Particle Physics School

5-Oct

- QFT
- QED
- QCD
- Symmetry Breaking
- $SU(2)_L \otimes U(1)_Y$

QFT: Best way to unify QM and SR

- QFT allows the non conservation of Particle number
- Normalization. (Renormalization).

SR: Lagrangians invariant under LT.

Remember that: $x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu \qquad \Lambda^\mu{}_\nu \in SO^+(1,3)$

$$\eta_{\mu\nu} x^\mu x^\nu = \eta_{\mu\nu} x'^\mu x'^\nu$$

- Scalar field $\phi(x)$ is defined as a function that transforms like:

$$\phi(x) \to \phi'(x) = \phi(\Lambda^{-1} x')$$

- Vector field: $A^\mu(x) \to A'^\mu(x') = \Lambda^\mu{}_\nu A^\nu(\Lambda^{-1} x')$

- $\Lambda^\mu{}_\nu = \exp\left[-\frac{i}{2} \Omega_{\alpha\beta} M^{\alpha\beta}\right]^\mu{}_\nu$

$\Rightarrow$ $\psi_\alpha(x) \to \psi'_\alpha(x') = S[\Lambda]_{\alpha\beta} \psi_\beta(\Lambda^{-1} x')$

$$S[\Lambda] = \exp\left[-\frac{i}{2} \Omega_{\mu\nu} S^{\mu\nu}\right]$$

$$S^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$S = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \frac{1}{2} m^2 \phi^2\right]$$

Klein Gordon

$$S = \int d^4x\, \bar{\Psi}(x)\,(i\not{\partial} - m)\,\psi(x)$$ Dirac.

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu}\right)$$ Maxwell.

QM Locality: $[\,O_1(x), O_2(y)\,] = 0 \qquad (x-y)^2 < 0$

- Propagators:

Momentum Conservation.

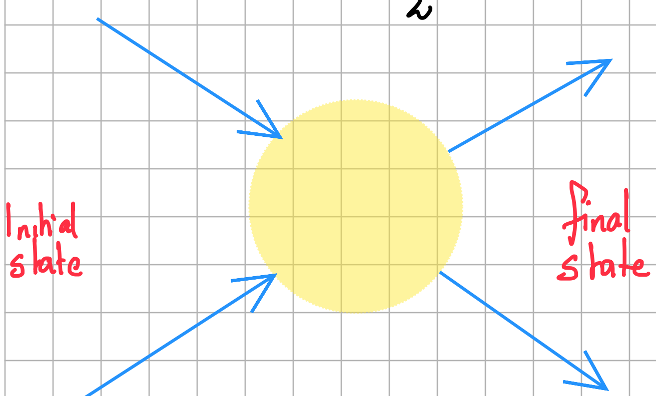
$$\langle 0 | T[\phi(x)\phi(y)] | 0 \rangle = D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon}\, \overbrace{e^{ip.(x-y)}}$$

Feynman propagator is the Green function of the three operator:

$$(\partial_x^2 + m^2)\, D_F = \delta(x-y).$$

Interacting Field:

Yukawa: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 + \bar{\Psi}(i\not{\partial} - m)\psi - g_Y \bar{\Psi}\psi\phi$



QED

- Describes the interaction between fermions and massless vectors
- $U(1)$
- One generator:

$$U(x) = e^{-iqQ\lambda(x)}$$

For electron: $U(x) = e^{ieQ\lambda(x)}$ $\qquad \lambda(x)$: Parameter characterizing the transformation.

$$\to \quad \psi(x) \Rightarrow \psi'(x) = e^{ieQ\lambda(x)}\psi(x)$$

$$\bar{\psi}(x) \to \bar{\psi}'(x) = \bar{\psi}(x)\, e^{-ieQ\lambda(x)}.$$

Local :

$$i\bar{\psi}\not{\partial}\psi \to i\bar{\psi}\not{\partial}\psi + e\,[\bar{\psi}\gamma^{\mu}\psi]\,\partial_{\mu}\lambda(x)$$

Kinetic term is not invariant.

Let us add photons:

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e\,(\bar{\psi}\gamma^{\mu}\psi)A_{\mu}.$$

$$\Rightarrow \mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad \text{where } D_{\mu} = \partial_{\mu} - ieA_{\mu}.$$

Gauge Symmetry and QCD:

Going from $U(1)$ (non-abelian) to $SU(N)$ symmetries.

Invariance under such a transformation is the basis of Yang-Mills theories.

$\to$ $SU(2)$

$$\psi \to U\psi = \exp\left[i\frac{\vec{\tau}}{2}\cdot\vec{a}\right]\begin{pmatrix}\psi_1(x)\\ \psi_2(x)\end{pmatrix}$$

$\vec{\tau}$ Parameter Generators

$$\left[\frac{\tau^{\alpha}}{2}, \frac{T^{\beta}}{2}\right] = i\varepsilon^{\alpha\beta\gamma}\frac{T^{\gamma}}{2}$$

• Define: $D_{\mu} \equiv \partial_{\mu} + ig\frac{\tau^{\alpha}}{2}A_{\mu}^{a}$

To keep the lagrangian gauge invariant: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\mu A^a_\mu + g\,\epsilon^{abc} A^b_\mu A^c_\nu$

- $A^{\pm}_\mu = \frac{1}{\sqrt{2}} \left(A^1_\mu \mp i A^2_\mu \right.$

Symmetry Breaking:

Consider a scalar doublet

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\phi \rightarrow \exp[-i\theta J]\,\phi \qquad J = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\rightarrow \mathcal{L} = \frac{1}{2} \partial_\mu \phi^\dagger \partial^\mu \phi \underbrace{- \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4}_{-V(\phi)} \qquad \phi^2 \equiv \phi^T \phi$$

- Massless vectors have two polarizations : Two transverse modes

Parity :

$$x^\mu \longrightarrow (x^0, -\vec{x}) = x_\mu .$$

Scalars and Vectors :

$$\hat{P}^\dagger \phi(x^\mu)\hat{P} = \eta_\phi\, \phi(x_\mu) \qquad \hat{P}^\dagger V^\mu(x^\mu)\hat{P} = \eta_v V_\mu(x_\mu)$$

Intrinsic parities ± 1.

For fermions : $\psi(x^\mu) \rightarrow \eta_\psi \gamma^0 \psi(x_\mu)$.

$$\underbrace{\hat{P}^\dagger(\bar{\psi}\psi)\hat{P} = \bar{\psi}\psi}_{\text{scalar}} \qquad \underbrace{\hat{P}^\dagger(\bar{\psi}\gamma^5\psi)P = -\bar{\psi}\gamma^5\psi}_{\text{Pseudo-scalar}}.$$

The objects:

$$P_L = \frac{1}{2}(1-\gamma^5) \qquad P_R = \frac{1}{2}(1+\gamma^5)$$

We can construct chiral states for fermions $\psi_L = P_L \psi$, $\psi_R = P_R \psi$.
Irreps of Lorentz Group.

In QED $\frac{1}{2}\bar{\psi}\gamma^\mu(1-\gamma^5)\psi W_\mu \to \bar{\psi}_L \gamma^\mu \psi_L W_\mu$

$$e(\bar{\psi}\gamma^\mu \psi)A_\mu = e(\bar{\psi}_L \gamma^\mu \psi_L)A_\mu + e(\bar{\psi}_R \gamma^\mu \psi_R)A_\mu$$

$\Rightarrow$ Neutrinos (Left)
Anti-neutrinos (Right).

STANDARD MODEL:

Electrons and Neutrinos having V-A with the two charged massive mediators of the weak interaction.

$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ Complex fields $SU(2) \Rightarrow$ 3 Generators.

$\Rightarrow e_R$ as a singlet of $SU(2)$. The model has not right handed neutrinos.

Then:
$$\mathcal{L} = \bar{L}(i\not{D})L + \bar{e}_R(i\not{\partial})e_R - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}$$

$$D_\mu = \partial_\mu + ig\frac{T^a}{2}W^a_\mu$$

Consider a scalar doublet: $\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$

$$\phi = \frac{1}{\sqrt{2}}\exp\left[i\frac{\tau^\alpha}{2}\frac{\eta^\alpha}{v}\right]\begin{pmatrix} 0 \\ v+h^0 \end{pmatrix}$$