Lecture 04 Boolean Algebra

ECE09 – Digital Electronics: Logic Circuits and Switching Theory

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Introduction

- Boolean Algebra, named after its pioneer **George Boole** (1815-1864) is the algebra of logic presently applied to computer systems and devices.
- The rule of this algebra is based on human reasoning.
- Boolean algebra remained in the realm of philosophy till 1938 when Claude E. Shannon used it to solve relay logic problems.
- As compared to other mathematical tools of analysis and design, Boolean algebra has the advantages of simplicity, speed and accuracy.

Laws of Boolean Algebra

- OR LAWS

Law 1. $A + 0 = A$

Law 2. $A + 1 = 1$

Law 3. $A + A = A$

Law 4. $A + \overline{A} = 1$

- AND LAWS

Law 5. $A \,.\, 0 = 0$

Law 6. $A \,.\, 1 = A$

Law 7. $A \,.\, A = A$

Law 8. $A \,.\, \overline{A} = 0$

Laws of Boolean Algebra

- LAWS OF COMPLEMENTATION

Law 9. $\overline{0} = 1$

Law 10. $\overline{1} = 0$

Law 11. if $A = 0$, then $\overline{A} = 1$

Law 12. if $A = 1$, then $\overline{A} = 0$

Law 13. $A = A$

- COMMUTATIVE LAWS

These laws allow ***change in the position*** of variables in *OR* and *AND* expressions.

Law 14. $A + B = B + A$

Law 15. $A \, . \, B = B \, . \, A$

Laws of Boolean Algebra

- ASSOCIATIVE LAWS

These laws allow removal of brackets from logical expression and regrouping of variables.

Law 16. $A+(B+C)=(A+B)+C$

Law 17. $(A+B)+(C+D)=A+B+C+D$

Law 18. $A\,.\,(B\,.\,C)=(A\,.\,B)\,.\,C$

- DISTRIBUTIVE LAWS

These laws permit factoring or multiplying out of an expression.

Law 19. $A\,(B+C)=A\,B+A\,C$

Law 20. $A+BC=(A+B)(A+C)$

Law 21. $A+\bar{A}\,.\,B=A+B$

Laws of Boolean Algebra

- ABSORPTIVE LAWS

These enable us to reduce a complicated logic expression to a simpler form by absorbing some of the terms into existing terms.

Law 22. $A + AB = A$

Law 23. $A.(A + B) = A$

Law 24. $A.(\bar{A} + B) = AB$

- DE MORGAN'S THEOREM

Law 25. $\overline{A + B} = \bar{A}\,.\,\bar{B}$

Law 26. $\overline{A.B} = \bar{A} + \bar{B}$

Examples

1. Prove the following Boolean identity: $AC + ABC = AC$

Examples

2. Prove the following Boolean identity: $(\mathrm{A} + \mathrm{B})(A + C) = A + BC$

Examples

3. Prove the following Boolean identity: $A + \bar{A}B = A + B$

Examples

4. Prove the following Boolean identity: $\mathrm{ABC} + \mathrm{A}\bar{B}C + AB\bar{C} = A(B + C)$

Examples

5. Demorganize the expression: $\overline{(A+B)(C+D)}$

Examples

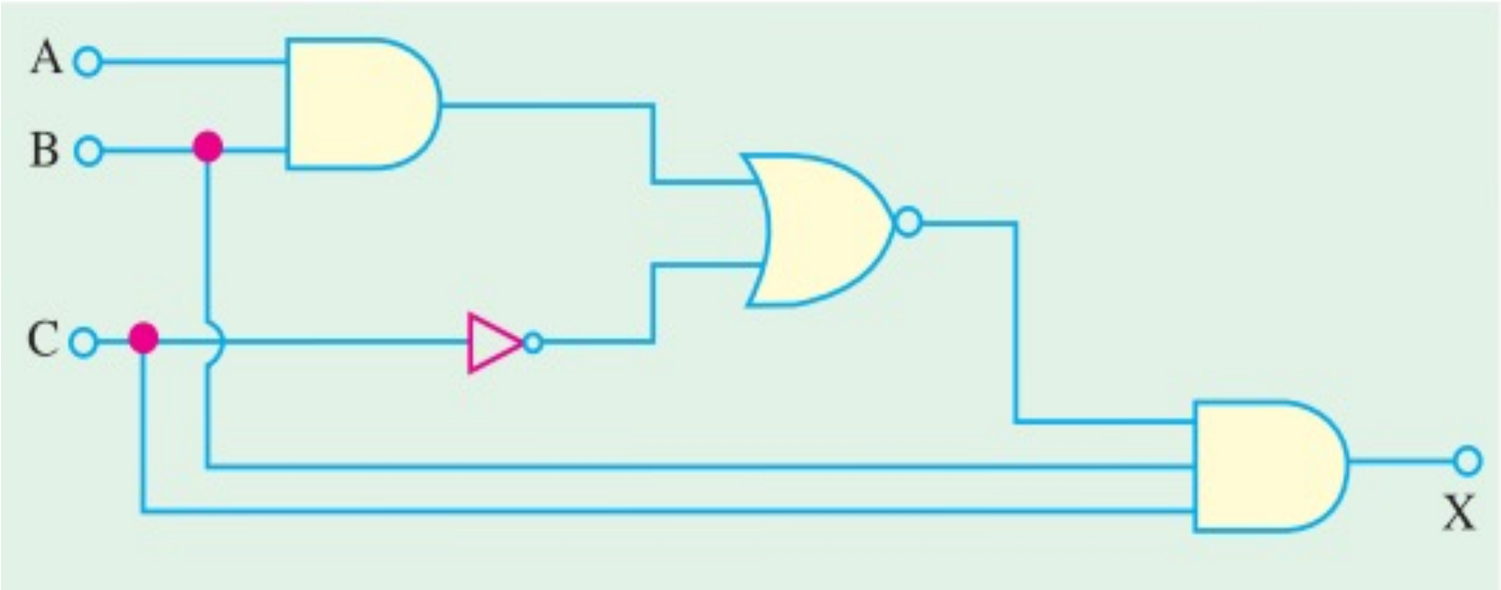
6. Simplify each of the following using De Morgan’s Theorem

   a. $\overline{A(\overline{B+\overline{C}})D}$

   b. $\overline{\overline{\overline{AB}\,\overline{C}}D}$

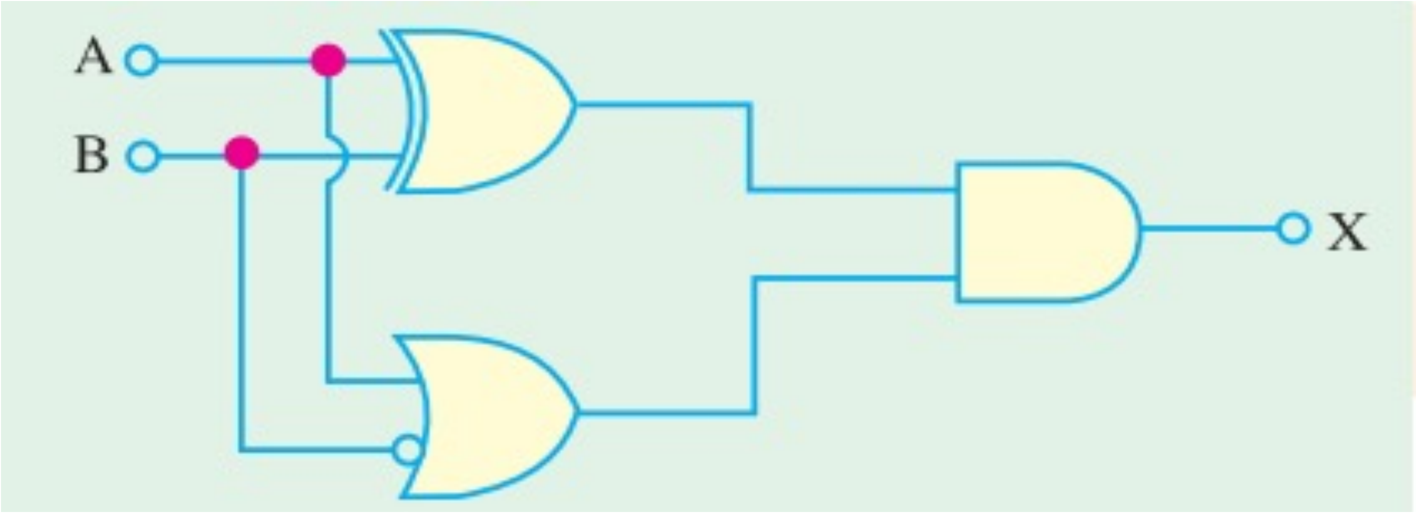
Examples

- Determine the Boolean expression for the logic circuit below. Simplify the Boolean expression and redraw the logic circuit using the simplified Boolean expression.



Example

- Determine the output X of a logic circuit shown below. Simplify the output expression using Boolean laws and theorems. Redraw the logic circuit with the simplified version.



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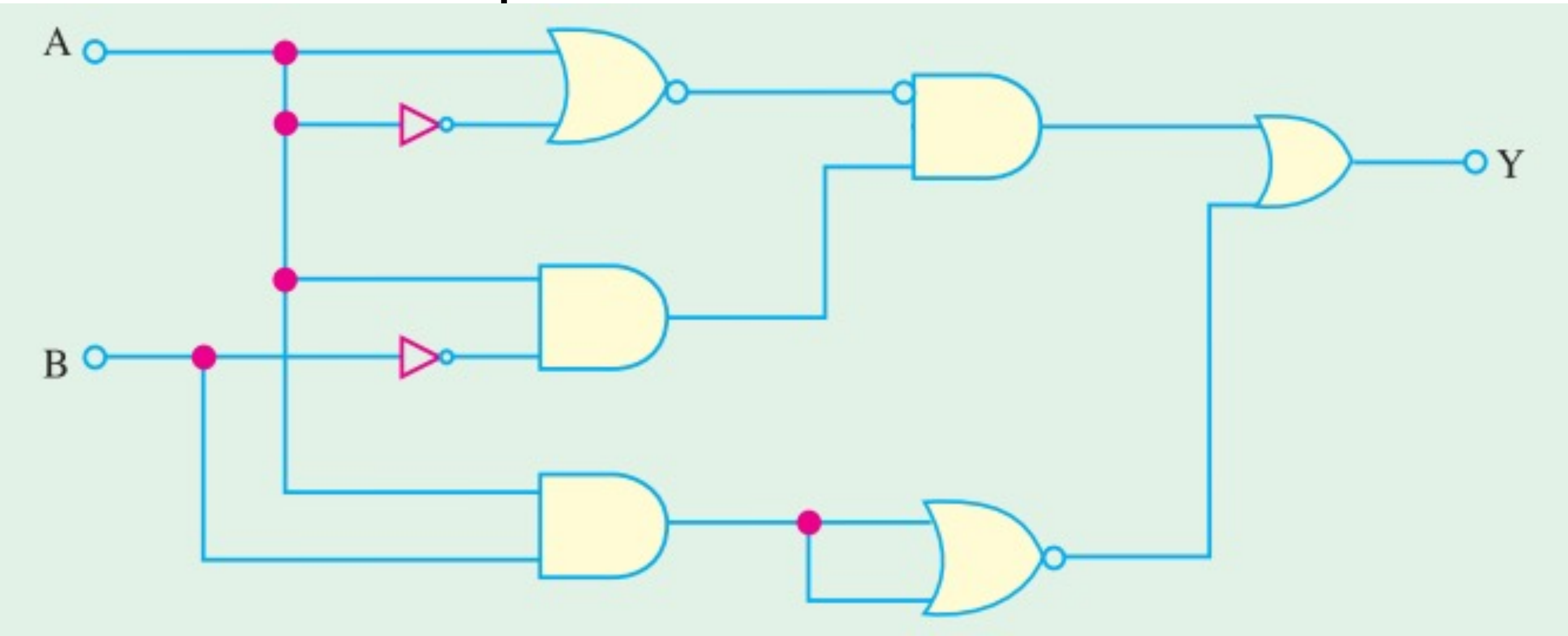

Design of Logic Circuit Using Boolean Algebra

1. SUM OF PRODUCT (SOP)

$$A\overline{B} + \overline{A}BC \qquad \overline{A}BC + A\overline{C}D + A\overline{B}CD, \qquad \overline{A}B + AC + \overline{A}\,\overline{B}\,C$$

2. PRODUCTS OF SUM (POS)

$$(A + \overline{B})(\overline{A} + B + C), \qquad (\overline{A} + \overline{B})(A + C)(\overline{A} + \overline{B} + C)$$

Conversion of Truth Table to Equivalent Boolean Expression

A	B	C	X	MINTERM	MAXTERM
0	0	0	0		
0	0	1	1		
0	1	0	0		
0	1	1	0		
1	0	0	1		
1	0	1	0		
1	1	0	0		
1	1	1	1		

Examples

1. Obtain the simplified Boolean expression based on the given truth table.

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Examples:

1. Design a logic circuit that has three inputs A, B and C whose output will be high only when the majority of the inputs are high.
2. Jon Snow’s castle has three doors. He wants to know when a white walker opens one of his door. Design a logic circuit that will be used to trigger an alarm when only one door is left open.