Let me try to clear up how to get the eigenvectors from the eigenvalues.

We first started with the equation $Cx = \lambda x$ and you performed operations to rewrite that equation into the form $(C - \lambda I)x = 0$. Now that you've found the eigenvalues, we need to find the corresponding eigenvectors that satisfy the equation. Rewriting the equation in vector components we get the following:

$$\begin{bmatrix} -1-\lambda & 2 \\ -6 & 6-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

If we substitute your first eigenvalue of 2 into the equation, we get:

$$\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

If we multiply this out we get two equations with two unknowns, $-3v_1 + 2v_2 = 0$ and $-6v_1 + 4v_2 = 0$. But this isn't very helpful, so we put the matrix in reduced row echelon form. You did this with the row operation $-2R_1 + R_2$, which corresponds to the transformation matrix $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$. So we apply this transformation to both sides of the equation above and we get this:

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

You then completed another transformation $-\frac{1}{3}R_1$, which corresponds to the transformation matrix $\begin{bmatrix} -\frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$. So again, we apply the transformation matrix to both sides of the equation above and we get this:

$$\begin{bmatrix} -\frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

If we multiply this equation out, we have two equations, $v_1 - \frac{2}{3}v_2 = 0$ and $0v_1 + 0v_2 = 0$, but the second equation no longer provides any information to the system, so we can rewrite the first one to $v_1 = \frac{2}{3}v_2$ and this describes the relationship of the two components that make up the vector. So the eigenspace corresponding to $\lambda = 2$ is the set of vectors of the form $\begin{bmatrix} \frac{2}{3}v_2 \\ v_2 \end{bmatrix}$. Since the span of the vector is all of the scalar multiples, we can factor out the fraction and write it as:

$$\xi_A(2) = \left\langle \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\rangle$$

You can perform the same analysis with the second eigenvalue.