University of Toronto
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Part 2: Functional Dependencies, Decompositions, Normal Forms

1. Relation R with attributes $LMNOPQRS$ and FDs $S = \{L \rightarrow NO, M \rightarrow P, N \rightarrow MQR, O \rightarrow S\}$

   (a) **FDs that Violate BCNF**
   Check LHS of each FDs whether it is a superkey or not.
   - ✓ $L^+ = LMNOPQRS$, so L is a superkey and $L \rightarrow NO$ does not violate BCNF.
   - ✗ $M^+ = MP$, so $M \rightarrow P$ violates BCNF.
   - ✗ $N^+ = MNPQR$, so $N \rightarrow MQR$ violates BCNF.
   - ✗ $O^+ = OS$, so $O \rightarrow S$ also violates BCNF.

   (b) **Employ BCNF Decomposition**
   - Decompose R using FD $N \rightarrow MQR$. $N^+ = MNPQR$, so this yields two relations: $R_1 = MNPQR$ and $R_2 = LNOS$.
   - Project the FDs onto $R_1 = MNPQR$

M	N	P	Q	R	Closure	FDs
✓					$M^+ = MP$	$M \rightarrow P$: violates BCNF

   - Hence we decompose R_1 further using FD $M \rightarrow P$, this yields two relations: $R_{11} = MP$ and $R_{12} = MNQR$.
   - Project FDs onto $R_{11} = MP$

M	P	Closure	FDs
✓		$M^+ = MP$	$M \rightarrow P$: M is a superkey of R_{11}
	✓	$P^+ = P$	nothing

   This relation satisfies BCNF.
   - Project FDs on $R_{12} = MNQR$.

M	N	Q	R	Closure	FDs
✓				$M^+ = MP$	nothing
	✓			$N^+ = MNPQR$	$N \rightarrow MQR : N$ is superkey of R_{12}
		✓		$Q^+ = Q$	nothing
			✓	$R^+ = R$	nothing
supersets of N				irrelevant	can only generate weaker FDs
✓		✓		$MQ^+ = MPQ$	nothing
✓			✓	$MR^+ = MPR$	nothing
		✓	✓	$QR^+ = QR$	nothing
✓		✓	✓	$MQR^+ = MQRP$	nothing

   This relation satisfies BCNF.
   - BCNF decomposition on R_1 get $R_{11} = MP$ and $R_{12} = MNQR$. Return to $R_2 = LNOS$ and project FDs on it.

L	N	O	S	Closure	FDs
✓				$L^+ = LMNOPQRS$	$L \rightarrow NOS : N$ is superkey of R_2
	✓			$N^+ = MNPQR$	nothing
		✓		$O^+ = OS$	$O \rightarrow S$: violates BCNF

- Decompose R_2 further using FD $O \to S$ which yields two relations: $R_{21} = OS$ and $R_{22} = LNO$.

- Project FDs on $R_{21} = OS$

O	S	Closure	FDs
✓		$O^+ = OS$	$O \to S$: O is a superkey of R_{21}
	✓	$S^+ = S$	nothing

This relation satisfies BCNF.

- Project FDs on $R_{22} = LNO$.

L	N	O	Closure	FDs
✓			$L^+ = LMNOPQRS$	$L \to NO : L$ is superkey of R_{22}
	✓		$N^+ = MNPQR$	nothing
		✓	$O^+ = OS$	nothing
supersets of L			irrelevant	can only generate weaker FDs
	✓	✓	$NO^+ = MNOPQRS$	nothing

This relation satisfies BCNF.

- Hence the final decomposition is:

(a) $R_{22} = LNO$ with FD $L \to NO$.

(b) $R_{12} = MNQR$ with FD $N \to MQR$.

(c) $R_{11} = MP$ with FD $M \to P$.

(d) $R_{21} = OS$ with FD $O \to S$.

(c) **Dependency Preservation**

Yes, the schema preserves dependencies. For each of the original FDs in set S, there is a relation that includes all of the attributes. This ensures that they are preserved.

(d) **Chase Test to show Lossless-join**

The Chase Test demonstrates that it is a lossless-joint decomposition.
We start with:

L	**M**	**N**	**O**	**P**	**Q**	**R**	**S**
0	**m**	1	2	**p**	3	4	5
6	**m**	**n**	7	8	**q**	**r**	9
10	11	12	**o**	13	14	15	**s**
l	16	**n**	**o**	17	18	19	20

Then because of FDs $M \to P, N \to MQR, O \to S$, we make changes:

L	**M**	**N**	**O**	**P**	**Q**	**R**	**S**
0	**m**	1	2	**p**	3	4	5
6	**m**	**n**	7	~~8~~ **p**	**q**	**r**	9
10	11	12	**o**	13	14	15	**s**
l	~~16~~ **m**	**n**	**o**	~~17~~ **p**	~~18~~ **q**	~~19~~ **r**	~~20~~ **s**

We observe the tuple $< l, m, n, o, p, q, r, s >$ does occur. The Chase Test has succeeded.

2. Relation A with attributes $ABCDEFGH$ and FDs $B = \{ACD \rightarrow E, B \rightarrow CD, BE \rightarrow ACF, D \rightarrow AB, E \rightarrow AC\}$

   (a) **Minimal Basis**
   Step1: simplify FDs to singleton right-hand sides.

   1 $ACD \rightarrow E$
   2 $B \rightarrow C$
   3 $B \rightarrow D$
   4 $BE \rightarrow A$
   5 $BE \rightarrow C$
   6 $BE \rightarrow F$
   7 $D \rightarrow A$
   8 $D \rightarrow B$
   9 $E \rightarrow A$
   10 $E \rightarrow C$

   Step2: try reducing the LHS of FDs with multiple attributes on the LHS. Thus we only need to consider FDs 1, 4, 5, 6.

   1 The closure of A is $A^+ = A$, the closure of C is $C^+ = C$, the closure of D is $D^+ = DABCEF$, hence we reduce the LHS to D.
   4 The closure of B is $B^+ = BCDAEF = ABCDEF$, hence we reduce the LHS to B.
   5 The closure of B is $B^+ = BCDAEF = ABCDEF$, hence we reduce the LHS to B.
   6 The closure of B is $B^+ = BCDAEF = ABCDEF$, hence we reduce the LHS to B.

   The reduced FDs are:

   1 $D \rightarrow E$
   2 $B \rightarrow C$
   3 $B \rightarrow D$
   4 $B \rightarrow A$
   5 $B \rightarrow C$
   6 $B \rightarrow F$
   7 $D \rightarrow A$
   8 $D \rightarrow B$
   9 $E \rightarrow A$
   10 $E \rightarrow C$

   Step3: eliminate redundant FDs.

Eliminate FDs			
FD	Exclusions	Closure	Decision
1	1	There's no way to get E without this FD	keep
2	2	Duplicated FD to (5)	discard
3	2, 3	There's no way to get D without this FD	keep
4	2, 4	$B^+ = BDCAFE = ABCDEF$	discard
5	2, 4, 5	$B^+ = BDAFEC = ABCDEF$	discard
6	2, 4, 5, 6	There's no way to get F without this FD	keep
7	2, 4, 5, 7	$D^+ = DBFEAC = ABCDEF$	discard
8	2, 4, 5, 7, 8	There's no way to get B without this FD	keep
9	2, 4, 5, 7, 9	There's no way to get A without this FD	keep
10	2, 4, 5, 7, 10	There's no way to get C without this FD	keep

   So the following set is a minimal basis

1 $B \to D$
2 $B \to F$
3 $D \to B$
4 $D \to E$
5 $E \to A$
6 $E \to C$

(b) **All Keys**
Notice that GH does not appear anywhere is the FDs, hence they are in every keys of the relation. Notice that ACF appears only on the RHS of the FDs, hence they are not in any keys of the relation. Now we only need to check all combinations of BDE, and for each we add GH.

- The closure of BGH is $GHBDFEAC = ABCDEFGH$, hence BGH is a key for the relation.
- The closure of DGH is $GHDBFEAC = ABCDEFGH$, hence DGH is a key for the relation.
- The closure of EGH is $GHEAC = ACEGH$, hence EGH is not a key for the relation.
- All other possibilities include BGH or DGH.

Hence BGH, DGH are all the keys for the relation A.

(c) **Employ 3NF Synthesis**
I first combine the right hand sides to obtain the following FDs:

1 $B \to DF$
2 $D \to BE$
3 $E \to AC$

So the 3NF synthesis algorithm generates the following relations:

$$R1(B, D, E), \quad R2(B, D, F), \quad R3(A, C, E)$$

However, notice that no relation above is a superkey, so we need to add a new relation that is a key. From part(b), I got BGH is a key, hence the final set of relations are

$$R1(B, D, E), \quad R2(B, D, F), \quad R3(A, C, E), \quad R4(B, G, H)$$

(d) **Redundancy**

Project to BDE $B^+ = ABCDEF$ with FD $B \to DE$, $D^+ = ABCDEF$ with FD $D \to BE$, $E^+ = ACE$ without FD. Notice B and D are superkeys for this relation, hence we do not need to check their supersets, so we do not need to check any other sets. Since the FDs are $B \to DE$ and $D \to BE$ and both LHS are superkeys, this relation satisfies BCNF.

Project to BDF $B^+ = ABCDEF$ with FD $B \to DF$, $D^+ = ABCDEF$ with FD $D \to BF$, $F^+ = F$ without FD. Notice B and D are superkeys for this relation, hence we do not need to check their supersets, so we do not need to check any other sets. Since the FDs are $B \to DF$ and $D \to BF$ and both LHS are superkeys, this relation satisfies BCNF.

Project to ACE $A^+ = A$ without FDs, $C^+ = C$ without FDs, $E^+ = ACE$ with FD $E \to AC$. Notice E is a superkey for this relation hence we do not need to check it supersets. $AC^+ = AC$ without FDs. Since the FD is $E \to AC$ and its LHS is a superkey, this relation satisfies BCNF.

Project to BGH $B^+ = ABCDEF$ without FD, $G^+ = G$ without FD, $H^+ = H$ without FD, hence this relation satisfies BCNF.

Since all relations satisfies BCNF, we know our schema does not allow redundancies.