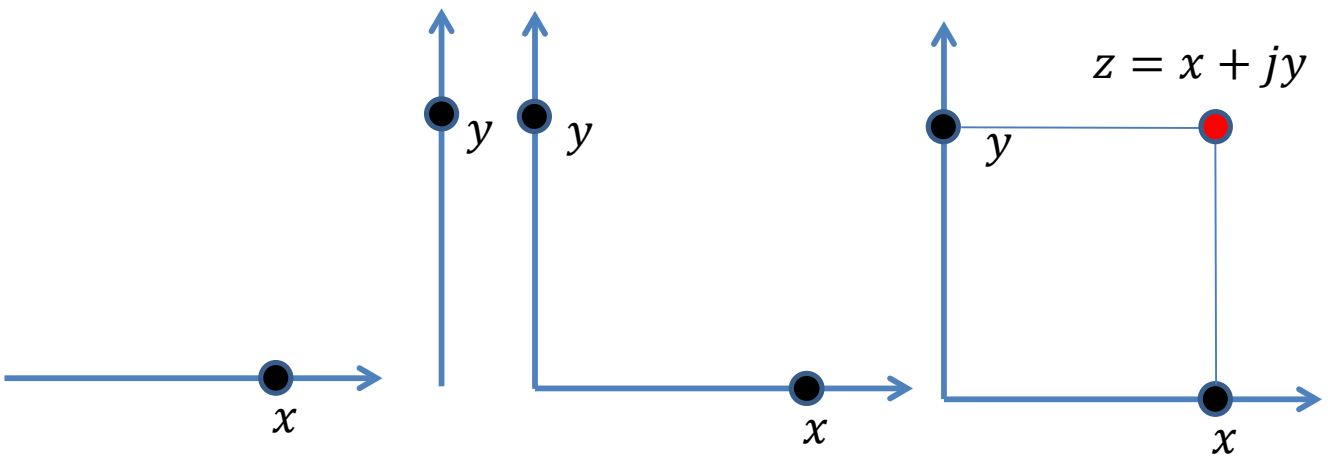
Números Complejos



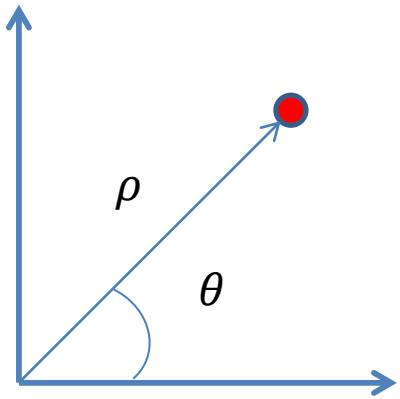
Notación cartesiana

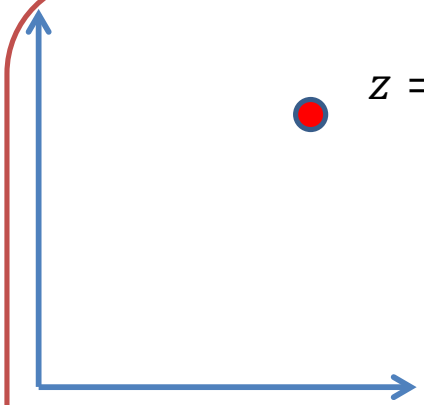
$z = x + jy$

$y = Im(z)$

$j = \sqrt{-1}$

$x = Re(z)$

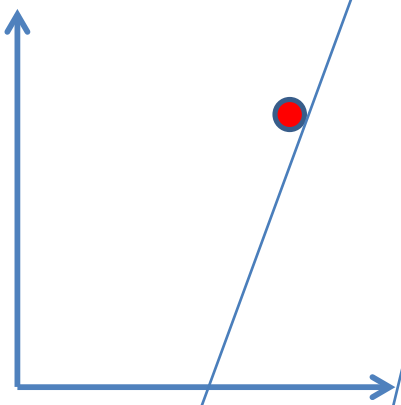




$$\rho = \sqrt{x^2 + y^2}$$

$$\theta = atan\left(\frac{y}{x}\right)$$

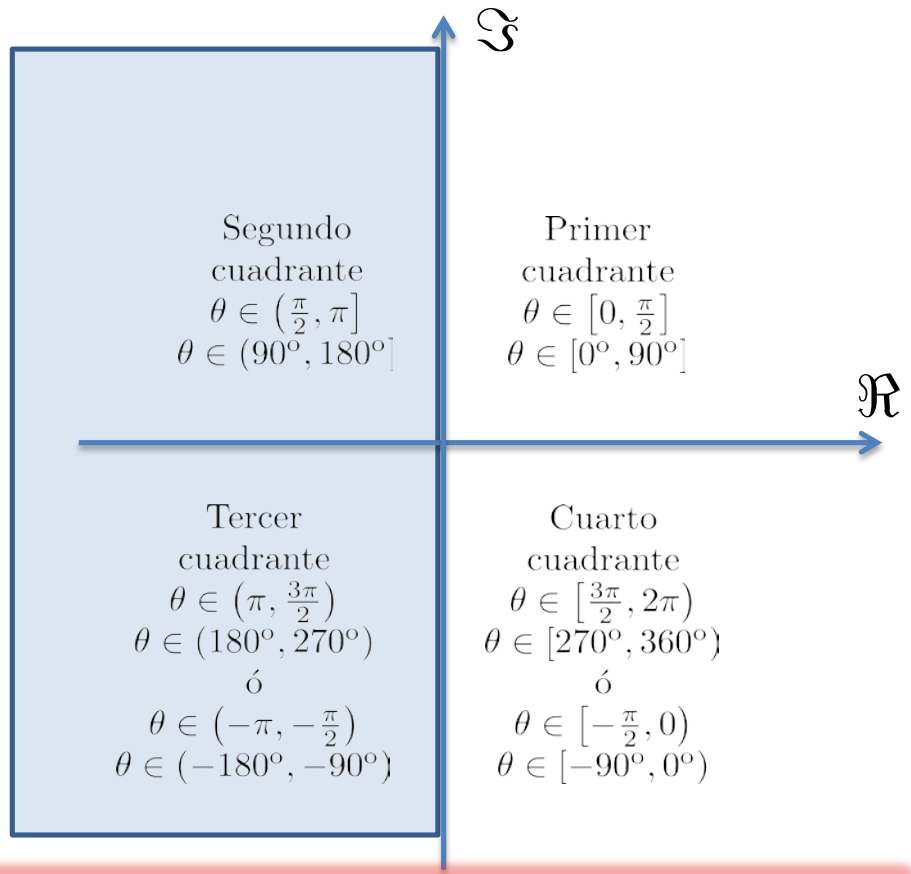
Notación polar



$$\rho e^{j\theta} = \rho\cos\theta + j\rho\,\mathrm{sen}\,\theta$$

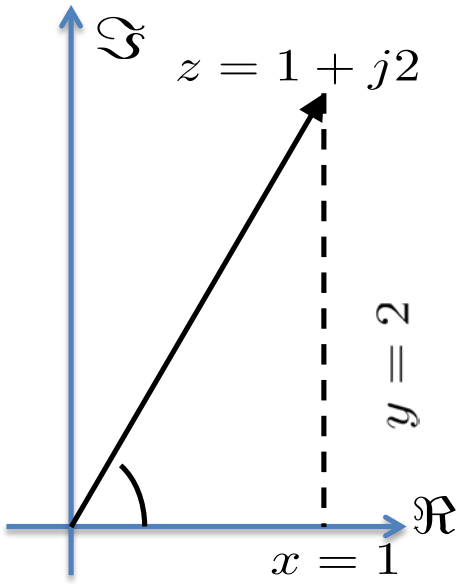
Relación de Euler
(exponencial compleja)

Ejemplo 1



Si la parte real es negativa, hay que sumar 180º

Notación cartesiana



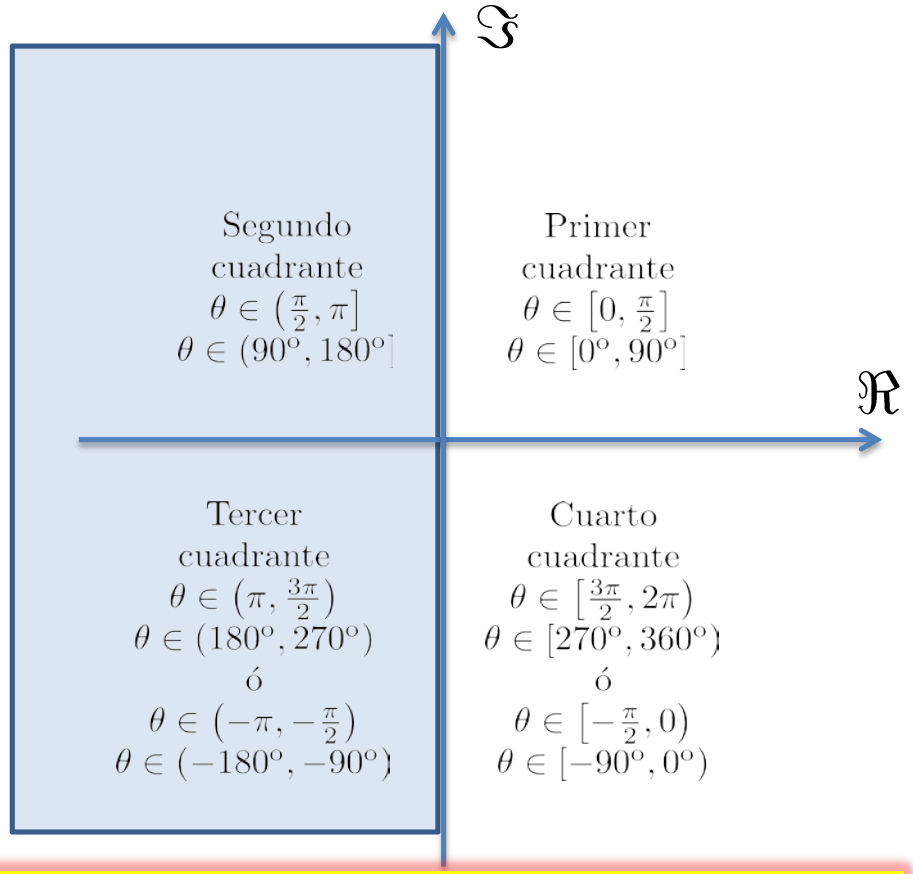
Notación polar

$$\rho = \sqrt{x^2 + y^2} = \sqrt{1 + 4} = \sqrt{5}$$

$$\theta = \text{atan}\left(\frac{2}{1}\right) = 1.1071 \ rad/seg$$

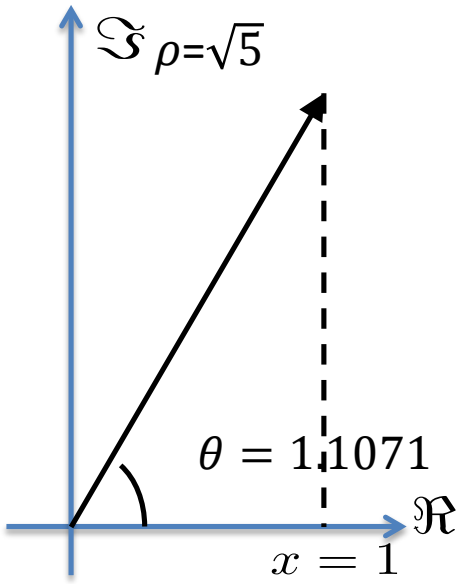
$$\theta = 1{,}1071 \ \frac{180º}{\pi} = 63.435º$$

Ejemplo 1 (cont.)



Si la parte real es negativa, hay que sumar 180º

Notación polar



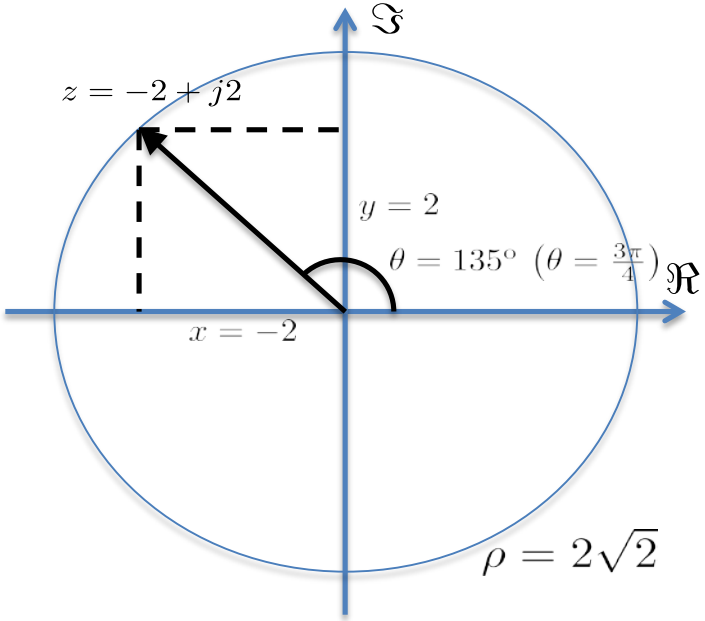
Notación cartesiana

$x = \rho \cos\theta = \sqrt{5} \cos(1.1071) = 1$

$y = \rho \operatorname{sen}\theta = \sqrt{5} \operatorname{sen}(1.1071) = 2$

Ejemplo 2

$z = -2 + j\,2$ pertenece al segundo cuadrante



- Cálculo del módulo:

$$\rho = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$$

- Cálculo de la fase:

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(-1\right)$$

- El resultado obtenido con la función `atan` es -45º, que pertenece al cuarto cuadrante. Debemos corregirlo añadiéndole 180º (π radianes) para obtener el resultado correcto

$$\arctan\left(-1\right) = -45^\circ \ (\text{ó} \ -\tfrac{\pi}{4}),$$

$$\theta = -45^\circ + 180^\circ = 135^\circ$$

Suma

$$z_1 = x_1 + jy_1 = |z_1|e^{j\theta_1}$$
$$z_2 = x_2 + jy_2 = |z_2|e^{j\theta_2}$$

Suma

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 + z_2 = |z_1|e^{j\theta_1} + |z_2|e^{j\theta_2}$$

$$\rho = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$$

$$\theta = atan\left(\frac{y_1 + y_2}{x_1 + x_2}\right)$$

$$z = \rho e^{j\theta}$$

Multiplicación y división

$$z_1 = x_1 + jy_1 = |z_1|e^{j\theta_1}$$
$$z_2 = x_2 + jy_2 = |z_2|e^{j\theta_2}$$

Multiplicación

$$z_1.z_2 = (x_1 + jy_1) * (x_2 + jy_2)$$

$$z_1.z_2 = |z_1|e^{j\theta_1}|z_2|e^{j\theta_2} = |z_1 z_2|e^{j(\theta_1+\theta_2)}$$

$$z_1.z_2 = |z_1 z_2|e^{j(\theta_1+\theta_2)}$$

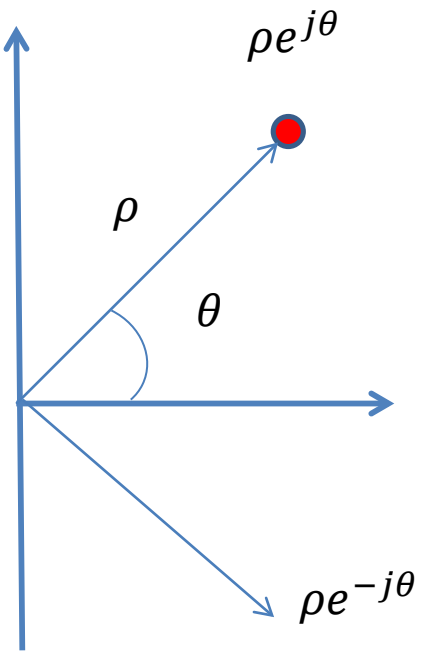
División

$$\frac{z_1}{z_2} = \frac{x_1 + jy_1}{x_2 + jy_2}$$

$$\frac{z_1}{z_2} = \frac{|z_1|e^{j\theta_1}}{|z_2|e^{j\theta_2}} = \frac{|z_1|}{|z_2|}e^{j(\theta_1-\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}e^{j(\theta_1-\theta_2)}$$

Relación de Euler



$$\rho e^{\pm j\theta} = \rho\cos\theta \pm j\rho\,\text{sen}\,\theta$$

Relación de Euler
(exponencial compleja)

$$\rho e^{j\theta} = \rho\cos\theta + j\rho\,\text{sen}\,\theta$$
$$\rho e^{-j\theta} = \rho\cos\theta - j\rho\,\text{sen}\,\theta$$

Si sumamos:

$$\rho e^{j\theta} + \rho e^{-j\theta} = 2\rho\cos\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Si restamos:

$$\rho e^{j\theta} - \rho e^{-j\theta} = j2\rho\,\text{sen}\,\theta$$

$$\text{sen}\,\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$