$\underline{n=1}$

$$\max_{\|x\|\le 1} |x^T D^2 g(u)\, x| = \max_{\substack{x=\begin{pmatrix} x_1 \\ \vdots \\ x_m\end{pmatrix} \\ \sum_j \|x_j\|^2 \le 1}} \frac{1}{m} \Big| \sum_j x_j^T D^2\phi(u_j)\, x_j \Big|$$

$$\le \max_{\sum_j \|x_j\|^2 \le 1} \frac{1}{m} \sum_j \underbrace{|x_j^T D^2\phi(u_j)\, x_j|}_{\le \|D^2\phi(u_j)\|_{op} \|x_j\|^2}$$

$$\le \max_{\sum_j \|x_j\|^2\le 1} \frac{1}{m} \sum_{j=1}^{m} \|D^2\phi(u_j)\|_{op} \|x_j\|^2 = \max_j \|D^2\phi(u_j)\|_{op}$$

$$f(u,x) = \begin{bmatrix} f(u,x_1) \\ \vdots \\ f(u,x_n)\end{bmatrix} \in \mathbb{R}^n$$

valeurs pour x_n = n-ième donnée

$\underline{n \ge 2}$:

$$\max_{\|x\|^2\le 1} \| D^2 g(u).(x,x) \|$$

$$\le \max_j \max_{\|x_j\|^2\le 1} \frac{1}{m} \| D^2\phi(u_j).(x_j,x_j)\|$$

$g : \mathbb{R}^{dm} \to \mathbb{R}^n$

$Dg(u) : \mathcal{L}(\mathbb{R}^{dm} \to \mathbb{R}^n) \simeq \mathbb{R}^{dm\times n}$

$D^2 g(u) : \mathcal{L}_2(\mathbb{R}^{dm} \times \mathbb{R}^{dm} \to \mathbb{R}^n)$