

**COMP 2711H Discrete Mathematical Tools for Computer Science**  
**2021 Fall Semester**  
**Homework 1: Logic and Proofs**  
**Handed out: Sep 15**  
**Due: Sep 29**

**Problem 1.** Write the converse and contrapositive of each statement.

- (a) If the hose is 60 ft long, then the hose will reach the tomatoes.
- (b) George goes for a walk only if Mary goes for a walk.
- (c) Pamela recites a poem if Andre asked for a poem.

**Problem 2.** Show that

$$\neg(s \vee t) \vee \neg(s \vee \neg t)$$

is logically equivalent to  $\neg s$ . Present two proofs of this fact—one, using truth tables, and the other without truth tables.

**Problem 3.** Show that  $(\forall x P(x)) \vee (\forall x Q(x))$  and  $\forall x (P(x) \vee Q(x))$  are not logically equivalent.

**Problem 4.** Let  $P(x, y)$  be a propositional function. Is the following implication a tautology:

$$\forall y \exists x P(x, y) \rightarrow \exists x \forall y P(x, y)$$

Justify your answer fully.

**Problem 5.** Is the statement  $p \rightarrow q$  logically equivalent to the statement  $\neg p \rightarrow \neg q$ ? Justify your answer fully.

**Problem 6.** Answer each part of this question with full justification.

- (a) Does  $\vee$  distribute over  $\wedge$ ?
- (b) Does  $\vee$  distribute over  $\oplus$ ?
- (c) Does  $\wedge$  distribute over  $\oplus$ ?

**Problem 7.** Construct a contrapositive proof that for all real numbers  $x$ , if  $x^2 - 2x \neq 8$ , then  $x \neq 4$ .

**Problem 8.** Construct a proof by contradiction that for all real numbers  $x$ , if  $x^2 - 2x \neq 8$ , then  $x \neq 4$ .

**Problem 9.** Prove or disprove the following statement: For every positive integer  $n$ , if  $n$  is prime, then 12 and  $n^3 - n^2 + n$  have a common factor greater than 1.

**Problem 10.** Prove that there is no largest prime number.

**Problem 11.** Prove that between every two rational numbers there is an irrational number.

**Problem 12.** Using the shorthand notation for quantifiers, how would you rewrite Euclid's division theorem taught in class. Note that your formula will involve four variables representing the divisor, dividend, quotient and remainder. You may use  $Z$  to stand for the set of integers,  $Z^+$  to stand for the set of positive integers, and  $N$  to stand for the set of nonnegative integers.