IE 539 Convex Optimization Assignment 5

Fall 2023

Out: 30th November 2023

Due: 10th December 2023 at 11:59pm

Instructions

- Submit a PDF document with your solutions through the assignment portal on KLMS by the due date. Please ensure that your name and student ID are on the front page.
- Late assignments will be subject to a penalty. Special consideration should be applied for in this case.
- It is **required** that you typeset your solutions in LaTeX. Handwritten solutions will not be accepted.
- Spend some time ensuring your arguments are **coherent** and your solutions **clearly** communicate your ideas.

Question:	1	2	Total
Points:	50	50	100

1. (50 points) Let $f(x) = \|x\|_1 : \mathbb{R}^d \to \mathbb{R}$. Prove that the Moreau-Yosida smoothing of f is given by

$$f_\eta(x) = \sum_{i=1}^{d} \frac{1}{\eta} L_\eta(x_i)$$

where

$$L_\eta(c) = \begin{cases} \eta|c| - \eta^2/2, & \text{if } |c| \geq \eta, \\ |c|^2/2, & \text{if } |c| \leq \eta. \end{cases}$$

2. Consider

$$h(\mu) = f^*(-A^\top \mu) + g^*(\mu)$$

for some convex functions f and g.

(a) (20 points) Prove that

$$\mu_{t+1} = \mu_t - \eta_t g_t \quad \text{where } g_t \in \partial h(\mu_t)$$

if and only if

$$\mu_{t+1} = \mu_t + \eta(Ax_t - y_t)$$

for some

$$x_t \in \arg\min_x \left\{ f(x) + \mu_t^\top Ax \right\},$$
$$y_t \in \arg\min_y \left\{ g(y) - \mu_t^\top y \right\}.$$

(b) (20 points) Prove that

$$\mu_{t+1} = \text{prox}_{\eta h}(\mu_t)$$

if and only if

$$\mu_{t+1} = \mu_t + \eta(Ax_t - y_t)$$

for some

$$(x_t, y_t) \in \arg\min_{(x,y)} \left\{ f(x) + g(y) + \mu_t^\top (Ax - y) + \frac{\eta}{2}\|Ax - y\|_2^2 \right\}.$$

(c) (10 points) Prove that $\text{prox}_{\eta h}(\mu) = \mu - \eta \nabla h_\eta(\mu)$.