ELEC 2400 ELECTRONIC CIRCUITS
Final Exam Solution

22 Dec 2020 Online

Q1. [AC analysis] Refer to the AC circuit in Fig. 1. Given $v_s(t) = 8\cos(1000t)$.

(a) Find Z_R, Z_L and Z_C (the impedance of the resistor, inductor, capacitor, respectively).

(b) Find the source current I_S in phasor form and find the source current $i_S(t)$ in the time domain.

(c) Find the voltage across each element V_R, V_L, and V_C in phasor form.

(d) Plot the voltages V_R, V_L, V_C and V_S in a single phasor diagram.

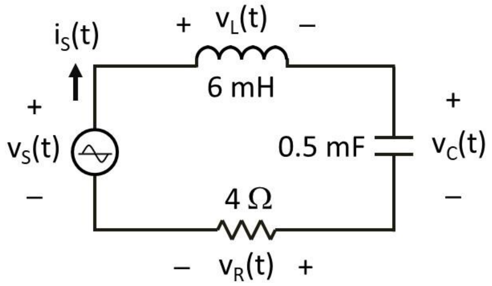


Fig. 1 Question 1

Solution:

(a) Given $\omega = 1000$ rad/s.

$$Z_R = 4\ \Omega$$

$$Z_L = j1000 \times 6\text{m} = 6\text{j}\ \Omega$$ 3 pt

$$Z_C = \frac{1}{j1000 \times 0.5\text{m}} = -2\text{j}\ \Omega$$

(b)

$$I_S = \frac{V_S}{Z_L + Z_C + Z_R} = \frac{8}{4 + 4j} = \sqrt{2}\angle(-45°)\ \text{A}$$

2 pt

$$i_S(t) = \sqrt{2}\cos(1000t - 45°)\ \text{A}$$

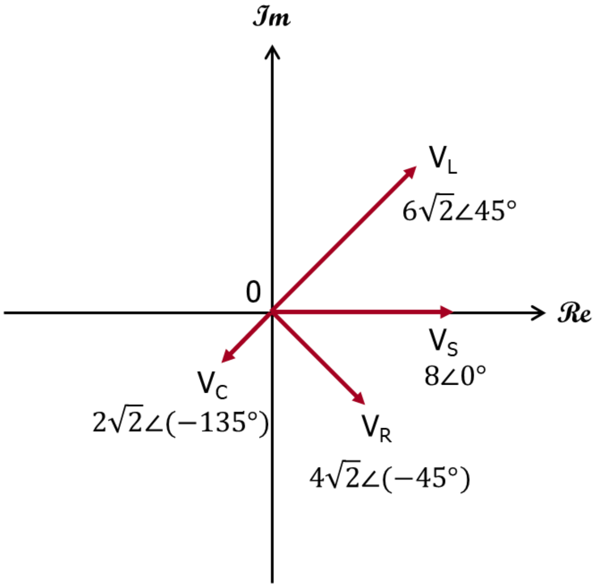
(c)

$$V_R = I_S Z_R = 4\sqrt{2}\angle(-45°)\ \text{V}$$

$$V_L = I_S Z_L = \sqrt{2}\angle(-45°) \times 6j = 6\sqrt{2}\angle 45°\ \text{V}$$ 3 pt

$$V_C = I_S Z_C = \sqrt{2}\angle(-45°) \times (-2j) = 2\sqrt{2}\angle(-135°)\ \text{V}$$

(d) Phasor plot is as shown in below.



2 pt

Q2. [Frequency Response] Sketch the Bode plots of $\frac{s(s+100000)}{(s+1000)^2}$ on the separate graph paper provided. Label the y-axis and y-scale clearly.

Solution:

$$H(s) = \frac{s(s\ +\ 100000)}{(s+1000)^2} = \frac{s(1\ +\ s/100000)}{10(1+s/1000)^2}$$

There are two zeros at $\omega = 0, 100000$ rad/s and a double pole at $\omega = 1000$ rad/s.

We would need one point to nail down the magnitude plot. Consider, e.g., $\omega = 10$ rad/s

$$H(j10) = \frac{j10(1\ +\ j10/100000)}{10(1+j10/1000)^2} \approx j$$

Magnitude is

$$20\log_{10}|H(j10)| = 20\log_{10}1 = 0 \text{ dB}$$

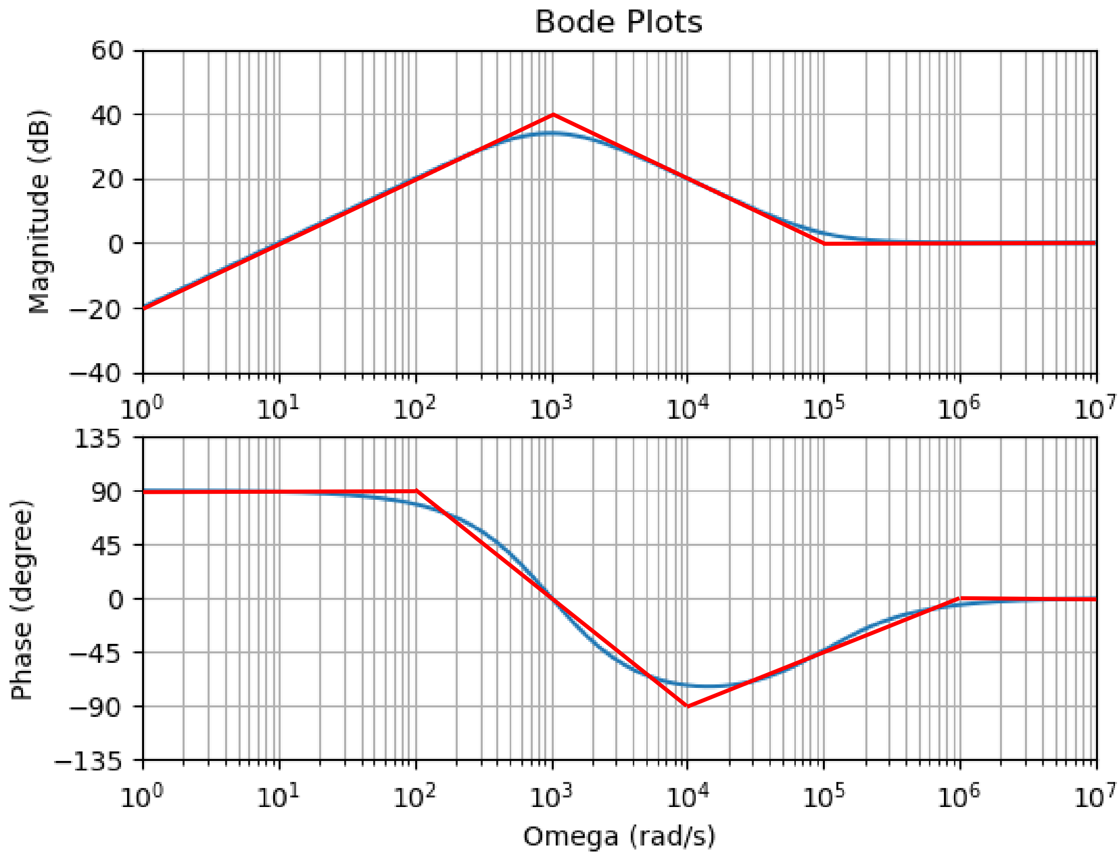
and phase is 90°.

Alternatively, we can also consider $\omega \to \infty$ at which

$$H(j\infty) \to \frac{j\infty(j\infty\ +\ 100000)}{(j\infty+1000)^2} \approx 1$$

So magnitude is 0 dB and phase is 0°.

The Bode plots are as shown below. Only the red sketches are required. The blue computer-generated plots are provided only as a reference.



5 pt

5 pt

Q3. [Op Amps] Refer to the op amp circuit in Fig. 2. Assume the op amps are ideal.

(a) Find V_1, V_o and I_o in terms of V_{in}.

(b) Find V_o when $V_{in} = -1$ V.

(c) Find V_o when $V_{in} = 2$ V.

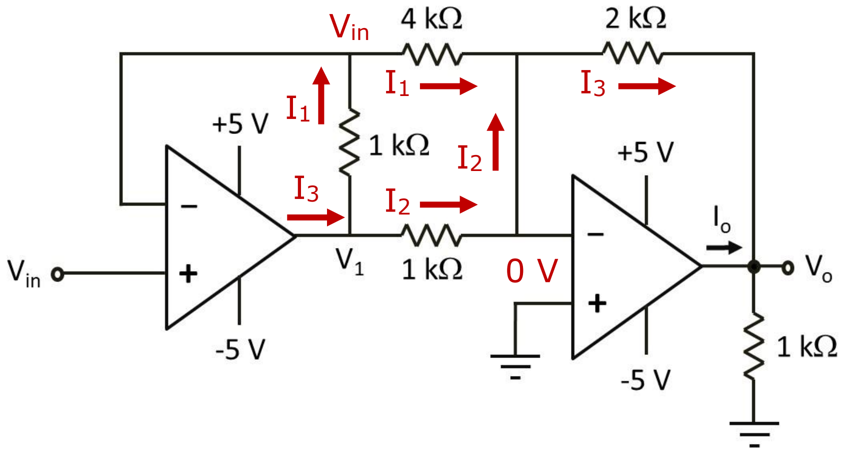


Fig. 2 Question 3

Solution:

(a) With the voltages and currents marked in red above, we can see that

$$I_1 = \frac{V_1 - V_{in}}{1\text{k}} = \frac{V_{in} - 0}{4\text{k}} = \frac{V_{in}}{4\text{k}}$$

Hence

$$V_1 = \frac{5}{4} V_{in}$$ 3 pt

Therefore

$$I_2 = \frac{V_1 - 0}{1\text{k}} = \frac{5V_{in}}{4\text{k}}$$

$$I_3 = I_1 + I_2 = \frac{3V_{in}}{2\text{k}}$$

Finally

$$V_o = -I_3 \times 2\text{k} = -3V_{in}$$ 3 pt

$$I_o = -I_3 + \frac{V_o}{1\text{k}} = -\frac{3V_{in}}{2\text{k}} - \frac{3V_{in}}{1\text{k}} = -\frac{9V_{in}}{2\text{k}}$$ 2 pt

(b) When $V_{in} = -1$ V, $V_o = 3$ V. 2 pt
(c) When $V_{in} = 2$ V, V_o is saturated at -5 V,

Q4. [Diodes] Refer to the diode circuit in Fig. 3. Assume the diodes are ideal. Find V_1, V_2, I_1, I_2, and I_3.

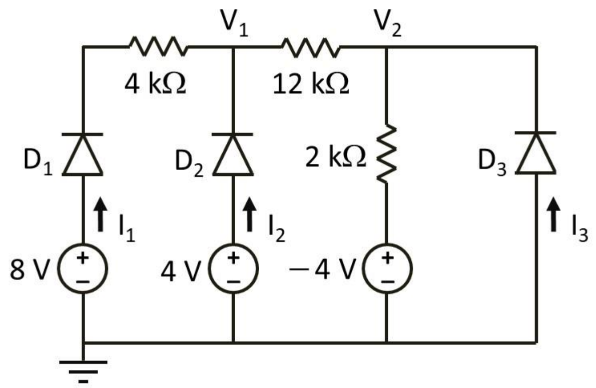


Fig. 3 Question 4

Solution:

From an examination of the supply voltages, we can assume D_1, D_3 to be ON and D_2 OFF. Under this assumption

$$V_2 = 0 \text{ V}$$ 2 pt

$$V_1 = 8\left(\frac{12\text{k}}{4\text{k} + 12\text{k}}\right) = 6 \text{ V}$$ 2 pt

This confirms that D_2 is OFF.

$$I_1 = \frac{8 - V_1}{4\text{k}} = \frac{8 - 6}{4\text{k}} = 0.5 \text{ mA}$$ 2 pt

Since D_2 is OFF

$$I_2 = 0 \text{ mA}$$ 2 pt

$$I_3 = \frac{V_2 - (-4)}{2\text{k}} - I_1 = \frac{0 + 4}{2\text{k}} - 0.5 = 1.5 \text{ mA}$$ 2 pt

Q5. [Transient Analysis] Refer to the circuit having a voltage-controlled current source in Fig. 4.

(a) Consider the sub-circuit (shown inside the dotted box) all by itself, find an expression for the open-circuit voltage V_{AB} when both terminals A and B are left open-circuited.

(b) For the sub-circuit alone, find an expression for the short-circuit current I_{AB} when A and B are short-circuited.

(c) For the sub-circuit alone, find an expression for the equivalent resistance R_{EQ} across terminals A and B.

(d) Back to the entire circuit, assume that the switch has been open for a long time. The switch is closed at t = 0. Find an expression for the time constant of the transient response for t > 0.

(e) Find an expression for the capacitor voltage $v_c(t)$ for t > 0.

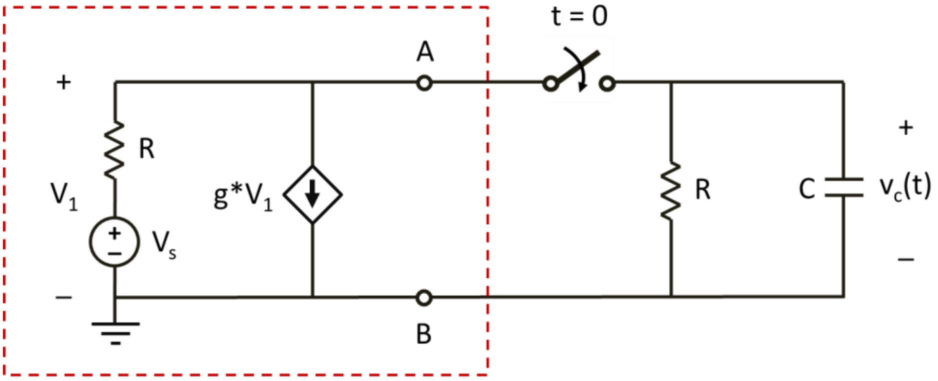


Fig. 4 Question 5

Solution:

(a) For the sub-circuit alone with A and B open-circuited

$$V_1 = V_S - gV_1R$$

yielding

$$V_{AB} = V_1 = \frac{V_S}{1 + gR} \qquad \text{2 pt}$$

(b) For the sub-circuit alone with A and B shorted, $V_{AB} = V = 0$. The dependent current source has zero current, i.e., open-circuited, leaving just the independent voltage source driving a current through R. Hence

$$I_{AB} = \frac{V_S}{R} \qquad \text{2 pt}$$

(c) For the sub-circuit alone, the equivalent resistance is the same for both Thevenin and Norton equivalent circuits, which is equal to the open-circuit voltage divided by the short-circuit current. Hence

$$R_{EQ} = \frac{V_{AB}}{I_{AB}} = \frac{R}{1 + gR} \qquad \text{2 pt}$$

(d) For the entire circuit, the overall equivalent resistance seen by the capacitor is

$$R_{Overall} = R||R_{EQ} = \frac{\frac{R^2}{1+gR}}{R + \frac{R}{1+gR}} = \frac{R}{2+gR}$$

Time constant is therefore

$$\tau = R_{Overall}C = \frac{RC}{2+gR}$$ 2 pt

(e) Initial state: the capacitor behaves as an open circuit and because of the pull-down resistor

$$v_c(0^+) = 0$$

Final state: the capacitor again behaves as an open circuit. Apply KCL to node A

$$\frac{V_1 - V_s}{R} + \frac{V_1}{R} + gV_1 = 0$$

Yielding

$$V_1 = \frac{\frac{V_s}{R}}{\frac{1}{R} + \frac{1}{R} + g} = \frac{V_s}{2+gR} = v_c(\infty)$$

Therefore, for $t > 0$

$$v_c(t) = v_c(\infty)(1 - e^{-\frac{t}{\tau}}) = \frac{V_s}{2+gR}\left(1 - e^{-\frac{2+gR}{RC}t}\right)$$ 2 pt

Q1. [Diodes] Refer to the diode circuit in Fig. 1. Assume the diodes are ideal. Plot V_o as a function of V_{in} from 0 V to 6 V.

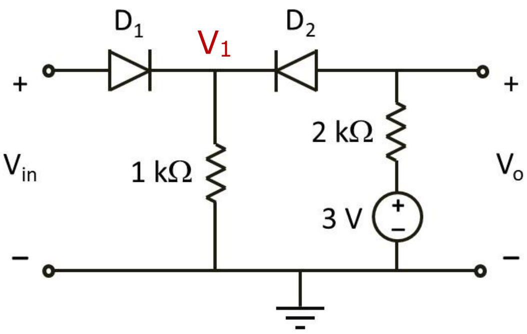


Fig. 1 Question 1

Solution:

(a) For $0 \text{ V} \leq V_{in} \leq 1$ V, D_1 is OFF and D_2 is ON, and

$$V_o = V_1 = 1 \text{ V}$$ 2 pt

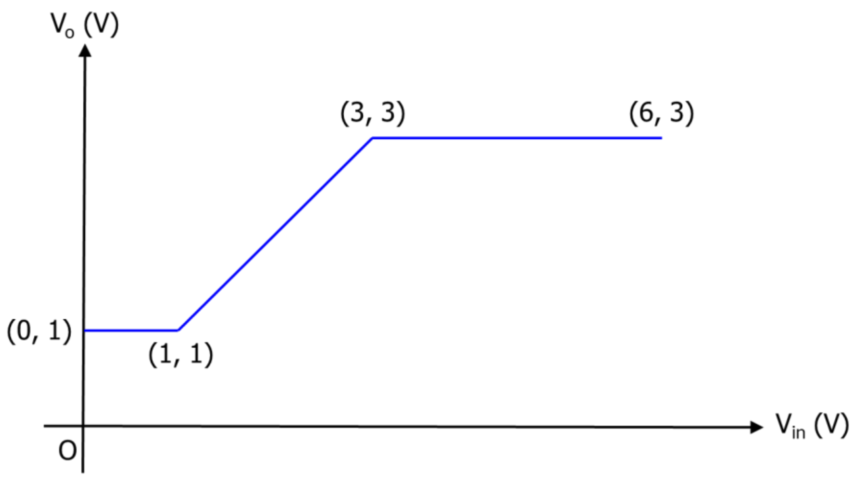
(b) For $1 \text{ V} \leq V_{in} \leq 3$ V, D_1 and D_2 are both ON, and

$$V_o = V_1 = V_{in}$$ 2 pt

(c) For $V_{in} > 3$ V, D_1 is ON and D_2 is OFF. Hence

$$V_o = 3 \text{ V}$$ 2 pt

(d) The plot of V_o vs. V_{in} is shown below.



4 pt

Q2. [Transient Analysis] Refer to the circuit in Fig. 2. Assume the switch has been open for a long time. The switch is closed at t = 0.

(a) Find the values of $v_1(t)$ and $v_2(t)$ for t < 0.

(b) Find the values of $v_1(t)$ and $v_2(t)$ for t = 0^+.

(c) Find the values of $v_1(t)$ and $v_2(t)$ for t → ∞.

(d) Find the value of the time constant for the transient response in this circuit for t > 0.

(e) Find the expressions for $v_1(t)$ and $v_2(t)$ for t > 0.
(f) Plot $v_1(t)$ and $v_2(t)$ as a function of time on the same chart starting from t < 0.

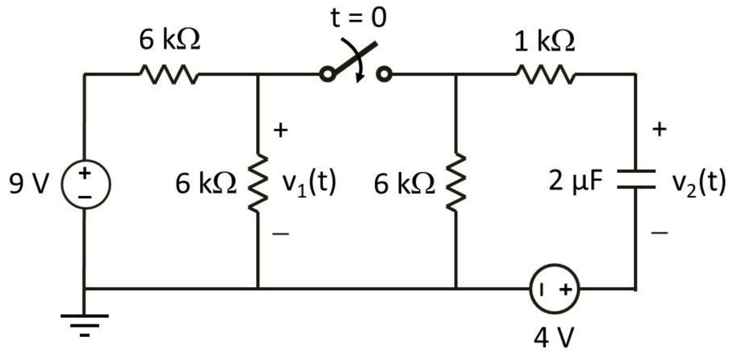


Fig. 2 Question 2

Solution:
(a) For $t < 0$

$$v_1(t<0) = 4.5\text{ V}$$
$$v_2(t<0) = -4\text{ V}$$

2 pt

(b) At $t = 0^+$, the capacitor voltage, v_2, is momentarily unchanged. The capacitor behaves as an –4 V battery. Apply KCL to the node containing v_1

$$\frac{v_1 - 9}{6\text{k}} + \frac{v_1}{6\text{k}} + \frac{v_1}{6\text{k}} + \frac{v_1 - (-4) - 4}{1\text{k}} = 0$$

$$v_1(0^+) = 1\text{ V}$$
$$v_2(0^+) = -4\text{ V}$$

2 pt

(c) As $t \to \infty$, the capacitor behaves as an open circuit

$$v_1(\infty) = 3\text{ V}$$
$$v_2(\infty) = 3 - 4 = -1\text{ V}$$

2 pt

(d) The equivalent resistance as seen by the capacitor is

$$R_{eq} = 1\text{k} + 6\text{k}||6\text{k}||6\text{k} = 1\text{k} + 2\text{k} = 3\text{ k}\Omega.$$

The time constant is

$$\tau = R_{eq}C = 3\text{k} \times 2\mu = 6\text{ ms}$$

1 pt

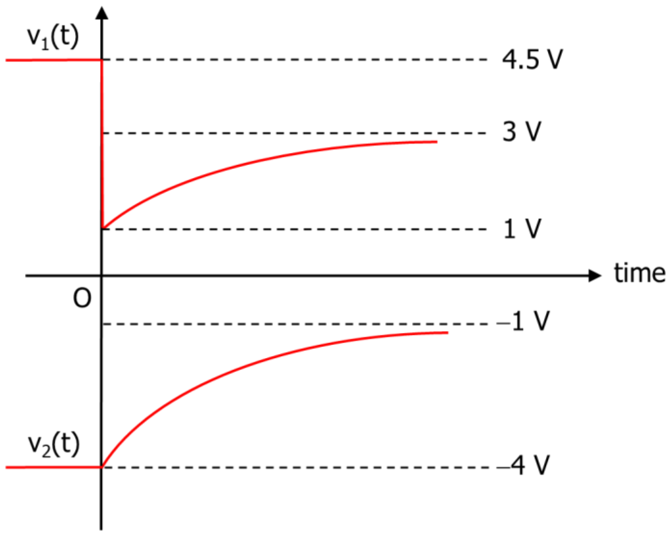
(e) For $t > 0$, the equations for $v_1(t)$ and $v_2(t)$ are therefore

$$v_1(t) = 3 + (1-3)e^{-\frac{t}{6\text{m}}} = 3 - 2e^{-167t}\text{ V}$$

$$v_2(t) = -1 + (-4+1)e^{-\frac{t}{6\text{m}}} = -1 - 3e^{-167t}\text{ V}$$

2 pt

(f) The plots for $v_1(t)$ and $v_2(t)$ vs. time are shown below.



1 pt

Q3. [Op Amp] Refer to the op amp circuit in Fig. 3. Assume the op amp is ideal.

(a) Find the expression of $v_o(t)$ in terms of $v_{in}(t)$.

(b) Plot the waveform of $v_o(t)$ when $v_{in}(t) = v_{inA}(t)$. Label the key turning point values.

(c) Plot the waveform of $v_o(t)$ when $v_{in}(t) = v_{inB}(t)$. Label the key turning point values.

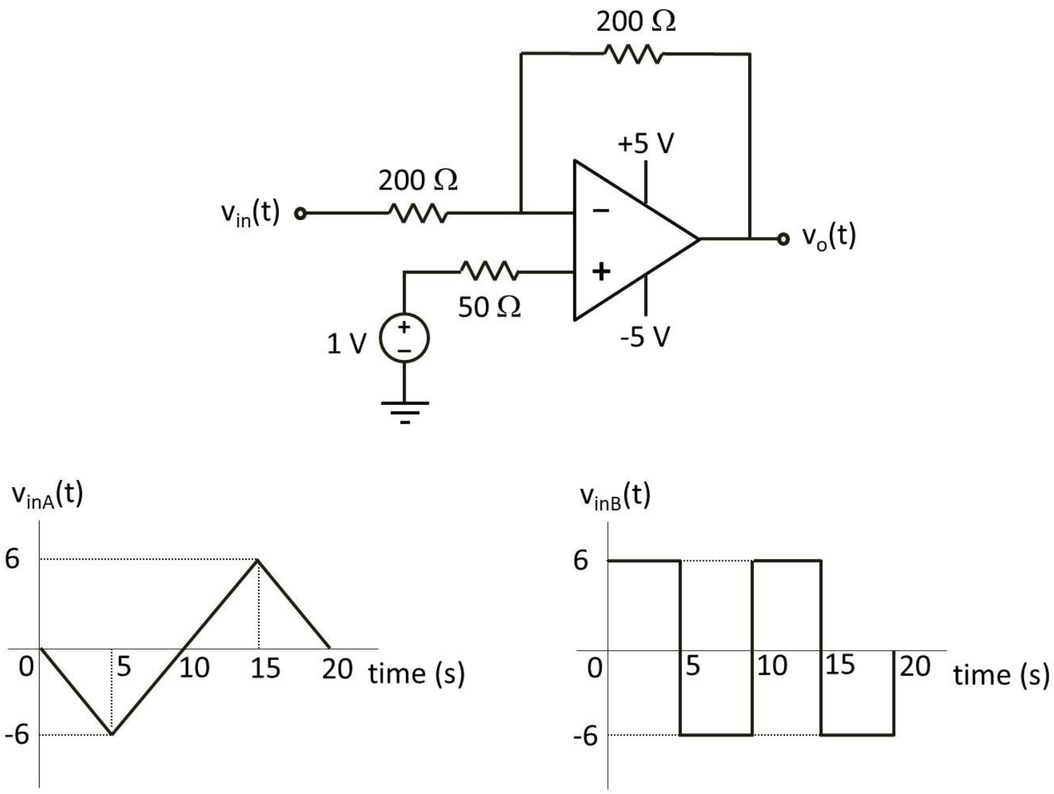


Fig. 3 Question 3

Solution:

(a) Assume the op amp is not saturated

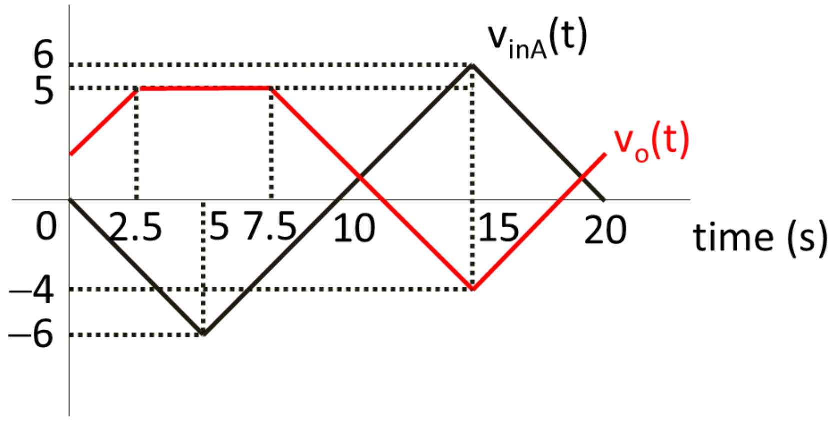
$$v_+ = 1\text{ V} = v_-$$

Apply KCL at V_-

$$\frac{v_{in} - v_-}{200} = \frac{v_- - v_o}{200}$$

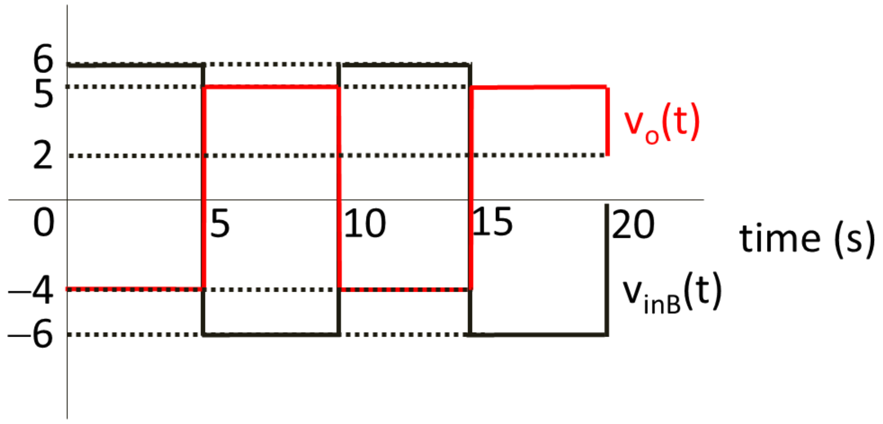
$$v_o = -v_{in} + 2\text{ V}$$ 3 pt

(b) The plot of $v_o(t)$ when $v_{in}(t) = v_{inA}(t)$ is shown below.



4 pt

(c) The plot of $v_o(t)$ when $v_{in}(t) = v_{inB}(t)$ is shown below.



3 pt

Q4. [AC Analysis] Refer to the AC circuit having a current-controlled current source in Fig. 4.
(a) Find I and V_1 and express the answers in phasor form.
(b) Compute the average AC power for *each* of the circuit elements: 2 resistors, capacitor, inductor, independent voltage source, and dependent current source.
(c) Specify whether each circuit element is supplying AC power, absorbing AC power (dissipating power), or neither.

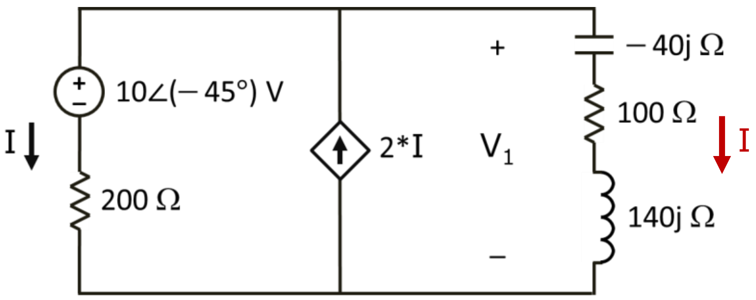


Fig. 4 Question 4

Solution:

(a) First, we notice that the current coming down the right branch is also I. For both the left and right branches

$$V_1 = 10\angle(-45°) + 200I = (-40j + 100 + 140j)I = (100 + 100j)I$$

Hence

$$I = \frac{10\angle(-45°)}{-100 + 100j} = \frac{10\angle(-45°)}{100\sqrt{2}\angle 135°} = \frac{1}{10\sqrt{2}}\angle(-180°) = -\frac{1}{10\sqrt{2}}\ \text{A}$$

2 pt

$$V_1 = (100 + 100j)I = 100\sqrt{2}\angle 45° \times \frac{1}{10\sqrt{2}}\angle(-180°) = 10\angle(-135°)\ \text{V}$$

(b) (c) Average AC power calculation:

(i) Capacitor and inductor: average AC power is zero. They are neither generating nor absorbing power. 2 pt

(ii) 100 Ω resistor:

$$\text{Average AC power} = \frac{1}{2}I^2 \times 100 = \frac{1}{2}(-\frac{1}{10\sqrt{2}})^2 100 = 0.25\ \text{W, absorbing}$$ 1 pt

(iii) 200 Ω resistor:

$$\text{Average AC power} = \frac{1}{2}I^2 \times 200 = \frac{1}{2}(-\frac{1}{10\sqrt{2}})^2 200 = 0.5\ \text{W, absorbing}$$ 1 pt

(iv) Independent voltage source:

$$\text{Average AC power} = \frac{1}{2} \times 10 \times \frac{1}{10\sqrt{2}} \cos[(-45° - (-180°))]$$

$$= \frac{1}{2\sqrt{2}}\cos(135°) = \frac{1}{2\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right) = -0.25\ \text{W, generating}$$ 2 pt

(v) Dependent current source: the $2I$ current is opposite to our reference direction. So we flipped it with a minus sign.

$$\text{Average AC power} = \frac{1}{2} \times 10 \times \left(-\frac{2}{10\sqrt{2}}\right) \cos[(-135° - (-180°))]$$

$$= -\frac{1}{\sqrt{2}}\cos(45°) = -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = -0.5\ \text{W, generating}$$ 2 pt

Lastly, we notice that the total average AC power for the entire circuit is zero. Hence the AC power balance is satisfied.

Q5. [Op Amp Frequency Response] Refer to the op amp circuit in Fig. 5. All voltages are referenced to ground.

(a) Derive and simplify the transfer function $E(s) = \frac{V_o(s)}{V_p(s)}$. Your expression should not contain any other voltage variables.

(b) Derive and simplify the transfer function $F(s) = \frac{V_o(s)}{V_q(s)}$. Your expression should not contain any other voltage variables.

(c) Derive and simplify the transfer function $G(s) = \frac{V_q(s)}{V_{in}(s)}$. Your expression should not contain any other voltage variables.

(d) Derive and simplify the transfer function $H(s) = \frac{V_o(s)}{V_{in}(s)}$. Your expression should not contain any other voltage variables.

(e) What is the order of the overall system? What type of circuit is this?

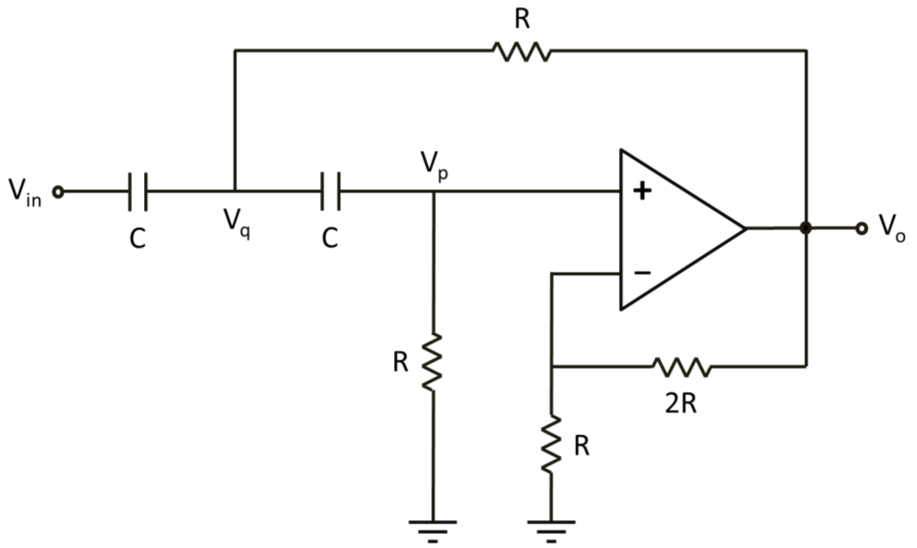


Fig. 5 Question 5

Solution:

(a) The op amp is a non-inverting amplifier with V_p as input and V_o as output

$$E(s) = \frac{V_o(s)}{V_p(s)} = \frac{3R}{R} = 3 \qquad \text{2 pt} \qquad (1)$$

(b) V_p is the voltage divider output, with the voltage V_j being divided between the R and C connected in series. Hence

$$\frac{V_p(s)}{V_q(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{1 + sRC}$$

Substituting (1)

$$F(s) = \frac{V_o(s)}{V_q(s)} = \frac{V_o(s)}{V_p(s)}\frac{V_p(s)}{V_q(s)} = \frac{3sRC}{1+sRC} \qquad \text{2 pt} \qquad (2)$$

(c) Apply KCL to the node containing V_q

$$\frac{V_o - V_q}{R} + \frac{V_{in} - V_q}{\frac{1}{sC}} - \frac{V_q}{R + \frac{1}{sC}} = 0$$

$$\frac{V_o}{R} + \left(-\frac{1}{R} - sC - \frac{sC}{1+sRC}\right)V_q = -sCV_{in}$$

Substituting (2)

$$\frac{3sRC}{R(1+sRC)}V_q + \left(-\frac{1}{R} - sC - \frac{sC}{1+sRC}\right)V_q = -sCV_{in}$$

$$\frac{3sRC - 1 - sRC - sRC - s^2R^2C^2 - sRC}{R(1+sRC)}V_q = -sCV_{in}$$

$$-\frac{1+s^2R^2C^2}{R(1+sRC)}V_q = -sCV_{in}$$

$$G(s) = \frac{V_q(s)}{V_{in}(s)} = \frac{sRC(1+sRC)}{1+s^2R^2C^2} \qquad \text{3 pt} \qquad (3)$$

(d) Combining (2) and (3)

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{V_o(s)}{V_q(s)}\frac{V_q(s)}{V_{in}(s)} = \left(\frac{3sRC}{1+sRC}\right)\frac{sRC(1+sRC)}{1+s^2R^2C^2} = \frac{3s^2R^2C^2}{1+s^2R^2C^2} \qquad \text{2 pt}$$

(e) This is a second order high-pass filter. 1 pt