COMP 2711H Discrete Mathematical Tools for Computer Science
2021 Fall Semester
Homework 3: Combinatorics
Handed out: Nov 1
Due: Nov 12

Problem 1. How many people are needed to guarantee that at least three were born on the same day of the week and in the same month (perhaps in different years)?

Problem 2. For all parts of this problem, assume that any two people are either friends or enemies. One can show that in any group of six people, there are either three mutual friends or three mutual enemies. Assuming this fact, now show the following:

(a) Show that in any group of ten people, there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends. (*Hint:* To prove the first claim, focus on a particular member of the group. Show that this member has either four friends or six enemies in the group. Then try to apply the fact given above for six people.)

(b) Show that in any group of 20 people, there are either four mutual friends or four mutual enemies.

Problem 3. Give a formula for the coefficient of x^k in the expansion of $(x^2 + 1/x)^{20}$, where k is an integer.

Problem 4. Show that in any sequence of m integers there exists one or more consecutive terms with a sum divisible by m.

Problem 5. Give a combinatorial proof of the following fact: Any set has the same number of subsets with an odd number of elements as it does subsets with an even number of elements.

Problem 6. How many terms are there in the expansion of $(x + y + z)^{20}$?

Problem 7. How many ways are there to travel in $xyzw$ space from the origin $(0, 0, 0, 0)$ to the point $(3, 6, 8, 5)$ by taking steps one unit in the positive x, positive y, positive z, or positive w direction? (Moving in the negative x, y, z, or w direction is prohibited, so that no backtracking is allowed.)

Problem 8. How many bit strings of length n, where $n \geq 5$, contain exactly two occurrences of 10.

Problem 9. Using the inclusion-exclusion principle, determine the number of onto functions from a set with n elements to one with m elements?