Lab Assignments Computational Finance

Lecturer: Prof. dr. Drona Kandhai
Office: C3.142
Address: Section Computational Science, Science Park 403, 1098 XH, Amsterdam
Email: B.D.Kandhai@uva.nl

Teaching assistants: Dr. Sumit Sourabh and Ioannis Anagnostou MSc
Office: C3.146b
Address: Section Computational Science, Science Park 403, 1098 XH, Amsterdam
Email: S.Sourabh@uva.nl and Ioannis.Anagnostou@uva.nl

Submission guidelines
These assignments can be done in groups of three students. Reports with a *clear description of the assignment, the methods, the results and discussion* should be submitted before the deadlines. You are free to choose the programming language/environment in which you would like to write your computer programs. If you have questions about the assignments do not hesitate to contact the teaching assistant or the lecturer.

Grading scheme

- Each of the three assignments carries equal weight of 20% and the exam is worth 40%;
- The score of the exam should be 5 points (on the scale of 1 to 10) and higher for passing the course;
- The fourth assignment is a bonus assignment and has to be submitted before the exam. With the bonus assignment the final grade can be increased by at most 1 point (on the scale of 1 to 10), but only if this assignment has been graded sufficient ($\geq 50\%$).

Assignment 1	Assignment 2	Assignment 3	Exam	Assignment 4 (Bonus)
20%	20%	20%	40%	10%

Assignment 1: Black-Scholes Model and Binomial Tree Methods

Part I

Option Valuation

A commonly used approach to compute the price of an option is the so-called binomial tree method. Suppose that the maturity of an option on a non-dividend-paying stock is divided into N subintervals of length δt . We will refer to the j^{th} node at time $i^{\delta t}$ as the (i, j) node. The stock price at the (i, j) node is $S_{i,j} = S_0 u^j d^{i-j}$ (with u and d the upward and down-ward stock price movements, respectively). In the binomial tree approach, option prices are computed through a back-ward induction scheme:

1. The value of a call option at its expiration date is $MAX(0, S_{N,j} - K)$;
2. Suppose that the values of the option at time $(i+1)^{\delta t}$ is known for all j. There is a probability p of moving from the (i, j) node at time $i^{\delta t}$ to the $(i+1, j+1)$ node at time $(i+1)^{\delta t}$, and a probability $1-p$ of moving from the (i, j) node at time $i^{\delta t}$ to the $(i+1, j)$ node at time $(i+1)^{\delta t}$. Risk-neutral valuation gives

$$f_{i,j} = e^{-r\delta t}(pf_{i+1,j+1} + (1-p)f_{i+1,j})$$

Consider a European call option on a non-dividend-paying stock with a maturity of one year and strike price of €99. Let the one year interest rate be 6% and the current price of the stock be €100. Furthermore, assume that the volatility is 20%.

1. Write a binomial tree program to approximate the price of the option. Take a tree with 50 steps. How does your estimate compare to the analytical value? Experiment for different values of the volatility.
2. Study the convergence of the method for increasing number of steps in the tree. What is the computational complexity of this algorithm as a function of the number of steps in the tree?
3. Compute the hedge parameter from the Binomial Tree model. Compare with the analytical values. Experiment for different values of the volatility.
4. Now suppose that the option is American. Change the code such that it can handle early exercise opportunities. What is the value of the American put and call for the corresponding parameters? Experiment for different values of the volatility.

Part II
Hedging Simulations

The fundamental idea behind the Black-Scholes model is that of dynamic replication of the claim by taking positions in the underlying. In practice this means that a trader should apply a dynamic hedging strategy in order to ensure that the claim is replicated at expiry. In this part of the assignment we will apply a delta hedging of a European Call option.
We make the following assumptions:

- The dynamics of the stock price S is given by the following equation

$$dS = rSdt + \sigma SdZ$$

- The option and the corresponding delta sensitivities is based on the Black-Scholes model.

Consider again a short position in a European call option on a non-dividend-paying stock with a maturity of one year and strike price of €99. Let the one year interest rate be 6% and the current price of the stock be €100. Furthermore, assume that the volatility is 20%. Perform hedging simulation where the volatility in the stock price process is matching the volatility used in the option valuation (set both equal to 20%). Vary the frequency of the hedge adjustment (from daily to weekly) and explain the results. Perform numerical experiments where the volatility in the stock price process is not matching the volatility used in the option valuation. Experiment for various levels and explain the results.