ELEC210 Spring 2011 Midterm Answer Sheet

Section Ⅰ

bbcbd

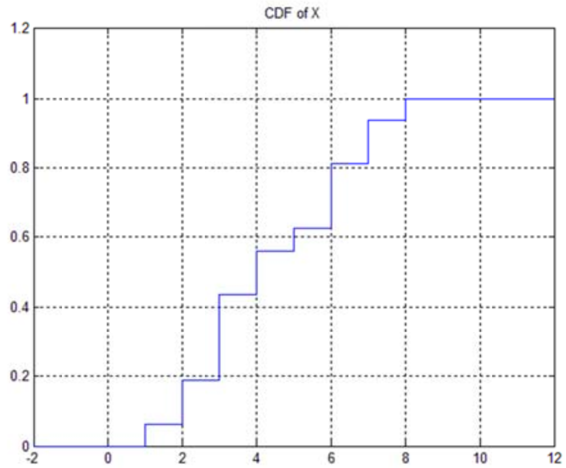
Section Ⅱ

Q1.

a) $P[3 < X \leq 6] = P[X = 4 \text{ or } 5 \text{ or } 6] = \frac{6}{16} = \frac{3}{8}$

$P[1 < X < 6|X > 2] = P[X = 3 \text{ or } 4 \text{ or } 5|X > 2] = \frac{P[X=3 \text{ or } 4 \text{ or } 5]}{P[X>2]} = \frac{\frac{7}{16}}{\frac{13}{16}} = \frac{7}{13}$

b) The cdf of X



c) $E(X) = \sum_{k=1}^{8} kp_X(k) = \frac{35}{8} = 4.375$

$E(X^2) = \sum_{k=1}^{8} k^2 p_X(k) = 23.25$

$D(X) = E(X^2) - E(X)^2 = 4.1094$

d) $P[3 \leq X \leq 7] = \frac{12}{16} = \frac{3}{4}$, therefore the conditional pmf of X is listed as follows,

K	3	4	5	6	7
$P_X(k\|A)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{6}$

e) The probability of Y

Y	1	2	3	4
$P_Y(y)$	$\frac{3}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{3}{16}$

$E(Y) = \frac{39}{16}$, $E(Y^2) = \frac{111}{16}$, $D(Y) = E(Y^2) - E(Y)^2 = \frac{255}{256} = 0.9961$

Q2.

a) Total probability is equal to 1, therefore we have the area under the function $f(x)$ is 1.

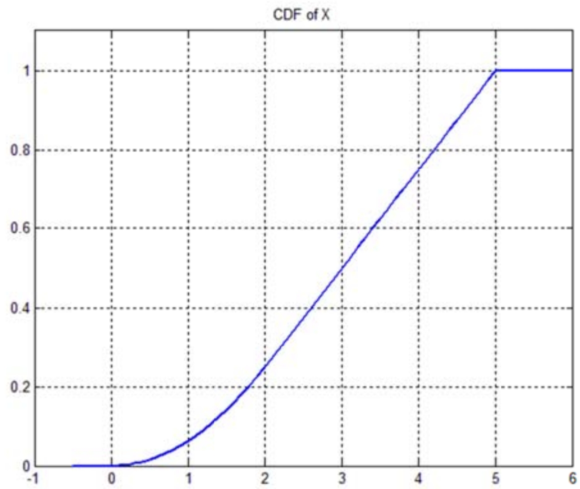
$$(c + c - 2) \times \frac{1}{4} \div 2 = 1$$

Then $c = 5$.

b) $P[1 < X \le 3] = 1 - P[0 < X \le 1 \cup 3 < X \le 5] = 1 - 1 \times \frac{1}{8} \times \frac{1}{2} - (5-3) \times \frac{1}{4} = \frac{7}{16}$

c) The function $f(x) = \begin{cases} \frac{1}{8}x, & 0 < x \le 2 \\ \frac{1}{4}, & 2 < x \le 5 \\ 0, & \text{otherwise} \end{cases}$, therefore

$$F(X) = \begin{cases} 0, & x \le 0 \\ \frac{1}{16}x^2, & 0 < x \le 2 \\ \frac{1}{4}(-1 + x), & 2 < x \le 5 \\ 1, & 5 < x \end{cases}$$



d) Expectation and Variance

$E(X) = \int_0^5 xf(x)dx = \int_0^2 \frac{x}{8} \cdot xdx + \int_2^5 \frac{1}{4} \cdot xdx = \frac{x^3}{24}\Big|_0^2 + \frac{x^2}{8}\Big|_2^5 = \frac{1}{3} + \frac{21}{8} = \frac{71}{24}$

$E(X^2) = \int_0^5 x^2 f(x)dx = \int_0^2 \frac{x}{8} \cdot x^2 dx + \int_2^5 \frac{1}{4} \cdot x^2 dx = \frac{x^4}{32}\Big|_0^2 + \frac{x^3}{12}\Big|_2^5 = \frac{1}{2} + \frac{117}{12} = \frac{123}{12}$

$D(X) = E(X^2) - E(X)^2 = \frac{123}{12} - \left(\frac{71}{24}\right)^2 = 1.4983$

e) $P[X > 1] = 1 - P[X \le 1] = 1 - \frac{1}{16} = \frac{15}{16}$

$$f(x|A) = \begin{cases} \frac{2}{15}x, & 1 < x \le 2 \\ \frac{4}{15}, & 2 < x \le 5 \\ 0, & otherwise \end{cases}$$

Q3.

a) $0.9 \times 0.7 \times 0.5 = 0.315$

b) $0.9 \times 0.7 = 0.63$

c) Let us define the event "get the \$100" as A and the event "answer the question 2 incorrectly" as B, then

$$P[A^c] = 1 - P[A] = 1 - 0.315 = 0.685$$
$$P[B] = 0.9 \times (1 - 0.7) = 0.27$$
$$P[B|A^c] = \frac{P[A^c B]}{P[A^c]} = \frac{P[B]}{P[A^c]} = \frac{0.27}{0.685} = 0.3942$$

Since B is a subcase of A^c, therefore $P[A^c B] = P[B]$.

d) The probability of \$

\$	0	10	20	100
P[\$]	0.1	0.27	0.315	0.315

$E[\$] = \sum_{\$=0}^{\$=100} \$P[\$] = 2.7 + 6.3 + 31.5 = 40.5$

Q4.

a) $\frac{200}{300} \times \frac{199}{299} = \frac{398}{897}$

b) $\frac{200}{300} \times \frac{100}{299} + \frac{100}{300} \times \frac{200}{299} = \frac{400}{897}$

c) P[Vote for C2]

= P[Vote for C2|A group]P[A group] + P[Vote for C2|B group]P[B group]

$= 0.2 \times \frac{2}{3} + 0.9 \times \frac{1}{3} = \frac{13}{30}$

d) According to Bayes' Theorem

$$P[\text{in Group B}|\text{vote for C1}] = \frac{P[\text{in Group B \& vote for C1}]}{P[\text{Vote for C1}]} = \frac{\frac{1}{3}\times 0.1}{1-P[\text{Vote for C2}]} = \frac{1/30}{17/30} = \frac{1}{17}$$

e) $P[C1] = \sum_{\substack{k+j\geq 151 \\ j\leq 100}} \binom{200}{k} 0.8^k 0.2^{200-k} \times \binom{100}{j} 0.1^j 0.9^{100-j}$