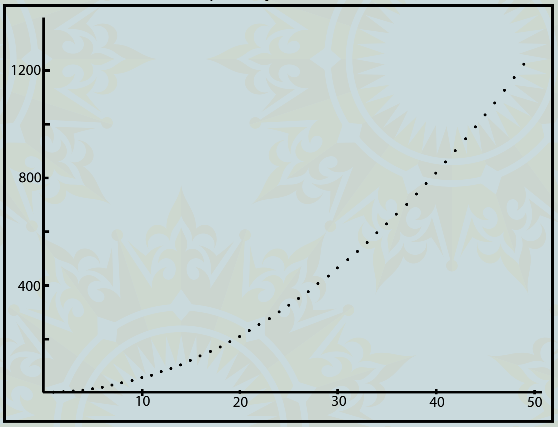
As I was working out how to find the list, I first thought of a solution like this, $\mathbf{n_x= (x)^2 - n_{x-1}}$.Where x is the location in the sequence, and n_x is the value at location x. I used this to find the next number in the sequence. For example, $\mathbf{n_{36}= (36)^2 - n_{35} = 1296 - 630 = 666}$. As I was finding the numbers in the sequence, I immediately noticed how slow it was. As I was writing the sequence out, I noticed that the difference between each element in the sequence was always increasing by one. Which made me realize that the second differences were increasing at a constant rate. Since the difference of the differences, or the second difference in this sequence were increasing by a constant factor of 1, meaning that the sequence is quadratic or a sequence of degree two. I remembered back to the reading, that the triangular numbers are also a sequence of degree 2. In addition, when you square the first differences, they are perfect squares.

Graph of: $y = 0.5n^2 + 0.5n$



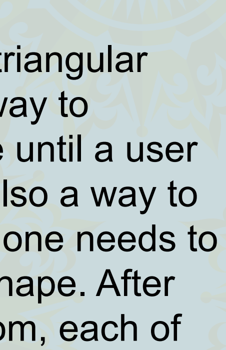
When I first saw that the sequence was quadratic, I decided to input the sequence as a list in my TI-84, and got a quadratic equation: $y = 0.5n^2 + 0.5n$. When I plugged in a number to the equation, the result was always a number in the sequence. This equation verified that the sequence I had originally made. To the right is a visualization of the equation.

Another realization I came across was that if you want to find the next number (n) in the sequence, you can take the last number in the sequence you have, and then add n to it. Therefore, if you only have seven numbers in your sequence, the last one in the sequence will be twenty-eight. Adding eight to twenty-eight will give you thirty-six. This is the correct number of the eighth element in sequence set.

So, in order to find the sequence, I used two tools. As I was brainstorming how the sequence should go, I unconsciously made a tree, which is the first tool I used. I took the first two elements and made sure they were equal to the perfect square of two(which is four). It was then that I discovered that the difference, of those elements was two (three minus one), and that got me wondering if that is how the sequence went. And sure enough, the next two elements were equal to the square of three; in addition, the difference between them was three. I think that trees are a good way to “visualize” and think out how a sequence should go; however, there is a much more efficient way to find the sequence of size n.

The tool that I believe is the most efficient is the closed-form expression for triangular numbers. $S_n = \frac{n(n+1)}{2}$ gives the n^{th} element in the sequence. Given that you have a way to automate the process; for example, a simple program that prints out the sequence until a user entered ‘n’. In this way, you will be able to get the set quick as possible. There is also a way to visualize triangle numbers. I did this in the first part of this assignment, but all that one needs to do is make a single dot. Next, add two more dots, arranging them in a triangular shape. After that, the process becomes simpler as you only need to add more rows to the bottom, each of them one dot longer than the one before.

In conclusion, to find this sequence I used two different structures, one of them being trees and the other being an expression for the triangular number sequence. I found that the latter was the better tool in this particular instance. Finally, I went through my thoughts when I was trying to solve the sequence, and then the steps I took after to further my understanding of this sequence and others.



S_4