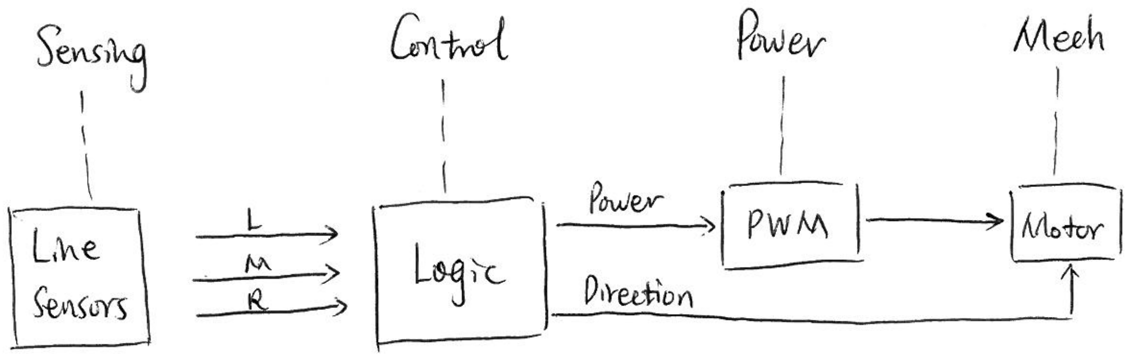
L11: logic Gates

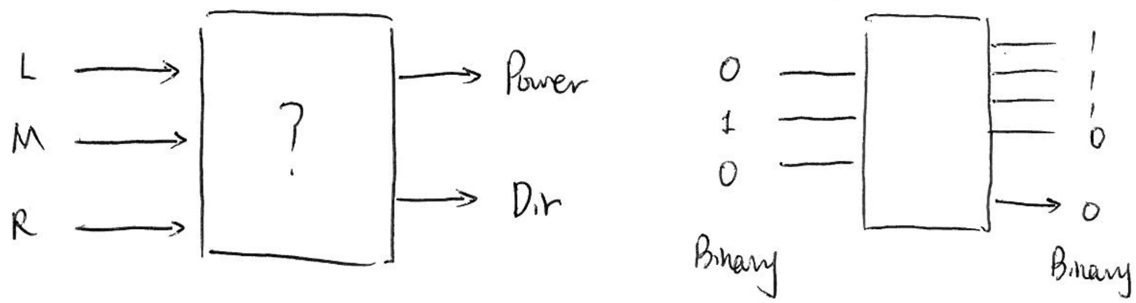
1. Review: By far, we have completed the "power", "Mech" and "sensing" sub-system



From this lecture on, we talk about the control system.

The question we want to answer is:

Given the sensing outputs "L, M, R", what should I do?



There are two things we need to consider

1) Math rules to do the computation [Math]

2) Hardware devices that can execute the rules. [Physics]

2. Logic: (Math foundation)

In daily life, we make decision by logic, say true and false.

Can we give this ability to our Robot, so that Robot can make decision?

For that purpose, we need Math theory that deals with True/False, or 0/1. (Binary System).

For Binary system, we use "Boolean Algebra" to do operations.

① In Boolean Algebra, we only deal with "True" & "False" represented by "1" & "0".

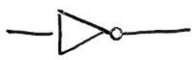
② The basic operations include "AND" "OR" "NOT".

There are derived operations like "NAND" "NOR" "XOR" "XNOR".

3. Logic Operations:

The logic operations are represented by logic gates, which is defined by

Truth table: A tabular summary for all possible outputs of a logic gate, when all possible input values are given.

1) NOT Gate: A  $\bar{A}$

input output

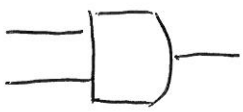
Truth Table:

A	$\bar{A}$
0	1
1	0

2) AND Gate

A

B

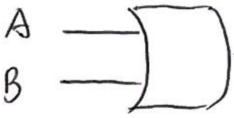


$A \cdot B = 0$

A	B	"0"
0	0	0
0	1	0
1	0	0
1	1	1

True when both A & B are true.

3) OR Gate :



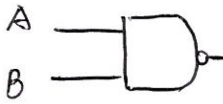
$O = A+B$

A	B	"O"
0	0	0
0	1	1
1	0	1
1	1	1

Output true when at least one input is true.

"NOT, AND, OR" are the basic operations. We can construct other operations from them.

4) NAND



$O = \overline{AB}$

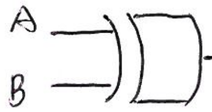
A	B	"O"
0	0	1
0	1	1
1	0	1
1	1	0

5) NOR

$O = \overline{A+B}$

A	B	"O"
0	0	1
0	1	0
1	0	0
1	1	0

6) XOR

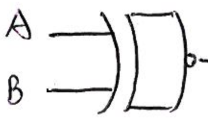


$$O = A \oplus B$$
$$= A\bar{B} + \bar{A}B$$

A	B	"O"
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive OR, output true when inputs are different.

7) XNOR



$$O = \overline{A \oplus B}$$
$$= AB + \bar{A}\bar{B}$$

A	B	"O"
0	0	1
0	1	0
1	0	0
1	1	1

Output true when inputs are the same

4. Law of Boolean Algebra [Logic simplification : the simpler the logic, the less gates needed.]

$0 + X = X$ $\quad$ $1 \cdot X = X$

$1 + X = 1$ $\quad$ $X \cdot X = X$

$\bar{X} + X = 1$ $\quad$ $X \cdot \bar{X} = 0$

$0 \cdot X = 0$ $\quad$ $\bar{\bar{X}} = X$

Exchange : $\begin{cases} X + Y = Y + X \\ X \cdot Y = Y \cdot X. \end{cases}$

Associative : $\begin{cases} (X + Y) + Z = X + (Y + Z) \\ X(Y \cdot Z) = (X \cdot Y) \cdot Z \end{cases}$

Distributive : $X(Y + Z) = XY + XZ$

$X + XZ = X(1 + Z) = X \cdot 1 = X$

$X(X + Y) = X \cdot X + X \cdot Y = X + X \cdot Y = X(1 + Y) = X$

$(X+Y)(X+Z) = X \cdot X + X \cdot Z + Y \cdot X + YZ = X(1+Z) + XY + YZ$

$= X + XY + YZ = X + YZ$

$\bar{X} + XY = \bar{X} + Y$

X	Y	$\bar{X}+XY$	$\bar{X}+Y$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

Demorgan's Law:

$$\begin{cases} \overline{(X+Y)} = \bar{X} \cdot \bar{Y} \\ \overline{XY} = \bar{X} + \bar{Y}. \end{cases}$$

$$\overline{XY} + \overline{YZ} + \overline{XZ} = \bar{X} + \bar{Y} + \bar{Y} + \bar{Z} + \bar{X} + \bar{Z}$$

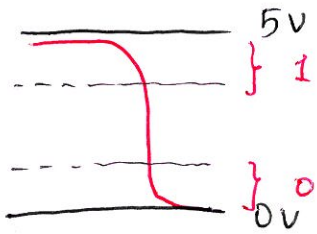
$$= \bar{X} + \bar{Y} + \bar{X} + \bar{Z}$$

$$= \overline{XY} + \overline{XZ}.$$

5. Implementation of Boolean Algebra (Physics foundation)

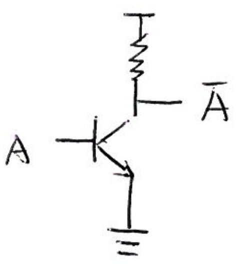
1) Number Representation:

Logic "0" and "1" are represented by high and low voltages. In fact it is a range:

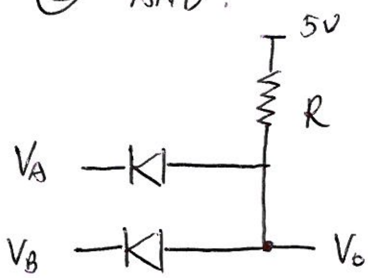


2) Hardware implementation of logic gates.

① NOT



② AND.

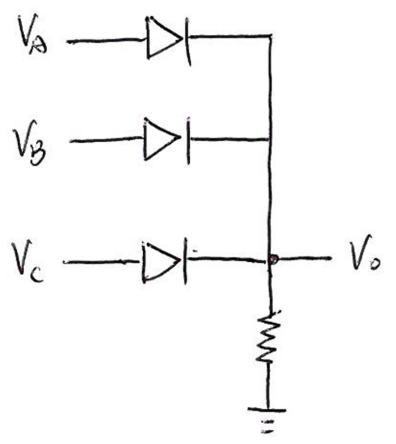


V_A	V_B	V_o
0	0	0
0	1	0
1	0	0
1	1	1

V_o is high when both V_A V_B are high.

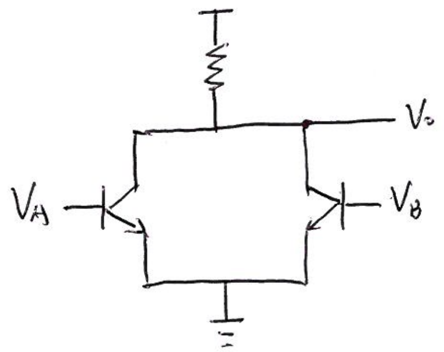
Any low voltage will turn the diode on and pull V_o down.

③ OR



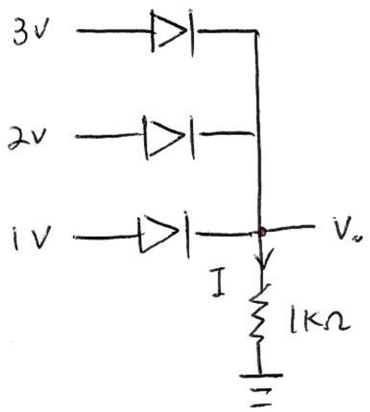
Any high input (V_A, V_B, V_C) will turn the diode on and pull V_o up.

V_A	V_B	V_o
0	0	0
0	1	1
1	0	1
1	1	1



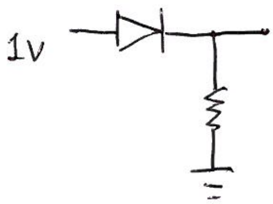
$O = \overline{A + B}$ NOR gate.

\# Exercise : Determine V_o & I. FYI



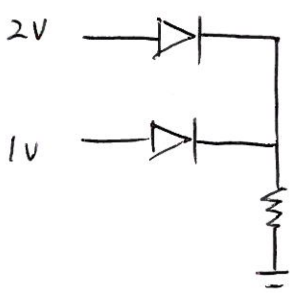
By assuming different modes (ON, OFF) for 3 diodes, we can for sure solve the circuit. But, that takes time.

Let assume we put three inputs in sequence.

① 

$\Rightarrow V_o = 1 - 0.7 = 0.3\,V.$

$I = 0.3/1000 = 0.3\,mA$

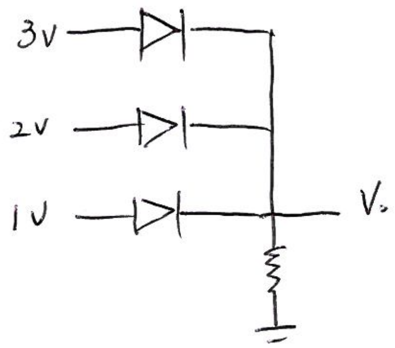
② 

If we put 2V in, then the upper diode will be ON.

Thus $V_o = 2 - 0.7 = 1.3\,V$

$I = 1.3/1000 = 1.3\,mA$

③ Similarly,

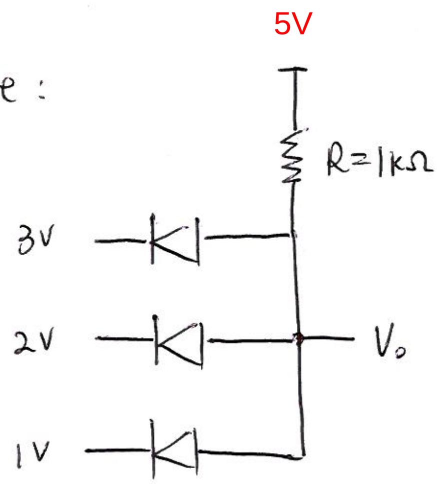


$V_o = 3 - 0.7 = 2.3\,V$

$I = 2.3/1000 = 2.3\,mA$.

V_o is determined by the highest input.
(OR Gate).

Exercise:



By following a similar procedure, we know V_o is determined by the "lowest" voltage "1V" (AND Gate).

$V_o = 1 + 0.7 = 1.7\,V$

$I = \frac{5-1.7}{1000} = 3.3\,mA$