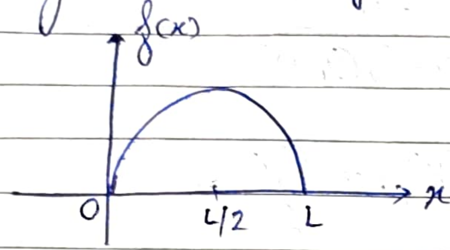
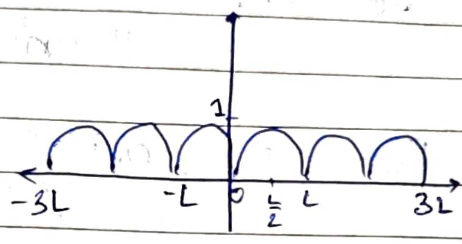
Math OBA - 2

(2GI19CS175)

1.) In $0<x<L$, $f(x)$ is neither periodic nor odd or even.
Construct, $g(x) = -\sin\left(\frac{\pi x}{L}\right)$ in $L<x<0$
$g(x)$ is even, periodic with perion $2L$.





Fourier cosine series of $g(x)$ is,

$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$ where,

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)dx$$

$$= \frac{2}{L}\frac{L}{\pi}\left(-\cos\frac{\pi x}{L}\right)\Big|_0^L$$

$$a_0 = \frac{-2}{\pi}[-1-1] = \frac{4}{\pi}$$

For $n \neq 1$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)\cdot\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{L}\cdot\frac{1}{2}\int_0^L\left[\sin\frac{(1+n)\pi x}{L} + \sin\frac{(1-n)\pi x}{L}\right]\cdot dx$$

$$a_n = \frac{1}{L}\left[\frac{-L}{\pi(n+1)}\cdot\cos\frac{(1+n)\pi x}{L} - \frac{L}{\pi(1-n)}\cdot\cos\frac{(1-n)\pi x}{L}\right]_0^L$$

GoodLuck Page No.
Date

(2GI19CS175)

1.) $\frac{1}{\pi}\left[\frac{1}{n+1} - \frac{1}{n-1}\right]\left[(-1)^n+1\right]$

$= \frac{4}{\pi} \cdot \frac{1}{n^2-1}$ if n is even

$= 0$ if n is odd

i.e., $a_{2n} = \frac{4}{\pi} \frac{1}{4n^2-1} = \frac{-4}{\pi(2n-1)(2n+1)}$

For $n=1$, $a_1 = \frac{2}{L} \int_0^L \sin\left(\frac{\pi x}{L}\right) \cdot \cos\left(\frac{\pi x}{L}\right) dx = 0$

Thus,

$$g(x) = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} \cdot \cos 2n \frac{\pi x}{L}$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{1}{1\cdot 3}\cos\frac{2\pi x}{L} + \frac{1}{3\cdot 5}\cos\frac{4\pi x}{L} + \frac{1}{5\cdot 7}\cos\frac{6\pi x}{L} + \dots\right)$$

Hence the fourier cosine series representation of $f(x)$ in $(0, L)$ is

$$f(x) = g(x) = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} \cdot \cos\left(\frac{2n\pi x}{L}\right).$$

GoodLuck Page No.
Date

2.)

x (in °)	$f(x)$	x (in rad)	$f(x)\cos x$	$f(x)\sin x$
30	7.976	$\pi/6$	6.90741	3.988
60	8.026	$\pi/3$	4.013	6.9507
90	7.204	$\pi/2$	0	7.204
120	5.676	$2\pi/3$	-2.838	4.9155
150	3.674	$5\pi/6$	-3.1817	1.837
180	1.764	π	-1.764	0
210	0.552	$7\pi/6$	-0.4780	-0.276
240	0.262	$4\pi/3$	-0.131	-0.22689
270	0.904	$3\pi/2$	0	-0.904
300	2.492	$5\pi/3$	1.246	-2.1581
330	4.736	$11\pi/6$	4.1014	-2.368
360	6.284	2π	6.284	0
	$\Sigma = 49.55$		$\Sigma = 14.15911$	$\Sigma = 18.96221$

$(0, 2\pi) = (\alpha, \alpha + 2c)$

$\rightarrow \alpha_1 = 0, \quad c = \pi$

$a_0 = 2\,[\text{mean of } f(x) \text{ in } [0, 2\pi)]$

$a_1 = 2\,[\text{mean of } f(x) \text{ in } (0, 2\pi)]$

$b_1 = 2\,[\text{mean of } f(x) \text{ in } (0, 2\pi)]$

$$a_0 = 2\left[\frac{49.55}{12}\right] = \underline{8.2583}$$

$$a_1 = 2\left[\frac{14.15911}{12}\right] = \cancel{2.359} \; \underline{2.35985}$$

$$b_1 = 2\left[\frac{18.96221}{12}\right] = \underline{3.16036}$$

$$\boxed{y = 4.12915 + 2.35985\cos x + 3.16036\sin x}$$

GoodLuck Page No.
Date

(2GI19CS175)

3.) i.)

$$P = \begin{array}{c c} & \begin{array}{ccc} a_0 & a_1 & a_2 \end{array} \\ \begin{array}{c} a_0 \\ a_1 \\ a_2 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1/8 & 1/2 & 3/8 \\ 0 & 1/2 & 1/2 \end{bmatrix} \end{array}$$

ii.) The system starts in state a_0, so that $p^{(0)} = (1,0,0)$ is the initial state.

Now,

$$p^{(1)} = p^{(0)} P = (1\ 0\ 0) \begin{bmatrix} 0 & 1 & 0 \\ 1/8 & 1/2 & 3/8 \\ 0 & 1/2 & 1/2 \end{bmatrix} = (0\ 1\ 0)$$

$$p^{(2)} = p^{(1)} P = (0\ 1\ 0) \begin{bmatrix} 0 & 1 & 0 \\ 1/8 & 1/2 & 3/8 \\ 0 & 1/2 & 1/2 \end{bmatrix} = (1/8, 1/2, 3/8)$$

$$p^{(3)} = p^{(2)} P = \left(\frac{1}{8}, \frac{1}{2}, \frac{3}{8}\right) \begin{bmatrix} 0 & 1 & 0 \\ 1/8 & 1/2 & 3/8 \\ 0 & 1/2 & 1/2 \end{bmatrix} = \left(\frac{1}{16}, \frac{9}{16}, \frac{6}{16}\right)$$

Probability that there are 2 red in urn A, i.e in state a_2 after 3 steps is $\frac{6}{16} = \frac{3}{8}$

iii.) Let t be (x, y, z) or $(x, y, 1-x-y)$

Then $tP = t$

$$(x\ y\ z) \begin{bmatrix} 0 & 1 & 0 \\ 1/8 & 1/2 & 3/8 \\ 0 & 1/2 & 1/2 \end{bmatrix} = (x\ y\ z)$$

Solving $\frac{1}{8} \cdot y = x$ or $y = 8x$

$x + \frac{y}{2} + \frac{z}{2} = y$ or $2x - y + z = 0$

3.) $\frac{3y}{8} + \frac{z}{2} = z$ or $3y = 4z$

Now, $3y = 4z = (4(1-x-y)$

$7y = 4 - 4x$ or $56x + 4x = 4$

$\therefore x = \frac{4}{60}$, $y = \frac{8}{15}$, $z = \frac{6}{15}$

$\therefore$ The fixed version, $t = \left(\frac{1}{15}, \frac{8}{15}, \frac{6}{15}\right)$

Therefore, system in long run stays in ~~stable~~ state a_2, 40% of the time $\left(\frac{6}{15} = \frac{2}{5}\right)$ (i.e. there will be 2 red in A, 40% of time)