DATA605: Fundamentals of Computational Mathematics

Assignment 13

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05/07/2022

Problem 1

Use integration by substitution to solve the integral, $\int 4e^{-7x}dx$.

Let $u = -7x$ and $du = -7dx$.

$$\begin{aligned}
\int 4e^{-7x}dx &= \int \frac{4}{-7}e^{-7x}(-7dx) \\
&= \frac{-4}{7}\int e^{u}du \\
&= \frac{-4}{7}e^{u} + C \\
&= \frac{-4}{7}e^{-7x} + C
\end{aligned}$$

Problem 2

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = \frac{-3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

$$\begin{aligned}
\frac{dN}{dt} &= \frac{-3150}{t^4} - 220 \\
dN &= (\frac{-3150}{t^4} - 220)dt \\
\int dN &= \int (\frac{-3150}{t^4} - 220)dt \\
N(t) &= \int (\frac{-3150}{t^4} - 220)dt \\
&= \frac{-3150}{-3t^3} - 220t + C \\
&= \frac{1050}{t^3} - 220t + C
\end{aligned}$$

Use the initial condition to find the value C.

$$\begin{aligned}
N(1) &= \frac{1050}{1^3} - 220(1) + C \\
6530 &= 1050 - 220 + C \\
5700 &= C
\end{aligned}$$

So, $N(t) = \frac{1050}{t^3} - 220t + 5700$.

Problem 3

Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$.

$\sum_{x=5}^{8}(1)(2x-9) = 1+3+5+7 = 16$

Problem 4

Find the area of the region bounded by the graphs of the given equations.

$$y = x^2 - 2x - 2, y = x + 2$$

Find the values of x where these equations intersect.

$$\begin{aligned} x+2 &= x^2 - 2x - 2 \\ 0 &= x^2 - 3x - 4 \\ 0 &= (x-4)(x+1) \end{aligned}$$

Calculate the definite integral on the interval $(-1, 4)$.

$$\begin{aligned} \int_{-1}^{4} (x+2) - (x^2 - 2x - 2)dx &= \int_{-1}^{4} -x^2 + 3x + 4dx \\ &= \frac{-x^3}{3} + \frac{3x^2}{2} + 4x\Big|_{-1}^{4} \\ &= \frac{-64}{3} + \frac{48}{2} + 16 - \frac{1}{3} - \frac{3}{2} + 4 \\ &= 20\frac{5}{6} \end{aligned}$$

Problem 5

A beauty supply store expects to sell 110 flat irons during the next year. It costs \$3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

Inventory Cost = Holding Cost + Ordering Cost

Assumptions

1. Sales are made at a relatively uniform rate. This allows us to assume that we are on average holding half the lot size over the year.
2. Lot size, x, of each reorder is the same.
3. As inventory falls to 0, another order arrives immediately.

$$f(x) = 3.75\frac{x}{2} + 8.25\frac{110}{x}$$

To find the minimum value of $f(x)$, we find the value of x to satisfy $f'(x) = 0$.

$$
\begin{aligned}
f'(x) &= 1.875 - 907.5x^{-2} \\
0 &= 1.875 - 907.5x^{-2} \\
907.5x^{-2} &= 1.875 \\
\frac{907.5}{1.875} &= x^2 \\
484 &= x^2 \\
22 &= x
\end{aligned}
$$

With a lot size of 22, we can minimize inventory costs by making 5 orders per year.

Problem 6

Use integration by parts to solve the integral below.

$$\int \ln(9x)x^6 dx$$

Integration by parts: $\int udv = uv - \int vdu$.

Let $u = ln(9x)$ and $dv = x^6dx$. Then $du = \frac{1}{x}dx$ and $v = \frac{x^7}{7}$.

$$
\begin{aligned}
\int \ln(9x)x^6 dx &= \int udv \\
&= uv - \int vdu \\
&= \frac{ln(9x)x^7}{7} - \int \frac{x^7}{7} * \frac{1}{x}dx \\
&= \frac{ln(9x)x^7}{7} - \int \frac{x^6}{7}dx \\
&= \frac{ln(9x)x^7}{7} - \frac{x^7}{49} + C
\end{aligned}
$$

Problem 7

Determine whether $f(x)$ is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral.

$$f(x) = \frac{1}{6x}$$

A probability density function must be greater than or equal to 0 over the interval and the integral over the interval must be equal to 1. The function $f(x)$ is strictly greater than 0 on the interval $[1, e^6]$, which satisfies the first requirement.

$$
\begin{aligned}
\int_1^{e^6} f(x)dx &= \int_1^{e^6} \frac{1}{6x}dx \\
&= \frac{1}{6}\int_1^{e^6} \frac{1}{x}dx \\
&= \frac{1}{6}(\ln(e^6) - \ln(1)) \\
&= \frac{1}{6}(6 - 0) \\
&= 1
\end{aligned}
$$

Since the integral on the interval is 1, the function is a probability density function.