DATA605: Fundamentals of Computational Mathematics

Discussion 14

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Chapter 8.8 Question 29

Use the Taylor series given in Key Idea 8.8.1 to find the first 4 terms of the Taylor series of the function $f(x) = e^x \sin(x)$.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + ...$$

Multiplying term by term to find coefficients up to the x^4 term.

$$e^x \sin(x) = x - \frac{x^3}{3!} + x^2 - \frac{x^4}{3!} + \frac{x^3}{2!} + \frac{x^4}{3!}$$

Combining like terms we get:

$$e^x \sin(x) = x + x^2 + \frac{2x^3}{3!}$$

Another method for solving this problem would be to use the chain rule and calculate the first 4 derivatives.

$$\begin{aligned}
f(x) &= e^x \sin(x) & \Rightarrow f(0) = 0 \\
f'(x) &= e^x \cos(x) + e^x \sin(x) & \Rightarrow f'(0) = 1 \\
f''(x) &= 2e^x \cos(x) & \Rightarrow f''(0) = 2 \\
f'''(x) &= -2e^x \sin(x) + 2e^x \cos(x) & \Rightarrow f'''(0) = 2 \\
f''''(x) &= -4e^x \sin(x) & \Rightarrow f''''(0) = 0
\end{aligned}$$

This gives us:

$$e^x \sin(x) = \frac{0}{0!} + \frac{1}{1!}x + \frac{2}{2!}x^2 + \frac{2}{3!}x^3 + \frac{0}{4!}x^4 = x + x^2 + \frac{2x^3}{3!}$$