

WeBWork Homework-3 due 10/30/2020 at 05:00pm HKT

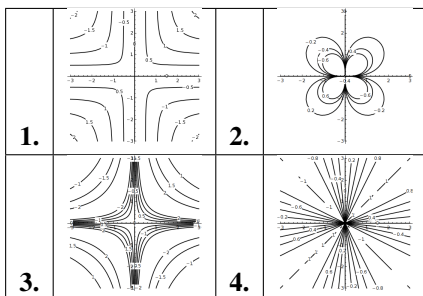
You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

1. (2 points) Consider the function

$$f(x,y) = \begin{cases} \frac{2xy}{(x^2+y^2)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(a) Use a computer to draw a contour diagram for f . Which of the following is the contour diagram?

- ?
- figure 1
- figure 2
- figure 3
- figure 4



(b) Is f differentiable at all points $(x,y) \neq (0,0)$? [?/yes/no]

(c) Calculate the partial derivatives of f for $(x,y) \neq (0,0)$:

$$f_x = \underline{\hspace{2cm}}$$

$$f_y = \underline{\hspace{2cm}}$$

Do the partial derivatives f_x and f_y exist and are they continuous at all points $(x,y) \neq (0,0)$?

- ?
- they don't exist at at least one point
- they exist but aren't continuous at at least one point
- they exist and are continuous at all points

(d) A first test for whether f is differentiable at $(0,0)$ is to see if it is continuous there. Calculate each of the following limits to determine if f is continuous at $(0,0)$:

$$\lim_{h \rightarrow 0} f(0,h) = \underline{\hspace{2cm}}$$

$$\lim_{h \rightarrow 0} f(h,0) = \underline{\hspace{2cm}}$$

$$\lim_{h \rightarrow 0} f(h,h) = \underline{\hspace{2cm}}$$

(In each case, enter **DNE** if the limit does not exist.)

Is f continuous at $(0,0)$?

- ?
- yes
- no
- it is not possible to tell

Is f differentiable at $(0,0)$?

- ?
- yes
- no
- it is not possible to tell

(e) Find the partial derivative f_x at $(0,0)$ by calculating it directly with a limit:

$$f_x = \lim_{h \rightarrow 0} \frac{1}{h} (f(\underline{\hspace{1cm}}, \underline{\hspace{1cm}}) - f(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})) = \underline{\hspace{2cm}}$$

Do the partial derivatives f_x and f_y exist and are they continuous at $(0,0)$? (Hint: to test continuity, you may want to use a similar calculation as you used to test the continuity of f)

- ?
- yes
- no
- it is not possible to tell
- they exist but are not continuous

(Be sure that you can justify all of your answers in this problem.)

Answer(s) submitted:

- figure 2
- yes
- $(-2(3x^2 - y^2)y) / (x^2 + y^2)^3$
- $(-2(3y^2 - x^2)x) / (x^2 + y^2)^3$
- they exist and are continuous at all points
- 0
- 0
- DNE
- no
- no
- h
- 0
- 0
- 0
- they exist but are not continuous

(correct)

2. (1 point) Find the limit of the function

$$f(x,y) = \frac{\sin(5\sqrt{x^2+y^2})}{5\sqrt{x^2+y^2}}$$

as $(x,y) \rightarrow (0,0)$. Assume that polynomials, exponentials, logarithmic, and trigonometric functions are continuous. [Hint: $\lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$.]

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(5\sqrt{x^2+y^2})}{5\sqrt{x^2+y^2}} = \underline{\hspace{2cm}}$$

(Enter **DNE** if the limit does not exist.)

Answer(s) submitted:

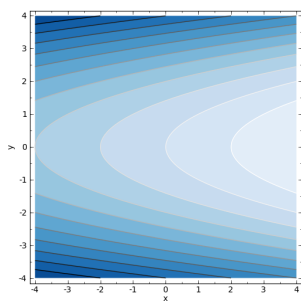
- 1

(correct)

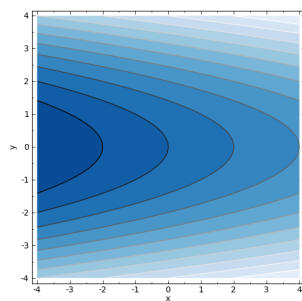
3. (2 points)

Match each function with its contour plot. Click on a graph to make it larger. Darker areas represent lower elevations and lighter areas represent higher elevations.

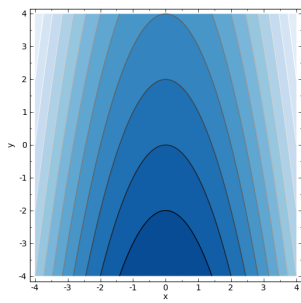
- ☐ 1. $f(x, y) = y + x^2$
- ☐ 2. $f(x, y) = x - y^2$
- ☐ 3. $f(x, y) = y - x^2$
- ☐ 4. $f(x, y) = x + y^2$



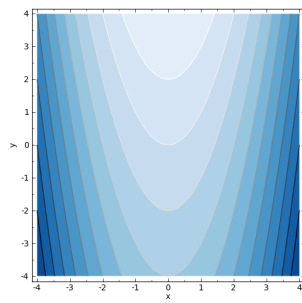
A



B



C



D

Answer(s) submitted:

- C
- A
- D
- B

(correct)

4. (1 point) Find the partial derivatives indicated. Assume the variables are restricted to a domain on which the function is defined.

$$z = (x^3 + x - y)^6.$$

$$\frac{\partial z}{\partial x} = \underline{\hspace{2cm}}$$

$$\frac{\partial z}{\partial y} = \underline{\hspace{2cm}}$$

Answer(s) submitted:

- $6(x^3 + x - y)^5(3x^2 + 1)$

- $6(x^3 + x - y)^5(-1)$

(correct)

5. (1 point)

Find the linear approximation of the function $z = x\sqrt{y}$ at the point $(-7, 4)$.

$$L(x, y) = \underline{\hspace{2cm}}$$

Answer(s) submitted:

- $-7\sqrt{4} + (x+7)\sqrt{4} + (y-4) \cdot (-7(1/2)/\sqrt{4})$

(correct)

6. (1 point) If

$$z = \cos\left(\frac{y}{x}\right), \quad x = 4t, \quad y = 2 - t^2,$$

find dz/dt using the chain rule. Assume the variables are restricted to domains on which the functions are defined.

$$\frac{dz}{dt} = \underline{\hspace{2cm}}$$

Answer(s) submitted:

- $-\sin((2-t^2)/(4t))(-2t \cdot 4t - (2-t^2) \cdot 4)/(4t)^2$

(correct)

7. (2 points) Let $F(u, v)$ be a function of two variables. Let $F_u(u, v) = G(u, v)$, and $F_v(u, v) = H(u, v)$. Find $f'(x)$ for each of the following cases (your answers should be written in terms of G and H).

(a) $f(x) = F(x, 2)$: then

$$f'(x) = \underline{\hspace{2cm}}$$

(b) $f(x) = F(1, x)$: then

$$f'(x) = \underline{\hspace{2cm}}$$

(c) $f(x) = F(x, x)$: then

$$f'(x) = \underline{\hspace{2cm}}$$

(d) $f(x) = F(2x, x^4)$: then

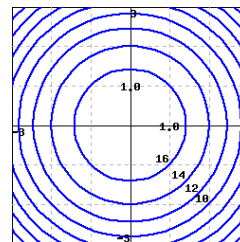
$$f'(x) = \underline{\hspace{2cm}}$$

Answer(s) submitted:

- $G(x, 2)$
- $H(1, x)$
- $G(x, x) + H(x, x)$
- $2 \cdot G(2x, x^4) + 4x^3 \cdot H(2x, x^4)$

(correct)

8. (2 points) Use the contour diagram of f in the figure below to decide if the specified directional derivatives below are positive, negative, or approximately zero.



(a) At point $(-2, 2)$, in direction \vec{i} : $f_{\vec{i}}$ is

- ?
 - positive
 - negative
 - approximately zero
- (b) At point $(0, -2)$, in direction $-\vec{j}$: $f_{\vec{u}}$ is
- ?
 - positive
 - negative
 - approximately zero
- (c) At point $(-1, 1)$, in direction $-\vec{i} + \vec{j}$: $f_{\vec{u}}$ is
- ?
 - positive
 - negative
 - approximately zero
- (d) At point $(-1, 1)$, in direction $\vec{i} + \vec{j}$: $f_{\vec{u}}$ is
- ?
 - positive
 - negative
 - approximately zero
- (e) At point $(0, -2)$, in direction $\vec{i} + 2\vec{j}$: $f_{\vec{u}}$ is
- ?
 - positive
 - negative
 - approximately zero
- (f) At point $(0, -2)$, in direction $-\vec{i} - 2\vec{j}$: $f_{\vec{u}}$ is
- ?
 - positive
 - negative
 - approximately zero

Answer(s) submitted:

- positive
- negative
- negative
- approximately zero
- positive
- negative

(correct)

9. (1 point) If the gradient of f is $\nabla f = 3y\vec{i} + x^2\vec{j} + 2zx\vec{k}$ and the point $P = (-8, -1, 2)$ lies on the level surface $f(x, y, z) = 0$, find an equation for the tangent plane to the surface at the point P .

$z =$ _____

Answer(s) submitted:

- $(-3/32)x + 2y + (104/32)$

(correct)

10. (1 point) You are standing above the point $(2, 3)$ on the surface $z = 25 - (3x^2 + 3y^2)$.

(a) In which direction should you walk to descend fastest? (Give your answer as a unit 2-vector.)

direction = _____

(b) If you start to move in this direction, what is the slope of your path?

slope = _____

Answer(s) submitted:

- $\langle (12)/\sqrt{468}, 18/\sqrt{468} \rangle$
- $-\sqrt{468}$

(correct)