Knowledge Representation Chapter 3. Relational Representation and Reasoning

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- Deductive Databases
- 2 Answer Set Programming
- Actions and change
- Diagnosis

Relational Representation

Atoms = instead of propositions, we have now predicates.
 They represent relations among entities:

```
neighbour(france, spain).
exports(germany, france, cars).
```

 Herbrand Domain = set of individuals, each one uniquely identified by a (lowercase) constant name. E.g.

```
D = \{\text{germany}, \text{france}, \text{spain}, \text{cars}, \dots \}.
```

Unique Names Assumption (UNA) =
 different terms represent different individuals.
 spain ≠ france, spain ≠ cars, spain ≠ españa

We can use unary predicates to represent types:

```
country(spain). country(france). country(germany).
tradegood(cars). tradegood(food).
```

```
country(spain; france; germany).
tradegood(cars; food).
```

Relational Representation

A set of facts becomes the extensional database (EDB)!

```
neighbour(spain, france).
neighbour(france, germany).
exports(spain, germany, food).
exports(spain, france, food).
exports(germany, france, cars).
exports(france, spain, cars).
```

Table neighbour

	_
C1	C2
spain	france
france	germany

Table exports

ida da angaras					
FROM	TO	GOOD			
spain	germany	food			
spain	france	food			
germany	france	cars			
france	spain	cars			

Relational Representation

A query to the EDB becomes a rule with variables.
 Variable = name with upcase initial (x, y, Country, ...) universally quantified and denoting arbitrary individuals.
 '_' = anonymous variable (different each time it occurs)

```
 \begin{array}{lll} & \texttt{exgood}(\texttt{G}) : - \texttt{exports}(\_,\_,\texttt{G}) . \texttt{exgood}(\texttt{G}) : - \texttt{exports}(\texttt{X1},\texttt{X2},\texttt{G}) . \\ & \forall \texttt{X1}, \texttt{X2}, \texttt{G} \left( \textit{exports}(\texttt{X1},\texttt{X2},\texttt{G}) \rightarrow \textit{exgood}(\texttt{G}) \right) \end{array}
```

• Ex.: "neighbours of France and goods she imports from them"

```
answer(N,G) :- neighbour(france,N), exports(N, france,G).
```

SQL equivalent is more verbose

```
SELECT neighbour.C2, exports.GOOD FROM neighbour INNER JOIN exports ON neighbour.C2=exports.FROM WHERE neighbour.C1=france AND exports.TO=france;
```

Problem: we get no goods from Spain using our previous data! We had neighbour (spain, france) but not the opposite!

Predicate neighbour should be symmetric! We add a rule

```
neighbour(X,Y) :- neighbour(Y,X).
```

- Deductive database: some predicates are intensional or (partially) deduced from rules, rather than extensional (list of facts).
- Ground atom = predicate + constants, no variables.
 Grounding = replacing variables by all their possible instances.
 (although it is actually more intelligent than that)

Example: the grounding of program

```
neighbour(spain,france). neighbour(france,germany).
neighbour(X,Y) :- neighbour(Y,X).
```

would potentially yield the rules

```
neighbour(spain, france). neighbour(france, germany).
neighbour(spain, france) :- neighbour(france, spain).
neighbour(spain, germany) :- neighbour(germany, spain).
neighbour(france, spain) :- neighbour(spain, france).
neighbour(france, germany) :- neighbour(germany, france).
neighbour(germany, spain) :- neighbour(spain, germany).
neighbour(germany, france) :- neighbour(france, germany).
```

Example: the grounding of program

```
neighbour(spain,france). neighbour(france,germany). neighbour(X,Y):- neighbour(Y,X).
```

would potentially yield the rules, but in practice ...

```
neighbour(spain, france) . neighbour(france, germany) .
neighbour(spain, france) :- neighbour(france, spain) .
neighbour(spain, germany) :- neighbour(germany, spain) .
neighbour(france, spain) :- neighbour(spain, france) .
neighbour(france, germany) :- neighbour(germany, france) .
neighbour(germany, spain) :- neighbour(spain, germany) .
neighbour(germany, france) :- neighbour(france, germany) .
```

Example: the grounding of program

```
neighbour(spain,france). neighbour(france,germany).
neighbour(X,Y) :- neighbour(Y,X).
```

would potentially yield the rules, but in practice ...

```
neighbour(spain, france). neighbour(france, germany).
neighbour(france, spain).
neighbour(germany, france).
```

- Datalog: deductive database paradigm using normal logic programs (under stratified negation) with predicates and variables.
- Remember: stratified implies a unique stable model.
 - Datalog is more expressive than SQL, but less expressive than logic programs without the stratification limitation.
 - It allows, for instance, defining recursive relations, such as:

```
\begin{array}{c} \text{connected}(X,Y) : - \text{ neighbour}(X,Y) \, . \\ \text{connected}(X,Z) : - \text{ neighbour}(X,Y) \, , \, \text{connected}(Y,Z) \, . \end{array}
```

so that we would get connected (spain, germany) even though they are not neighbours.

Bodies can add conditions on variables X!=Z, X>Z* (Y+1), etc.

```
connected(X, Z) :- neighbour(X, Y), connected(Y, Z), X!=Z.
```

 Domain independence: answers shouldn't change if we just augment the Herbrand Domain

```
switch(1..3). p(X,Y) := X < Y. \quad \text{% ordered pairs of different switches}  \text{returns } p(1,2) \text{, } p(1,3) \text{, } p(2,3) \text{ if } D = \{1,2,3\}   \text{but for } D = \{1,2,3,4\} \text{ we miss } p(1,4) \text{, } p(2,4) \text{, } p(3,4).   \text{The set of possible pairs of integers is infinite!}   p(X) := \text{not switch}(X). \quad \text{% anything that is not a switch}
```

The potential *D* with non-switches is even worse!

All variable occurrences in a rule must be safe

Definition (Safety: guarantees domain independence)

A variable is safe if it occurs in a non-negated predicate in the body.

```
p(X,Y) := X < Y, switch(X), switch(Y).

q(X) := object(X), not switch(X). % define valid objects!
```

- Deductive Databases
- 2 Answer Set Programming
- Actions and change
- Diagnosis

- Answer Set Programming (ASP) = we allow normal logic programs (unstratified negation) with predicates and variables.
- In ASP, the stable models are called answer sets.
- Example:

```
pacifist(X) :- quaker(X), not bellicous(X).
bellicous(X) :- republican(X), not pacifist(X).
quaker(nixon). republican(nixon).
republican(reagan).
```

Two answer sets:

```
Answer: 1
... bellicous(reagan) bellicous(nixon)
Answer: 2
... bellicous(reagan) pacifist(nixon)
```

Definition (HAMILT)

The Hamiltonian Cycle problem, *HAMILT*, consists in deciding whether a graph contains a cyclic path in a graph that visits each vertex exactly once. *HAMILT* is an **NP**-complete problem.



• extensional database mygraph.gph with the graph

```
vtx(1). vtx(2). vtx(3). vtx(4).
edge(1,2). edge(2,3). edge(2,4).
edge(3,1). edge(3,4). edge(4,3). edge(4,1).
```

• Examples of medium sized graphs (200 nodes, 1250 edges):

```
http://www.cs.uky.edu/ai/benchmark-suite/
hamiltonian-cycle.html
```

• Predicate in(X,Y) points out that an edge $X \to Y$ is in the cycle. We generate arbitrary choices

```
\{in(X,Y)\}:-edge(X,Y).
```

Only one outgoing vertex, only one incoming vertex:

```
:- in(X,Y), in(X,Z), Y!=Z.
:- in(X,Z), in(Y,Z), X!=Y.
```

• Disregard disconnected cycles. We use reached (X) meaning that X can be reached from an arbitrary fixed vertex, say 1.

and we forbid unreached vertices:

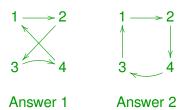
```
:- vtx(X), not reached(X).
```

• Making the call:

clingo O hamilt.lp

We obtain two answers:

```
Answer: 1
in(4,3) in(3,1) in(2,4) in(1,2)
Answer: 2
in(4,1) in(3,4) in(2,3) in(1,2)
SATISFIABLE
```

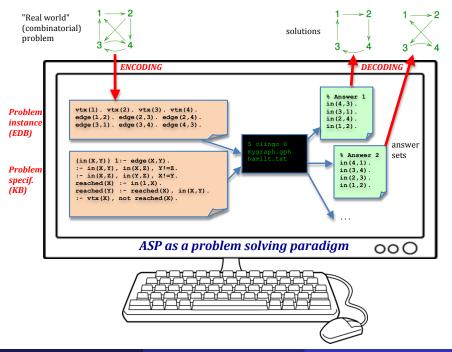


- We can split clingo in two steps: grounder gringo + propositional solver clasp.
- Download gringo from potassco.org and make the call

```
$ gringo hamilt.txt | clasp 0
```

To display the ground program, try the following

```
$ gringo -t hamilt.txt
...
:-in(1,2),in(1,3).
:-in(1,3),in(1,2).
:-in(2,1),in(2,3).
...
reached(2):-in(1,2).
reached(3):-in(2,3),reached(2).
reached(3):-in(1,3),reached(1).
...
```



	ASP	Prolog
semantics	several $n \ge 0$ answer sets	unique (canonical)
		model
problem solving	1 answer set = 1 solution	1 var. instantiation
		=
		1 solution
		?- graph(G), hamilt(G,X).
		X=[(4,3),(3,1),(2,4),(1,2)];
		X = [(4,1), (3,4), (2,3), (1,2)]
computational power	NP-complete	Turing-complete
language	specification	programming
type	(execution)	(flow control: ordering, cut,)

8 Queens revisited



Example (8-queens problem)

- \bullet Arrange 8 queens in a 8 \times 8 chessboard so they do not attack one each other.
- Exercise: encode the problem in ASP. (Use cardinality atoms).

Explicit negation

 We can sometimes be interested in a second negation, strong or explicit negation (originally called "classical"). Example:

```
fill :- empty, not fire.
```

risky! we fill when no information on fire, but no guarantee.

We could use auxiliary atom no_fire ("I'm sure there is no fire")

```
fill :- empty, no_fire.
:- fire, no_fire.
no_fire :- wet.
```

• Explicit negation '-' makes this same effect.

```
fill :- empty, -fire.
-fire :- wet.
```

and the constraint :- fire, -fire is implicit.

Einstein's 5 houses riddle: who keeps fishes as pets?

- The Brit lives in the red house.
- The Swede keeps dogs as pets.
- The Dane drinks tea.
- The green house is on the immediate left of the white house.
- The green house's owner drinks coffee.
- The owner who smokes Pall Mall rears birds.
- The owner of the yellow house smokes Dunhill.
- The owner living in the center house drinks milk.
- The Norwegian lives in the first house.
- The Blends smoker is neighbor of the one who keeps cats.
- The horse keeper is neighbor of the one who smokes Dunhill.
- The owner who smokes Bluemasters drinks beer.
- The German smokes Prince.
- The Norwegian lives next to the blue house.
- The Blends smoker lives next to the one who drinks water.

Pooling: abbreviate several facts in a same atom

```
house(1..5).
color(red;green;blue;white;yellow).
```

is the same than

```
house(1). house(2). house(3). house(4).house(5).
color(red). color(green). color(blue).
color(white). color(yellow).
```

Constants: can be defined in the file

```
#const numhouses=5.
house(1..numhouses).
```

or passed as arguments in command line

```
$ clingo -c numhouses=5 einstein.txt
```

New features

Function symbols as constructors.

```
owner( person(bill,gates), microsoft ).
owner( person(jeff,bezos), amazon ).
owner( company(inditex), zara).
family(Y) :- owner( person(X,Y), Z).
```

- Aggregate = function on sets of values.
- We may have #sum, #max, #min, #avg, #count. Example:

```
income(jan,5). income(feb,3).
income(mar,-2). income(apr,10).
total(S) :- #sum{X: income(M,X)} = S.
```

• Problem: if we have repeated values, they count once

```
income(may,10). income(jun,10).
```

the set is still $\{5, 3, -2, 10\}$ and S=16.

• We use tuples (the sum applies to the first component):

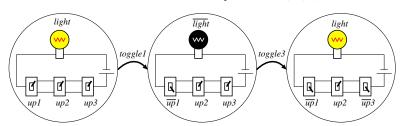
```
total(S) :- #sum{X,M: income(M,X)} = S.

{X,M: income(M,X)} = {(5,jan), (3,feb), (-2,mar), (10,apr), (10,may), (10,jun)}
```

- Deductive Databases
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Back to our simple example

- Lamp and switches revisited
- Fluents: *up*1, *up*2, *up*3, *light* (Boolean).
- Actions: toggle1, toggle2, toggle3.
- State: a possible configuration of fluent values. Example: $\{\overline{up1}, up2, up3, \overline{light}\}$.
- Situation: a moment in time. We can just use 0, 1, 2, ...



Reasoning about actions with ASP

- Download system telingo (temporal clingo)
- We can make groups of rules

```
#program initial. % At timepoint t=0
...
#program dynamic. % Transition from t-1 to t
...
#program always. % Any timepoint t=0..n-1
...
#program final. % Last timepoint t=n-1
...
```

- Predicate names preceded by 'refer to timepoint t-1
- Predicate names preceded by _ refer to timepoint t=0

```
% File: switches.lp (domain description)
switch(1..3).
action(tog(X)) :- switch(X).
#program dynamic.
% Effect axioms
 h(sw(X), up) := 'h(sw(X), down), o(tog(X)).
 h(sw(X), down) := 'h(sw(X), up), o(tog(X)).
 h(light, off) := 'h(light, on), o(tog(_)).
 h(light, on) := 'h(light, off), o(tog(_)).
% Executability constraints: none in this case
% Inertia: c(F) = fluent F has changed
 h(F,V) := 'h(F,V), \text{ not } c(F).
 c(F) := 'h(F,V), h(F,W), V!=W.
% Action generation
  1 { o(A): _action(A) } 1.
```

RAC goals

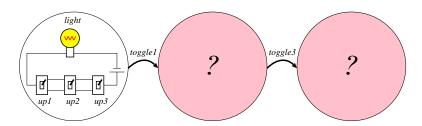
We want to solve some typical reasoning problems.

The most usual ones:

- Simulation (aka prediction, aka temporal projection):
 run a sequence of actions on an initial state
- Temporal explanation (aka postdiction): fill gaps from partial observations
- Planning: obtain sequence of actions to reach some goal
- Diagnosis: explain unexpected observed results
- Verification: check system properties

Prediction (simulation, or temporal projection)

- Knowing: initial state + sequence of actions
- Find out: final state (alternatively sequence of intermediate states)



Reasoning about actions with ASP

Prediction example

```
% File: switches-predict.lp (instance of prediction problem)
#program initial.
h(light,off).
h(sw(X),up) :- switch(X).
```

We assert a sequence of facts using:

```
% Sequence of performed actions
&tel{
         &true
; > o(tog(3))
; > o(tog(1))
; > o(tog(2))
; > o(tog(2))
}.
#show h/2.
#show o/1.
```

where ; > is a sequence operator

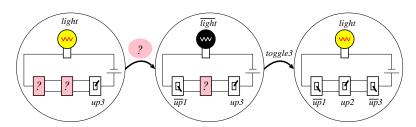
Prediction example

Calling telingo switches.txt switches-predict.txt

```
Answer: 1
 State 0:
  h(light, off) h(sw(1), up) h(sw(2), up) h(sw(3), up)
 State 1:
 o(tog(3))
 h(light, on) h(sw(1), up) h(sw(2), up) h(sw(3), down)
 State 2:
 o(tog(1))
  h(light, off) h(sw(1), down) h(sw(2), up) h(sw(3), down)
 State 3:
 o(toq(2))
  h(light, on) h(sw(1), down) h(sw(2), down) h(sw(3), down)
 State 4:
 o(tog(2))
  h(light, off) h(sw(1), down) h(sw(2), up) h(sw(3), down)
```

Postdiction (or temporal explanation)

- Knowing: partial observations of states and performed actions
- Find out: complete information on states and performed actions



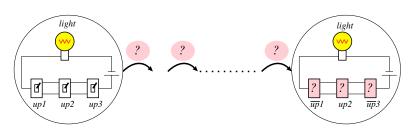
Postdiction example:

```
% switches-postdict.lp
#program initial.
% Completing unknown facts
 1 \{h(sw(X), up); h(sw(X), down)\} 1 :- switch(X).
 1 {h(light,on); h(light,off)} 1.
% Observations: we use a constraint!
 :- not &tel{
     h(sw(3),up) & h(light,on)
  ;> h(light,off) & h(sw(1),down) & h(sw(3),up)
  ;> o(tog(3))
 } .
```

Calling telingo 0 switches.txt switches-postdict.txt we get 4 possible explanations

Planning

- **Knowing**: initial state + goal (partial description of final state)
- Find out: plan (sequence of actions) that guarantees reaching the goal



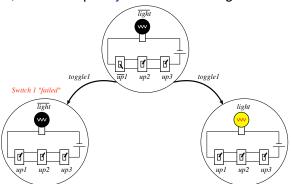
Reasoning about actions with ASP

Planning example

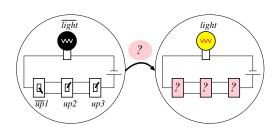
Calling telingo 0 switches.txt switches-plan.txt we get two minimal plans of length 2 toggling 1 and 3 or vice versa.

Planning vs Postdiction

- Note that planning seems a type of postdiction. For deterministic systems, this is true, but . . .
- Nondeterministic transition system: fixing current state + performed action --> several possible successor states.
- For instance, switch 1 up may fail to turn the light on...



Planning vs Postdiction



- For postdiction, one valid explanation is: we performed toggle1, and it succeeded to turn the light on.
- For planning, toggle1 is not a valid plan: it does not guarantee reaching the goal light. Possible plans are toggle2 or toggle3.

- Knowing: a model distinguishing between normal and abnormal transitions + a partial set of observations (usually implying abnormal behavior).
- Find out: the minimal set of abnormal transitions that explains the observations.
- We will see an ASP example later on.
- Similar to postdiction, but we are additionally interested in minimality of explanations.

Exercise

"Elaborating Missionaries and Cannibals Problem" [J. McCarthy]

3 missionaries and 3 cannibals come to a river and find a boat that holds two. If the cannibals ever outnumber the missionaries on either bank, the missionaries will be eaten. How shall they cross?



We will use the following fluents:

- \bigcirc n (G, B) = is the number of persons of group G at bank B.
 - Ex.: h (n (mis, 1), 3) = "there are 3 missionaries in the left bank"
- ② boat points out the boat bank. Ex. h (boat, 1) = "the boat is at left bank"

Exercise: missionaries and cannibals

We will use action:

- move (M, C) = move M missionaries and C cannibals.
- For simplicity, we include two action attributes moved (mis, N)
 and moved (can, N) that point out separatedly how many persons
 of each group are moved.

We begin with types and initial state

```
#program initial.
% Some types
group(mis;can).
bank(l;r).
opposite(l,r). opposite(r,l).
action(move(M,C)) :- M=0..2, C=0..2, M+C<3, M+C>0.
% Initial state
h(n(G,l),3) :- group(G).
h(n(G,r),0) :- group(G).
h(boat,l).
```

Rules for transitions

```
#program dynamic.
% Action generation
1 {o(A) : _action(A) } 1.
% Auxiliary (action attributes)
moved(mis, M) := o(move(M, C)).
moved(can, C) := o(move(M, C)).
% Executability axioms
:- moved(G, N), 'h(boat, B), 'h(n(G, B), M), N>M.
% Effect axioms (no inertia needed)
h(n(G,B),M+N) := 'h(n(G,B),M), h(boat,B), moved(G,N).
h(n(G,B),M-N) := 'h(n(G,B),M), 'h(boat,B), moved(G,N).
h(boat, B1) :- 'h(boat, B), _opposite(B, B1).
```

Inertia not needed because all fluents are changed

Rules for transitions

```
#program always.
% Missionaries not outnumbered by cannibals
:- h(n(mis,B),M), h(n(can,B),C), C>M, M>0.

#program final.
:- not goal.
goal :- h(n(mis,r),3), h(n(can,r),3).

#show o/1. % We only show performed actions
```

- We execute telingo 0 mc.txt and it will try length t = 1, 2, ... until a solution is found.
- Four solutions of length t = 11 are eventually found.

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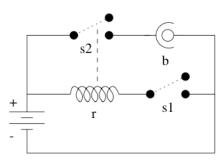
Diagnosis

- An agent acts in a dynamic environment and observes the results of her actions.
- ullet Sometimes she gets discrepancies: observations \neq expected result

Diagnosis

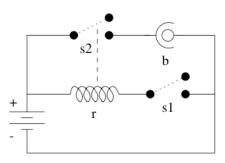
• Example [Balduccini & Gelfond 03]

We have a circuit with lightbulb b and a relay r. The agent can close s1 causing s2 to close (if r is not damaged). The bulb emits light if s2 is closed and b is not damaged.



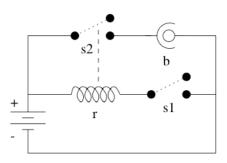
Example [Balduccini & Gelfond 03]

Exogenous action break damages the relay. Action power-surge damages r, and b too, if the latter is not protected (prot).



Example [Balduccini & Gelfond 03]

We close s1 but b does not emit light: what has happened?



Types and domains

```
#program initial.
switch(s1;s2).
component (relay; bulb) .
fluent (relay; light; b_prot).
fluent(S):-switch(S).
fluent(ab(C)) :- component(C).
value(relay, (on; off)).
value(light, (on; off)).
value(S, (open; closed)) :- switch(S).
hasvalue(F) :- value(F, V).
% Fluents are boolean by default
domain(F,(true;false)) :- fluent(F), not hasvalue(F).
% otherwise, they take the specified values
domain(F,V) := value(F,V).
```

Fluents ab(C) point out that a component is damaged

• Actions are exogenous *exog* or agent's *agent*:

```
agent(close(s1)).
exog(break; surge).
action(Y):-exog(Y).
action(Y):-agent(Y).
```

```
#program dynamic.
% Tnertia
h(F,V) := 'h(F,V), \text{ not } c(F).
c(F) := 'h(F,V), h(F,W), V!=W.
% Direct effects
h(s1, closed) := o(close(s1)).
#program always.
% Indirect effects
h(relay, on) :- h(s1, closed), h(ab(relay), false).
h(relay, off) :- h(s1, open).
h(relay, off) :- h(ab(relay), true).
h(s2, closed) :- h(relay, on).
h(light,on) :- h(s2,closed), h(ab(bulb),false).
h(light, off) :- h(s2, open).
h(light,off) :- h(ab(bulb),true).
```

```
#program dynamic.
% Executability
:- o(close(S)), 'h(S,closed).

% Malfunctioning
h(ab(bulb),true) :- o(break).
h(ab(relay),true) :- o(surge).
h(ab(bulb),true) :- o(surge), not 'h(b_prot,true).
```

We use predicates obs_o and obs_h to denote observations

```
% Observed actions actually occur
o(A) :- obs_o(A).

#program always.
% Check that observations hold
:- obs_h(F,V), not h(F,V).

#program initial.
% Completing the initial state
1 {h(F,V):_domain(F,V)} 1 :- _fluent(F).
```

• These are the observations:

```
% A history
&t.el {
     obs_h(s1,open) & obs_h(s2,open) &
     obs_h(b_prot,true) &
     obs_h(ab(bulb), false) &
     obs_h(ab(relay), false)
  ; > obs_o(close(s1)) &
     obs_h(light,off)
} .
#program dynamic.
% Generate exogenous actions
\{ o(Z) : \underline{-exoq(Z)} \}.
cause(X) :- o(X), _exoq(X).
#show cause/1.
```

 This will provide all possible explanations, but not minimal diagnoses.

```
$ telingo 0 diag.lp
Answer: 1
State 0:
 State 1:
 cause(break)
Answer: 2
State 0:
 State 1:
 cause (break) cause (surge)
Answer: 3
State 0:
State 1:
 cause (surge)
SATISFIABLE
```

- Optimization problems: we can use maximize/minimize
- One possible notation is:

```
#minimize <numerical_expr>: <condition>.
```

Example

```
numcauses(N) :- #count{X:cause(X)}=N.
#minimize {N:numcauses(N)}.
```

means "get minimal number of exogenous actions"

• To obtain all minimal solutions we use the options:

```
$ telingo --opt-mode=optN -n0 diag.lp
```

Two minimal solutions are found:

```
Answer: 1
State 0:
State 1:
cause(surge)
Optimization: 1
Answer: 2
State 0:
State 1:
cause(break)
Optimization: 1
OPTIMUM FOUND
```