DATA605: Fundamentals of Computational Mathematics

Assignment 7

Donald Butler

03/12/2022

Problem 1.

Let X_1, X_2, ..., X_n be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the X_is. Find the distribution of Y.

The distribution of Y is discrete on the interval $[1, k]$ and can be calculated by finding the probability that the minimum value of the X_is each value. There are k^n ways of selecting the X_i values. For $1 \leq j \leq k$, the number of ways that each X_i can be selected so that the minimum value is j, is the number of ways that $X_i \geq j$ minus the ways that $X_i > j$. So, the distribution of Y is:

$$\frac{(k-j+1)^n - (k-j)^n}{k^n}, 1 \leq j \leq k$$

Problem 2.

Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years.

Part a. (geometric)

What is the probability that the machine will fail after 8 years?

```r
p <- .1

(prob_fail_after_8 <- (1-p)^8)
```

```
## [1] 0.4304672
```

```r
(ev <- 1/p)
```

```
## [1] 10
```

```r
(sd <- sqrt((1-p)/p^2))
```

```
## [1] 9.486833
```

Part b. (exponential)

What is the probability that the machine will fail after 8 years?

```
p <- .1

(prob_fail_after_8 <- 1 - pexp(8,p))
```

```
## [1] 0.449329
```

```
(ev <- 1/p)
```

```
## [1] 10
```

```
(sd <- sqrt(1/p^2))
```

```
## [1] 10
```

Part c. (binomial)

What is the probability that the machine will fail after 8 years?

```
p <- .1

(prob_fail_after_8 <- dbinom(0,8,p))
```

```
## [1] 0.4304672
```

```
(ev <- 8 * p)
```

```
## [1] 0.8
```

```
(sd <- sqrt(8 * p * (1-p)))
```

```
## [1] 0.8485281
```

Part d. (poisson)

What is the probability that the machine will fail after 8 years?

```
p <- .1

(prob_fail_after_8 <- dpois(0,8 * p))
```

```
## [1] 0.449329
```

```r
(ev <- 8 * p)
```

```
## [1] 0.8
```

```r
(sd <- sqrt(8 * p))
```

```
## [1] 0.8944272
```