MATH2011 Intro to Multivariable Calculus (Fall 2013)

Final Examination Name:__________________ Student I.D.:__________________

20 Dec 2013 *4:30–7:30pm* Seat Number:__________________

DIRECTIONS:

- Do **NOT** open the exam until instructed to do so.
- All mobile phones and pagers should be switched **OFF** during the examination.
- You must show the steps in order to receive full credits.
- Electronic calculators are **NOT** allowed.
- This is a closed book examination.
- Answer **ALL** questions.

THE HKUST ACADEMIC HONOR CODE

Honesty and integrity are central to the academic work of HKUST. Students of the University must observe and uphold the highest standards of academic integrity and honesty in all the work they do throughout their program of study.

As members of the University community, students have the responsibility to help maintain the academic reputation of HKUST in its academic endeavors.

Sanctions will be imposed on students, if they are found to have violated the regulations governing academic integrity and honesty.

Declaration of Academic Integrity

I confirm that I have answered the questions using only materials specified approved for use in this examination, that all the answers are my own work, and that I have not received any assistance during the examination.

Student's Signature: ______________________

Question No.	Points	Out of
1		4
2		4
3		8
4		8
5		8
6		8
7		20
Total		60

Answer all questions. Show all your work for full credit.

1. Let $\mathbf{u} = -3\,\mathbf{i} + 4\,\mathbf{j} + 0\,\mathbf{k}$ and $\mathbf{v} = 0\,\mathbf{i} + 2\,\mathbf{j} - 5\,\mathbf{k}$. Compute $\mathbf{u} \cdot \mathbf{v}$ and $\mathbf{u} \times \mathbf{v}$.

$\mathbf{u} \cdot \mathbf{v} = \boxed{}$ and $\mathbf{u} \times \mathbf{v} = \boxed{}$.

2. Let $\mathbf{r}(t) = (\sin t)\,\mathbf{i} + (t^2 - \cos t)\,\mathbf{j} + e^t\,\mathbf{k}$. Find the equation of the tangent line of $\mathbf{r}(t)$ at $t = 0$.

The equation of the tangent line at $t = 0$ is $\boxed{}$.

3. Consider the function $f(x,y) = 3x^2 - 2y^3 + 6xy + 5$.

   (a) Find all critical points of this function.

   The critical points of $f(x,y)$ are $\boxed{}$.

   (b) Find the relative maximum, relative minimum and saddle points of the function, if any.

4. Let $\mathcal{D}$ be the region in the first octant (i.e. $x \geq 0, y \geq 0$ and $z \geq 0$) bounded by the plane $x + y = 4$ and the paraboloid $z = 16 - x^2 - y^2$.

   (a) Write down a triple integral to determine the volume of the region $\mathcal{D}$ by filling in the limits of the integrals.

$$\text{Volume} = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} 1 \, dz \, dy \, dx$$

   (b) Evaluate the above integral.

5. (a) Find a parametrization of the cone $z = 2\sqrt{x^2 + y^2}$, i.e. find a vector-valued function which has two variables that traces out the surface of the given cone.

(b) Using (a), or otherwise, find the surface area of the portion of the cone $z = 2\sqrt{x^2 + y^2}$ between the planes $z = 2$ and $z = 6$.

The surface area is $\boxed{}$.

6. Let

$$\mathbf{F} = (\sin z)\,\mathbf{i} + (y + \cos x)\,\mathbf{j} + (xe^y + z)\,\mathbf{k}\,.$$

(a) Compute $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.

$$\nabla \cdot \mathbf{F} = \boxed{}\,.$$

$$\nabla \times \mathbf{F} = \boxed{}\,.$$

(b) Let $\mathcal{S}$ by the closed surface of a solid region bounded above by $z = 1$, bounded below by $z = 0$ and bounded laterally by the cylinder $x^2 + y^2 = 1$. Using (a) and an appropriate theorem, or otherwise, evaluate the flux

$$\iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n} \, d\sigma$$

where $\mathbf{F}(x, y, z) = (\sin z)\,\mathbf{i} + (y + \cos x)\,\mathbf{j} + (xe^y + z)\,\mathbf{k}$ and $\mathbf{n}$ is the unit outward normal on S.

$\iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n} \, d\sigma = \boxed{}$.

7. (a) Determine if the following vector fields are conservative. If so, find a potential of the vector field.

$$\mathbf{E}(x,y) = (-2y+e^x \sin y)\,\mathbf{i}+(e^x \cos y)\,\mathbf{j} \ \text{ and } \ \mathbf{F}(x,y) = (-2y+e^x \sin y)\,\mathbf{i}+(-2x+e^x \cos y)\,\mathbf{j}\,.$$

(b) Let $\mathcal{C}_1$ be the upper half unit circle centered at $(0,0)$ lying on the xy-plane oriented in the counterclosewise direction from $(1,0)$ to $(-1,0)$. Evaluate

$$\int_{\mathcal{C}_1} \mathbf{G} \cdot d\mathbf{r}$$

where $\mathbf{G}(x,y) = 0\,\mathbf{i} + x\,\mathbf{j}$.

$\int_{\mathcal{C}_1} \mathbf{G} \cdot d\mathbf{r} =$ $\boxed{}$.

(c) Using (a) and (b), or otherwise, calculate the line integral

$$\int_{\mathcal{C}_1} \mathbf{E} \cdot d\mathbf{r}$$

where $\mathbf{E}(x, y) = (-2y + e^x \sin y)\,\mathbf{i} + (e^x \cos y)\,\mathbf{j}$ and $\mathcal{C}_1$ is the upper half unit circle centered at $(0, 0)$ lying on the xy-plane oriented in the counterclosewise direction from $(1, 0)$ to $(-1, 0)$.

- continue -

$\int_{\mathcal{C}_1} \mathbf{E} \cdot d\mathbf{r} = \boxed{}$.

(d) Using the Green's Theorem, or otherwise, evaluate

$$\oint_{\mathcal{C}_2} \mathbf{G} \cdot d\mathbf{r}$$

where $\mathbf{G}(x, y) = 0\,\mathbf{i} + x\,\mathbf{j}$ and $\mathcal{C}_2$ is the unit circle centered at $(0, 0)$ lying on the xy-plane oriented in the counterclosewise direction.

$\oint_{\mathcal{C}_2} \mathbf{G} \cdot d\mathbf{r} =$ $\boxed{}$.

(e) Let S be the portion of the paraboloid where $z = 1 - x^2 - y^2$ which lies above the xy-plane with unit normal $\mathbf{n}$ pointing upward. Using (a) and (d), or otherwise, compute

$$\iint_S (\nabla \times \mathbf{H}) \cdot \mathbf{n} \, d\sigma = \iint_S (\operatorname{curl} \mathbf{H}) \cdot \mathbf{n} \, d\sigma \,,$$

where $\mathbf{H}(x, y, z) = (-2y + e^x \sin y)\,\mathbf{i} + [(e^x \cos y) + \sin z]\,\mathbf{j} + z\,\mathbf{k}$.

- continue -

$$\iint_S (\nabla \times \mathbf{H}) \cdot \mathbf{n} \, d\sigma = \iint_S (\operatorname{curl} \mathbf{H}) \cdot \mathbf{n} \, d\sigma = \boxed{}.$$