Coronavirus Pandemic Modeling Deliverable 2

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Executive Summary

In this project, I have used simulation data for over 200 US metropolitan areas to determine how medical supplies should be strategically distributed in order to both minimize unmet demand and to minimize the total number of supplies needed. My findings for how resources should be distributed are summarized in the table below.

Population per 1 Unit of Medical Supplies	Total Medical Supplies Used
~150,000	~2,000
~13,000	~14,000
~5,000	~21,000
~3,600	~30,000

Intro

The goal of this project is to optimize the distribution of medical supplies around the United States during the Coronavirus pandemic. The type of medical supplies is not specified, but likely refers to things such as ventilators, gloves, masks, etc. Given the peak medical supply demand for over 200 US metropolitan areas in 200 simulations, I will be determining where supplies should be sent to both minimize the unmet demand as well as minimize the total amount of equipment required.

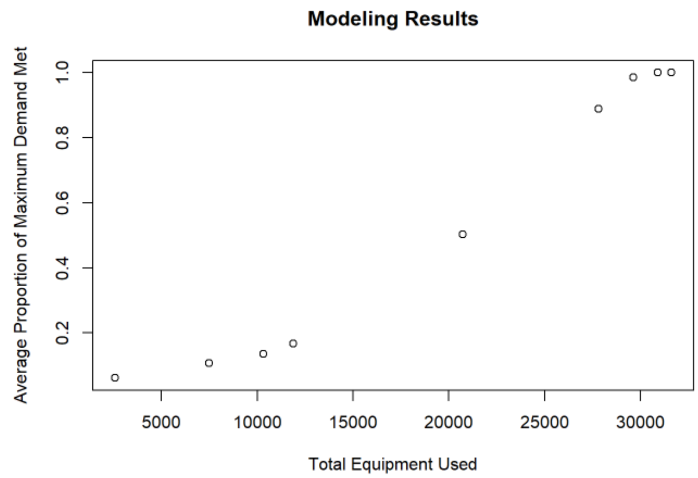
Exploring the Data

From these simulations, I have determined that 75% of cities will likely require under 116 medical supplies, but the other 25% of cities have significantly higher demands, some even surpass a demand of 3000 medical supplies.

In fact, all but 10 cities will likely not have a peak demand exceeding 500. These are the cities which can be expected to get hit hardest by the Coronavirus. The 10 cities with average demand exceeding 500 can be found in Appendix A.

Solving the Model

In order to solve this problem, I created a model which can be found in Appendix B. However, a decision must be made on how to weigh the importance of having minimized unmet demand versus a minimized number of total supplies required. For example, if minimizing total equipment used is the only thing that matters, then no cities should be sent any supplies. On the other extreme, all cities should be sent their maximum expected demand if minimizing unmet demand is all that matters.



Keeping this balance in mind, I was able to create the figure to the right which shows what amount of supplies needed will correspond to what national average proportion of maximum demand will be met:

Accounting for Equipment Divisibility

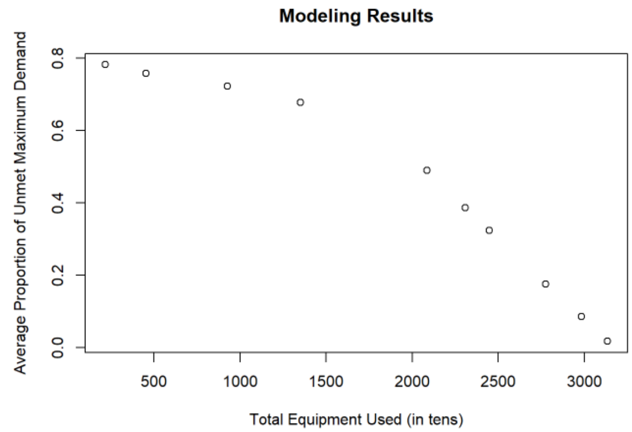
Considering that the equipment shipped may not be perfectly divisible, I have altered the pervious model to account for the supplies having to be shipped in set multiples of ‘z.’ This new model can be found in Appendix C. The following work is done assuming that equipment can only be shipped in multiples of 10 units.

Fairness

In order for the distribution of supplies to be fair, the amount any city gets should be based on how many supplies the city is expected to need, which the model described above accounts for. However, it would not be entirely fair to for cities which need very little to receive nothing while some cities receive a lot. For this reason, I have created another model which can be found in Appendix D which guarantees that each metropolitan area will receive at least 10 medical supplies, meaning that every city will at least have some resources to work with.

Solution

Using the fairer model described above, I generated the figure to the right based on various weightings of the two goals.



Rule of Thumb

In order to keep things simple, I have created a rule of thumb for how supplies should be distributed based on a given metropolitan area's population. See Appendix E for its formulation. The figure in Appendix H compares how many total supplies would be used with how large a metropolitan area's population would have to be to merit receiving 1 unit of medical equipment.

This figure can be generalized further through the following table:

Population per 1 Unit of Medical Supplies	Total Medical Supplies Used
~150,000	~2,000
~13,000	~14,000
~5,000	~21,000
~3,600	~30,000

Most and Least At-Risk Metropolitan Areas

Using the above rule of thumb to distribute supplies, most cities will receive about the right number of supplies. However, using this rule of thumb, some areas will receive more supplies than they will likely need, and some will likely receive not enough. In general, a metropolitan area can be considered more at-risk if it will likely need more medical supplies than indicated by its population. The opposite is true for less at-risk metropolitan areas. The figure below shows the top 5 most and least at-risk areas in the country.

Most At-Risk	Santa Maria-Santa Barbara, CA	Santa Fe, NM	Sioux Falls, SD	Santa Rosa, CA	Portland-South Portland, ME
Least At-Risk	San Francisco-Oakland-Hayward, CA	San Diego-Carlsbad, CA	San Antonio-New Braunfels, TX	Seattle-Tacoma-Bellevue, WA	Portland-Vancouver-Hillsboro, OR-WA

APPENDIX A: Metropolitan Areas with Expected Demand Exceeding 500

Metro Area	Expected Peak Demand
New York-Newark-Jersey City, NY-NJ-PA	3064.99
Los Angeles-Long Beach-Anaheim, CA	2732.57
Dallas-Fort Worth-Arlington, TX	1115.4
Chicago-Naperville-Elgin, IL-IN-WI	1107.27
Phoenix-Mesa-Scottsdale, AZ	975.462
Santa Maria-Santa Barbara, CA	925.241
Miami-Fort Lauderdale-West Palm Beach, FL	812.367
Houston-The Woodlands-Sugar Land, TX	721.412
Riverside-San Bernardino-Ontario, CA	710.762
Santa Fe, NM	677.09

APPENDIX B: Formulating a Linear Program

When creating a linear program, the first step I took was to determine the objective function. Because we would like to minimize both unmet demand and total equipment required, we are faced with a bi-objective linear program. While total equipment required is easy to represent as the summation of the equipment sent to each city, unmet demand is a bit trickier.

Unmet demand can be represented as max{0, demand – supply}. To convert this non-linear max function to something linear, I chose to create a decision variable y which represents the unmet demand. This new decision variable is then constrained so that it must be greater than 0 and also greater than (demand – supply) which effectively serves the role of the max function. To account for uncertainty, I chose to represent demand as the city's demand for any given simulation, but supply is constant regardless of which simulation you consider. To model this, I chose to create another decision variable which represents the supply sent to each metropolitan area. In other words, by having demand vary by simulation but supply stay constant, I am able to have a model which is based on the data of various simulations but ultimately must make some decision about supply prior to knowing which simulation is the most accurate representation of the pandemic. This allows for the model to account for the fact that the optimal supply amount may be greater than what is demanded in some simulations. I then divide the unmet demand by the number of simulations so that the unmet demand objective represents the average unmet demand in the country.
Next, I needed to determine the constraints of the problem. In addition to the ones already mentioned, there have to be non-negativity constraints. While having as many medical supplies as demanded is beneficial, having excess is not. Additionally, the amount of supplies sent to a given city should not exceed that of its expected maximum demand which was determined earlier.
After this, I created the necessary variables for sets and data. The linear program based on this methodology is shown in the figure below.

Data-Independent LP:

Indicies and Sets:

$i \in I$ Cities, $j \in J$ Simulations

Data:

m_j Max demand in city $i \in I$, $d_{i,\,j}$ Demand in city $i \in I$ in simulation $j \in J$

Variables: x_i Supply to city $i \in I$, $y_{i,\,j}$ Unmet demand in city $i \in I$ in similation $j \in J$

Constraints: $x_j \leq m_j \; \forall i \in I$ <- Avoid sending more than max needed, $x_i \geq 0$ <- Non-negativity, $0 \leq y_{i,\,j} \leq d_{i,\,j}$ <- Non-negativity & Avoid sending more than max needed

Formulation:

$$min \sum_{i \in I} x_i$$

$$min \, \frac{\sum_{i \in I} \sum_{j \in J} y_{i,\,j}}{\sum_{j \in J} 1}$$

subject to:

$$y_{i,\,j} \geq 0 \; \forall i \in I, \; j \in J$$

$$y_{i,\,j} \geq (d_{i,\,j} - x_i) \; \forall i \in I, \; j \in J$$

$$x_i \leq m_i \; \forall i \in I$$

$$y_{i,\,j} \leq d_{i,\,j} \; \forall i \in I, \; j \in J$$

$$x_i \geq 0 \; \forall i \in I$$

APPENDIX C: Handling Divisibility

To account for the equipment only being able to be shipped in multiples of 10, I modified my linear program into a mixed integer program by changing the decision variable x into an integer which represents the tens of equipment being shipped to each city. See MIP below:

Data-Independent MIP:

Indicies and Sets:

$i \in I$ Cities, $j \in J$ Simulations

Data:

m_j Max demand in city $i \in I$, $d_{i,\,j}$ Demand in city $i \in I$ in simulation $j \in J$ z The divisibility of equipment.

Variables: x_i Supply to city $i \in I$ in z - ($z * x_i$ total items shipped), $y_{i,\,j}$ Unmet demand in city $i \in I$ in similation $j \in J$

Constraints: $z * x_j \leq m_j \; \forall i \in I$ <- Avoid sending more than max needed, $x_i \geq 0$ <- Non-negativity, $0 \leq y_{i,\,j} \leq d_{i,\,j}$ <- Non-negativity & Avoid sending more than max needed

Formulation:

$$min \;\; sum_{i \in I} z * x_i$$

$$min \frac{\sum_{i \in I} \sum_{j \in J} y_{i,\,j}}{\sum_{j \in J} 1}$$

subject to:

$$y_{i,\,j} \geq 0 \; \forall i \in I, \; j \in J$$

$$y_{i,\,j} \geq (d_{i,\,j} - x_i) \; \forall i \in I, \; j \in J$$

$$z * x_i \leq m_i \; \forall i \in I$$

$$y_{i,\,j} \leq d_{i,\,j} \; \forall i \in I, \; j \in J$$

$$x_i \geq 0 \; \forall i \in I$$

APPENDIX D: Accounting for Fairness

To make the model fairer, I ensured that all metropolitan areas are shipped at least z medical supplies. By constraining the decision variable x (representing the number of supplies sent to a given city in z units) to be greater than 1, I ensure all cities receive some supplies. See model below.

Data-Independent MIP:

Indicies and Sets:

$i \in I$ Cities, $j \in J$ Simulations

Data:

m_j Max demand in city $i \in I$, $d_{i,\,j}$ Demand in city $i \in I$ in simulation $j \in J$ z The divisibility of equipment.

Variables: x_i Supply to city $i \in I$ in z - ($z * x_i$ total items shipped), $y_{i,\,j}$ Unmet demand in city $i \in I$ in similation $j \in J$

Constraints: $z * x_j \leq m_j \; \forall i \in I$ <- Avoid sending more than max needed, $x_i \geq 1$ <- Non-negativity and ensures all cities are sent at least z medical supplies, $0 \leq y_{i,\,j} \leq d_{i,\,j}$ <- Non-negativity & Avoid sending more than max needed

Formulation:

$$min \;\; sum_{i \in I} z * x_i$$

$$min \; \frac{\sum_{i \in I} \sum_{j \in J} y_{i,\,j}}{\sum_{j \in J} 1}$$

subject to:

$$y_{i,\,j} \geq 0 \; \forall i \in I, \; j \in J$$

$$y_{i,\,j} \geq (d_{i,\,j} - x_i) \; \forall i \in I, \; j \in J$$

$$z * x_i \leq m_i \; \forall i \in I$$

$$y_{i,\,j} \leq d_{i,\,j} \; \forall i \in I, \; j \in J$$

$$x_i \geq 1 \; \forall i \in I$$

APPENDIX E: Rule of Thumb Formulation

To find a simple rule of thumb for how supplies should be distributed, I created a best fit line through my data using a loss function based on the 1-norm. The decision variable m is the slope of the best fit line which represents the number of supplies distributed per person. With some unit conversions, this can be converted to the number of people needed to receive one unit of medical supplies. The linear program formulation is shown below.

Data Independent LP:

Indicies and Sets: $i \in I$ Cities

Data: Pop_i: Population of city $i \in I$, $Shipment_i$: How many supplies should be shipped to city $i \in I$

Variables: m: The slope of the best fit line, $eplus_i$, $eminus_i$

Formulation:

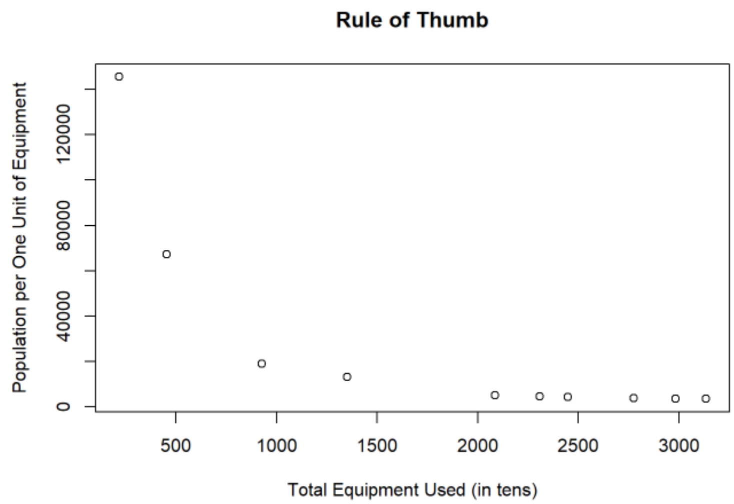
$$min \sum_{i \in I} eplus_i + eminus_i$$

Subject to:

$$eplus_i - eminus_i = Shipment_i - m(Pop_i) \; \forall i \in I$$

$$eplus_i, \; eminus_i \geq 0$$

APPENDIX F: Rule of Thumb Graph



APPENDIX G: Estimating Maximum Demand

Using the average means, I calculated the one-sided 95% confidence interval for each city. In other words, I found the demand value that would be greater than or equal to that of the true demand of the city 95% of the time. This helps to account for variations in simulations by considering the standard deviation. Because there is only a 5% chance of a city needing more than its confidence interval value, I can use these values as the maximum demand of the city, meaning the city shouldn't need more medical supplies.

APPENDIX H: Fair and Divisible AMPL Code .mod

```
# Daniel Barnett
# Deliverable 2
# 6/3/20


set METRO_AREAS;
set SIMS;

param DATA{METRO_AREAS, SIMS};
param MaxDemand{METRO_AREAS};
param lambda;
param z;


var x{i in METRO_AREAS} >= 1, <= (MaxDemand[i]/z)+1, integer;
var y{i in METRO_AREAS, j in SIMS} >= 0;

minimize objective: (lambda)*sum{i in METRO_AREAS}(sum{j in SIMS}(y[i,j]))/(sum{k in
SIMS}(1))+(1-lambda)*(sum{i in METRO_AREAS}(x[i]*z));

subject to constraint_1{i in METRO_AREAS, j in SIMS}: y[i,j] >= DATA[i, j]-x[i]*z;
subject to constraint_2{i in METRO_AREAS, j in SIMS}: y[i,j] <= DATA[i, j];
```

APPENDIX I: Fair and Divisible AMPL Code .dat

Too large to display. Attached to this report as a separate file called project.dat.

APPENDIX J: Fair and Divisible AMPL Code .run

```
# Daniel Barnett
# Deliverable 2
# 6/3/20


reset;
option solver gurobi;
model Deliverable2.mod;
data project.dat;
solve;
display sum{i in METRO_AREAS}(x[i]) > output1.txt;
display sum{i in METRO_AREAS}((MaxDemand[i] - x[i]*10)/MaxDemand[i])/217 >
output1.txt;
display {i in METRO_AREAS}(x[i]) > output1.txt;
#display {i in METRO_AREAS}((MaxDemand[i] - x[i]*10)/MaxDemand[i]) > output1.txt;
```

APPENDIX K: Rule of Thumb AMPL Code .mod

```
# Daniel Barnett
# Deliverable 2 - rule of thumb
# 6/3/20


set METRO_AREAS;

param Pop{METRO_AREAS};
param SHIPMENT{METRO_AREAS};

var m >= 0;
var e_plus{i in METRO_AREAS} >= 0;
var e_minus{i in METRO_AREAS} >= 0;

minimize objective: sum{i in METRO_AREAS}(e_plus[i] + e_minus[i]);

subject to constraint_1{i in METRO_AREAS}: e_plus[i] - e_minus[i] = SHIPMENT[i] - m *
Pop[i];
```

APPENDIX L: Rule of Thumb AMPL Code .dat

Too large to display. Attached to this report as a separate file called Rule_of_thumb.dat.

APPENDIX M: Rule of Thumb AMPL Code .run

```
# Daniel Barnett
# Deliverable 2 - Rule of thumb
# 6/3/20


reset;
option solver gurobi;
model Rule_of_thumb.mod;
data Rule_of_thumb.dat;
solve;
display m > RoT.txt;
display 1/(10*m) > RoT.txt;
display sum{i in METRO_AREAS}(SHIPMENT[i] - m*Pop[i])/217 > RoT.txt;
display {i in METRO_AREAS}(SHIPMENT[i] - m*Pop[i]) > RoT.txt;
display {i in METRO_AREAS}(m*Pop[i]) > RoT.txt;
```