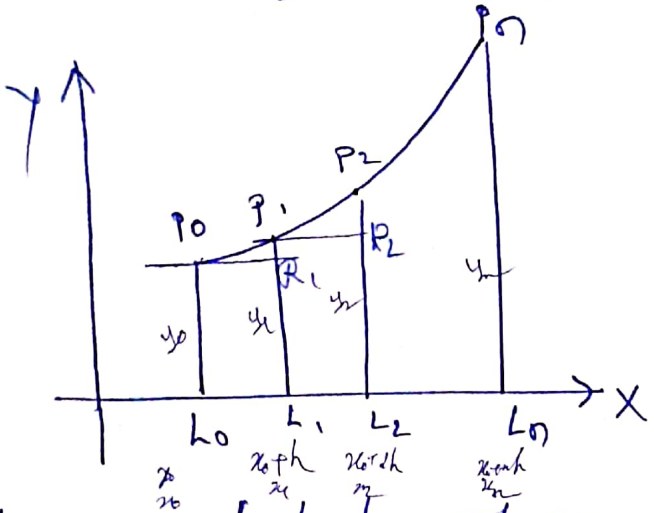
EULER'S METHOD



The general first order diff eqn is $\frac{dy}{dx} = f(x, y)$.

with $y(x_0) = y_0$.

Let us divide L_0L_n in to n subintervals each of width 'h' at $L_1\ L_2\ L_3 - - - L_n - -$

In the interval L_0L_1 we approximate the curve by the tangent at $P_0(x_0, y_0)$.

The co-ords of P_1 are (x_0+h, y_1)

$$y_1 = P_1L_1 = R_1L_1 + P_1R_1$$

$$= P_0L_0 + P_0R_1 \tan\theta = y_0 + h(\tan\theta)$$

$$= y_0 + h\left(\frac{dy}{dx}\right)_{P_0}$$

$$y_1 = y_0 + h f(x_0, y_0).$$

III^{ly} $y_2 = y_1 + h f(x_0+h, y_1)$ or $y_1 + h f(x_1, y_1)$

Repeating this n-times,

$$y_n = y_{n-1} + h f\{x_0 + (n-1)h, y_{n-1}\}$$

OR

$$\boxed{y_{n+1} = y_n + h f(x_0 + nh, y_n)}$$

$$y_{n+1} = y_n + h f(x_n, y_n).$$

Solve the eq^n $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ for $x = 1$ taking $h = 0.2$. Compare the value with actual value obtained from analytical sol^n $y = \sqrt{2x+1}$

$y(0) = 1 \Rightarrow x_0 = 0,\ y_0 = 1$ $\quad n = 5$ $\quad x_n = 1$, $x_0 = 0$

$h = 0.2$. $\quad h = \frac{1-0}{5} = 0.2$

By Euler's method $\quad n = \frac{x_n - x_0}{h} = \frac{1-0}{0.2} = \frac{10}{2} = 5$

$y_1 = y_0 + h f(x_0, y_0) = 1 + 0.2 f\left(y_0 - \frac{2x_0}{y_0}\right)$

$y_1 = 1 + 0.2\left[1 - \frac{2(0)}{1}\right] = 1.2$

$y_2 = y_1 + h f(x_0 + h, y_1) = 1.2 + 0.2\left[f(0.2, 1.2)\right]$

$= 1.2 + 0.2\left[1.2 - \frac{2(0.2)}{1.2}\right]$

$= 1.37333333$

$y_3 = y_2 + h f(x_0 + 2h, y_2)$

$= 1.37333333 + 0.2\left[1.37333333 - \frac{2(0.4)}{1.3733333}\right]$

$y_3 = 1.53149545$

$y_4 = 1.53149545 + 0.2\left(1.53149545 - \frac{2(0.6)}{1.53149545}\right)$

$y_4 = 1.68108458$

$y_5 = 1.82826948129$

For exact value

Given $\frac{dy}{dx} = y - \frac{2x}{y}$ ——— (1)

$\frac{dy}{dx} - y = -\frac{2x}{y}$ is of the type $\frac{dy}{dx} + py = Qy^n$

$p = -1$, $Qy^n = \frac{-2x}{y}$

$\frac{1}{y^n}\frac{dy}{dx} + py^{1-n} = Q$

From (1)

$y\frac{dy}{dx} - y^2 = -2x$ put $y^2 = z$

$2y\frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \frac{1}{2}\frac{dz}{dx} - z = -2x$ $\quad y\frac{dy}{dx} = \frac{1}{2}\frac{dz}{dx}$

$\frac{dz}{dx} - 2z = -4x$ which is now linear eqn in z

$I.F = e^{\int p\,dx} = e^{\int -2dx} = e^{-2x}$

$z(e^{-2x}) = \int -4x(e^{-2x})\,dx + C$

$= -4\int xe^{-2x}\,dx + C$

$= -4\left\{x\left(\frac{e^{-2x}}{-2}\right) - 1\frac{e^{-2x}}{4}\right\} + C$

$= -4\left[-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4}\right] + C$

$z(e^{-2x}) = 2xe^{-2x} + e^{-2x} + C$

$y^2(e^{-2x}) = e^{-2x}(2x+1) + C$

$y^2 = 2x + 1 + Ce^{2x}$

Given $y=1$ when $x=0$

$1 = 0+1+Ce^{2(0)} \Rightarrow \underline{C=0}$

$y^2 = 2x+1+0(\ ) = 2x+1$

$y = \sqrt{2x+1}$

when $x=1$ $y = \sqrt{3} = \underline{1.732050808}$

which is the exact value

Error = exact ~~calculated~~ value

$= 1.732050808 - 1.826948129$

Error $= -0.094897371$

$\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y(0)=1$ find y approximately for $x=0.1$ by EM. (5 steps) (12)

Sol Given $\frac{dy}{dx} = \frac{y-x}{y+x} = f(x, y)$

$x_0 = 0 \quad y_0 = 1 \quad h = \frac{x_n - x_0}{n} = \frac{0.1-0}{5} = \underline{0.02}$

By Euler's method

$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.02\left[\frac{y_0 - x_0}{y_0 + x_0}\right]$

$= 1 + 0.02\left[\frac{1-0}{1+0}\right]$

$\boxed{y_1 = 1.02}$

$y_2 = y_1 + hf(x_1, y_1) = y_1 + hf(x_0+h, y_1)$

$= y_1 + 0.02\,[f(0.02, 1.02)]$

$= y_1 + 0.02\left[\frac{1.02 - 0.02}{1.02+0.02}\right]$

$= 1.02 + 0.02\left(\frac{1.02 - 0.02}{1.02 + 0.02}\right)$

$y_2 = \underline{1.039230769}$

$y_3 = y_2 + hf(x_0 + 2h, y_2)$

$= y_2 + 0.02 f[0.04, y_2]$

$= 1.0577482332$

$y_4 = y_3 + hf(x_0 + 3h, y_3)$

$= y_3 + 0.02 f(0.6, y_3)$

$= 1.075601058$