COMP 2711H Discrete Mathematical Tools for Computer Science
Tutorial Problems: Combinatorics

Problem 1. An ice cream parlor has 28 different flavors, 8 different kinds of sauce, and 12 different toppings.

(a) In how many different ways can a dish of three scoops of ice cream be made where each flavor can be used more than once and the order of the scoops does not matter?

(b) How many different kinds of small sundaes are there if a small sundae contains one scoop of ice cream, a sauce, and a topping?

(c) How many different kinds of large sundaes are there if a large sundae contains three scoops of ice cream, where each flavor can be used more than once and the order of the scoops does not matter; two kinds of sauce, where each sauce can be used only once and the order of the sauces does not matter; and three toppings, where each topping can be used only once and the order of the toppings does not matter?

Problem 2. Show that given any set of 10 distinct positive integers not exceeding 50, there exist at least two different five-element subsets of this set that have the same sum.

Problem 3. Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

Problem 4. Show that in any set of $n+1$ positive integers not exceeding $2n$ there must be two that are relatively prime.

Problem 5. Give a combinatorial proof that $\sum_{k=1}^{n} kC(n,k) = n2^{n-1}$. (*Hint:* Count in two ways the number of ways to select a committee and to then select a leader of the committee.)

Problem 6. How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 10,$$

where x_1, x_2, and x_3 are nonnegative integers?

Problem 7. A professor packs her collection of 30 issues of a mathematics journal in three boxes with 10 issues per box. How many ways can she distribute the journals if

(a) each box is numbered, so that they are distinguishable?

(b) the boxes are identical, so that they cannot be distinguished?

Problem 8. How many ways are there to seat six boys and eight girls in a row of chairs so that no two boys are seated next to each other?

Problem 9. Suppose that p and q are distinct primes. Use the principle of inclusion-exclusion to find $\phi(pq)$, the number of integers not exceeding pq that are relatively prime to pq.