COMP 2711H Discrete Mathematical Tools for Computer Science
Solutions to Tutorial 7

EP3-19. In how many different ways can $2n$ students be paired up?

Solution

$2n$ students form n pairs. If these pairs have order, this problem is equivalent to **distributing objects into different boxes so that each box has 2 objects, (L9-P27)**, which has number of cases:

$$\frac{2n!}{(2!)^n}$$

However, the order of n pairs does not matter. For each set of n pairs, we have $n!$ different ways to give it an order. So that different ways to pair up $2n$ students is:

$$\frac{2n!}{(2!)^n} \cdot \frac{1}{n!} = \frac{2n!}{(2)^n n!}$$

EP3-30. An n-input, m-output boolean function is a function from $\{\text{TRUE, FALSE}\}^n$ to $\{\text{TRUE, FALSE}\}^m$.

(a) How many n-input, 1-output boolean functions are there?

(b) How many n-input, m-output boolean functions are there?

Solution

(a) The input domain has 2^n elements. The output codomain has 2^1 elements. Each input can be mapped to 2 output, so the total number of functions is $2^{(2^n)}$

(b) The input domain has 2^n elements. The output codomain has 2^m elements. Each input can be mapped to 2^m output, so the total number of functions is $(2^m)^{(2^n)}$

EP3-44. Twelve contestants are divided into four teams of three contestants each, in which Ann and Bob want to be in the same team. How many ways are there to choose the teams so that Ann and Bob are together?

First, we form the group including Ann and Bob, which is to choose 1 member from 10 rest persons $= \binom{10}{1} = 10$.

To form the rest 9 persons into 3 groups, the idea is similar to EP3-19. If the order of groups matters, we have $\frac{9!}{3!3!3!}$ cases. Each case of 3 groups repeated in 3! different orders. So the total number of cases is

$$10 \cdot \frac{9!}{3!3!3!} \cdot \frac{1}{3!} = 2800$$

EP3-48. There are 20 books arranged in a row on a shelf. How many ways are there to select six books so that no two adjacent books are selected?

Solution

We can view the 6 selected books as 6 bars. As the 6 bars cannot be adjacent, we can bound each of the first 5 selected books with the book after it (the last book does not need that). So the total number of objects shrinks from 20 to $20 - 5 = 15$. The problem is equivalent to selecting 6 bars from 15 objects $= \binom{15}{6}$.

EP3-54. How many ways are there to put 14 identical objects in three different boxes with at least eight objects in one box?

Solution

First we choose the box which has at least 8 objects, 3 ways.

Then we assign 8 objects into this box, $14 - 8 = 6$ objects left.

6 identical objects into 3 boxes is equivalent to select 2 bars from $6 + 3 - 1 = 8$ objects. The number of cases is $\binom{8}{2}$.

The total number of cases is $3\binom{8}{2}$