

COMP 2711H Discrete Mathematical Tools for Computer Science
Tutorial Problems: Logic and Proofs

Problem 1. Let p, q , and r be the following propositions “you get an A on the final exam”, “you do every exercise in this book”, and “you get an A in this class”, respectively. Write the following propositions using p, q , and r and logical connectives.

- (a) You get an A in this class, but you do not do every exercise in this book.
- (b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
- (c) To get an A in this class, it is necessary for you to get an A on the final.
- (d) You get an A on the final, but you don’t do every exercise in this book; nevertheless, you get an A in this class.
- (e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
- (f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Problem 2. Prove that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent. Present two different proofs of this fact, only one of which should use truth tables.

Problem 3. Let $P(x, y)$ be a propositional function. Show that the implication $\exists x \forall y P(x, y) \rightarrow \forall y \exists x P(x, y)$ is a tautology.

Problem 4. Use existential and universal quantifiers to express the statement “Everyone has exactly two biological parents” using the propositional function $P(x, y)$ which represents “ x is the biological parent of y .”

Problem 5. Prove that the square of an integer not divisible by 5 leaves a remainder of 1 or 4 when divided by 5. (*Hint:* Use a proof by cases, where the cases correspond to the possible remainders for the integer when it is divided by 5.)

Problem 6. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Problem 7. Prove that there is no rational number r for which $r^3 + r + 1 = 0$.

Problem 8. A *triangle* is a set of three people such that either every pair has shaken hands or no pair has shaken hands. Prove that among every six people there is a triangle.