For boundary Conditions Given

Effect of gamma

1. SOR N=150
   1. Gamma=pi
      1. timedoc = 14.7931
      2. Iterations = 11130
   2. Gamma=-pi
      1. timedoc = 0.2304
      2. iterations= 200
   3. Gamma=0
      1. timedoc =2.4554
      2. iterations= 2080

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**Numerical Solution of the 2D Helmholtz Equation**

**MECE 5397: Scientific Computing for Mechanical Engineers**

**Professors:**

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# Abstract

# Mathematical statement of the project

The time-independent wave response over a rectangular region, is described by a 2-dimensinal partial differential equation as shown below. This equation is known as the 2D Helmholtz equation. Dirichlet boundary conditions have been prescribed on the boundaries 1, 2 and 4, while a Neumann boundary condition is applied on the bottom edge. The wave constant in the equation is given as, and a forcing function described below is applied to the system. Numerical solvers are used to approximate the solution to this differential equation. In this report the Gauss-Seidel method and the Successive Over Relaxation method are used to approximate the solution. There is a special case of the Helmholtz equation, where, and F(x,y) =0. This form of the equation is known as the 2-dimensional Laplace equation. Approximations for the Laplace equation will be provided in addition to the Helmholtz approximation.

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The differential equation above can be approximated through discretization of the second derivatives in x and y. Discretizing the Helmholtz equation must be in order for the numerical methods to be applied. The second order centered-difference formula is used to approximate the second derivative terms in the equation above.

|  |  |
| --- | --- |
| Centered-difference Formula |  |

These equation are found using the Taylor series expansion to determine an approximation of the second derivative that is described by three different consecutive points in the region. Substituting these equations into the main differential equation yields:

This is the discretized version of the given partial differential equation. The next step it is to collect all the similar terms in order to obtain a form of the discretized equation that allows the iterative approximation to the solution.

# Numerical Solvers

## Gauss-Seidel Method

The Gauss-Seidel method is a numerical solver that