DATA605: Fundamentals of Computational Mathematics

Assignment 9

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Problem 1.

The price of one share of stock in the Pilsdorff Beer Company is given by Y_n on the nth day of the year. Finn observes that the differences $X_n = Y_{n+1} - Y_n$ appear to be independent random variables with a common distribution having mean $\mu = 0$ and variance $\sigma^2 = 1/4$. If $Y_1 = 100$, estimate the probability the $Y_3 65$ is:

The price of a stock on day N is the sum of the previous days differences.

1a. ≥ 100.

```
n <- 364
mu <- 0
var <- 1/4

Sn <- 100 - 100 #Y_365 - Y_1
sd <- sqrt(n * var)

pnorm(Sn, mu, sd, lower.tail = FALSE)
```

```
## [1] 0.5
```

1b. ≥ 110.

```
n <- 364
mu <- 0
var <- 1/4

Sn <- 110 - 100 #Y_365 - Y_1
sd <- sqrt(n * var)

pnorm(Sn, mu, sd, lower.tail = FALSE)
```

```
## [1] 0.1472537
```

1c. ≥ 120.

```r
n <- 364
mu <- 0
var <- 1/4

Sn <- 120 - 100 #Y_365 - Y_1
sd <- sqrt(n * var)

pnorm(Sn, mu, sd, lower.tail = FALSE)
```

```
## [1] 0.01801584
```

Problem 2.

Calculate the expected value and variance of the binomial distribution using the moment generating function.

The moment generating function for the binomial distribution is $g(t) = (pe^t + (1-p))^n$. To calculate the mean and variance, we need the first and second derivatives of $g(t)$.

$$g'(t) = n(pe^t + (1-p))^{n-1}(pe^t)$$

$$g''(t) = n(n-1)(pe^t + (1-p))^{n-2}(pe^t)^2 + n(pe^t + (1-p))^{n-1}(pe^t)$$

$$\begin{aligned}
\mu &= g'(0) \\
&= n(pe^0 + (1-p))^{n-1}(pe^0) \\
&= n(p + (1-p))^{n-1}(p) \\
&= n(1^{n-1})p \\
&= np
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= g''(0) - \mu^2 \\
&= n(n-1)(pe^0 + (1-p))^{n-2}(pe^0)^2 + n(pe^0 + (1-p))^{n-1}(pe^0) - (np)^2 \\
&= (n^2 - n)(p + (1-p))^{n-2}p^2 + n(p + (1-p))^{n-1}p - (np)^2 \\
&= (np)^2 - np^2 + np - (np)^2 \\
&= np - np^2 \\
&= np(1-p)
\end{aligned}$$

Problem 3.

Calculate the expected value and variance of the exponential distribution using the moment generating function.

$$g(t) = \frac{1}{1 - t/\lambda}$$

$$g'(t) = (1/\lambda)(1 - t/\lambda)^{-2}$$

$$g''(t) = (2/\lambda)^2(1 - t/\lambda)^{-3}$$

$$\begin{aligned}\mu &= g'(0)\\ &= 1/\lambda\end{aligned}$$

$$\begin{aligned}\sigma^2 &= g''(0) - \mu^2\\ &= (2/\lambda)^2 - (1/\lambda)^2\\ &= 1/\lambda^2\end{aligned}$$