ELEC2400 | **ELECTRONIC CIRCUITS** | FALL 2021-22

HOMEWORK 2 SOLUTION

Q1. Compute and write the answers in polar form, i.e., $r\angle\theta°$.

a. $\dfrac{2j}{10\angle 30°} = \dfrac{2\angle 90°}{10\angle 30°} = 0.2\angle 60°$

Note: $j = e^{j\frac{\pi}{2}} = \angle 90°$

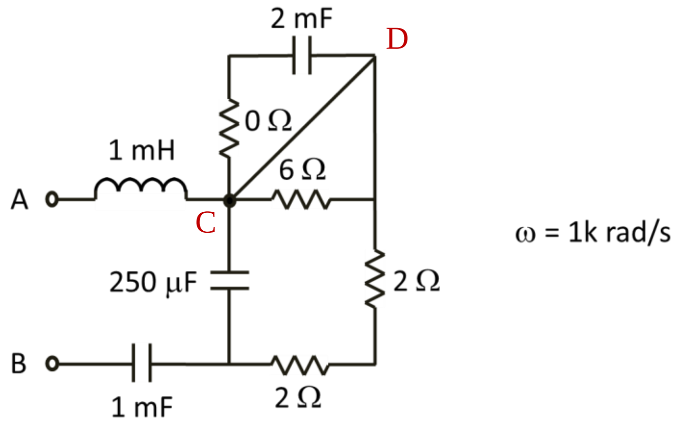
$-1 = e^{\pm j\pi} = \angle(\pm 180°)$

b. $\dfrac{15e^{j\frac{\pi}{2}}}{-3} = \dfrac{15\angle 90°}{-3} = -5\angle 90° = 5\angle(-90°)$

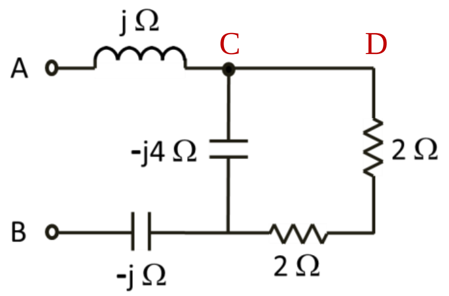
c. $\dfrac{5\angle 60°}{j+2} = \dfrac{5\angle 60°}{2.236\angle 26.57°} = 2.24\angle 33.4°$

d. $j^j = \left(e^{j\frac{\pi}{2}}\right)^j = e^{j^2\frac{\pi}{2}} = e^{-\frac{\pi}{2}} = 0.208\angle 0°$

Q2. Find the impedance Z_{AB} and write the answer in rectangular form.



The diagonal short reduces the impedance between C and D to zero. The circuit is then simplified to



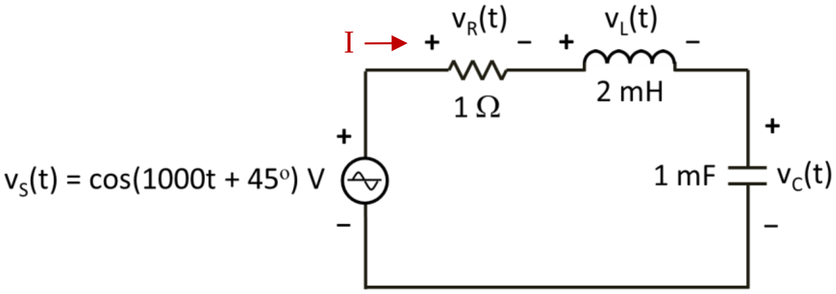
in which the impedances of the inductor and capacitors are computed and as indicated.

By direct observation,

$$Z_{AB} = j + (-j4)||(2+2) - j = -j4||4 = \frac{-j16}{4-j4} = \frac{-j4}{1-j}$$

$$= \left(\frac{-j4}{1-j}\right)\left(\frac{1+j}{1+j}\right) = \frac{4-j4}{2} = 2 - j2\,\Omega$$

Q3. Consider the following circuit.



(a) Draw all the impedances (Z_R, Z_L and Z_C) in one impedance diagram.
(b) Draw all the phasor voltages (V_S, V_R, V_L and V_C) in one phasor diagram.
(c) Evaluate $v_S(t)$, $v_R(t)$, $v_L(t)$ and $v_C(t)$ at t = 1 ms.

(a) $\omega = 1000$ rad/s. Hence

$$Z_R = 1\,\Omega,\ Z_L = j1000 \times 2\text{m} = j2\,\Omega,\ Z_C = 1/(j1000 \times 1\text{m}) = -j\,\Omega$$

$$Z_{Total} = 1 + j2 - j = 1 + j\,\Omega$$

(b) Since $V_S = 1\angle 45° = 0.7071(1+j)$ V, $I = V_S/Z_{Total} = 0.7071$ A

Hence $V_R = IZ_R = 0.7071$ V, $V_L = IZ_L = j1.4142$ V, $V_C = IZ_C = -j0.7071$ V

(c) At $t = 1$ ms,

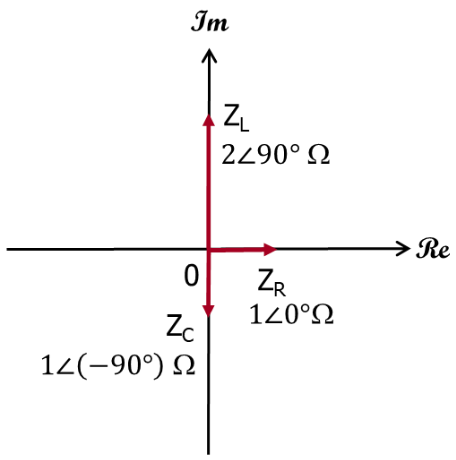
$$v_S = \cos(1000t + 45°) = \cos(1 + \pi 45°/180°) = -0.213\text{ V}$$

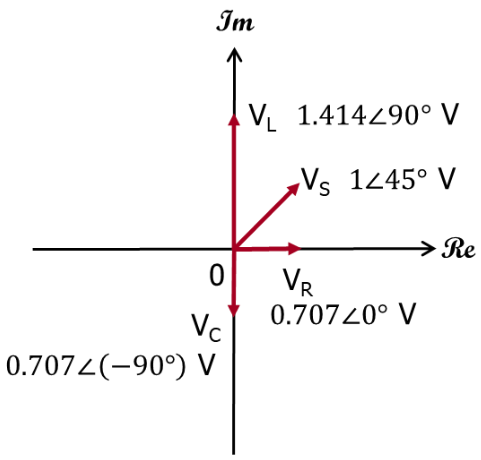
$$v_R = 0.7071\cos(1000t) = 0.7071\cos(1) = 0.382\text{ V}$$

$$v_L = 1.4142(1000t + 90°) = 1.4142\cos(1 + \pi 90°/180°) = -1.19\text{ V}$$

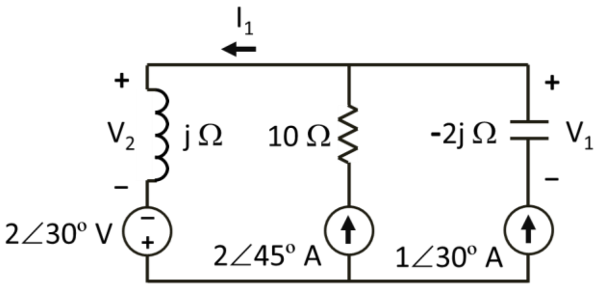
$$v_C = 0.7071(1000t - 90°) = 0.7071\cos(1 - \pi 90°/180°) = 0.595\text{ V}$$

Note that $v_S = v_R + v_L + v_C$. The impedance and phasor diagrams are shown below.





Q4. Find I_1, V_1 and V_2, and write the answers in polar form.

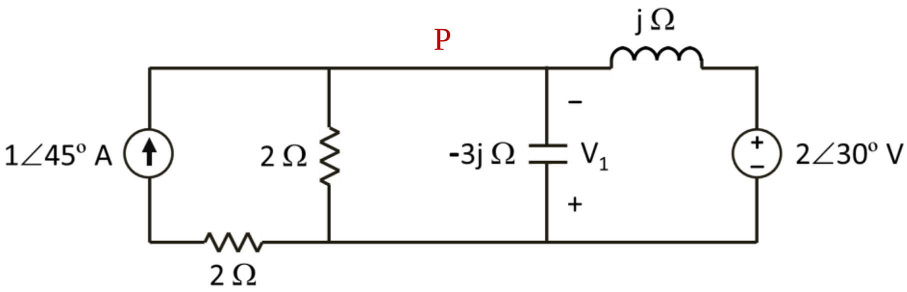


$$I_1 = 2\angle 45° + 1\angle 30° = \sqrt{2}(1 + j) + 0.5(\sqrt{3} + j) = 2.98\angle 40.0° \text{ A}$$

$$V_1 = -1\angle 30°\ (-j2) = 2\angle 30° \angle 90° = 2\angle 120° \text{ V}$$

$$V_2 = I_1 j = 2.98\angle 40.0° \angle 90° = 2.98\angle 130° \text{ V}$$

Q5. Find V_1 and write the answer in polar form.



Apply KCL to node P:

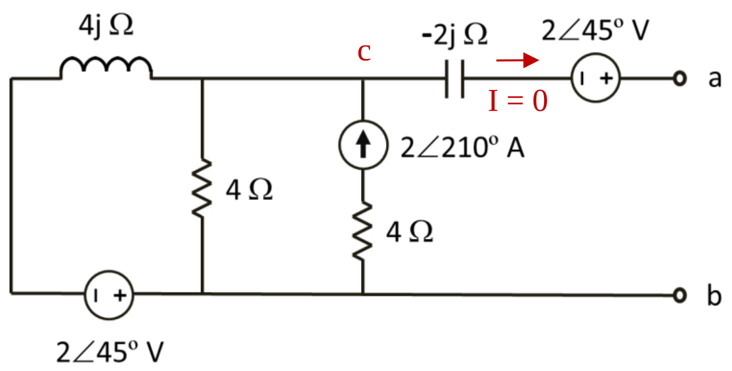
$$1\angle 45° + \frac{V_1}{2} + \frac{V_1}{-j3} + \frac{V_1 + 2\angle 30°}{j} = 0$$

$$3\sqrt{2}(1 + j) + 3V_1 + j2V_1 - j6V_1 - j6(\sqrt{3} + j) = 0$$

$$(3 - j4)V_1 = (-6 - 3\sqrt{2}) + j(6\sqrt{3} - 3\sqrt{2}) = -10.243 + j6.150$$

$$V_1 = \frac{-10.243 + j6.150}{3 - j4} = \frac{11.95\angle 149.0°}{5\angle(-53.13°)} = 2.39\angle(-158°) \text{ V}$$

Q6. Find and draw the Thevenin's equivalent circuit with respect to the terminals a and b, and write the answers in polar form.



Apply KCL at node c:

$$\frac{V_{cb}}{4} + \frac{V_{cb} + 2\angle 45^\circ}{j4} = 2\angle 210^\circ$$

$$jV_{cb} + V_{cb} + 2\angle 45^\circ = j8\angle 210^\circ$$

$$V_{cb} = \frac{8\angle 300^\circ - 2\angle 45^\circ}{\sqrt{2}\angle 45^\circ} = 4\sqrt{2}\angle 255^\circ - \sqrt{2}$$

Open-circuit voltage:

$$V_{OC} = V_{ab} = V_{ac} + V_{cb} = 2\angle 45^\circ + \left(4\sqrt{2}\angle 255^\circ - \sqrt{2}\right)$$

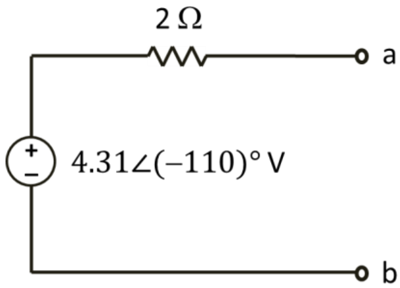
$$= -1.464 - j4.050 = 4.31\angle(-110)^\circ \text{ V}$$

With the voltage sources shorted and the current source opened,

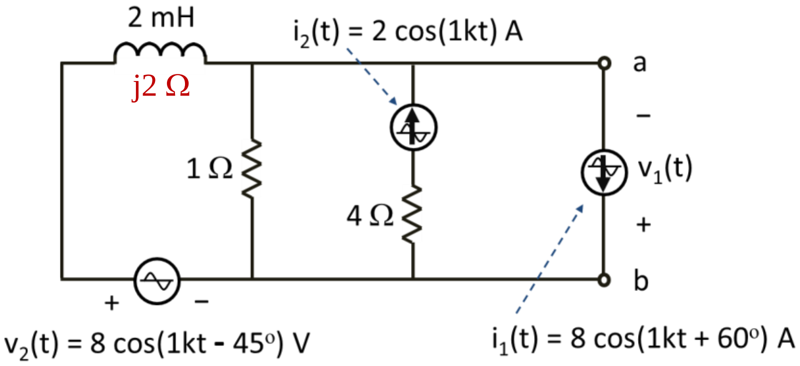
$$Z_{EQ} = Z_{ab} = -j2 + 4||j4 = -j2 + \frac{j16}{4 + j4} = -j2 + \frac{j4}{1 + j} = -j2 + \left(\frac{j4}{1 + j}\right)\left(\frac{1 - j}{1 - j}\right)$$

$$= -j2 + \frac{4 + j4}{2} = -j2 + 2 + j2 = 2\ \Omega$$

The Thevenin's equivalent circuit is therefore



Q7. Use Norton's theorem to find $v_1(t)$ in the network.



This problem can be solved directly by applying KCL at node a:

$$\frac{V_1 + 8\angle(-45°)}{j2} + \frac{V_1}{1} + 2\angle 0° = 8\angle 60° \qquad (1)$$

$$V_1 + 8\angle(-45°) + j2V_1 + j4 = j16\angle 60°$$

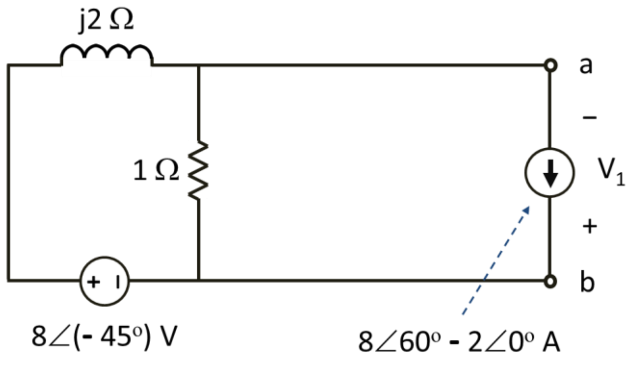
$$V_1 = \frac{16\angle 150° - 8\angle(-45°) - j4}{1 + j2} = \frac{21.77\angle 153.7°}{2.236\angle 63.43°} = 9.74\angle 90.2°$$

Hence $$v_1(t) = 9.74\cos(1kt + 90.2°)\ \text{V}$$

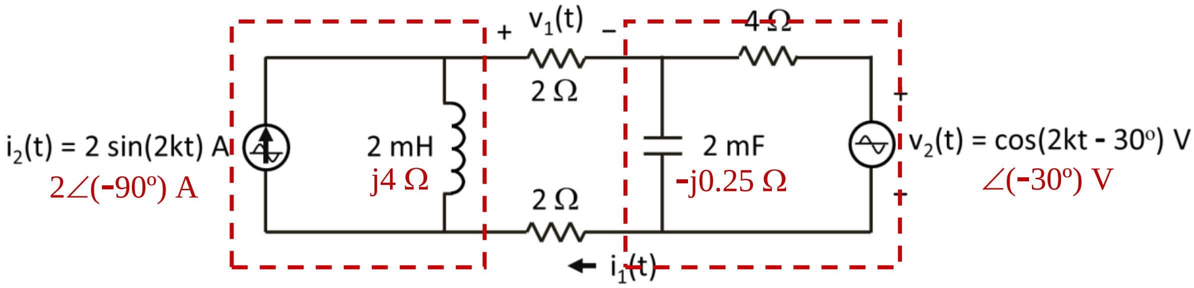
However, the question specifically asks for using Norton's theorem. We can easily meet this requirement by combining the two current sources on the right to their Norton's equivalent, for which

$$I_{SC} = 8\angle 60° - 2\angle 0°\ \text{A} \quad \text{and} \quad Z_{EQ} = \infty\ \Omega$$

The equivalent circuit is as shown below. Applying KCL at node a yields equation (1) and the same answer.

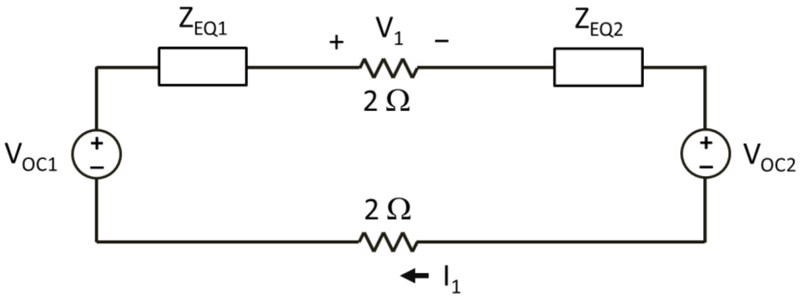


Q8. Use source transformation(s) to find $i_1(t)$ and $v_1(t)$ in the network.



The left and right parts of the circuit (shown inside the dotted boxes) can be replaced by their respective Thevenin's equivalences. The impedances of the capacitor and inductor, and the phasor current and voltage are as marked. Note that we must convert 2sin(2kt) into 2cos(2kt – 90º).

The phasor diagram of the equivalent circuit is shown below,



in which

$$Z_{EQ1} = j4\ \Omega, \qquad Z_{EQ2} = 4||(-j0.25) = \frac{-j}{4-j0.25} = 0.015564 - j0.249027\ \Omega$$

$$V_{OC1} = 2\angle(-90°) \times j4 = 8\ \text{V}$$

$$V_{OC2} = \angle(-30°) \times \frac{-j0.25}{4-j0.25} = -0.027759 - j0.055862\ V$$

Hence

$$I_1 = \frac{V_{OC1} - V_{OC2}}{Z_{EQ1} + Z_{EQ2} + 2 + 2} = \frac{8.027759 + j0.055862}{4.015564 + j3.750973} = 1.46\angle(-42.7°)\ \text{A}$$

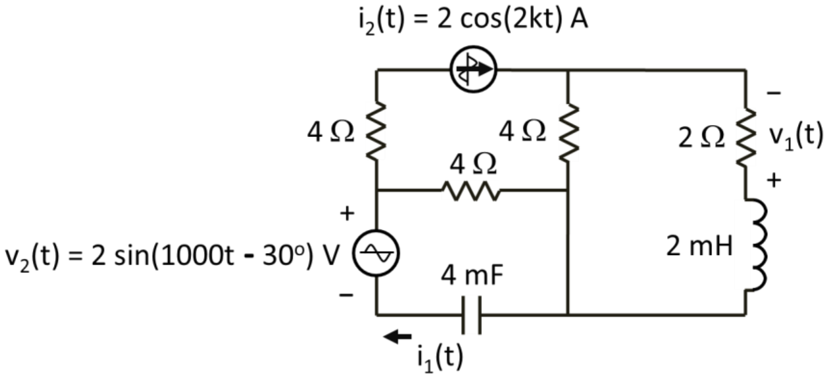
and

$$V_1 = 2I_1 = 2.92\angle(-42.7°)\ \text{V}$$

Finally,

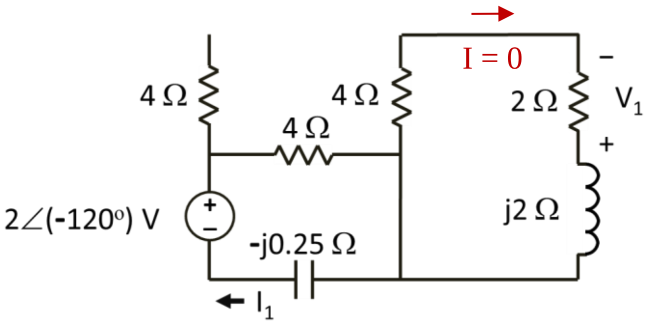
$$i_1(t) = 1.46\cos(2kt - 42.7°)\ \text{A}$$
$$v_1(t) = 2.92\cos(2kt - 42.7°)\ \text{V}$$

Q9. Find $i_1(t)$ and $v_1(t)$.



Note that the voltage and current sources are at different frequencies. We have no option but to use superposition.

Part 1: with only the voltage source, the circuit looks like this

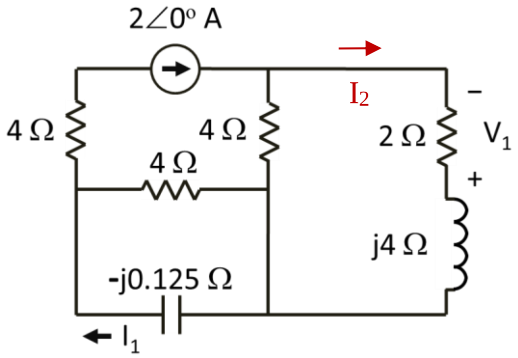


in which the voltage source 2sin(1000t – 30°) is converted to 2cos(1000t – 120°). Note that there is no current in the right mesh since there is no excitation. Hence

$$V_1 = 0$$

$$I_1 = \frac{2\angle(-120°)}{4 - j0.25} = \frac{2\angle(-120°)}{4.008\angle(-3.58°)} = 0.499\angle(-116°)$$

Part 2: with only the current source, the circuit looks like this



The middle and right vertical branches are connected in parallel. They form a current divider on the incoming current 2∠0° A. Hence

$$V_1 = -2I_2 = -2 \times 2\angle 0° \left(\frac{4}{4+2+j4}\right) = \frac{-8}{3+j2} = 2.22\angle 146°$$

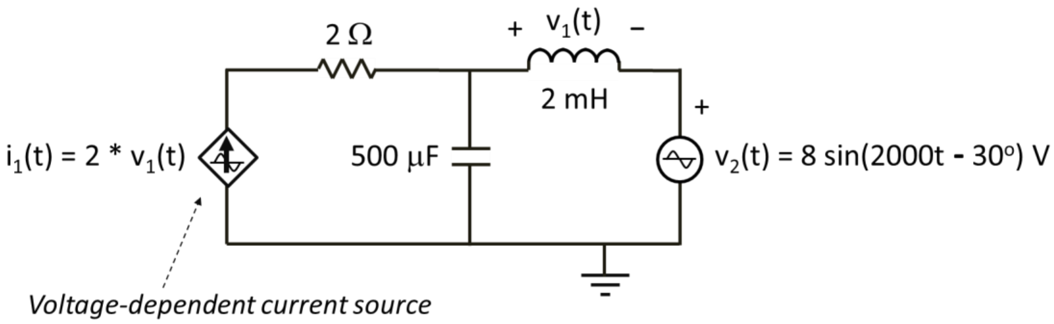
The bottom resistor and capacitor are also connected in parallel. They form a current divider on the outgoing current 2∠0° A. Hence

$$I_1 = 2\angle 0° \left(\frac{4}{4-j0.125}\right) = \frac{8}{4-j0.125} = 2.00\angle 2°$$

Finally, by superposition:

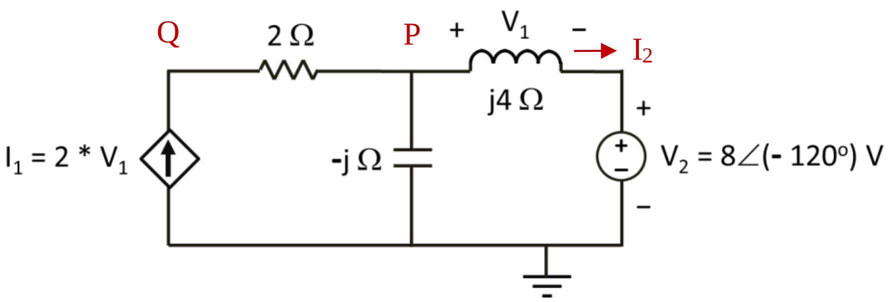
$$i_1(t) = 0.50\cos(1000t - 116°) + 2.00\cos(2000t + 2°) \text{ A}$$
$$v_1(t) = 2.22\cos(2000t + 146°) \text{ V}$$

Q10. Consider the following circuit.



(a) Compute the average AC power for each circuit element.
(b) Specify whether each circuit element is supplying AC power, absorbing AC power (dissipating power), or neither.
(c) Verify that the AC power balance is satisfied.

The voltage source 8sin(2000t – 30º) is converted to 8cos(2000t – 120º). The phasor circuit diagram is as shown below:



Apply KCL to node P:

$$2V_1 - \frac{8\angle(-120°) + V_1}{-j} - \frac{V_1}{j4} = 0$$

$$j8V_1 + 32\angle(-120°) + 4V_1 - V_1 = 0$$

$$V_1 = \frac{-32\angle(-120°)}{3 + j8} = 3.7453\angle(-9.444°)$$

Hence

$$I_1 = 2V_1 = 7.4906\angle(-9.444°)$$

$$I_2 = \frac{V_1}{j4} = 0.9363\angle(-99.444°)$$

$$V_Q = V_2 + V_1 + 2 \times 2V_1 = 8\angle(-120°) + 5 \times 3.7453\angle(-9.444°)$$

$$= 17.5920\angle(-34.645°)$$

Average power calculations:
Capacitor: average power is zero. It is neither supplying nor absorbing power.

Inductor: average power is zero. It is neither supplying nor absorbing power.

Resistor: average power

$$= \frac{|I_1|^2}{2} \times 2 = (7.4906)^2 = 56.11 \text{ W, absorbing.}$$

Independent voltage source: average power

$$= \frac{\mathcal{R}e(V_2 I_2^{\,*})}{2} \qquad (\mathcal{R}e\text{: real part,} \qquad * \text{: complex conjugate})$$

$$= \frac{\mathcal{R}e[8\angle(-120°) \times 0.9363\angle(+99.444°)]}{2}$$

$$= \frac{8 \times 0.9363\cos(-120° + 99.444°)}{2} = 3.51 \text{ W, absorbing.}$$

Dependent current source: average power

$$= \frac{\mathcal{R}e\left(-V_Q I_1^{\,*}\right)}{2}$$

$$= \frac{\mathcal{R}e[-17.5920\angle(-34.645°) \times 7.4906\angle(+9.444°)]}{2}$$

$$= \frac{-17.5920 \times 7.4906\cos(-34.645° + 9.444°)}{2} = -59.62 \text{ W, supplying.}$$

The sum of all the average powers is zero. Hence the AC power balance is satisfied.