HOMEWORK 1 SOLUTION

Q1. Determine the power consumed by each circuit element (P_A, P_B, P_C, P_D, P_E and P_F), respectively, and specify whether each circuit component is supplying power or absorbing power (dissipated power) shown in Fig. 1 and Fig. 2.

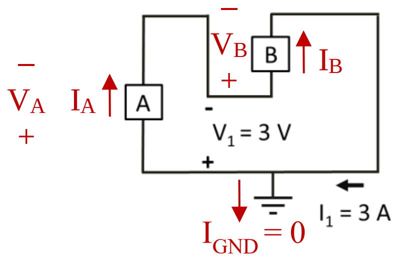


Fig. 1

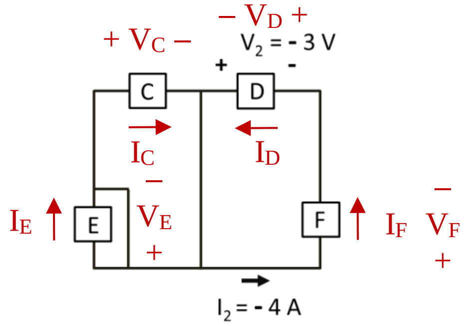
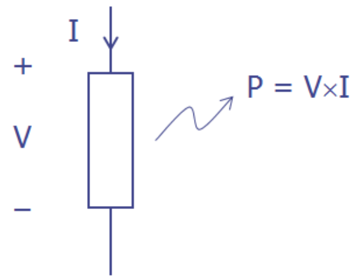


Fig. 2

First label all the voltages and currents according to our reference direction (shown below). Note that there is no ground current for the single ground connection. Then create a table for the circuit elements as follows

Circuit Element	*V* (V)	*I* (A)	*P = VI* (W)	Power Status
A	3	3	9	Absorbing
B	–3	3	–9	Supplying
C	0	Don't Care	0	Neither
D	3	–4	–12	Supplying
E	0	Don't Care	0	Neither
F	–3	–4	12	Absorbing



Finally, we can see that the net power for each circuit is zero. Thus, the electric power balance is maintained.

Q2. Find the resistance R_{AB} between node A and node B in the circuit shown in Fig. 3.

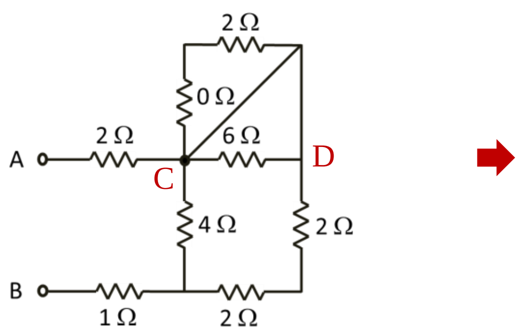
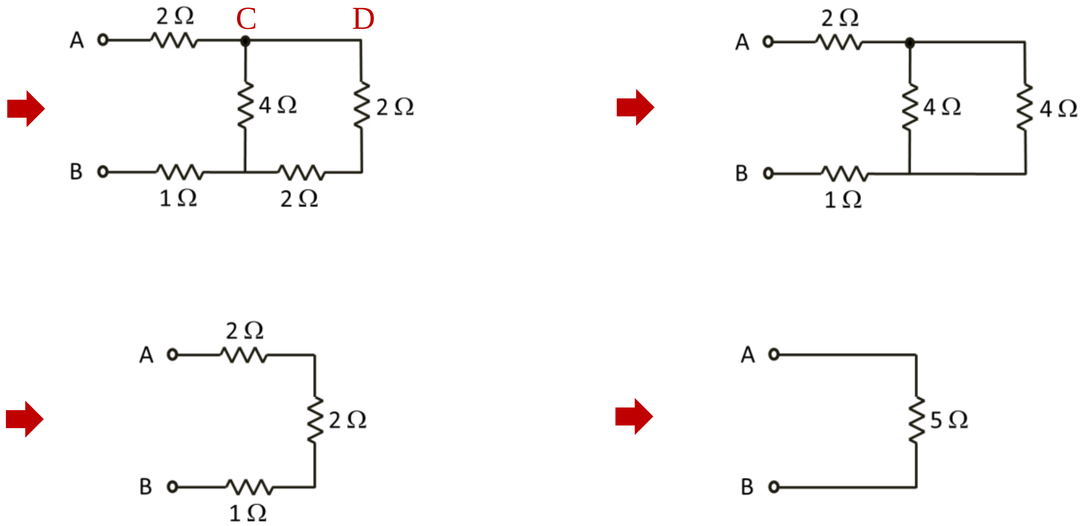


Fig. 3



Note that between nodes C and D, the (0 + 2) Ω, diagonal short (0 Ω), and 6 Ω are all connected in parallel. Their equivalent resistance is 0 Ω, i.e., a short circuit.

Q3. Find I_1, V_1 and V_2 for the circuit shown in Fig. 4.

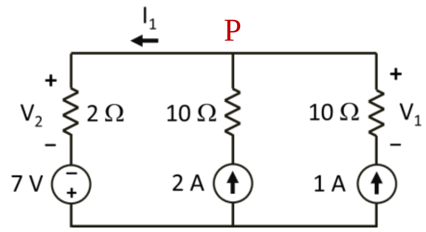


Fig. 4

Apply KCL to node P

$$I_1 = 2 + 1 = 3 \text{ A}$$

In the right branch, Ohm's law requires

$$V_1 = -1 \times 10 = -10 \text{ V}$$

In the left branch, Ohm's law requires

$$V_2 = I_1 \times 2 = 3 \times 2 = 6 \text{ V}$$

Q4. Use superposition to find I_1 and V_1 in Fig. 5.

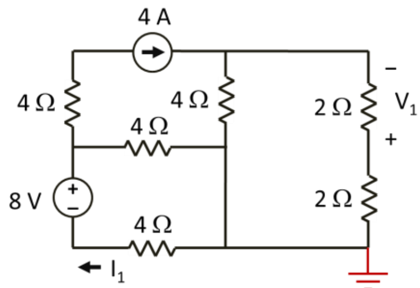
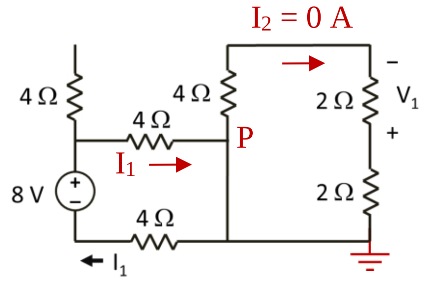


Fig. 5

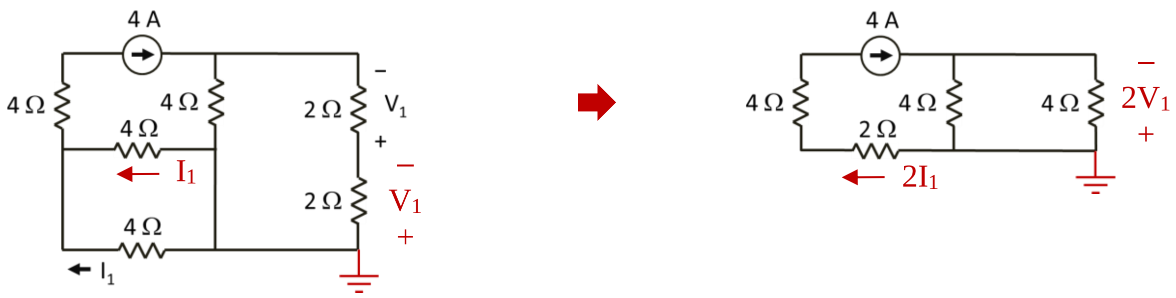
Make the bottom node our ground reference. Note we simply designate this node 0 V. We are not physically connecting this node to Mother Earth!

Part 1: with only the 8 V source, the circuit looks like this



For the right mesh, since V_P = 0 V, I_2 = 0 A. Hence V_1 = 0 V.
For the left mesh, $I_1 = 8\,V/(4+4)\,\Omega = 1$ A.

Part 2: with only the 4 A source, the circuit looks like the left figure below, in which we made two useful observations shown in red.



The left circuit can then be simplified to that on the right. By direct observations, the middle and right branches have equal resistances. Hence, they split the incoming 4 A current into two equal halves, which then recombine to form $2I_1$. Therefore I_1 = 2 A, and that is also the current that comes down the right branch. Thus,

$$-2V_1 = 2\text{ A} \times 4\,\Omega$$
$$V_1 = -4\text{ V}$$

From parts 1 and 2, by superposition

$$I_1 = 1 + 2 = 3\text{ A}$$
$$V_1 = 0 - 4 = -4\text{ V}$$

Q5. Use nodal analysis to find V_1 in the circuit shown in Fig. 6.

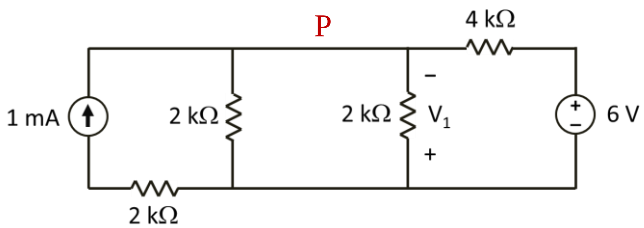


Fig. 6

Apply KCL to node P

$$1\text{m} + \frac{V_1}{2\text{k}} + \frac{V_1}{2\text{k}} + \frac{V_1 + 6}{4\text{k}} = 0$$

$$4 + 2V_1 + 2V_1 + V_1 + 6 = 0$$
$$V_1 = -2\text{ V}$$

Q6. Use source transformation(s) to find V_1 and I_1 in Fig. 7.

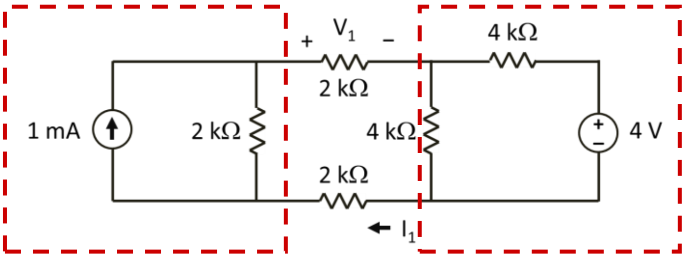
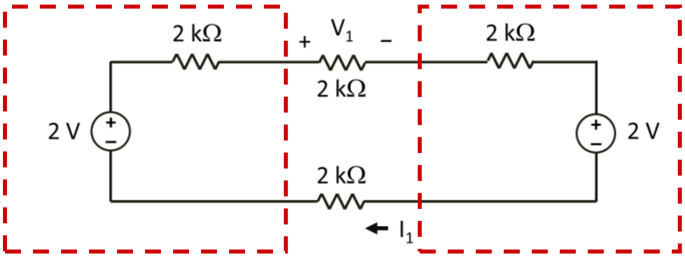


Fig. 7

The left and right parts of the circuit (shown inside the dotted boxes) can be replaced by their respective Thevenin's equivalences.



The simplified circuit has only one mesh. The left and right voltage sources cancel each other, resulting in $V_1 = 0$ V, $I_1 = 0$ A.

Q7. Find V_1 and I_1 in Fig. 8.

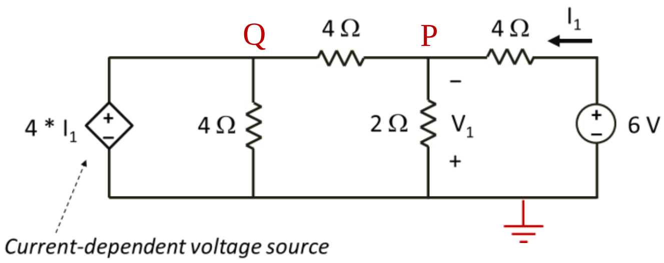


Fig. 8

Apply KVL to the right mesh

$$V_1 + 6 = 4I_1 \quad (1)$$

Apply KCL to node P. Note that the voltage at node Q is $4I_1$

$$\frac{4I_1 - (-V_1)}{4} + \frac{V_1}{2} + I_1 = 0$$

$$4I_1 + V_1 + 2V_1 + 4I_1 = 0$$

$$4I_1 = -1.5V_1 \quad (2)$$

Substituting into (1)

$$V_1 = -\frac{6}{2.5} = -2.4 \text{ V}$$

Substituting into (1)

$$I_1 = \frac{6-2.4}{4} = 0.9 \text{ A}$$

Q8. Determine the power consumed by each circuit element in Fig. 9. Specify whether each circuit component is supplying or absorbing power (dissipating power), and show the power balance.

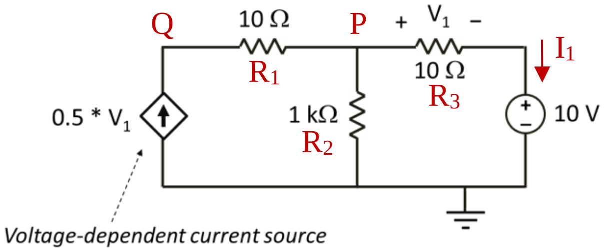


Fig. 9

Apply KCL at node P

$$0.5V_1 - \frac{V_1 + 10}{1\text{k}} - \frac{V_1}{10} = 0$$

$$500V_1 - V_1 - 10 - 100V_1 = 0$$

$$V_1 = \frac{10}{399} = 0.02506266 \text{ V}$$

$$I_1 = \frac{V_1}{10} = 0.002506266 \text{ A}$$

$$V_Q = 0.5V_1 \times 10 + V_1 + 10 = 10.15037594 \text{ V}$$

Power calculations:
R_1 power $= (0.5V_1)^2 \times 10 = 0.0015703$ W, absorbing
R_2 power $= (V_1 + 10)^2/1\text{k} = 0.1005019$ W, absorbing
R_3 power $= {V_1}^2/10 = 0.0000628$ W, absorbing
10 V independent voltage source power $= 10I_1 = 0.0250627$ W, absorbing
$0.5V_1$ dependent current source power $= V_Q(-0.5V_1) = -0.1271977$ W, supplying

The dependent current source is the only one supplying power. Since the net power for the entire circuit is zero, power balance is maintained.

Q9. Find and draw the Norton's equivalent circuit with respect to terminals a, b in Fig. 10.

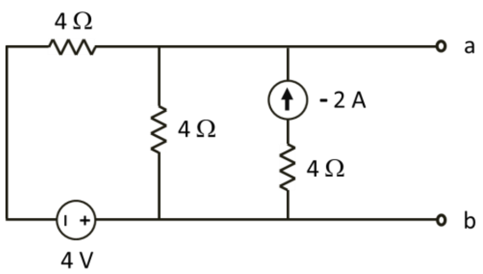
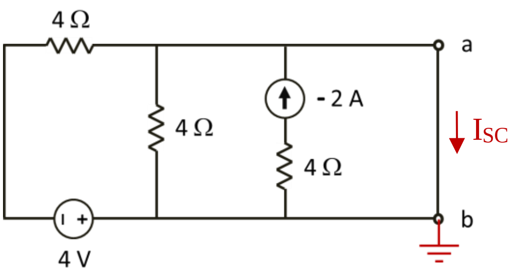


Fig. 10

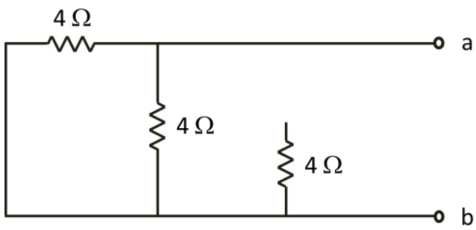
To determine the short-circuit current, we use the circuit diagram below



Make node b our ground reference. Note that node a is also at 0 V. Apply KCL at node a

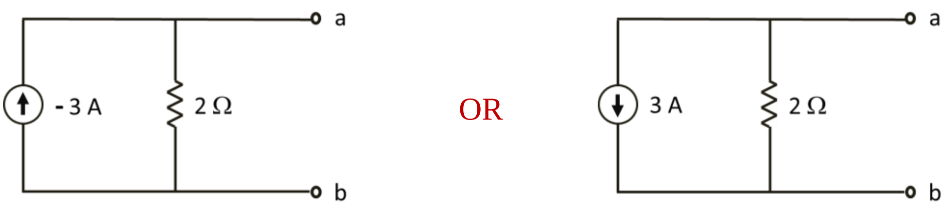
$$I_{SC} = -\frac{4}{4} + \frac{0}{4} - 2 = -3\text{ A}$$

To determine the equivalent resistance, we use the circuit diagram below



Between nodes a and b, the equivalent resistance $R_{eq} = 2\ \Omega$.

The Norton's equivalent circuit is therefore



Q10. Use the Norton's equivalent circuit in Q9 to find V_1 in Fig. 11.

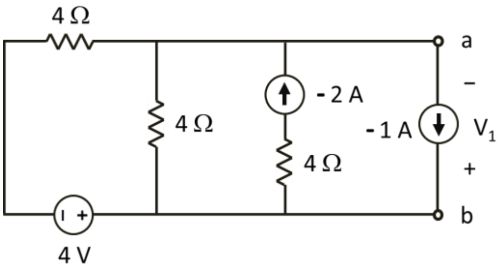
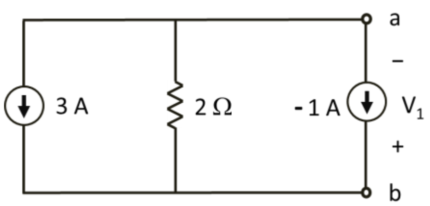


Fig. 11

Replace the sub-circuit to the left of nodes a and b with its Norton's equivalent obtained from Q9.



The net current that goes up the middle branch is 3 – 1 = 2 A. Hence $V_1 = 2 \times 2 = 4$ V.

Q11. Use the Norton's equivalent circuit in Q9 to find and draw the Thevenin's equivalent circuit with respect to terminals a, b in Fig. 12.

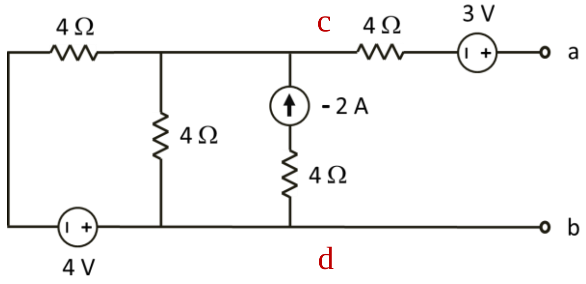
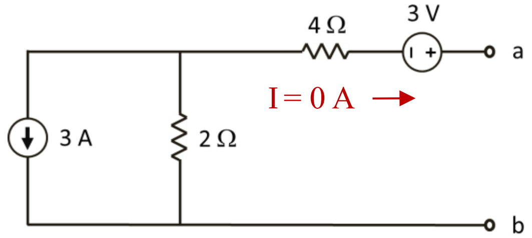
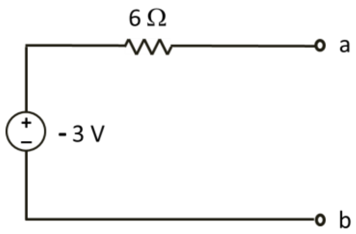


Fig. 12

Replace the sub-circuit to the left of nodes c and d with its Norton's equivalent obtained from Q9.



The open-circuit voltage $V_{OC} = V_{ab} = -3 \times 2 + 0 \times 4 + 3 = -3$ V.
The equivalent resistance $R_{eq} = R_{ab} = 2 + 4 = 6\ \Omega$.
The Thevenin's equivalent circuit is therefore



OR