# (T)EE2026

# Digital Fundamentals

## Number Systems

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**Modified from original slides by Prof. XU Yong Ping**

# Outline

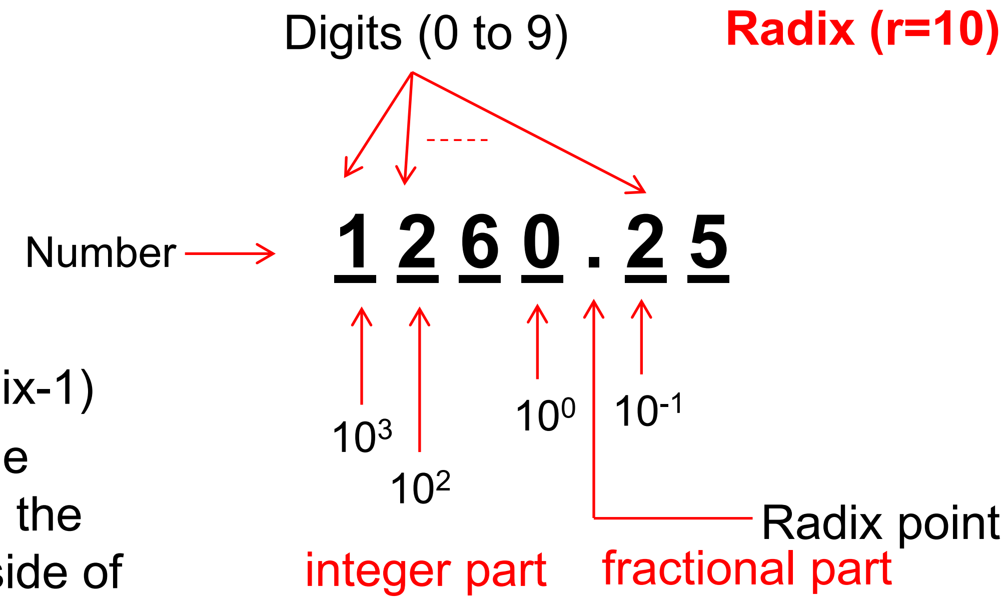
- Positional number system
- Radix conversion
- Binary arithmetic
- Binary signed representation
- Binary-coded decimal (BCD)

# Positional Number System

**Decimal number:**

**Radix (r=10)**

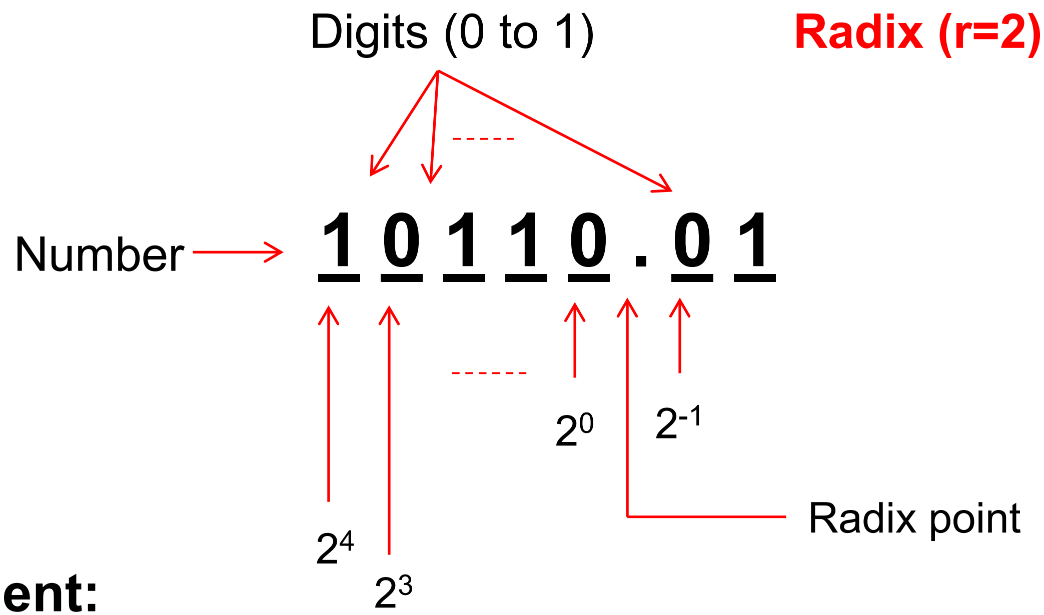
- **Terminologies**
  - Radix (or base)
  - Radix point
  - Digits and a numeral (0 → radix-1)
  - Place value (or weight) is in the power of the base (positive on the left and negative on the right side of the radix point




$$N = 1\times10^3 + 2\times10^2 + 6\times10^1 + 0\times10^0 + 2\times10^{-1} + 5\times10^{-2} = 1260.25$$

*Weighted sum of each digit (each digit is weighted by its place value)

# Binary number




**Decimal Equivalent:**

$$N_{10} = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 + 0\times 2^{-1} + 1\times 2^{-2}$$

$$= 16 + 0 + 4 + 2 + 0 + 0 + \frac{1}{4}$$

$$= 22.25$$

$(10110.01)_2 = (22.25)_{10}$

# Hexadecimal number

**Radix (r=16)**

Digits (0 to 15)

Number → **1 8 F 4 . 2 A**

$16^3$ $16^2$ $16^0$ $16^{-1}$ $16^{-2}$

Radix point

**Decimal Equivalent:**

$$N_{10} = 1\times16^3 + 8\times16^2 + F\times16^1 + 4\times16^0 + 2\times16^{-1} + 10\times16^{-2}$$

$$= 4096 + 2048 + 240 + 4 + \frac{2}{16} + \frac{10}{256}$$

$$= 6388 + \frac{21}{128}$$

$$\approx 6388.16$$

$(18F4.2A)_{16} \cong (6388.16)_{10}$

| Hex | Dec |
|---|---|
| **0** | 0 |
| **1** | 1 |
| **2** | 2 |
| **3** | 3 |
| **4** | 4 |
| **5** | 5 |
| **6** | 6 |
| **7** | 7 |
| **8** | 8 |
| **9** | 9 |
| **A** | 10 |
| **B** | 11 |
| **C** | 12 |
| **D** | 13 |
| **E** | 14 |
| **F** | 15 |

# Octal number

**Radix (r=8)**

Digits (0 to 7)

Number → $\underline{7}\ \underline{5}\ \underline{4}\ .\ \underline{2}$

$8^2$ ------ $8^0$ $8^{-1}$

Radix point

$(754.2)_8 = (520.25)_{10}$

| Oct | Dec |
|---|---|
| **0** | 0 |
| **1** | 1 |
| **2** | 2 |
| **3** | 3 |
| **4** | 4 |
| **5** | 5 |
| **6** | 6 |
| **7** | 7 |
| **?** | 8 |
| **?** | 9 |
| **?** | 10 |

**Decimal Equivalent:**

$$N_{10} = 7\times 8^2 + 5\times 8^1 + 4\times 8^0 + 2\times 8^{-1}$$

$$= 448 + 40 + 4 + \frac{2}{8}$$

$$= 492.25$$

# General form of A Number of radix r and its Decimal Equivalent

**General form of Number of radix r:**

Radix point

$$A_r = (a_n a_{n-1} \ldots a_o . a_{-1} \ldots a_{-m})_r$$

$$where \;\; a_n, a_{n-1}, \ldots, a_0, \ldots a_{-m} \in \{0, \ldots (r-1)\}$$ (Integer only)

**Decimal equivalent:**

Radix point is here

$$\begin{aligned} A_r &= (a_n a_{n-1} \ldots a_o . a_{-1} \ldots a_{-m})_r \\ &= a_n \times r^n + a_{n-1} \times r^{n-1} + \ldots a_o \times r^0 + a_{-1} \times r^{-1} + \ldots a_{-m} \times r^{-m} \\ &= \sum_{i=-m}^{n} a_i r^i \end{aligned}$$

***Weighted sum of all digits**

# Radix Conversion

**Three types of conversions:**

- Radix r ($r \neq 10$) → Decimal
- Decimal → Radix r ($r \neq 10$)
- Conversion among Binary, Octal and Hex numbers

# Radix r ($r \neq 10$) → Decimal

**Binary → Decimal** $(10110.01)_2 = (??)_{10}$

$$(10110.01)_2 \rightarrow 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 + 0\times 2^{-1} + 1\times 2^{-2} = (22.25)_{10}$$

**Hex → Decimal** $(18F4.2A)_{16} = (??)_{10}$

$$(18F4.2A)_{16} = 1\times 16^3 + 8\times 16^2 + F\times 16^1 + 4\times 16^0 + 2\times 16^{-1} + 10\times 16^{-2}$$
$$\approx (6388.16)_{10}$$

***Compute the weighted sum of all digits**

$$\begin{aligned} A_r &= (a_n a_{n-1} \dots a_o . a_{-1} \dots a_{-m})_r \\ &= a_n \times r^n + a_{n-1} \times r^{n-1} + \dots a_o \times r^0 + a_{-1} \times r^{-1} + \dots a_{-m} \times r^{-m} \\ &= \sum_{i=-m}^{n} a_i r^i \end{aligned}$$

# Decimal → Radix r (r ≠ 10)

**Decimal → Binary** $\quad (102)_{10} = (??)_2$

$$(102)_{10} = A_2 = (a_n a_{n-1} \ldots a_o . a_{-1} \ldots a_{-m})_r$$

$$= a_n \times 2^n + a_{n-1} \times 2^{n-1} + \ldots + a_1 \times 2^1 + a_o$$

(Assume integer)

$$= \underbrace{(a_n \times 2^n + a_{n-1} \times 2^{n-1} + \ldots + a_1 \times 2^1)}_{\text{Integer multiple of 2}} + a_o$$

$$\frac{(102)_{10}}{2} \to$$

Continue dividing <u>quotient</u> by 2

quotient $a_n \times 2^{n-1} + a_{n-1} \times 2^{n-2} + \ldots + a_1$

$$2 \overline{\smash{)}\, a_n \times 2^n + a_{n-1} \times 2^{n-1} + \ldots + a_1 \times 2 + a_o}$$

$$\underline{a_n \times 2^n + a_{n-1} \times 2^{n-1} + \ldots + a_1 \times 2}$$

$$a_o$$

Remainder is $a_0$

$$a_n \times 2^{n-2} + a_{n-1} \times 2^{n-3} + \ldots + a_1$$

$$2 \overline{\smash{)}\, a_n \times 2^{n-1} + a_{n-1} \times 2^{n-2} + \ldots + a_1}$$

$$\underline{a_n \times 2^{n-1} + a_{n-1} \times 2^{n-2} + \ldots}$$

$$a_1$$

Remainder is $a_1$

# Decimal → Radix r ($r \neq 10$) – cont.

**Decimal → Binary** $(102)_{10} = (??)_2$

| Division | Quotient | Remainder |
|---|---|---|
| 102/2 | 51 | 0 → $a_0$ |
| 51/2 | 25 | 1 → $a_1$ |
| 25/2 | 12 | 1 → $a_2$ |
| 12/2 | 6 | 0 → $a_3$ |
| 6/2 | 3 | 0 → $a_4$ |
| 3/2 | 1 | 1 → $a_5$ |
| 1/2 | 0 | 1 → $a_6$ |

**Stop when the quotient = 0**

$(102)_{10} = (1100110)_2$

**Check:**

$$
\begin{aligned}
N_{10} &= a_6 \times 2^6 + a_5 \times 2^5 + a_4 \times 2^4 + a_3 \times 2^3 \\
&\qquad + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0 \\
&= 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 \\
&\qquad + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
&= 64 + 32 + 0 + 0 + 4 + 2 + 0 \\
&= 102
\end{aligned}
$$

# How about Fractional Numbers?

**Decimal → Binary** **$(0.58)_{10} = (??)_2$**

$$
\begin{aligned}
(0.58)_{10} &= A_2 = (0.a_{-1}a_{-2}\ldots a_{-m+1}a_{-m})_r \\
&= a_{-1}\times 2^{-1} + a_{-2}\times 2^{-2} + \ldots + a_{-m+1}\times 2^{-m+1} + a_{-m}\times 2^{-m}
\end{aligned}
$$

**Multiply by 2:**

$$
(0.58)_{10}\times 2 = \underset{\text{Integer part is } a_{-1}}{\underset{\uparrow}{a_{-1}}} + \underbrace{a_{-2}\times 2^{-1} + \ldots + a_{-m+1}\times 2^{-m+2} + a_{-m}\times 2^{-m+1}}_{\text{fractional part}}
$$

# How about Fractional Numbers? – cont.

**Decimal → Binary** $(0.58)_{10} = (??)_2$

| Multiply by 2 | Product | Integer Part |
|---|---|---|
| 0.58x2 | 1.16 | 1 → $a_{-1}$ |
| 0.16x2 | 0.32 | 0 → $a_{-2}$ |
| 0.32x2 | 0.64 | 0 → $a_{-3}$ |
| 0.64x2 | 1.28 | 1 → $a_{-4}$ |
| 0.28x2 | 0.56 | 0 → $a_{-5}$ |
| 0.56x2 | 1.12 | 1 → $a_{-6}$ |
| 0.12x2 | 0.24 | 0 → $a_{-7}$ |
| 0.24x2 | 0.48 | 0 → $a_{-8}$ |

$(0.58)_{10} = (0.100101)_2$

**Check:**

$$
\begin{aligned}
N_{10} &= 1\times 2^{-1} + 1\times 2^{-4} + 1\times 2^{-6} \\
&= \frac{1}{2} + \frac{1}{16} + \frac{1}{64} \\
&= 0.578125 \\
&\approx 0.58
\end{aligned}
$$

- The conversion process may never end.
- Where to stop depends on the required precision
- The process only ends when fractional part = 0

# Numbers with Different Radixes: Summary

## Numbers with Different Radixes

| Decimal (radix 10) | Binary (radix 2) | Octal (radix 8) | Hexadecimal (radix 16) |
|---|---|---|---|
| 00 | 0000 | 00 | 0 |
| 01 | 0001 | 01 | 1 |
| 02 | 0010 | 02 | 2 |
| 03 | 0011 | 03 | 3 |
| 04 | 0100 | 04 | 4 |
| 05 | 0101 | 05 | 5 |
| 06 | 0110 | 06 | 6 |
| 07 | 0111 | 07 | 7 |
| 08 | 1000 | 10 | 8 |
| 09 | 1001 | 11 | 9 |
| 10 | 1010 | 12 | A |
| 11 | 1011 | 13 | B |
| 12 | 1100 | 14 | C |
| 13 | 1101 | 15 | D |
| 14 | 1110 | 16 | E |
| 15 | 1111 | 17 | F |

# Conversion among Hex, Octal and Binary

- Hex ←→ Binary
  - Each Hex digit → 4 Binary bits (digits)
  - Or each 4 Binary bits → 1 Hex digit (starting from radix point)

- Octal ←→ Binary
  - Each octal digit → 3 Binary bits
  - Or each 3 Binary bits → 1 Octal digit (starting from radix point)

- Hex ←→ Octal
  - Use Binary as an intermediate step
  - Hex → Binary → Octal
  - Octal → Binary → Hex

# Examples (Hex, Octal, Binary)

**Hex → Bin:**

$\mathbf{(A45F)_{16}}$

$(\underline{1010}\ \underline{0100}\ \underline{0101}\ \underline{1111})_2$

**Bin → Hex:**

$\mathbf{(\underline{11}\ \underline{1010}\ \underline{1101}\ \underline{0111})_2}$

$(3\ A\ D\ 7)_{16}$

**Oct → Bin:**

$\mathbf{(475)_8}$

$(\underline{100}\ \ \underline{111}\ \underline{101})_2$

**Bin → Oct:**

$\mathbf{(\underline{10}\ \underline{111}\ \underline{101}\ \underline{110})_2}$

$(2\ 7\ 5\ 6)_8$

# More Examples

**Hex → Oct:**

$$(\mathbf{A45F})_{16}$$

⇩

$(1\ \underline{010}\ \underline{010}\ \underline{001}\ \underline{011}\ \underline{111})_2$

$$(\mathbf{122137})_8$$

**Oct → Hex:**

$$(\mathbf{653})_8$$

⇩

$(\underline{1}\ \underline{1010}\ \underline{1011})_2$

$$(\mathbf{1\ A\ B})_{16}$$

For the fractional part: very similar, just group digits by starting from the position after the radix point

**Hex → Bin:**

$$(\mathbf{0\ .\ A45F})_{16}$$

⇩

$(0\ .\ \underline{1010}\ \underline{0100}\ \underline{0101}\ \underline{1111})_2$

# Radix Conversion: Generalization

- Radix r ($r \neq 10$) → Decimal
  - Compute weighted sum of all digits

- Decimal → Radix r ($r \neq 10$)
  - Integer → Divided by r and take the remainder
  - Fraction → Multiply by r and take the integer
  - Add integer and fraction parts

- Conversion among Binary, Octal and Hex numbers
  - 1 Hex digit = 4 Binary and 1 Oct = 3 Binary, vice versa
  - Hex→Oct: Hex→Binary→Octal, and vice versa. Binary is used as an intermediate step

# Binary Arithmetic

- Addition
- Multiplication
- Subtraction
- Division
- MSB and LSB
- Arithmetic using computer

# Addition

**Addition table:**

0 + 0 = 0
0 + 1 = 1
1 + 1 = 10

"1" is the carry to the next higher bit

Example:

10111 + 110 = 11101

```
    1  1      <---- Carry
   1 0 1 1 1
+      1 1 0
-------------
   1 1 1 0 1
```

# Multiplication

**Multiplication table:**

0 x 0 = 0
0 x 1 = 0
1 x 1 = 1

**Example:**

10111 x 110 = 10001010

```
         1 0 1 1 1     ← Multiplicand
    x        1 1 0     ← Multiplier
    ---------------
         0 0 0 0 0  }
       1 0 1 1 1    }  ← Partial products
  +  1 0 1 1 1      }
  -------------------
   1 0 0 0 1 0 1 0     ← Product
```

Multiplication:
→ Shift then Add
→ Only need “add” operation

# Subtraction

**Subtraction table:**

0 - 0 = 0
1 - 0 = 1
1 - 1 = 0
0 – 1 = 1 ← with a borrow from the next (higher) bit

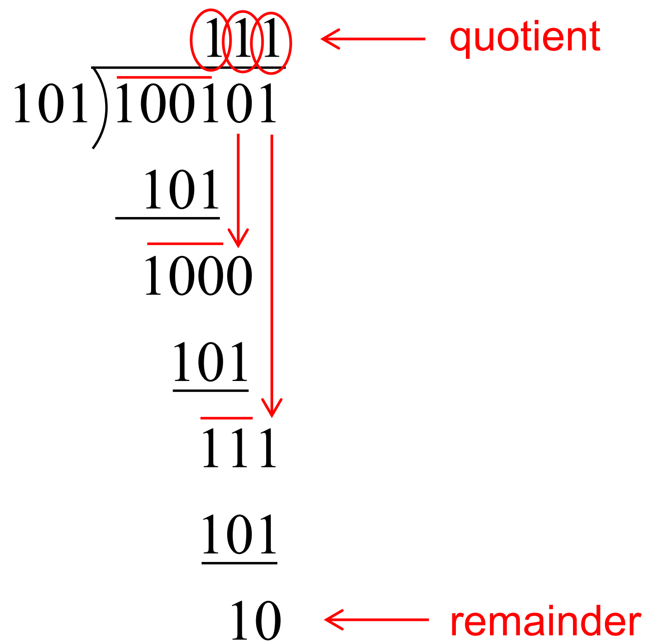
**Example:**

11011 - 110 = 10101

```
   1 1 0 1 1
-      1 1 0
--------------
   1 0 1 0 1
```

# Division

100101/101 = ?




Check in decimal

- **Set** quotient to 0
- Align leftmost digits in dividend and divisor
- **Repeat**
  - **If** that portion of the dividend above the divisor is greater than or equal to the divisor
    - **Then** subtract divisor from that portion of the dividend and
    - Concatenate 1 to the right hand end of the quotient
    - **Else** concatenate 0 to the right hand end of the quotient
  - Shift the divisor one place right
- **Until** dividend is less than the divisor
- quotient is correct, dividend is remainder
- **STOP**

Division (shift and subtract)
→ Shift then subtraction
→ Only need "subtract" operation

# Arithmetic using computer

- Only addition and subtraction are needed for 4 binary arithmetic operations
- Subtraction needs more elements than addition in hardware
- Subtraction can be performed by adding a negative number
- Thus, a computer may only use **adders** to perform all binary arithmetic operations
- This requires an appropriate representation of the negative binary numbers

# Signed Binary numbers

- Three ways to represent the signed binary numbers
  – Signed binary (Sign + magnitude)
  – 1's complement
  – 2's complement

# MSB and LSB of a Binary Number

- MSB
  - Most significant bit
- LSB
  - Least significant bit

**1 0 1 1 0 0 1**

(Left-most bit) MSB

LSB (Right-most bit)

***For integer binary number only**

# Unsigned Binary number

**Unsigned binary number (*n* bits)**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1** | **1** | **0** | **1** | **…** | **…** | **1** | **0** |

**Magnitude**

(No sign, always positive)

**Range of unsigned binary number:**

Max value of a 4-bit number:

$$\begin{array}{l} \quad\;\; 2^4\; 2^3\; 2^2\; 2^1\; 2^0 \\ 1111 = 1\;0\;0\;0\;0 - 1 \rightarrow (2^4)_{10} - 1 \end{array}$$

Example:

| Decimal | Unsigned binary |
|---|---|
| 0 | 000 |
| 1 | 001 |
| 2 | 010 |
| 3 | 011 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |

**Max value of *n*-bit unsigned number in decimal → $2^n - 1$. Range: 0 ~ $(2^n-1)$**

# Signed Binary – Signed Magnitude (S-M)

**Signed binary number (*n* bits)**

| | 1 | 0 | 1 | … | … | 1 | 0 |
|---|---|---|---|---|---|---|---|

Magnitude

Sign bit
- **0 – represents a positive number**
- **1 – represents a negative number**

**MSB is a sign bit**

**Example:**

| Decimal | S-M |
|---|---|
| 3 | 011 |
| 2 | 010 |
| 1 | 001 |
| +0 | 000 |
| -0 | **100** |
| -1 | **101** |
| -2 | **110** |
| -3 | **111** |

Note:
Two zeros

Negative numbers

"1" in MSB position for all negative numbers

# Signed Magnitude – cont.

**More examples:**

$00111010 = +0111010 = (58)_{10}$

$11100101 = -\ 1100101 = (-101)_{10}$

$10000001 = -\ 0000001 = (-1)_{10}$

$01111111 = +1111111 = (+127)_{10}$

**Range of binary number represented by S-M:**

For a *n*-bit Signed binary (S-M), its magnitude is (*n*-1) bits

Max magnitude: $(2^{n-1}-1)_{10}$

Range: $-(2^{n-1}-1)_{10} \sim +(2^{n-1}-1)_{10}$

# Arithmetic using Binary Numbers (S-M)

- Computer performs binary arithmetic operations using only
  - Adders
  - Multipliers
- Subtraction is performed by adding a negative number

**Examples of subtraction using S-M binary representation:**

```
    3          011
-   2   ⇨    + 110
-------      -------
    1         1001  (1)10 ✓
              ↑
          Discarded
```

```
    3          011
-   1   ⇨   +  101
-------      -------
    2         1000  (0)10 ✗
              ↑
          Discarded
```

```
    2          010
-   1   ⇨   +  101
-------      -------
    1          111  (-3)10 ✗
```

***S-M representation cannot be used for addition of two number with opposite signs or subtraction when using a simple adder (dedicated hardware is needed for all possible sign combinations)**

# Complement Representation

- **Complement representations of a number**
  - Radix complements
  - Diminished complements
- **Definitions:**

  \- ***Radix Complement***
  of a n-digit integer number A with radix (***r***):

$$A^* = r^n - A$$

  \- ***Diminished radix complement***
  of a n-digit integer number A with radix (***r***):

$$A^* = r^n - A - 1$$

# Diminished Radix Complement

$$A^* = r^n - A - 1 \quad \text{or} \quad A^* = (r^n - 1) - A$$

**Examples:**

**Decimal number:**

$A = 2375 \rightarrow A^* = (10000_{10} - 1) - 2375_{10} = 9999_{10} - 2375_{10} = 7624_{10}$

$A = 0919 \rightarrow A^* = (10000_{10} - 1) - 0919_{10} = 9080_{10}$

**Octal number:**

$A = 406 \rightarrow A^* = (1000_8 - 1) - 406_8 = 777_8 - 406_8 = 371_8$

$A = 0671 \rightarrow A^* = (10000_8 - 1) - 0671_8 = 7777_8 - 0671_8 = 7106_8$

**Hex number:**

$A = 4A09 \rightarrow A^* = (10000_{16} - 1) - 4A09_{16} = FFFF_{16} - 4A09_{16} = B5F6_{16}$

$A = 0A7F \rightarrow A^* = (10000_{16} - 1) - 0A7F_{16} = FFFF_{16} - 0A7F_{16} = F580_{16}$

**Binary number:**

$A = 1001 \rightarrow A^* = (10000_2 - 1) - 1001_2 = 1111_2 - 1001_2 = 0110_2$

$A = 1100 \rightarrow A^* = (10000_2 - 1) - 1100_2 = 1111_2 - 1100_2 = 0011_2$

Diminished **radix 2** complement is found by reversing the bits

# 1's Complement

- "1's Complement" is the *diminished radix complement* of binary numbers
- 1's complement of a *n-bit* number is $\mathbf{A^* = (2^n - 1) - A}$
- 1's complement of a binary number can be obtained by reversing the bits, i.e. "1" → "0" and "0" → "1", since

$$\mathbf{(2^n - 1)_{10} = \underbrace{1000\ldots000}_{n+1\ bits} - 1 = \underbrace{111\ldots111}_{n\ bits}}$$

Binary number (n=8): 01011100

1's Complement: 11111111 - 01011100 = 10100011

*Reversing the bits*

# 1's Complement representation of signed binary number

**No change for positive numbers and use 1's complement for negative numbers**

| Decimal | 1's Complement |
|---|---|
| 3 | 011 |
| 2 | 010 |
| 1 | 001 |
| +0 | 000 |
| -0 | **111** |
| -1 | **110** |
| -2 | **101** |
| -3 | **100** |

Still two zeros

Magnitude range: $-(2^{n-1}-1) \sim (2^{n-1}-1)$

**3 – 2 = 3+(-2)=1**

```
     011
  +  101
 --------
 (1)000
  +     1
 --------
     001
```

✓

**3 – 1 = 3+(-1) = 2**

```
     011
  +  110
 --------
 (1)001
  +     1
 --------
     010
```

✓

***It has no problem to perform subtraction, but needs to shift and add the carry**

# 2's Complement of a Binary Number

- "2's Complement" is the *radix complement* of binary numbers
- 2's complement of a *n-bit* number can be obtained by adding "1" to its **1's complement**, i.e.,

$$
\begin{aligned}
\mathbf{A^* } &= \mathbf{2^n - A} \\
&= \mathbf{(2^n - A - 1) + 1} \\
&= \mathbf{1\text{'s complement} + 1}
\end{aligned}
$$

Binary number (n=8): 01011100

2's Complement: $\underbrace{10100011}_{\text{1's complement}} + 1 = \underbrace{10100100}_{\text{2's complement}}$

# 2's Complement representation of signed binary number

**No change for positive numbers and use 2's complement for negative numbers**

| Decimal | 2's Complement |
|---|---|
| 3 | 011 |
| 2 | 010 |
| 1 | 001 |
| 0 | 000 |
| -1 | **111** |
| -2 | **110** |
| -3 | **101** |
| -4 | **100** |

Only one zero

3 – 2 = 3+(-2)=1

```
   011
+  110
--------
(1)001
```

Carry ignored

3 – 1 = 3+(-1) = 2

```
   011
+  111
--------
(1)010
```

Carry ignored

- **No problem in performing subtraction**
- **Carry is discarded (there is NO NEED to shift and add the carry, thus more hardware efficient)**

Magnitude range: $-(2^{n-1}) \sim (2^{n-1}-1)$

# Signed Binary Number (Recap)

- **Sign+Magnitude**
  - Two zero representations (+/- zeros)
  - It cannot correctly perform subtraction
  - Magnitude range: $-(2^{n-1}-1) \sim (2^{n-1}-1)$
- **1's Complement (Diminished radix complement)**
  - Defined as: $\mathbf{A^* = (2^n - 1) - A}$
  - 1's complement can be obtained by reversing the bits
  - Two zero representations (+/- zeros)
  - It can correctly perform subtraction, but needs to shift and add the carry
  - Magnitude range: $-(2^{n-1}-1) \sim (2^{n-1}-1)$
- **2's Complement (Radix complement)**
  - Defined as: $\mathbf{A^* = 2^n - A}$
  - One zero representation
  - It can correctly perform subtraction by just ignoring the carry
  - 2's complement can be obtained by adding "1" to its 1's complement
  - Magnitude range: $-(2^{n-1}) \sim (2^{n-1}-1)$

**Positive numbers are same in all 3 signed binary number representations**

# 4-bit Signed Binary Numbers Table

**Signed Representations of Binary Numbers**

| Decimal | Signed-2's Complement | Signed-1's Complement | Signed Magnitude |
|---|---|---|---|
| +7 | 0111 | 0111 | 0111 |
| +6 | 0110 | 0110 | 0110 |
| +5 | 0101 | 0101 | 0101 |
| +4 | 0100 | 0100 | 0100 |
| +3 | 0011 | 0011 | 0011 |
| +2 | 0010 | 0010 | 0010 |
| +1 | 0001 | 0001 | 0001 |
| +0 | 0000 | 0000 | 0000 |
| −0 | — | 1111 | 1000 |
| −1 | 1111 | 1110 | 1001 |
| −2 | 1110 | 1101 | 1010 |
| −3 | 1101 | 1100 | 1011 |
| −4 | 1100 | 1011 | 1100 |
| −5 | 1011 | 1010 | 1101 |
| −6 | 1010 | 1001 | 1110 |
| −7 | 1001 | 1000 | 1111 |
| −8 | 1000 | — | — |

# Binary-Coded Decimal (BCD)

- BCD is a code to represent ten decimal digits ( 0 – 9)
- Each decimal digit is represented by a 4-bit binary number

**Decimal → BCD:**

Decimal → $(5\ \ 9\ \ 8)_{10}$

BCD → 0101 1001 1000

Note:
Six numbers, from **1010 to 1111**, are not used in BCD.

**Binary-Coded Decimal (BCD)**

| Decimal Symbol | BCD Digit |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |

# Decimal number addition with BCD

**(Assume that BCD is used)**

```
   4           0100
 + 3    ⇨   +  0011
 -----      ----------
   7           0111 ✓
```

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```
   5           0101
 + 7    ⇨   +  0111
 -----      ----------
  12           1100 ✗
```

$(12)_{10}$ in BCD representation is
0001 0010

Not a legitimate BCD code (>9)

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```
   8           1000
 + 9    ⇨   +  1001
 -----      ----------
  17        1  0001 ✗
```

The result is in legitimate BCD code,
but the sum is wrong

$(17)_{10}$ in BCD representation is
0001 0111

(sum >15)

# What is the problem?

- Decimal addition is a modulo-10 scheme and a carry is generated when the sum > 9
- 4-bit binary addition is a modulo-16 scheme and the carry is only generated when the sum > 15
- **Need to generate carry when sum>9, so: what about adding 6 to the result?**

```
   5          0101
 + 7    ⇨   + 0111
 -----     ----------
  12          1100 ✗
            + 0110
           ----------
           1  0010
           1    2   ✓
```

```
   8            1000
 + 9    ⇨     + 1001
 -----        ----------
  17         1   0001 ✗
           +     0110
           -------------
             1   0111
             1    7   ✓
```

**The results are correct after adding 6!**

# What is the Rule?

- For decimal addition: S = A + B using BCD code

**If $S \leq 9$ → Sum = S and carry = 0**
(No correction is needed)

**If $S > 9$ → Sum = S + 6 and carry = 1**
(Need to be corrected by adding 6)

# Summary of the Lecture

- We have covered
  - Position number system (radix 10, 2, 8 and 16)
  - Conversion among decimal, binary, octal and hex)
  - Binary arithmetic
  - Signed binary number representations (S-M, 1's complement and 2's complement
  - Arithmetic using signed binary numbers
  - Binary-coded decimals
  - Decimal addition using BCD