The Hong Kong University of Science and Technology

Department of Electronic and Computer Engineering

ELEC 210: Probability and Random Processes in Electrical Engineering

2011 Spring Semester

Midterm Test

March 23, 2011

Name: _____

Student ID: _____

Instructions:

1. This is an 80-minute test.
2. You may use a non-programmable calculator.
3. There are **2 sections**:
   a. Section 1: 5 Multiple Choice Questions
   b. Section II: 4 Problems
4. Try to attempt all questions.
5. Answer each question in your **answer booklet**.
6. The distribution of marks is shown in the table below:

Question	Mark
Multi. Choice	20
1	20
2	20
3	20
4	20
Total	**100**

SECTION I: Multiple Choice [20 Marks]

Notes:

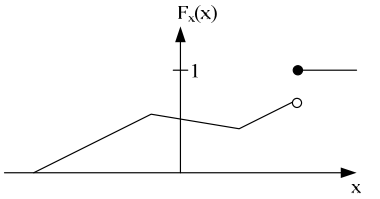
- **For each multiple-choice question below, select only ONE answer.**
- **Write your answer in your ANSWER BOOKLET (not this question sheet)**

1. [4'] Which of the following statements is true?
   a) Probability mass function (pmf) can be used for both discrete and continuous random variables
   b) Cumulative distribution function (cdf) can be used for both discrete and continuous random variables
   c) Not all the moments of *X* can be calculated from the probability density function (pdf)
   d) The value of the probability density function (pdf) cannot be greater than 1

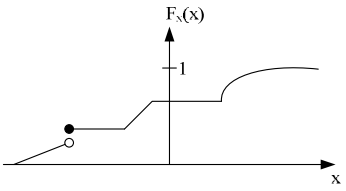
2. [4'] Which of the following statements about independence is true?
   a) For two events A and B, if P[B]=0, then A and B cannot be independent
   b) If A and B are independent, then A and B^c are also independent
   c) If A and B are mutually exclusive with non-zero probability, then A and B are independent
   d) A_1, A_2, and A_3 are independent if $P[A_1A_2A_3]=P[A_1]P[A_2]P[A_3]$

3. [4'] Let A and B be two sets. Which of the following is not true?
   a) $(A \cup B)^c = A^c \cap B^c$
   b) $A^c = (A^c \cap B) \cup (A^c \cap B^c)$
   c) $P[A \cup B] \geq P[A] + P[B]$
   d) $P[A \cap B] \leq P[A] + P[B]$

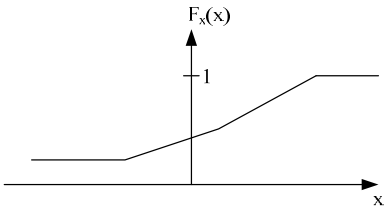
4. [4'] Which of the following functions is a valid cdf?



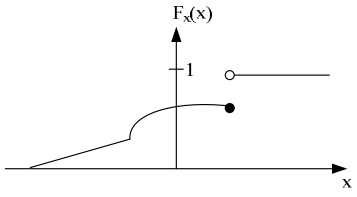
(a)



(b)



(c)



(d)

5. [4'] Which of the following is true?
   a) The expectation of a random variable cannot be negative
   b) Expectation is always one of the possible values a random variable can take
   c) The expectation of a function of a random variable equals the function of the expectation for that random variable
   d) For a discrete random variable X, E[g(X)] can be computed from the pmf of X

SECTION II: Problems [80 Marks]

Notes:

- **Write your solution in your ANSWER BOOKLET (not this question sheet)**

1. [20 Marks]

Suppose that a random variable X assumes integer values between 1 and 8 with the probability mass function (pmf) given in the table.

K	1	2	3	4	5	6	7	8
$p_X(k)$	1/16	2/16	4/16	2/16	1/16	3/16	2/16	1/16

a) [4'] Find P[3<X≤6] and P[1<X<6|X>2].

b) [4'] Plot the cdf of X. Label important values on your plot.

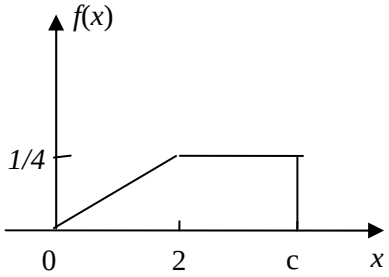
c) [4'] Find the expected value and variance of X.

d) [4'] Find the conditional pmf of X, conditioning on the event A={3≤X≤7}.

e) [4'] If X is the input of a quantizer and Y is the output given by $Y = \left\lceil \frac{X}{2} \right\rceil$, find the expected value and variance of Y. ($\lceil x \rceil$ is the smallest integer that is larger than x, e.g., $\lceil 10.3 \rceil = 11$)

2. [20 Marks]

A random variable X has the following pdf



a) [4'] Find the value of c.

b) [4'] Find the probability of P[1<X≤3].

c) [4'] Sketch the cdf of X. Label the important values on your sketch.

d) [4'] Find the expected value and variance of X.

e) [4'] For the event A={X>1}, find the conditional pdf f(x|A).

3. [20 Marks]

Consider the following quiz game with a maximum of 3 questions. If you answer question 1 correctly, then you will have the chance to answer question 2; otherwise you get nothing and the game terminates. If you answer question 2 correctly, then you will have the chance to answer question 3; otherwise you get $10 and the game terminates. If you answer question 3 correctly, then you get $100; otherwise you get $20. Suppose that the probabilities you answer correctly are 0.9, 0.7, and 0.5 for question 1, 2, 3, respectively.

a) [4'] What is the probability that you answer all the 3 questions correctly?

b) [4'] What is the probability that you get question 2 correctly?

c) [6'] Given that you didn't get $100, what is the conditional probability that you answer question 2 incorrectly?

d) [6'] What is the expected reward you can get from this quiz game? (The expected reward can be regarded as *the average payoff over a large number of trials*.)

4. [20 Marks]

300 students will vote to select the president for the student union among two candidates: Candidate "1" and "2". Suppose that the students can be divided into Group A and B, with 200 and 100 people respectively. Students in Group A are more likely to vote for Candidate 1 (with probability 0.8), while students in Group B are more likely to vote for Candidate 2 (with probability 0.9). Assume that all votes are independent. Answer the following questions and give your answers as fractions with two integers for the numerator and denominator.

a) [3'] Suppose that two different students are chosen at random. What is the probability that both belong to Group A?

b) [3'] Suppose that two different students are chosen at random. What is the probability that they belong to different groups?

c) [4'] Suppose that a student is selected at random. What is the probability that this student will vote for Candidate 2?

d) [5'] Suppose that a student is selected at random. If we know that the student voted for Candidate 1, what is the probability that the student is in Group B?

e) [5'] Suppose that every student will vote, what is the probability that Candidate 1 will be elected as the president? (One candidate is elected if he gets more votes than the other one. For this part, providing the expression is sufficient, and you do not need to calculate the numerical value.)