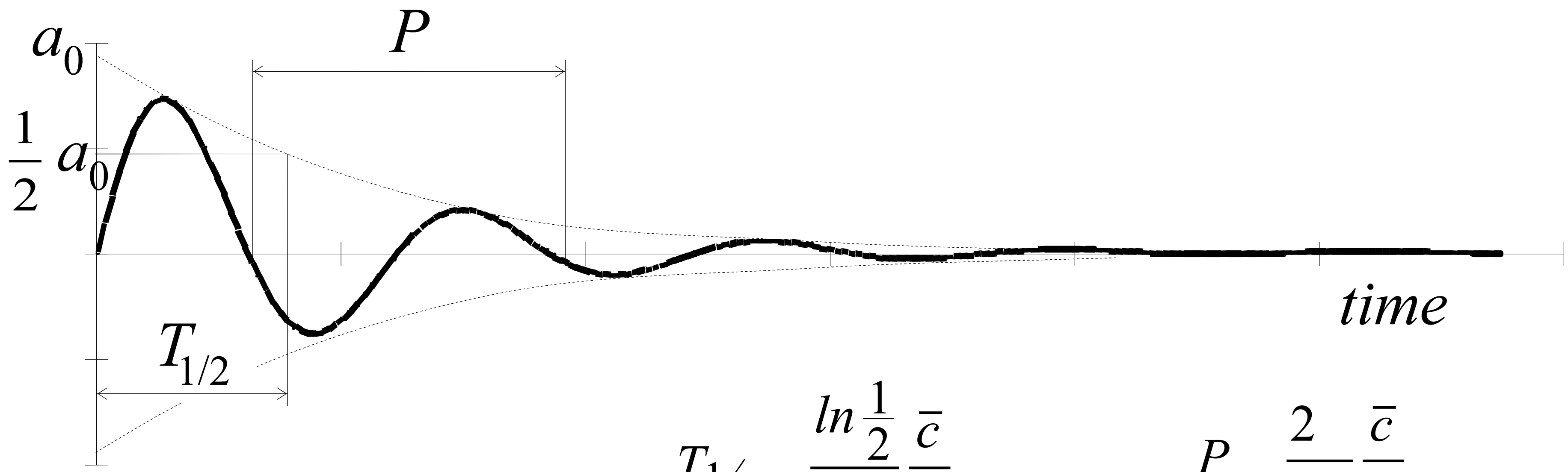


Asymmetric

- **A-periodic roll**
- **Spiral**
- **Dutch roll**

Checking the model

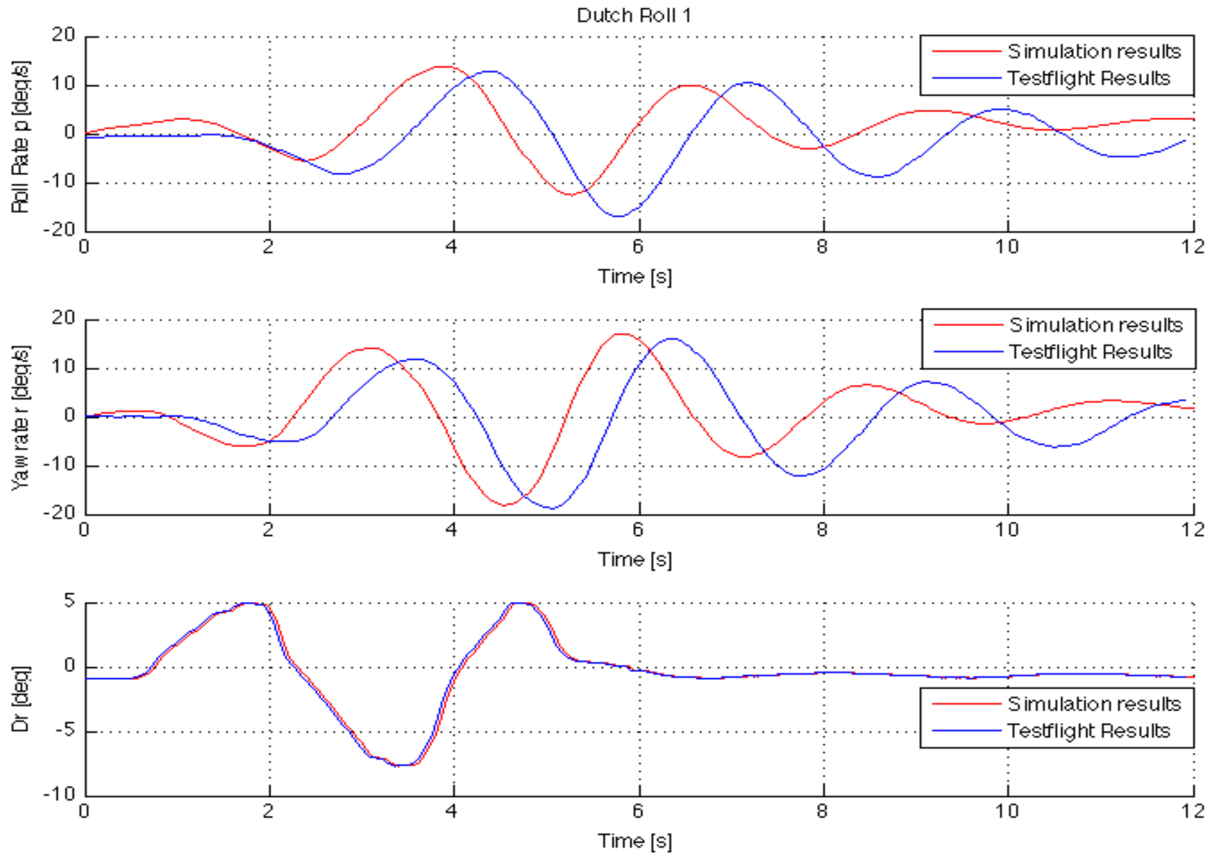
- Compare model properties with observed behaviour



$$T_{1\!/\!2} \qquad \frac{ln\frac{1}{2}}{c}\,\frac{\bar{c}}{V} \qquad\qquad P \qquad \frac{2}{c}\,\frac{\bar{c}}{V}$$

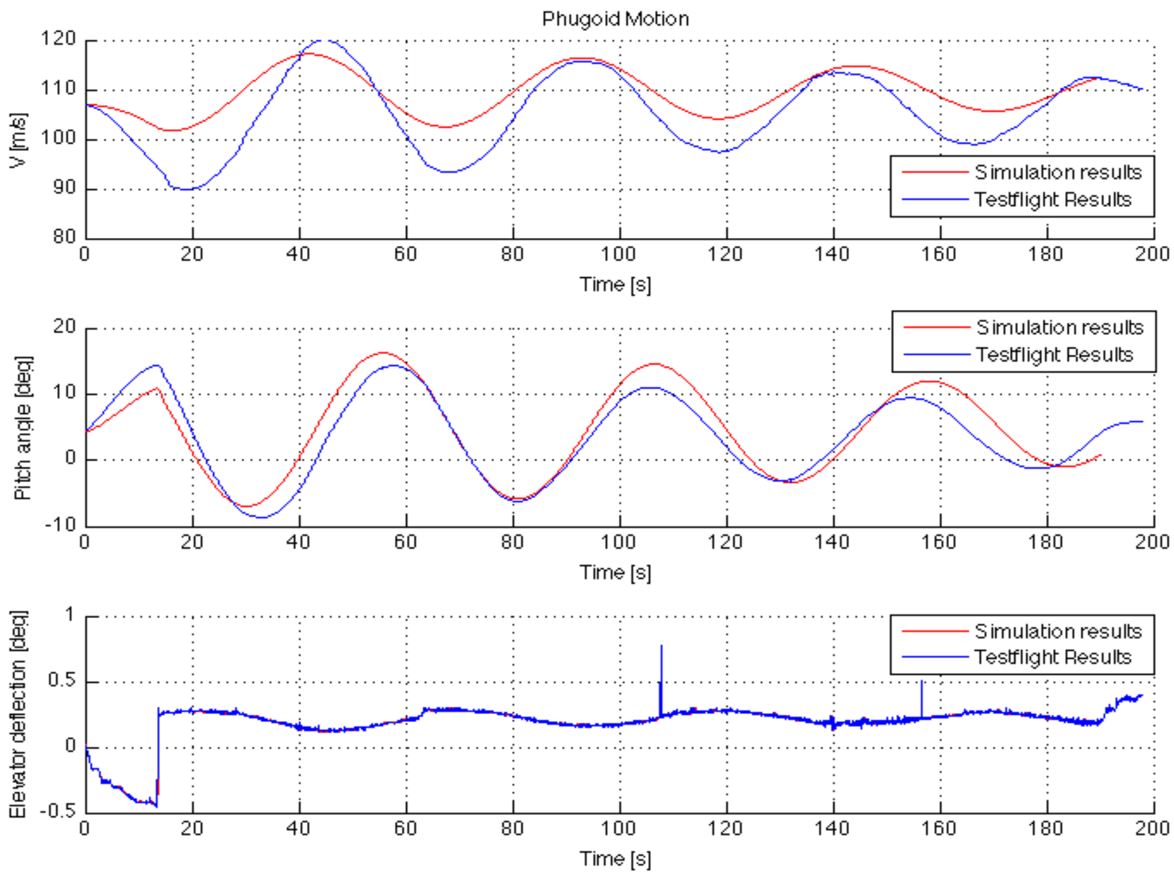
Checking the model

- Compare predicted behaviour to test data



Improving the model

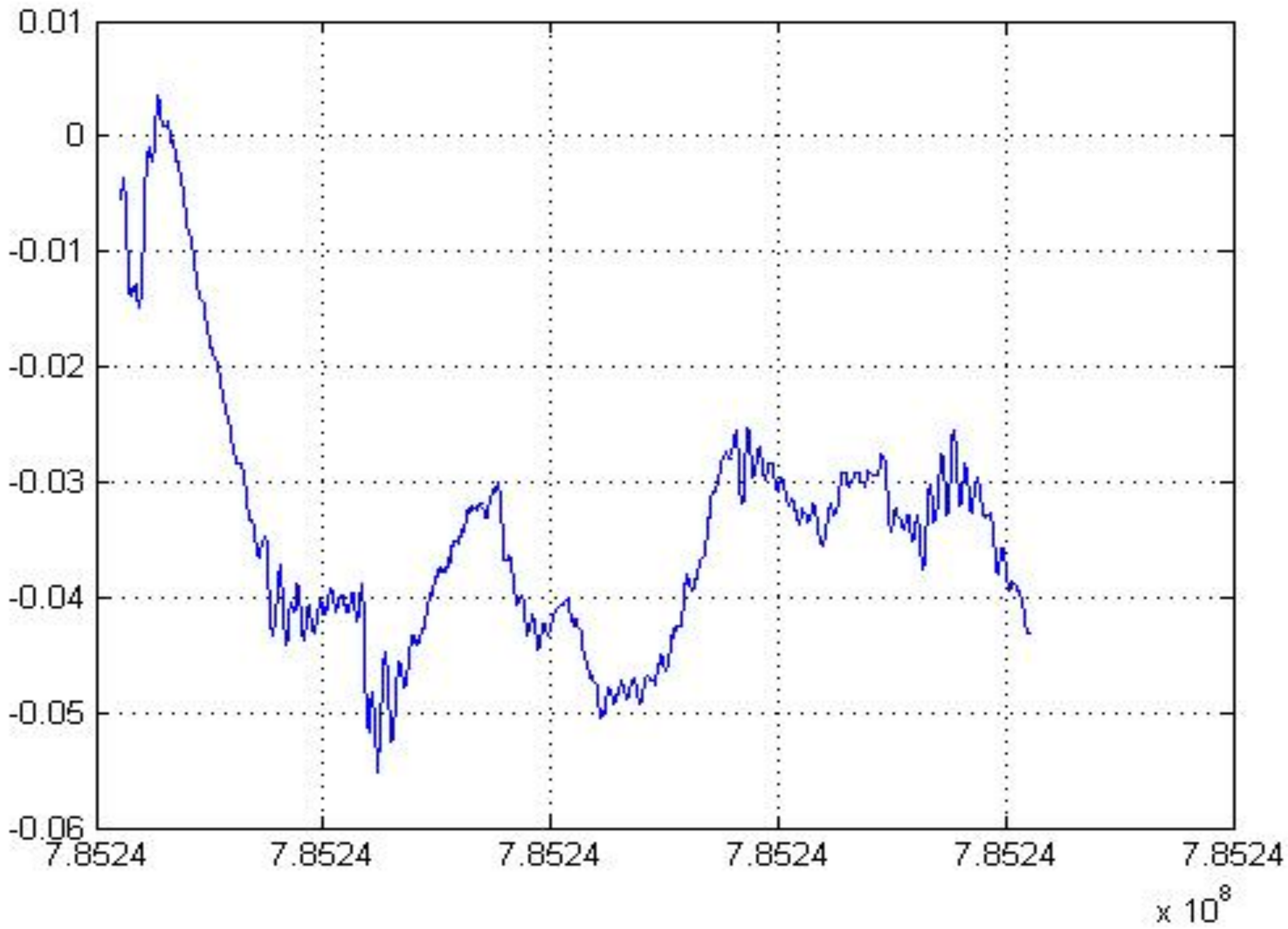
- Change model parameters to get a better match



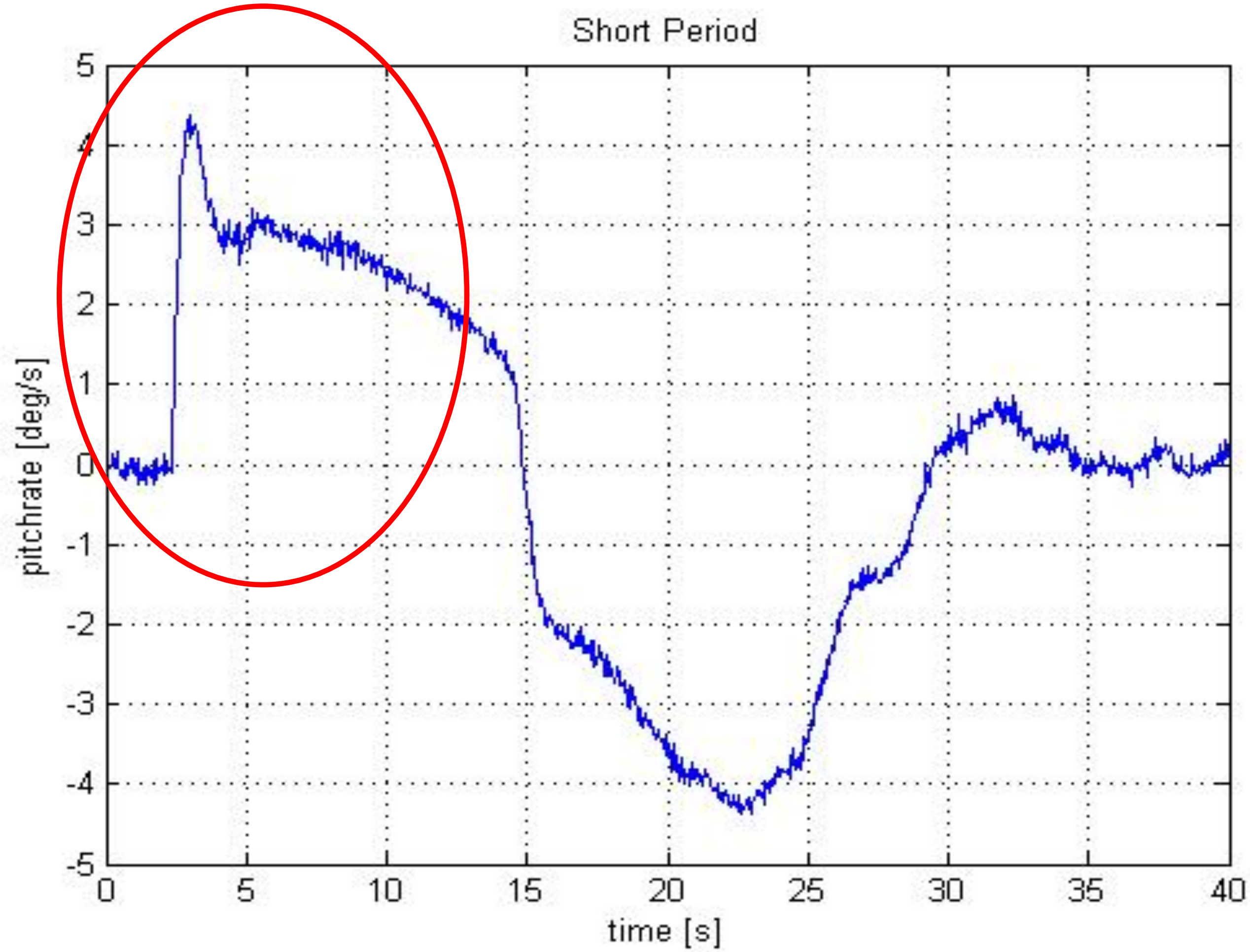
Pointers for the report

- Correct usage of units
- Use only relevant measurements
- Significant digits

How not to present your data



How not to present your data



AE3212-II

Flight Dynamics Assignment