MATH2011 Intro to Multivariable Calculus (Fall 2013)

Midterm Examination Name:_____

26 Oct 2013 Student I.D.:_____ Signature:_____

1:30–2:30pm LT (A/C/J/K): _____ Seat Number:_____

Directions:

- Do **NOT** open the exam until instructed to do so.
- All mobile phones and pagers should be switched **OFF** during the examination.
- You may write on both sides of the examination papers.
- You must show the steps in order to receive full credits.
- Electronic calculators are **NOT** allowed.
- This is a closed book examination.
- Answer **ALL** questions.

Question No.	Points	Out of
1(a)		4
1(b)		2
1(c)		2
2		5
3		5

Question No.	Points	Out of
4		5
5(a)		4
5(b)		3
Total		30

Answer all questions. Show all your work for full credit.

1. (a) Find an equation of the plane that passes through the three points $(1, 0, 0)$, $(1, 1, 1)$ and $(0, 0, 1)$.

The equation is given by .

(b) Write down a parametric equation of the line through $(-2, 0, 5)$ that is parallel to $3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$.

The equation is given by .

(c) Determine whether the plane in (a) and the line in (b) intersect; if so, find the coordinates of the intersection.

2. Let $\mathcal{S}_1$ be the surface of the equation $y = x^2$ and $\mathcal{S}_2$ be the surface of the equation $2x^2 + y^2 + 6z = 24$. Find the intersection curve of $\mathcal{S}_1$ and $\mathcal{S}_2$ and represent it using a vector-valued function $\mathbf{r}(t)$.

The equation is given by .

3. Given z implicitly defined as a function of x and y through the equation

$$x^2 + z \sin xyz = 0\,.$$

Use the implicit differentiation to derive $\dfrac{\partial z}{\partial x}$.

$\dfrac{\partial z}{\partial x} =$.

4. Find $\left.\dfrac{\partial z}{\partial u}\right|_{(u,v)=(1,1)}$ if

$$z = xe^y\,,\ x = u^2 + v^2 \text{ and } y = uv\,.$$

$\left.\dfrac{\partial z}{\partial u}\right|_{(u,v)=(1,1)} =$.

5. Let $\mathcal{S}$ be the surface of the equation $z - 3x^2 - y^2 = 0$.

(a) Find a point on the surface $\mathcal{S}$ at which the tangent plane is parallel to the plane $6x + 4y - z = 5$.

The point on the surface is $(x, y, z) =$.

(b) Determine the equation of the tangent plane at the point in (a).

The equation of the tangent plane is .

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Solutions

1.
(a) $x - y + z = 1$.
(b) $x(t) = -2 + 3t$, $y(t) = -4t$, $z(t) = 5 + t$.
(c) $(-11/4.1, 19/4)$.

2. $(t, t^2, \frac{1}{6}(24 - 2t^2 - t^4))$.

3.

$$\frac{\partial z}{\partial x} = -\frac{2x + yz^2 \cos xyz}{\sin xyz + xyz \cos xyz}$$

4. $4e$

5.
(a) $(1, 2, 7)$.
(b) $6x + 4y - z = 7$.