Junyu YIN **MATH 2011 (Fall 2014-2015)**

WeBWork Homework-3 due 10/30/2020 at 05:00pm HKT

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

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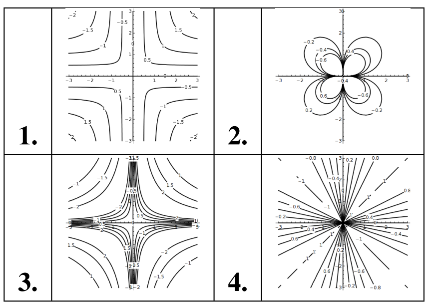
1. (2 points) Consider the function

$$f(x,y) = \begin{cases} \frac{2xy}{(x^2+y^2)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(a) Use a computer to draw a contour diagram for f. Which of the following is the contour diagram?

- ?
- figure 1
- figure 2
- figure 3
- figure 4



(b) Is f differentiable at all points $(x,y) \neq (0,0)$? [?/yes/no]

(c) Calculate the partial derivatives of f for $(x,y) \neq (0,0)$:

$f_x =$ ____________

$f_y =$ ____________

Do the partial derivatives f_x and f_y exist and are they continuous at all points $(x,y) \neq (0,0)$?

- ?
- `they don't exist at at least one point`
- `they exist but aren't continuous at at least one point`
- they exist and are continuous at all points

(d) A first test for whether f is differentiable at $(0,0)$ is to see if it is continuous there. Calculate each of the following limits to determine if f is continuous at $(0,0)$:

$\lim_{h\to 0} f(0,h) =$ ________

$\lim_{h\to 0} f(h,0) =$ ________

$\lim_{h\to 0} f(h,h) =$ ________

(In each case, enter **DNE** *if the limit does not exist.)*

Is f continuous at $(0,0)$?

- ?
- yes
- no
- it is not possible to tell

Is f differentiable at $(0,0)$?

- ?
- yes
- no
- it is not possible to tell

(e) Find the partial derivative f_x at $(0,0)$ by calculating it directly with a limit:

$f_x = \lim_{h\to 0} \frac{1}{h}(f(\_\_, \_\_) - f(\_\_, \_\_)) =$ ______

Do the partial derivatives f_x and f_y exist and are they continuous at $(0,0)$? *(Hint: to test continuity, you may want to use a similar calculation as you used to test the continuity of f)*

- ?
- yes
- no
- it is not possible to tell
- they exist but are not continuous

(Be sure that you can justify all of your answers in this problem.)

Answer(s) submitted:

- `figure 2`
- `yes`
- `(-2(3x^2-y^2)y)/(x^2+y^2)^3`
- `(-2(3y^2-x^2)x)/(x^2+y^2)^3`
- `they exist and are continuous at all points`
- `0`
- `0`
- `DNE`
- `no`
- `no`
- `h`
- `0`
- `0`
- `0`
- `0`
- `they exist but are not continuous`

(correct)

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2. (1 point) Find the limit of the function

$$f(x,y) = \frac{\sin(4\sqrt{x^2+y^2})}{4\sqrt{x^2+y^2}}$$

as $(x,y) \to (0,0)$. Assume that polynomials, exponentials, logarithmic, and trigonometric functions are continuous. *[Hint: $\lim_{t\to 0} \frac{\sin t}{t} = 1$.]*

$$\lim_{(x,y)\to(0,0)} \frac{\sin(4\sqrt{x^2+y^2})}{4\sqrt{x^2+y^2}} = \_\_\_\_\_\_\_\_$$

(Enter **DNE** *if the limit does not exist.)*

Answer(s) submitted:

- `1`

(correct)

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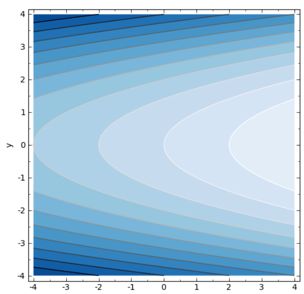
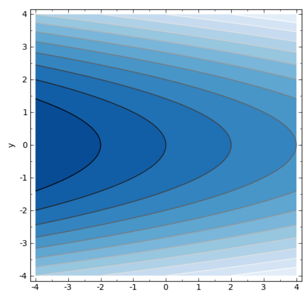
3. (2 points)

Match each function with its contour plot. Click on a graph to make it larger. Darker areas represent lower elevations and lighter areas represent higher elevations.

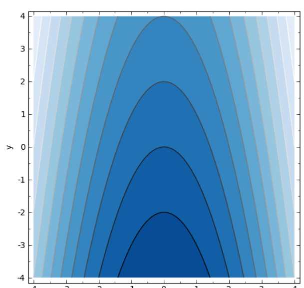
$\boxed{?}$ 1. $f(x,y) = x - y^2$
$\boxed{?}$ 2. $f(x,y) = y + x^2$
$\boxed{?}$ 3. $f(x,y) = x + y^2$
$\boxed{?}$ 4. $f(x,y) = y - x^2$



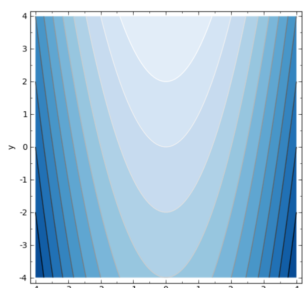
A



B



C



D

Answer(s) submitted:

- `A`
- `C`
- `B`
- `D`

(correct)

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4. (1 point) Find the partial derivatives indicated. Assume the variables are restricted to a domain on which the function is defined.

$$z = \left(x^5 + x - y\right)^5.$$

$\frac{\partial z}{\partial x} =$ ________________

$\frac{\partial z}{\partial y} =$ ________________

Answer(s) submitted:

- `5(x^5+x-y)^4*(5x^4+1)`
- `5(x^5+x-y)^4*(-1)`

(correct)

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5. (1 point)

Find the linear approximation of the function $z = x\sqrt{y}$ at the point (2, 64).

$L(x,y) =$ ____________________________

Answer(s) submitted:

- `2*sqrt(64)+(x-2)*sqrt(64)+(y-64)*(2*(1/2)/sqrt(64))`

(correct)

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6. (1 point) If

$$z = \cos\left(\frac{y}{x}\right), \qquad x = 5t, \qquad y = 5 - t^2,$$

find dz/dt using the chain rule. Assume the variables are restricted to domains on which the functions are defined.

$\frac{dz}{dt} =$ ____________________________

Answer(s) submitted:

- `-sin((5-t^2)/(5t))(-2t*5t-(5-t^2)*5)/(5t)^2`

(correct)

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7. (2 points) Let $F(u,v)$ be a function of two variables. Let $F_u(u,v) = G(u,v)$, and $F_v(u,v) = H(u,v)$. Find $f'(x)$ for each of the following cases (your answers should be written in terms of G and H).

(a) $f(x) = F(x,1)$: then
$f'(x) =$ ________________

(b) $f(x) = F(5,x)$: then
$f'(x) =$ ________________

(c) $f(x) = F(x,x)$: then
$f'(x) =$ ________________

(d) $f(x) = F(2x, x^4)$: then
$f'(x) =$ ________________

Answer(s) submitted:

- `G(x,1)`
- `H(5,x)`
- `G(x,x)+H(x,x)`
- `2*G(2x,x^4)+4x^3*H(2x,x^4)`

(correct)

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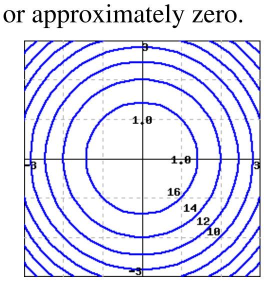
8. (2 points) Use the contour diagram of f in the figure below to decide if the specified directional derivatives below are positive, negative, or approximately zero.



(a) At point $(-2,2)$, in direction $\vec{i}$: $f_{\vec{u}}$ is

- ?
- positive
- negative
- approximately zero

(b) At point $(0,-2)$, in direction $-\overrightarrow{j}$: $f_{\overrightarrow{u}}$ is
- ?
- positive
- negative
- approximately zero

(c) At point $(-1,1)$, in direction $-\overrightarrow{i}+\overrightarrow{j}$: $f_{\overrightarrow{u}}$ is
- ?
- positive
- negative
- approximately zero

(d) At point $(-1,1)$, in direction $\overrightarrow{i}+\overrightarrow{j}$: $f_{\overrightarrow{u}}$ is
- ?
- positive
- negative
- approximately zero

(e) At point $(0,-2)$, in direction $\overrightarrow{i}+2\overrightarrow{j}$: $f_{\overrightarrow{u}}$ is
- ?
- positive
- negative
- approximately zero

(f) At point $(0,-2)$, in direction $-\overrightarrow{i}-2\overrightarrow{j}$: $f_{\overrightarrow{u}}$ is
- ?
- positive
- negative
- approximately zero

Answer(s) submitted:

- `positive`
- `negative`
- `negative`
- `approximately zero`
- `positive`
- `negative`

(correct)

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9. (1 point) If the gradient of f is $\nabla f = z\tilde{i} + y\tilde{j} - 2xy\tilde{k}$ and the point $P = (-8,1,-6)$ lies on the level surface $f(x,y,z) = 0$, find an equation for the tangent plane to the surface at the point P.

$z =$ ______

Answer(s) submitted:

- `6/16*x-1/16y-47/16`

(correct)

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10. (1 point) You are standing above the point $(5,4)$ on the surface $z = 25 - \left(3x^2 + y^2\right)$.

(a) In which direction should you walk to descend fastest? (Give your answer as a unit 2-vector.)

direction = ______

(b) If you start to move in this direction, what is the slope of your path?

slope = ______

Answer(s) submitted:

- `<3(2)(5)/sqrt(964),2(4)/sqrt(964)>`
- `-sqrt(964)`

(correct)