DATA605: Fundamentals of Computational Mathematics

Assignment 15

Donald Butler

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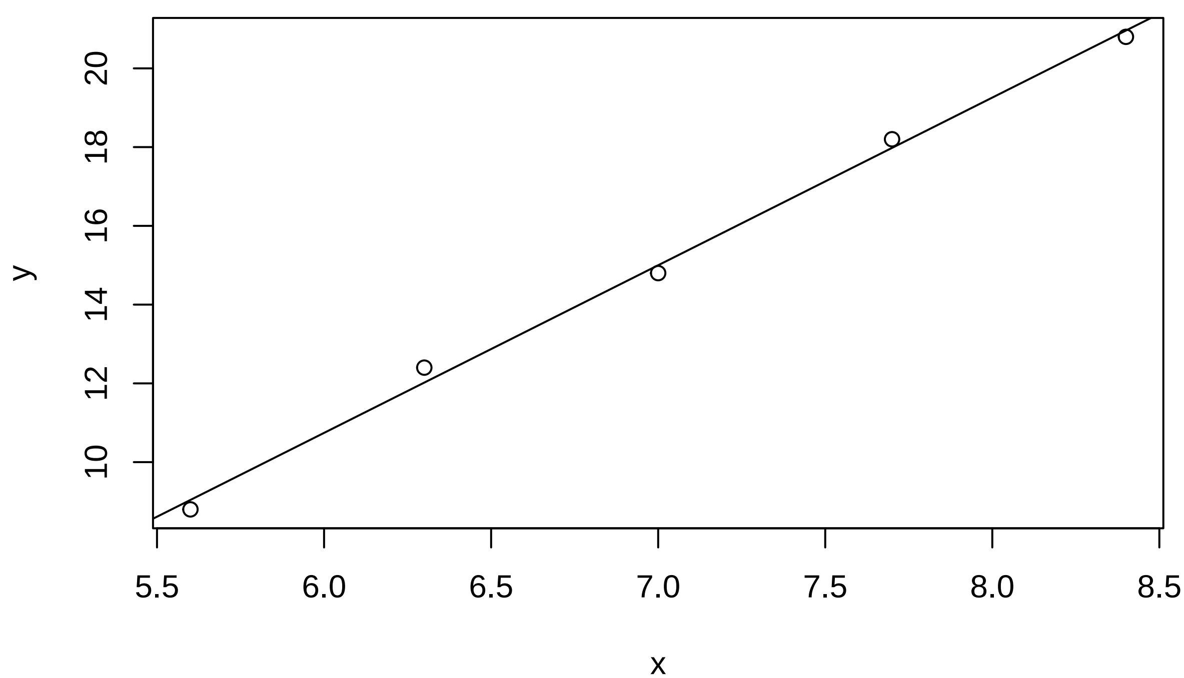
Problem 1

Find the equation of the regression line for the given points.

$$(5.6, 8.8), (6.3, 12.4), (7, 14.8), (7.7, 18.2), (8.4, 20.8)$$

```
df <- data.frame(x = c(5.6,6.3,7,7.7,8.4), y = c(8.8,12.4,14.8,18.2,20.8))
df.lm = lm(formula = y ~ x, data = df)
plot(df)
abline(df.lm)
```



```r
summary(df.lm)
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Residuals:
##     1     2     3     4     5
## -0.24  0.38 -0.20  0.22 -0.16
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -14.8000     1.0365  -14.28 0.000744 ***
## x             4.2571     0.1466   29.04 8.97e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3246 on 3 degrees of freedom
## Multiple R-squared:  0.9965, Adjusted R-squared:  0.9953
## F-statistic: 843.1 on 1 and 3 DF,  p-value: 8.971e-05
```

The equation for the regression line is: $y = 4.257x - 14.8$.

Problem 2

Find all local maxima, local minima, and saddle points for the function given below.

$$f(x, y) = 24x - 6xy^2 - 8y^3$$

Use first partial derivatives to identify critical points.

$$f_x = 24 - 6y^2$$

Evaluate $f_x = 0$.

$$\begin{aligned} 0 &= 24 - 6y^2 \\ 6y^2 &= 24 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

$$f_y = -12xy - 24y^2$$

Evaluate $f_y = 0$.

$$\begin{aligned} 0 &= -12xy - 24y^2 \\ 12xy &= -24y^2 \\ x &= -2y \end{aligned}$$

This gives the critical points, $(4, -2, 64)$ and $(-4, 2, -64)$.

Use the second derivative test to determine the type of critical points.

$$
\begin{aligned}
f_{xx} &= 0 \\
f_{xy} &= -12y \\
f_{yy} &= -12x - 48y
\end{aligned}
$$

Evaluate $D = f_{xx}(4,-2)f_{yy}(4,-2) - f_{xy}^2(4,-2)$ and $D = f_{xx}(-4,2)f_{yy}(-4,2) - f_{xy}^2(-4,2)$.

$$
\begin{aligned}
D &= f_{xx}(4,-2)f_{yy}(4,-2) - f_{xy}^2(4,-2) \\
&= 0 * (-48 + 96) - (24^2) \\
&= -(24^2) \\
&< 0
\end{aligned}
$$

The first critical point $(4, -2, 64)$ is a saddle point.

Evaluate $D = f_{xx}(-4,2)f_{yy}(-4,2) - f_{xy}^2(-4,2)$.

$$
\begin{aligned}
D &= f_{xx}(-4,2)f_{yy}(-4,2) - f_{xy}^2(-4,2) \\
&= 0 * (48 - 96) - ((-24)^2) \\
&= -(24^2) \\
&< 0
\end{aligned}
$$

The second critical point $(-4, 2, -64)$ is also saddle point.

Problem 3

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Part 1

Find the revenue function $R(x, y)$.

$$
\begin{aligned}
R(x,y) &= x(81 - 21x + 17y) + y(40 + 11x - 23y) \\
&= 81x - 21x^2 + 17xy + 40y + 11xy - 23y^2 \\
&= 81x - 21x^2 + 28xy + 40y - 23y^2
\end{aligned}
$$

Part 2

What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10?

```r
R <- function(x,y){81*x - 21*x^2 + 28*x*y + 40*y - 23*y^2}
R(2.30,4.10)
```

```
## [1] 116.62
```

Problem 4

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

Since we need to produce 96 units, $x + y = 96$. Substitute $y = 96 - x$ in $C(x, y)$ and evaluate in terms of just x.

$$\begin{aligned} C(x) &= \frac{1}{6}x^2 + \frac{1}{6}(96 - x)^2 + 7x + 25(96 - x) + 700 \\ &= \frac{1}{6}x^2 + \frac{1}{6}(9216 - 192x + x^2) + 7x + 2400 - 25x + 700 \\ &= \frac{1}{3}x^2 - 50x + 4636 \end{aligned}$$

Evaluate the first derivative to find the critical points.

$$\begin{aligned} C'(x) &= \frac{2}{3}x - 50 \\ 0 &= \frac{2}{3}x - 50 \\ 50 &= \frac{2}{3}x \\ 75 &= x \end{aligned}$$

Since $x = 75$, $y = 96 - 75 = 21$. Produce 75 units in Los Angeles and 21 units in Denver.

Problem 5

Evaluate the double integral on the given region.

$$\iint_R (e^{8x+3y})dA; R : 2 \leq x \leq 4 \text{ and } 2 \leq y \leq 4$$

$$\begin{aligned} \int_2^4 \int_2^4 e^{8x+3y} dx dy &= \int_2^4 \int_2^4 e^{8x} e^{3y} dx dy \\ &= \int_2^4 e^{8x} dx * \int_2^4 e^{3y} dy \\ &= \frac{e^{8x}}{8}\bigg|_2^4 * \frac{e^{3y}}{3}\bigg|_2^4 \\ &= \frac{e^{32} - e^{16}}{8} * \frac{e^{12} - e^6}{3} \\ &= \frac{1}{24}(e^{44} - e^{38} - e^{28} + e^{22}) \end{aligned}$$