2014-2015 ~~Sp~~ Fall.

MATH 2011 Mock Exam for Final

Study Rules : Do revision first.

Do exercise first.

When you start, start a timer and a stopwatch; switch the colour of your pen once ① you finish the paper first time

or ② the stopwatch ~~range~~ rings.

For ①, do it until the stopwatch rings & see how many more marks you get from the second colour.

For ②, once you finish the paper, stop the timer & see how much more time you need ⇒ how munch you need to speed up

⇓

keep exercise yourself & get improved

P.S. If you ~~tak~~ do wish to have the mock paper,

~~1. Let $\vec{u}$ =~~ then you should be reminded not to roll down to the solution before finishing all the above steps :).

Get everything ready before turning to the next page.

Q1. Let $\vec{u} = 5\vec{i} + 0\vec{j} - 2\vec{k}$ and $\vec{v} = 4\vec{i} - 3\vec{j} + 0\vec{k}$.
Compute $\vec{u} \cdot \vec{v}$ and $\vec{u} \times \vec{v}$.

Q2. Let $\vec{r}(t) = e^{-3t}\,\vec{i} + \cos 2t\,\vec{j} + (\sin t - t^3)\,\vec{k}$. Find the equation of the tangent line at $t=0$.

Q3. Let $\vec{F} = (x + \cos z)\,\vec{i} + (y + ze^x)\,\vec{j} + (\cos 2y)\,\vec{k}$.

~~(a)~~

(a) Compute $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$.

(b) Let S be the closed surface of a solid region bounded ~~by z=2,~~ above by $z=2$, bounded below by $z=0$ and bounded laterally by the cylinder $x^2+y^2 = \frac{1}{2}$.
Using (a) and an appropriate theorem, or otherwise, evaluate the flux $\iint_S \vec{F} \cdot \vec{n}\, d\sigma$ where $\vec{F}(x,y,z) = (x+\cos z)\vec{i} + (y+ze^x)\vec{j} + (\cos 2y)\vec{k}$ and $\vec{n}$ is the unit outward normal on S.

Q4. (a) Find a parametrization of the cone $z = 3\sqrt{x^2+y^2}$, i.e. find a ~~the~~ vector-valued function which has two variables that traces out the surface of the given cone.

(b) Using (a), or otherwise, find the surface area of the cone $z = 3\sqrt{x^2+y^2}$ between the planes $z=3$ and $z=6$.

Q5. Let D ~~(a)~~ be the region in the first octant (i.e. $x, y, z \geq 0$) bounded by the plane $x+y=5$ and the paraboloid $z = 25 - x^2 - y^2$.

(a) Write down a triple integral to determine the volume of the region D by filling in the limits of the integrals.

$$\text{Volume} = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} 1 \, dz \, dy \, dx$$

(b) Evaluate the above integral.

Q6. Consider the function $f(x,y) = 4x^3 + 6y^2 - 3xy - 7$.

(a) Find all the critical points of this function.

(b) Find the relative maximum, relative minimum and saddle points of the function, if any.

Q7. (a) Determine if the following vector fields are conservative. If so, find a potential of the vector field.

$$\begin{cases} \vec{E} = (6x^2y - e^{2y}\cos x)\,\vec{i} + 2e^{y}\sin x\,\vec{j}. \\ \vec{F} = (6x^2y - e^{2y}\cos x)\,\vec{i} + 2(x^3 + e^{2y}\sin x)\,\vec{j}. \end{cases}$$

(b) Let C_1 be the upper half unit circle centered at $(0,0)$ lying on the xy-plane oriented in the counterclockwise direction from $(1,0)$ to $(-1,0)$.
Evaluate $\int_{C_1} \vec{G} \, d\vec{r}$ where $\vec{G}(x,y) = x^3\,\vec{j}$.

Q7 (cont'd) (c) Using (a) and (b), or otherwise, calculate the line integral $\int_{C_1} \vec{E} \cdot d\vec{r}$ where $\vec{E}(x,y) = (6x^2y - e^{2y}\cos x)\vec{i} + 2e^{2y}\sin x\,\vec{j}$

and C_1 is the upper half unit circle centered at $(0,0)$ lying on the xy-plane oriented in the counterclockwise direction from $(1,0)$ to $(-1,0)$.

(d) Using the Green's Theorem, or otherwise, evaluate $\int_{C_2} \vec{G} \cdot d\vec{r}$ where $\vec{G}(x,y) = x^3\vec{j}$ and C_2 is the unit circle centered at $(0,0)$ lying on the xy-plane oriented in the counterclockwise direction.

(e) Let S be the portion of the paraboloid where $z = 1 - x^2 - y^2$ which lies above the xy-plane with unit normal $\vec{n}$ pointing upward. Using (a) and (d), or otherwise, compute

$$\iint_S (\nabla \times \vec{H}) \cdot \vec{n}\, d\sigma = \iint_S (\text{curl } \vec{H}) \cdot \vec{n}\, d\sigma$$

where $\vec{H}(x,y,z) = (6x^2y - e^{2y}\cos x)\vec{i} + 2(e^{y}\sin x + \cos z)\vec{j} + z^2\vec{k}$