COMP 2711H Discrete Mathematical Tools for Computer Science
Solutions to Tutorial 1

QB1-1. Let p, q, and r be the following propositions "you get an A on the final exam", "you do every exercise in this book", and "you get an A in this class", respectively. Write the following propositions using p, q, and r and logical connectives.

(a) You get an A in this class, but you do not do every exercise in this book.

(b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

(c) To get an A in this class, it is necessary for you to get an A on the final.

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Solution:

(a) $r \wedge \neg q$

(b) $p \wedge q \wedge r$

(c) $r \rightarrow p$

(d) $p \wedge \neg q \wedge r$

(e) $(p \wedge q) \rightarrow r$

(f) $(p \vee q) \leftrightarrow r$

QB1-2. Prove that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent. Present two different proofs of this fact, only one of which should use truth tables.

Solution: The proof using truth table,

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T	T	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

The proof without using truth table,

$$\begin{aligned}
& p \leftrightarrow q \\
\equiv & (p \to q) \wedge (q \to p) \\
\equiv & (\neg p \vee q) \wedge (\neg q \vee p) && (s \to t \equiv \neg s \vee t) \\
\equiv & ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p) && \text{(Distributive laws)} \\
\equiv & (\neg p \wedge \neg q) \vee (q \wedge \neg q) \vee (\neg p \wedge p) \vee (p \wedge q) && \text{(Distributive laws)} \\
\equiv & (\neg p \wedge \neg q) \vee F \vee F \vee (p \wedge q) && \text{(Negation laws)} \\
\equiv & (\neg p \wedge \neg q) \vee (p \wedge q) && \text{(Identity laws)}
\end{aligned}$$

EP1-1. Construct a truth table for the statement $\neg((p \vee q) \wedge \neg p) \wedge \neg p$.

Solution:

p	q	$p \vee q$	$(p \vee q) \wedge \neg p$	$\neg((p \vee q) \wedge \neg p)$	$\neg((p \vee q) \wedge \neg p) \wedge \neg p$
T	T	T	F	T	F
T	F	T	F	T	F
F	T	T	T	F	F
F	F	F	F	T	T

EP1-4. Are the following statements logically equivalent:

$$(\neg p \wedge (\neg p \wedge q)) \vee (p \wedge (p \wedge \neg q))$$

$$(\neg p \wedge q) \vee (p \wedge \neg q)$$

Solution: They are logically equivalent,

$$\begin{aligned}
& (\neg p \wedge (\neg p \wedge q)) \vee (p \wedge (p \wedge \neg q)) \\
\equiv & ((\neg p \wedge \neg p) \wedge q) \vee ((p \wedge p) \wedge \neg q) && \text{(Associative laws)} \\
\equiv & (\neg p \wedge q) \vee (p \wedge \neg q) && \text{(Idempotent laws)}
\end{aligned}$$

EP1-5. Are the following statements logically equivalent:

$$(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$$

$$r$$

Solution: They are logically equivalent,

$$\begin{aligned}
& (p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \\
\equiv & ((p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge q)) \wedge r && \text{(Distributive laws)} \\
\equiv & ((p \wedge (q \vee \neg q)) \vee (\neg p \wedge (\neg q \vee q)) \wedge r && \text{(Distributive laws)} \\
\equiv & ((p \wedge \mathrm{T}) \vee (\neg p \wedge \mathrm{T}) \wedge r && \text{(Negation laws)} \\
\equiv & (p \vee \neg p) \wedge r && \text{(Identity laws)} \\
\equiv & \mathrm{T} \wedge r && \text{(Negation laws)} \\
\equiv & r && \text{(Identity laws)}
\end{aligned}$$