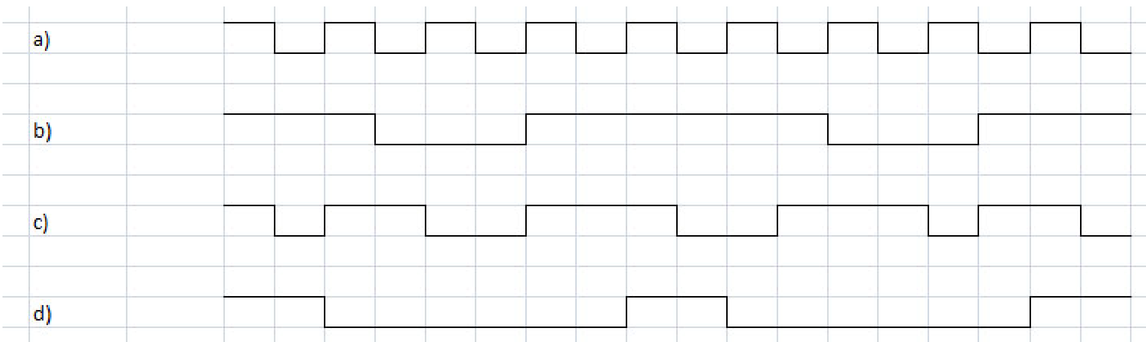
ELEC1200 Midterm Review Question

<u>Q1</u> The following figure shows the plots of several received waveforms. The transmitter is sending sequences of binary symbols (i.e., either 0 or 1) at some fixed symbol rate, using 0V to represent 0 and 1V to represent 1.The horizontal grid spacing is 1 microsecond (1e-6 sec).



Answer the following questions for each plot:
(a) Find the slowest symbol rate that is consistent with the transitions in the waveform.
(b) Use your answer in part (a) to write down the decoded bit string.

Solution

a) 1M symbols per second	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
b) 1/3 M symbols per second	1 0 1 1 0 1
c) 1 M symbols per second	1 0 1 1 0 0 1 1 1 0 0 1 1 1 0 1 1 0
d) 1/2 M symbols per second	1 0 0 0 1 0 0 0 1

<u>Q2</u> The input sequence to a linear time-invariant (LTI) system is given by $x_1[n]$

$x_1[0]=1$
$x_1[1]=1$
$x_1[2]=0$
$x_1[3]=0$ for all other values of n

and the output of the LTI system is given by $y_1[n]$

$y_1[0]=1$
$y_1[1]=2$
$y_1[2]=1$
$y_1[3]=0$ for all other values of n

What are the nonzero values of the output of this LTI system when the input is $x_2[n]$?

$x_2[0]=0$
$x_2[1]=1$
$x_2[2]=1$
$x_2[3]=1$
$x_2[4]=1$
$x_2[n]=0$ for all other values of n

Solution

$x_1[n]$ and $y_1[n]$ represent the first input and output of the system.
$x_2[n]$ and $y_2[n]$ represent the second input and output of the system.

$x_2[n]=x_1[n-1]+x_1[n-3]$ → $y_2[n]=y_1[n-1]+y_1[n-3]$

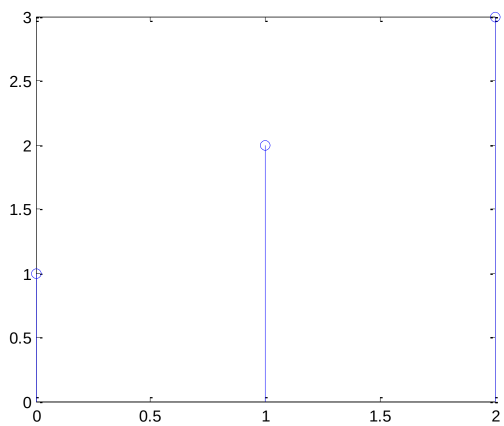
$y_2[1]=1$
$y_2[2]=2$
$y_2[3]=2$
$y_2[4]=2$
$y_2[5]=1$

Q3 Determine the output y[n] for a system with the input x[n] and unit step-response s[n] shown below.

Assume x[n] = 0 for all n not being shown.

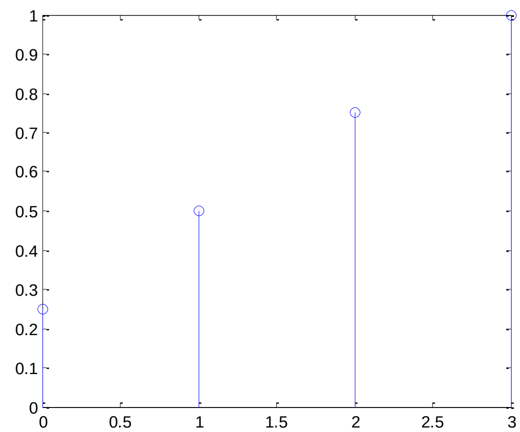
Assume s[n] = 0 for all n < 0 and s[n] = 1 for all n > 3.

x[n]



n

s[n]

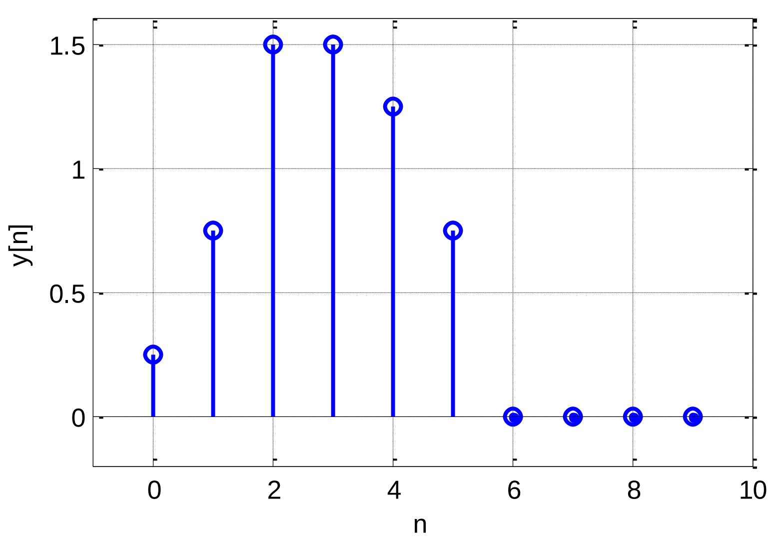


n

Solution

$$x[n] = u[n] - u[n-1] + 2\,u[n-1] - 2\,u[n-2] + 3\,u[n-2] - 3\,u[n-3]$$
$$= u[n] + u[n-1] + u[n-2] - 3u[n-3]$$

$$y[n] = s[n] + s[n-1] + s[n-2] - 3s[n-3]$$



Q4 The output of a particular communication channel is given by
$y[n] = \alpha x[n] + \beta x[n-1]$ where $\alpha > \beta$

(a) Is the channel linear? Is it time invariant?

(b) If the input is the following sequence of samples starting at time 0, x[n] = [1, 0, 0, 1, 1, 0, 1, 1] and x[n] = 0 for all n not being shown, what is the channel output assuming $\alpha = 7$ and $\beta = 3$?

Solution

(a) This is an LTI channel.

To be linear the channel must meet two criteria:
if we scale the inputs x[n] by some factor k, the outputs y[n] should scale by the same factor.
if we get y1[n] with inputs x1[n] and y2[n] with inputs x2[n], then we should get y1[n] + y2[n] if the input is x1[n] + x2[n].
It's easy to verify both properities given the channel response above, so the channel is linear.

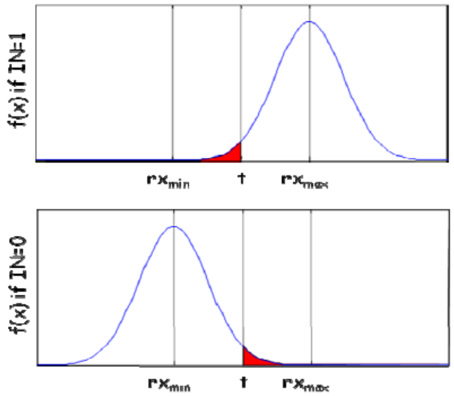
To be time invariant the channel must have the property that if we shift the input by some number of samples s, the output also shifts by s samples. Again that property is easily verified given the channel response above, so the channel is time invariant.

Show calculations or zero marks!

(b) $y[n] = 7\,x[n] + 3\,x[n-1]$

y[0] = 7
y[1] = 3
y[2] = 0
y[3] = 7
y[4] = 10
y[5] = 3
y[6] = 7
y[7] = 10
y[8] = 3
y[9] = 0
y[n] = 0 for all n not being shown

Q5 A noise probability density function is shown below.



Give an expression for BER in terms of the detection threshold T, $rx_{\min}$, $rx_{\max}$, the variance of the noise σ^2, and P(IN=0).

Solution

$$BER = P(IN = 0) \cdot Q\left(\frac{t - rx_{\min}}{\sigma}\right) + (1 - P(IN = 0)) \cdot Q\left(\frac{rx_{\max} - t}{\sigma}\right)$$

Q6 Consider the following (n, k, d) block code:

```
D0   D1   D2   D3   D4    | P0
D5   D6   D7   D8   D9    | P1
D10 D11 D12 D13 D14       | P2
-------------------------------
P3   P4   P5   P6   P7    |
```

where D0 - D14 are data bits, P0 - P2 are row parity bits and P3 - P7 are column parity bits.
The transmitted code word will be: D0 D1 D2 ... D13 D14 P0 P1 ... P6 P7

(a) What are the values of n, k and d for the code above?
(b) What are P0 through P7 if D0 D1 D2 ... D13 D14 = 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1?
(c) Now we receive the following four codewords:

M1: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 0 0 0 1 1 0 1 0
M2: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 0 0 1 1 1 0 1 0
M3: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 1 1 0 1 1 0 1 0
M4: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 1 0 0 1 1 0 1 0

For each of received codewords, indicate the number of errors. If there are errors, indicate if they are correctable, and if they are, what correction should be done.

Solution

(a) Length of data = 15 bits → k = 15
Length of codeword = 15 + 8 = 23 bits → n = 23

Referring to the table given in the question, it can be observed that 2 parity bits will be changed accordingly when 1 bit data is changed. Therefore, the minimum Hamming distance would be 3 (i.e. d = 3).

(b)

```
0  1  0  1  0   |  0
0  1  0  0  1   |  0
1  0  0  0  1   |  0
-------------------------
1  0  0  1  0
```

→ [P0 P1 P2 P3 P4 P5 P6 P7] = [0 0 0 1 0 0 1 0]

(c)
Calculate the syndrome bits, mark the syndrome bits which equals to 1 with red color, and highlight the data bits in error with yellow shade

```
0  1  0  1  0   |  0 0
0  1  0  0  1   |  0 0
1  0  0  0  1   |  0 0
-------------------------
1  1  0  1  0
0  1  0  0  0
```

M1 : Error occurs in P4 and no correction is needed.

```
0  1  0  1  0   |  0 0
0  1  0  0  1   |  0 0
1  0  0  0  1   |  1 1
-------------------------
1  1  0  1  0
0  1  0  0  0
```

M2 : Error occurs in D11 (0 →1)

```
0   1   0   1   0   |  1  1
0   1   0   0   1   |  1  1
1   0   0   0   1   |  0  0
-------------------------
1   1   0   1   0
0   1   0   0   0
```

M3 : More than 2 bits errors occur in the parity bits and correction cannot be done.

```
0   1   0   1   0   |  1  1
0   1   0   0   1   |  0  0
1   0   0   0   1   |  0  0
-------------------------
1   1   0   1   0
0   1   0   0   0
```

M4 : Error occurs in D1 (1 → 0)

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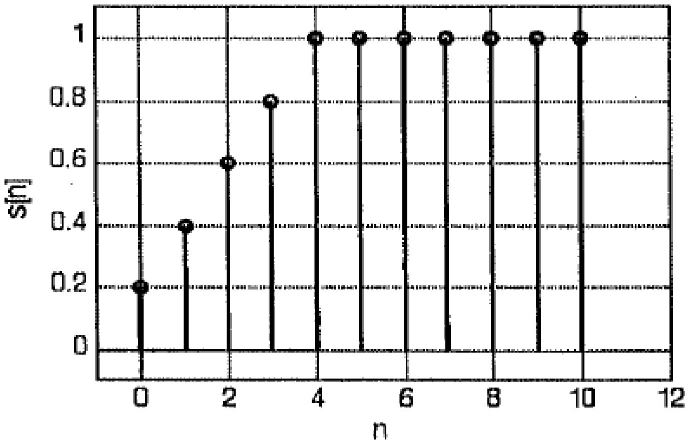
Q7 [20 Marks]

(a) [4] Given that the transmitted sampled waveform is [0 0 1 1 1 1 1 0 0 0 0 0 1 1 0 0 …], express it in terms of the step function u[n].

(b) [6] Explain what a linear time-invariant channel is.

(c) [10] Given that the step response s[n] of a linear time-invariant channel is as follows. Express the channel output waveform in terms of s[n] and sketch it for n = 0 to 24.

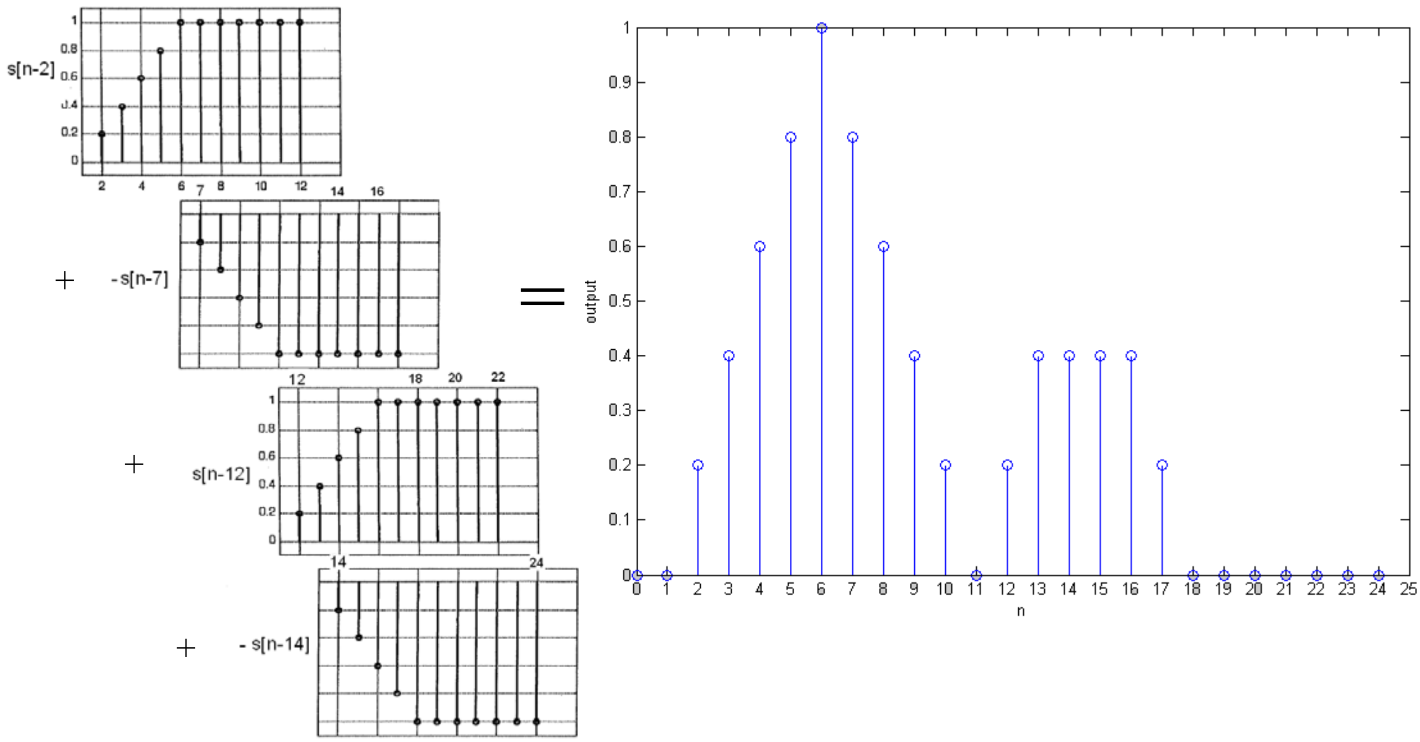


(a). u[n-2] -u[n-7]+u[n-12]-u[n-14]

(b). Linear time-invariant channel is
- Additive
- Homogeneity
- time-invariant

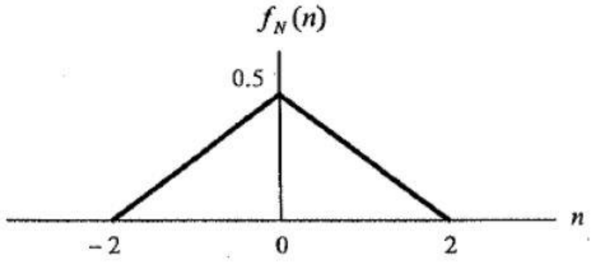
(c). Input = u[n-2] -u[n-7]+u[n-12]-u[n-14]

Output = s[n-2] -s[n-7]+s[n-12]-s[n-14]



Q8

Given that the probability density function of the additive channel noise is as follows.



The channel response to a bit input is the sum of the response to the input and noise. Without noise, the responses to bit "0" and bit "1" are 0V and 1V, respectively. Decoding the channel output is done by comparing the channel output to a threshold value of T. The decoded bit is "1" if the channel voltage is greater than T and "0" otherwise.

(a) [5] Sketch the probability density function of the received signal if the input bit is "0".

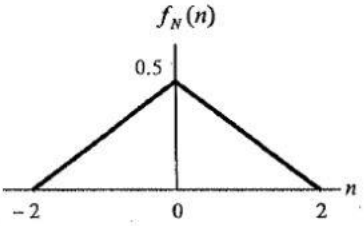
(b) [5] Sketch the probability density function of the received signal if the input bit is "1".

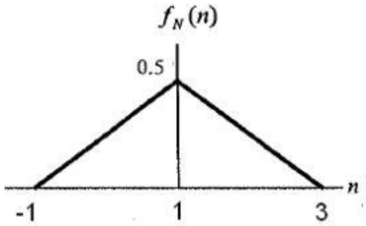
(c) [5] What is the optimal threshold if the input bits "0" and "1" are equally likely?

(d) [5] What is the bit error rate if we use the optimal threshold in 3(c)?

(e) [5] Re-do parts 3(c)&(d) if the input bit "0" is twice more likely to occur than the input bit "1".

Answer:

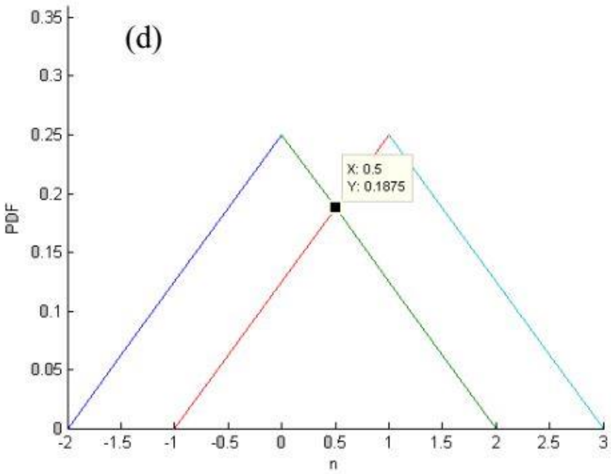
(a). P[IN=0], 

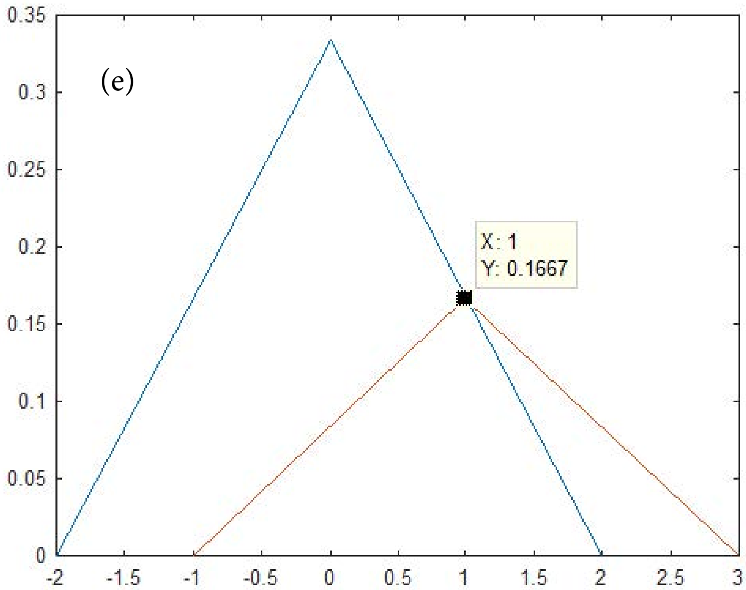
(b). P[IN=1], 

(c). optimal threshold = 0.5. (d).= 0.1875*1.5*0.5*2=0.28125.

(e). P[IN=0] = 2/3, P[IN=1] = 1/3.

Optimal threshold = 1. BER = 1/6 * 2*0.5+ 1/6 *0.5 = 0.25.





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Q9 [35 Marks]

Assume that (n, k, d) block code is used. Each block contains 9 data bits [D1 D2 … D9] and 6 parity bits [P1 P2 P3 P4 P5 P6]. The values of the parity bits are computed by arranging the data bits in a block as shown below and ensuring that each row and column containing data bits has even parity.

```
D1 D2 D3 P1
D4 D5 D6 P2
D7 D8 D9 P3
P4 P5 P6
```

The resulting codeword is [D1 D2 … D9 P1 P2 P3 P4 P5 P6].

(a) [6] What are the values of n, k and d?

(b) [3] If we wish to detect errors only (not correct), what is the maximum number of bit errors we can detect using this coding method?

(c) [3] What is the maximum number of bit errors in each codeword that we can detect and correct?

(d) [5] If we want to transmit the data bits [1 0 1 1 1 0 0 0 1], what is the resulting codeword?

(e) [9] Assume that each received codeword only contains either zero or one bit error. For each of the following received codewords, find the corrected codewords.

i. [0 1 1 0 1 1 1 1 1 0 1 1 1 0 1]

ii. [1 0 1 1 1 0 0 0 0 0 0 1 0 1 1]

iii. [1 1 0 0 1 0 0 0 1 0 1 1 1 0 1]

(f) [3] Assume that the received codeword in 4(e)i contains two bit errors. Find the corrected codeword.

(g) [6] Based on the results in 4(e)i and 4(f), argue if the block code can correct up to 2 bit errors. Explain your answer in detail.

(a). n=15, k=9,
d=3,
000 000 000 → 000 000 000 000 000
000 000 001 → 000 000 001 001 001
(n,k,d) = (15,9,3)

(b). Detect up to 2 errors. i.e.(d-1).

(c). Detect and correct 1 error, i.e. (d-1)/2.

(d). codeword = [101 110 001 001 010].

(ei). corrected codeword = [011 0**0**1 111 011 101]

0	1	1	0	S1=0
0	***1***	1	1	S2=1
1	1	1	1	S3=0
1	0	1		
S4=0	S5=1	S6=0		

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(eii). corrected codeword = [101 110 000 00**0** 011]

1	0	1	0	S1=0
1	1	0	0	S2=0
0	0	0	***1***	S3=1
0	1	1		
S4=0	S5=0	S6=0		

(eiii). no error, corrected codeword = [110 010 001 011 101]

1	1	0	0	S1=0
0	1	0	1	S2=0
0	0	1	1	S3=0
1	0	1		
S4=0	S5=0	S6=0		

(f). if 2 erros, P2 and P5 have errors.
corrected codeword = [011 011 111 0**0**1 1**1**1]

0	1	1	0	S1=0
0	1	1	***1***	S2=1
1	1	1	1	S3=0
1	***0***	1		
S4=0	S5=1	S6=0		

(g). No, there are cases that 2 errors can be detected but can't be corrected,

D1	D2	D3	P1	S1
D4	D5	D6	P2	S2
D7	D8	D9	P3	S3
P4	P5	P6		
S4	S5	S6		

for example, in case only S5=1, S6=1, S1,2,3,4=0, there may be bit errors in
- P5 and P6 or
- D2 and D3 or
- D5 and D6 or
- D8 and D9.