COMP 2711H Discrete Mathematical Tools for Computer Science
Solutions to Tutorial 3

QB2-1. Determine the digit in the unit's place of the following numbers: (a) 3^{70}, and (b) 9^{1573}.

Solution: The digit in the unit's place of number a is a mod 10. We work modulo 10.

(a) Since $3^2 \equiv -1$, we have that

$$3^{70} \equiv (3^2)^{35} \equiv (-1)^{35} \equiv -1 \equiv 9.$$

(b) Since $9^2 \equiv 1$, we have that

$$9^{1573} \equiv 9^{2 \cdot 786} \cdot 9 \equiv (1^{786} \cdot 9) \equiv 9.$$

EP2-7. (a) Let a be a positive integer. Show that $gcd(a, a-1) = 1$.
(b) Use the result of part (a) to solve the (Diophantine) equation $a + b = ab$, i.e., to find positive integers a, b solving the equation.

Solution:

(a) $gcd(a, a-1) = gcd(a-(a-1), a-1) = gcd(1, a-1) = 1$.

(b) $a = ab - b = (a-1) \cdot b$. We have $(a-1)|a$. With the result of (a), we know that $a - 1 = 1$ and the only solution is $a = 2, b = 2$.

EP2-8. Compute the results of the following statements:

(a) $5 \cdot 8 \mod 9$

(b) $(451 \cdot 25 + 7 \cdot 8) \mod 41$

(c) $2^{30} \mod 15$

Solution:

(a) $5 \cdot 8 \equiv 5 \cdot (-1) \equiv -5 \equiv 4 \mod 9$

(b) $(451 \cdot 25 + 7 \cdot 8) \equiv (41 \cdot 11 \cdot 25 + 56) \equiv 56 \equiv 15 \mod 41$

(c) $2^4 = 16 \equiv 1 \mod 15$
$2^{30} = (2^4)^7 \cdot 2^2 \equiv 4 \mod 15$

EP2-16. Given that $k \mod 4 = 3$, find $(9k^{333} + 22) \mod 4$.

Solution: $9k^{333} + 22 \equiv 9 \cdot 3^{333} + 22 \equiv 9 \cdot -1^{333} + 22 \equiv 9 \cdot -1 + 22 = 13 \equiv 1 \mod 4$