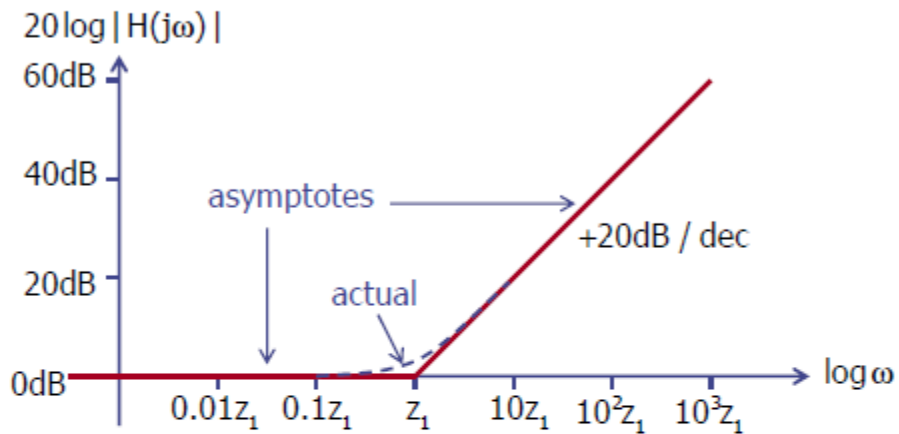


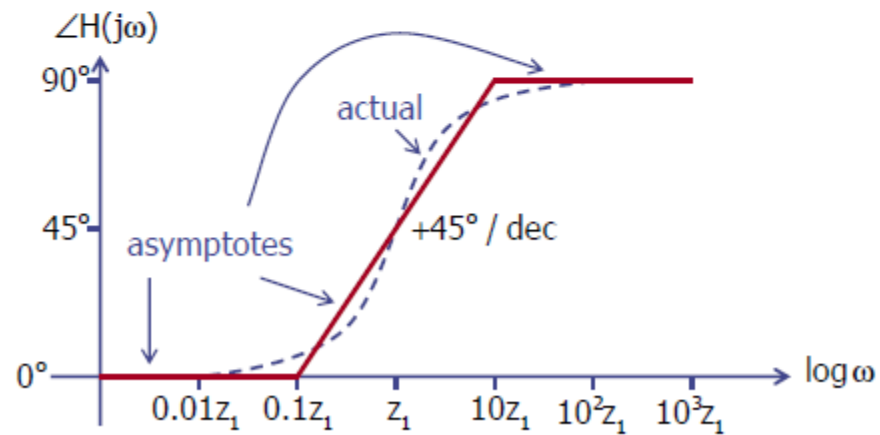
## Magnitude and Phase Plots of Real Zero

$$H(s) = 1 + \frac{s}{z_1}$$

Magnitude plot:



Phase plot:



4-71

## Magnitude and Phase of Real Zero

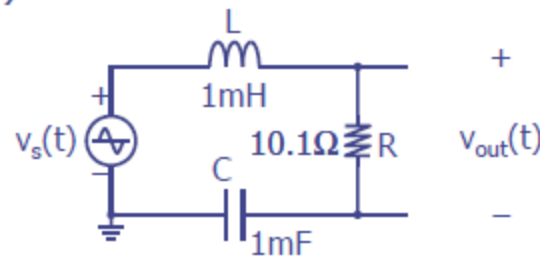
Consider  $H(s) = 1 + s/z_1$ .

$H(j\omega)$	$ H(j\omega) $	$20\log  H(j\omega) $	$\angle H(j\omega)$
$H(j0)$	$= 1$	0dB	$0^\circ$
$H(jz_1 / 100)$	$\sqrt{1 + 0.01^2} \approx 1$	0dB	$0^\circ$
$H(jz_1 / 10)$	$\sqrt{1 + 0.1^2} \approx 1$	0dB	$5.7^\circ$
$H(jz_1 / 3)$	$\sqrt{1 + 0.333^2} = 1.054 \approx 1$	0dB	$18.4^\circ$
$H(jz_1)$	$\sqrt{1 + 1} = \sqrt{2} \approx 1.414$	3dB	$45^\circ$
$H(j3z_1)$	$\sqrt{1 + 3^2} = \sqrt{10} \approx 3.162$	10dB	$71.6^\circ$
$H(j10z_1)$	$\sqrt{1 + 10^2} = \sqrt{101} \approx 10$	20dB	$84.3^\circ$
$H(j10^2z_1)$	$\sqrt{1 + 100^2} \approx 100$	40dB	$90^\circ$
$H(j10^3z_1)$	$\sqrt{1 + 1000^2} \approx 1000$	60dB	$90^\circ$

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## Example 4-27

**Example 4-27:** Consider an RLC circuit with its input connected to a signal generator that generates sine waves. Find the transfer function  $H(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$  and sketch the Bode plots of  $H(s)$ .

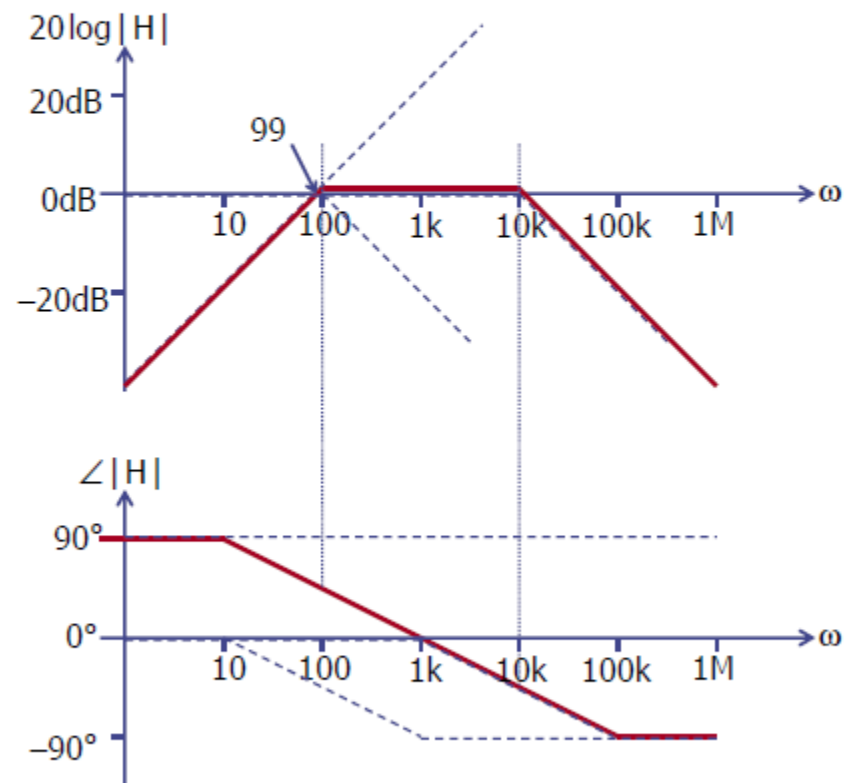


$$\text{Soln.: } V_{\text{out}}(s) = \frac{R}{sL + R + \frac{1}{sC}} V_{\text{in}}(s) = \frac{sRC}{s^2LC + sRC + 1} V_{\text{in}}(s)$$

$$\begin{aligned} H(s) &= \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{sRC}{s^2LC + sRC + 1} = \frac{s(10.1)(10^{-3})}{s^2(10^{-3})(10^{-3}) + s(10.1)(10^{-3}) + 1} \\ &= \frac{1.01 \times 10^{-2} s}{1 + 1.01 \times 10^{-2} s + 1 \times 10^{-6} s^2} = \frac{s}{99(1 + s/100)(1 + s/10^4)} \end{aligned} \quad 4-86$$

### Example 4-27 (cont.)

$$H(s) = \frac{s}{99(1 + s/100)(1 + s/10^4)}$$



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## Standard Form of Transfer Function

Recall a transfer function is written as

$$H(s) = H_1 \frac{(s + z_1)(s + z_2) \dots (s + z_m)}{(s + p_1)(s + p_2) \dots (s + p_n)}$$

For convenience in working out Bode plots, it is better to write the transfer function as

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = H_o \frac{(1 + s / z_1)(1 + s / z_2) \dots (1 + s / z_m)}{(1 + s / p_1)(1 + s / p_2) \dots (1 + s / p_n)}$$

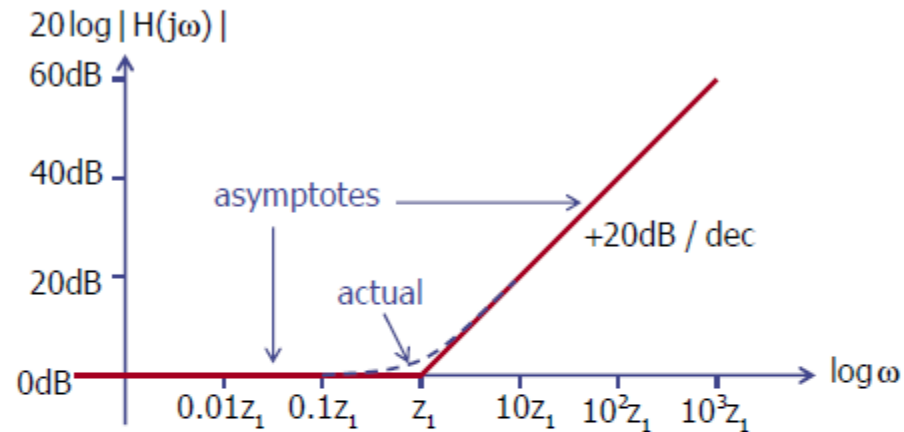
where  $H_o = H(j0)$  is the DC gain.

4-77

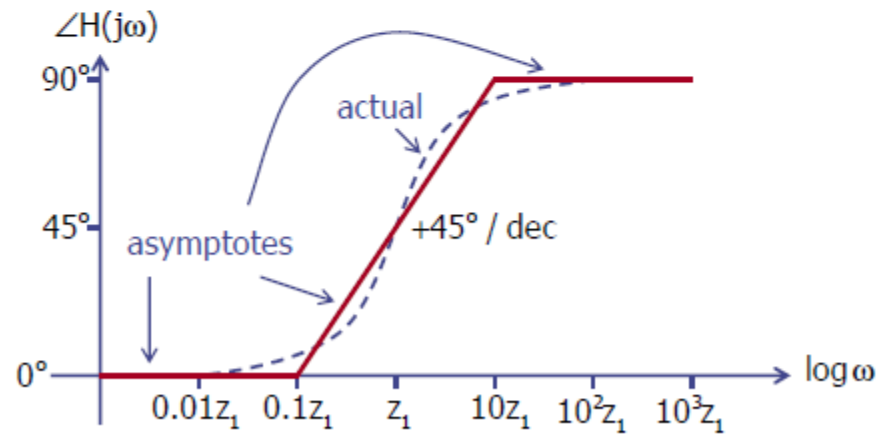
## Magnitude and Phase Plots of Real Zero

$$H(s) = 1 + \frac{s}{z_1}$$

Magnitude plot:



Phase plot:



4-71

## Magnitude and Phase Plots of Real Pole

$$H(s) = \frac{1}{1 + \frac{s}{p_1}}$$

Let

$$H'(s) = 1 + \frac{s}{p_1}$$

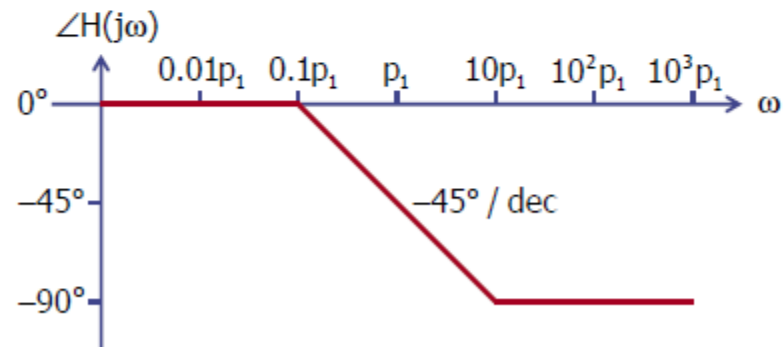
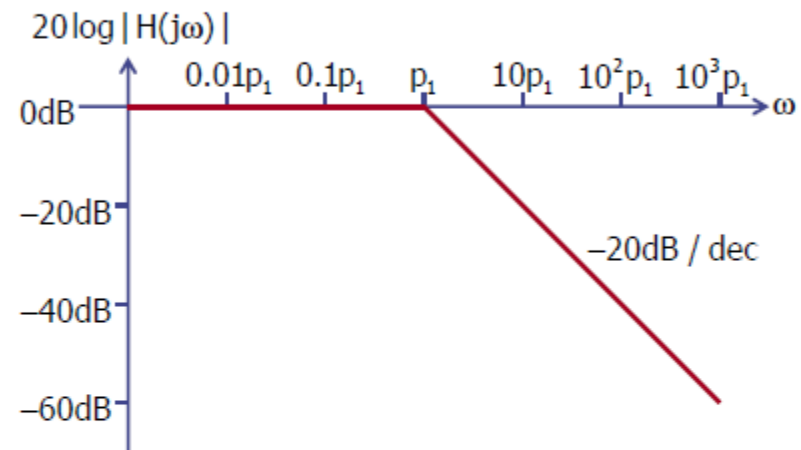
then

$$H(s) = \frac{1}{H'(s)}$$

and

$$20 \log |H(j\omega)| = -20 \log |H'(j\omega)|$$

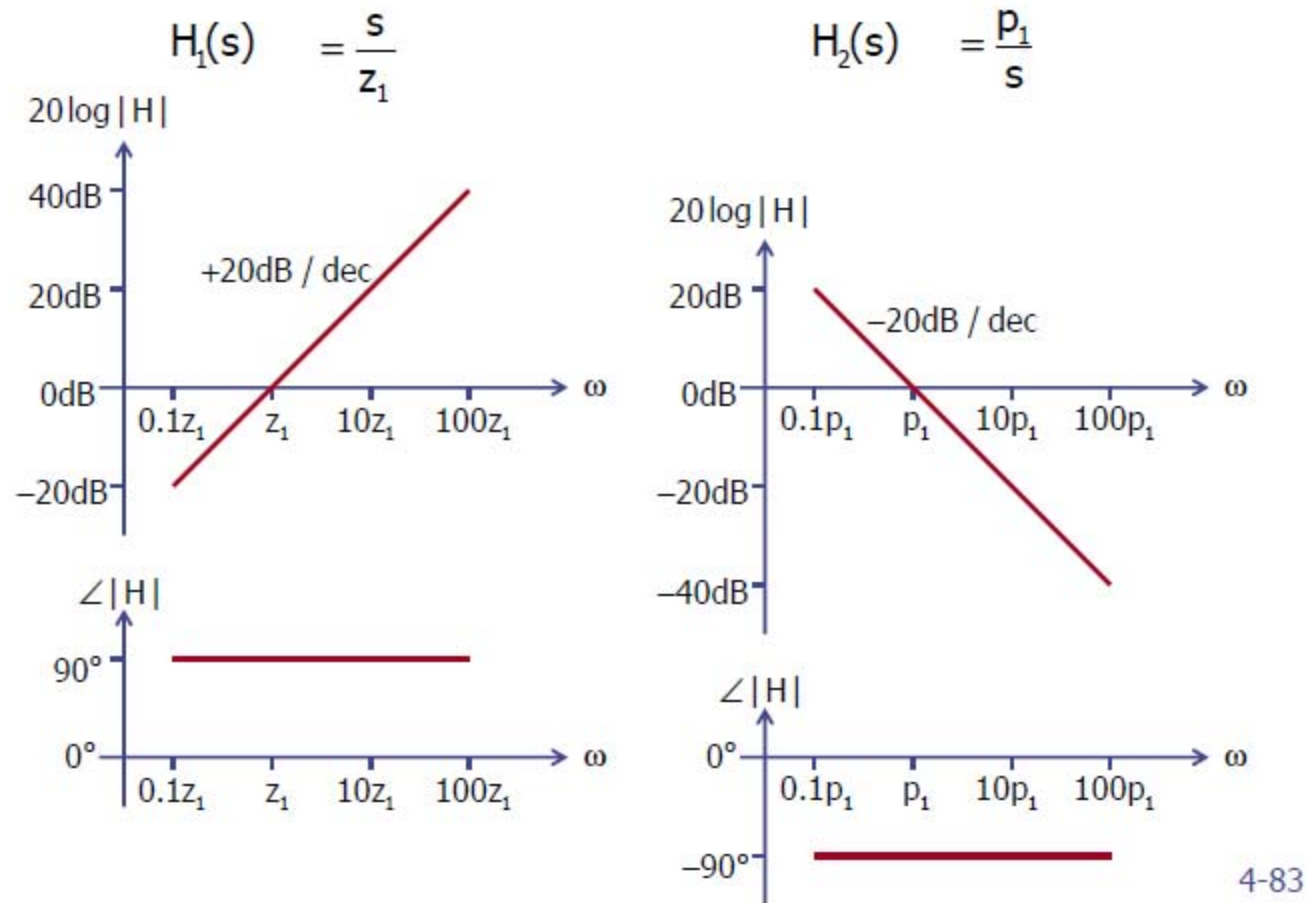
$$\angle H(j\omega) = \angle H'(j\omega)$$



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## Bode Plots of $s/z_1$ and $p_1/s$ (cont.)



## Example 4-24

Example 4-24: Sketch the Bode plots of

$$H(s) = \frac{1000(s + 10)}{(s + 1)(s + 1000)}$$

Soln.:

The magic of Bode plots is that if we have a complicated transfer function as the one given above, we can break it down into smaller pieces, and add up all the pieces together one by one.

(1) First of all, write  $H(s)$  in the standard form, i. e.,

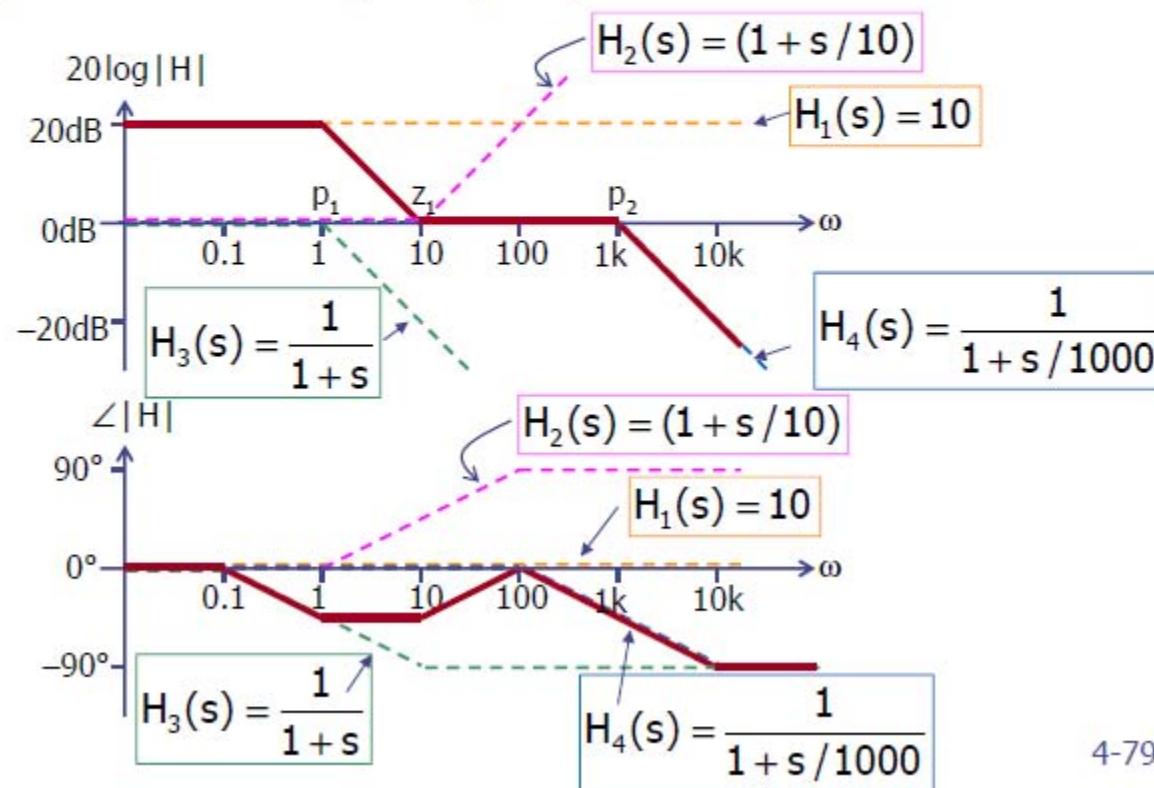
$$\begin{aligned} H(s) &= \frac{1000 \times 10(1 + s / 10)}{(1 + s) \times 1000 \times (1 + s / 1000)} \\ &= \frac{10(1 + s / 10)}{(1 + s)(1 + s / 1000)} \end{aligned}$$

(2) Identify all corner frequencies:  $z_1=10$  rad/s,  $p_1=1$  rad/s,  $p_2=1000$  rad/s.

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## Example 4-24 (cont.)

(3) Draw Bode plots of individual factor, and start adding the Bode plots from low to high frequency.



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