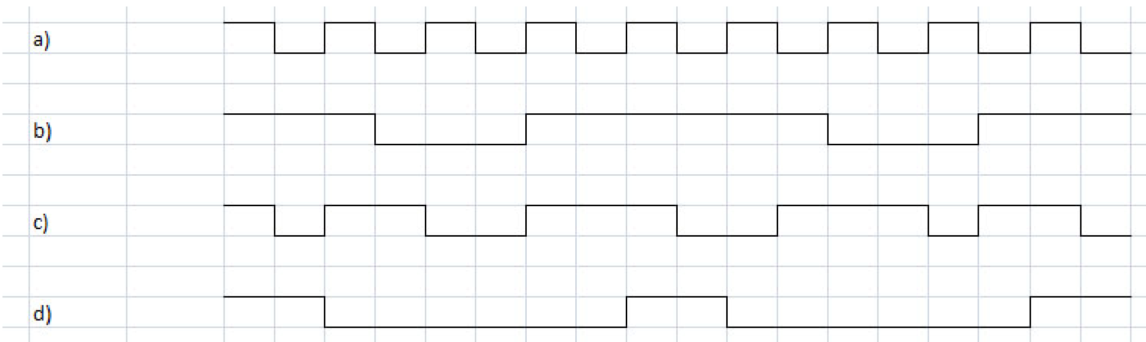
ELEC1200 Midterm Review Question

<u>Q1</u> The following figure shows the plots of several received waveforms. The transmitter is sending sequences of binary symbols (i.e., either 0 or 1) at some fixed symbol rate, using 0V to represent 0 and 1V to represent 1.The horizontal grid spacing is 1 microsecond (1e-6 sec).



Answer the following questions for each plot:
(a) Find the slowest symbol rate that is consistent with the transitions in the waveform.
(b) Use your answer in part (a) to write down the decoded bit string.

Solution

a) 1M symbols per second	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
b) 1/3 M symbols per second	1 0 1 1 0 1
c) 1 M symbols per second	1 0 1 1 0 0 1 1 1 0 0 1 1 1 0 1 1 0
d) 1/2 M symbols per second	1 0 0 0 1 0 0 0 1

Q2 The input sequence to a linear time-invariant (LTI) system is given by $x_1[n]$

$x_1[0]=1$
$x_1[1]=1$
$x_1[2]=0$
$x_1[3]=0$ for all other values of n

and the output of the LTI system is given by $y_1[n]$

$y_1[0]=1$
$y_1[1]=2$
$y_1[2]=1$
$y_1[3]=0$ for all other values of n

What are the nonzero values of the output of this LTI system when the input is $x_2[n]$?

$x_2[0]=0$
$x_2[1]=1$
$x_2[2]=1$
$x_2[3]=1$
$x_2[4]=1$
$x_2[n]=0$ for all other values of n

Solution

$x_1[n]$ and $y_1[n]$ represent the first input and output of the system.
$x_2[n]$ and $y_2[n]$ represent the second input and output of the system.

$x_2[n]=x_1[n-1]+x_1[n-3]$ → $y_2[n]=y_1[n-1]+y_1[n-3]$

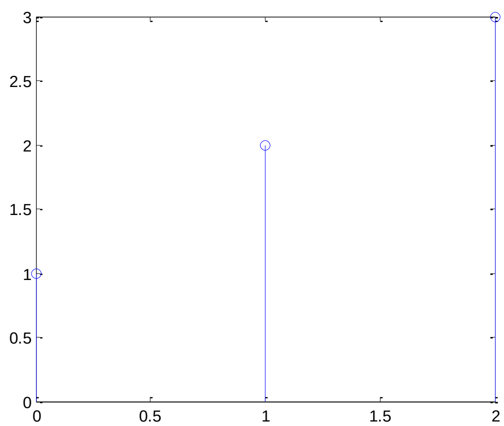
$y_2[1]=1$
$y_2[2]=2$
$y_2[3]=2$
$y_2[4]=2$
$y_2[5]=1$

<u>Q3</u> Determine the output y[n] for a system with the input x[n] and unit step-response s[n] shown below.

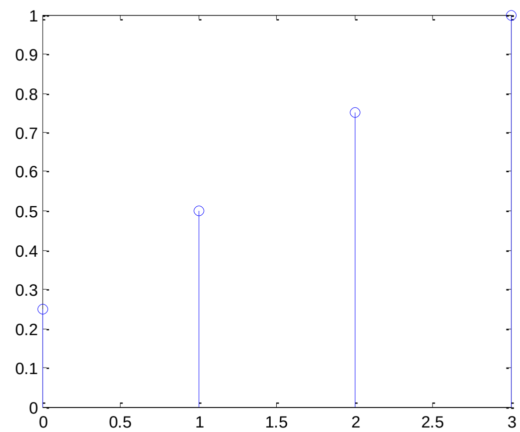
Assume x[n] = 0 for all n not being shown.

Assume s[n] = 0 for all n < 0 and s[n] = 1 for all n > 3.

x[n]



s[n]

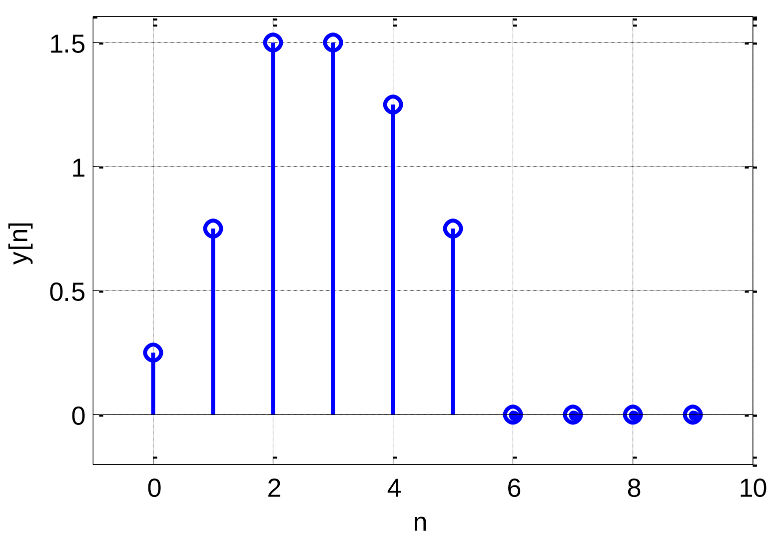


n n

Solution

$$x[n] = u[n] - u[n-1] + 2\,u[n-1] - 2\,u[n-2] + 3\,u[n-2] - 3\,u[n-3]$$
$$= u[n] + u[n-1] + u[n-2] - 3u[n-3]$$

$$y[n] = s[n] + s[n-1] + s[n-2] - 3s[n-3]$$



<u>Q4</u> The output of a particular communication channel is given by
$y[n] = \alpha x[n] + \beta x[n-1]$ where $\alpha > \beta$

(a) Is the channel linear? Is it time invariant?
(b) What is the unit step-response s[n]?
(c) If the input is the following sequence of samples starting at time 0, x[n] = [1, 0, 0, 1, 1, 0, 1, 1] and x[n] = 0 for all n not being shown, what is the channel output assuming $\alpha = 7$ and $\beta = 3$?

Solution

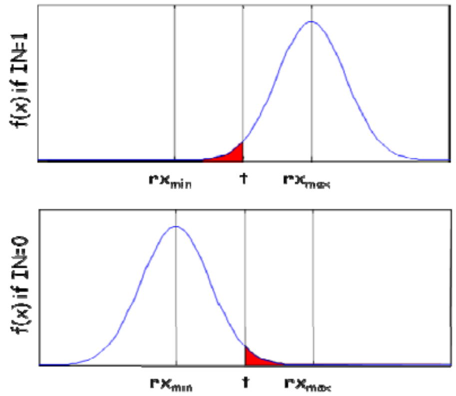
(a) This is an LTI channel.

(b) $y[n] = \alpha\, u[n] + \beta\, u[n-1] = \begin{cases} 0 & n < 0 \\ \alpha & n = 0 \\ \alpha + \beta & n > 0 \end{cases}$

(c) $y[n] = 7\, x[n] + 3\, x[n-1]$

y[0] = 7
y[1] = 3
y[2] = 0
y[3] = 7
y[4] = 10
y[5] = 3
y[6] = 7
y[7] = 10
y[8] = 3
y[9] = 0
y[n] = 0 for all n not being shown

Q5 A noise probability density function is shown below.



Give an expression for BER in terms of the detection threshold T, $rx_{\min}$, $rx_{\max}$, the variance of the noise σ^2, and P(IN=0).

Solution

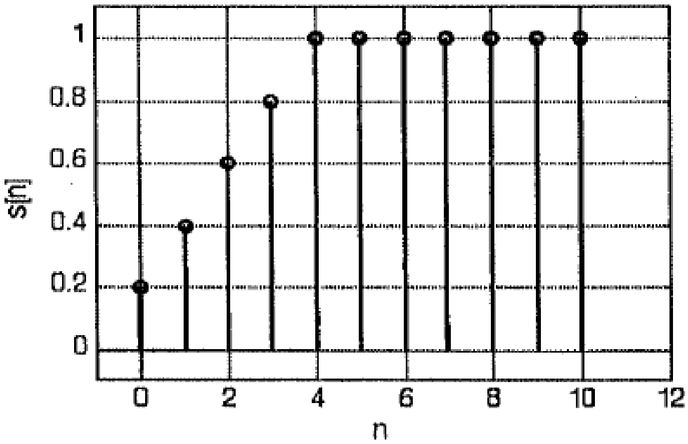
$$BER = P(IN=0)\cdot Q\left(\frac{t - rx_{\min}}{\sigma}\right) + (1 - P(IN=0))\cdot Q\left(\frac{rx_{\max} - t}{\sigma}\right)$$

Question 1 **[20 Marks]**

(a) [4] Given that the transmitted sampled waveform is [0 0 1 1 1 1 1 0 0 0 0 0 1 1 0 0 …], express it in terms of the step function u[n].

(b) [6] Explain what a linear time-invariant channel is.

(c) [10] Given that the step response s[n] of a linear time-invariant channel is as follows. Express the channel output waveform in terms of s[n] and sketch it for n = 0 to 24.

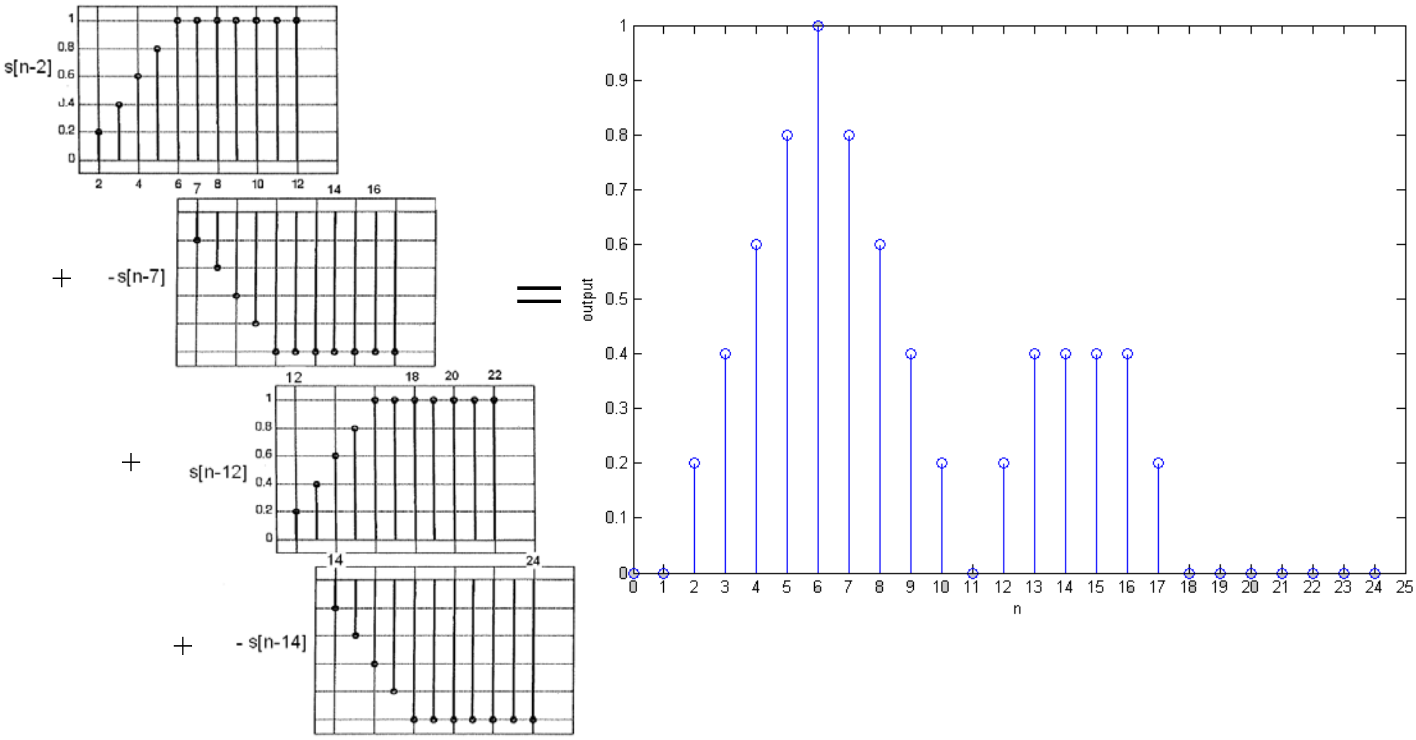


(a). u[n-2] -u[n-7]+u[n-12]-u[n-14]

(b). Linear time-invariant channel is
- Additive
- Homogeneity
- time-invariant

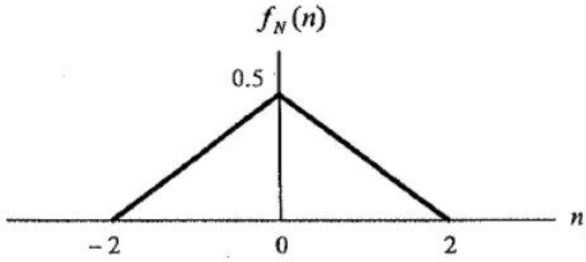
(c). Input = u[n-2] -u[n-7]+u[n-12]-u[n-14]

Output = s[n-2] -s[n-7]+s[n-12]-s[n-14]



Question 2

Given that the probability density function of the additive channel noise is as follows.



The channel response to a bit input is the sum of the response to the input and noise. Without noise, the responses to bit "0" and bit "1" are 0V and 1V, respectively. Decoding the channel output is done by comparing the channel output to a threshold value of T. The decoded bit is "1" if the channel voltage is greater than T and "0" otherwise.

(a) [5] Sketch the probability density function of the received signal if the input bit is "0".

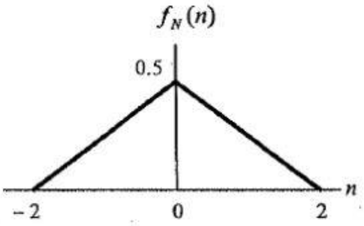
(b) [5] Sketch the probability density function of the received signal if the input bit is "1".

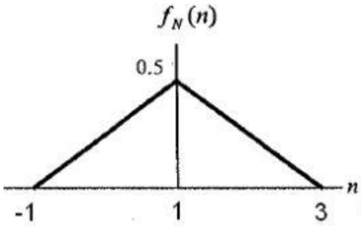
(c) [5] What is the optimal threshold if the input bits "0" and "1" are equally likely?

(d) [5] What is the bit error rate if we use the optimal threshold in 3(c)?

(e) [5] Re-do parts 3(c)&(d) if the input bit "0" is twice more likely to occur than the input bit "1".

Answer:

(a). P[IN=0], 

(b). P[IN=1], 

(c). optimal threshold = 0.5. (d).= 0.1875*1.5*0.5*2=0.28125.

(e). P[IN=0] = 2/3, P[IN=1] = 1/3.

Optimal threshold = 1. BER = 1/6 * 2*0.5+ 1/6 *0.5 = 0.25.