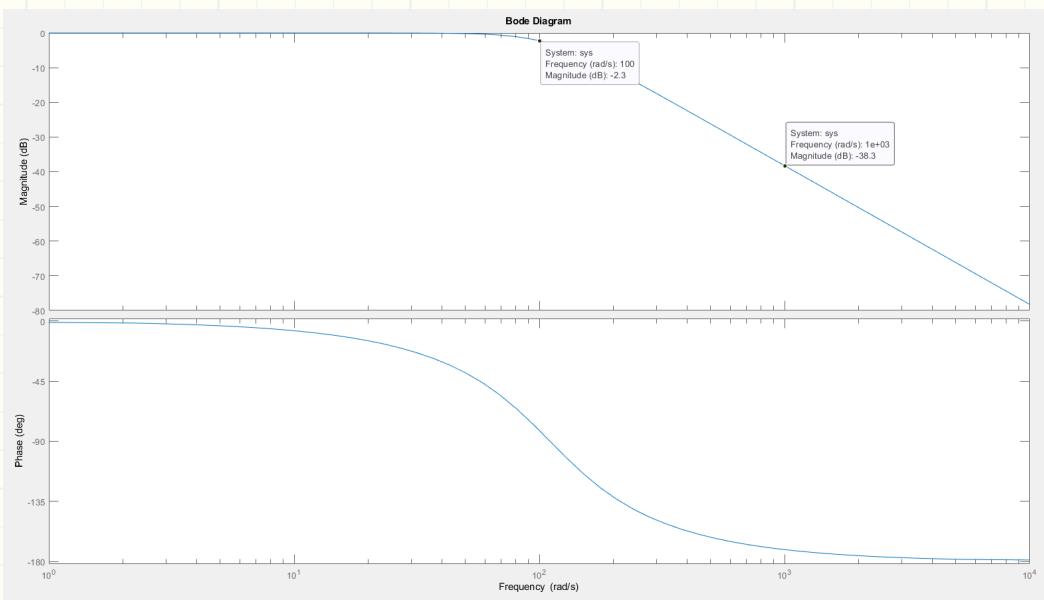


1) Design low pass filter $F(s)$ such that:

$$\begin{aligned} \text{for } f \in [0, 16] \text{ [Hz]} & , F(j\omega) \in (1, \frac{1}{\sqrt{2}}) \\ \text{for } f \in (16, \infty) \text{ [Hz]} & , F(j\omega) \in (0, 0.15) \end{aligned} \rightarrow |F(j\omega)|_{\text{dB}} \in (0, -3) \\ |F(j\omega)|_{\text{dB}} \in (-\infty, -16.47)$$

2nd Order Butter filter chosen: $F(s) = \frac{\omega_c^2}{s^2 + \frac{2}{\sqrt{2}}\omega_c s + \omega_c^2}$, $\omega_c = 32\pi(1+10\%) \text{ [rad/s]}$

Using Matlab's butter command:



2)

$$\ddot{y} + 0.2\dot{y} + 10y = 0.5 \sin(0.1t) + \sin(3t + \frac{\pi}{2})$$

$$H_{yu}(s) = \frac{1}{s^2 + 0.2s + 10}$$

$$H_{yu}(0+0.1j) = 0.1 e^{-0.2j}$$

$$H_{yu}(0+3j) = 0.857 e^{-0.540j}$$

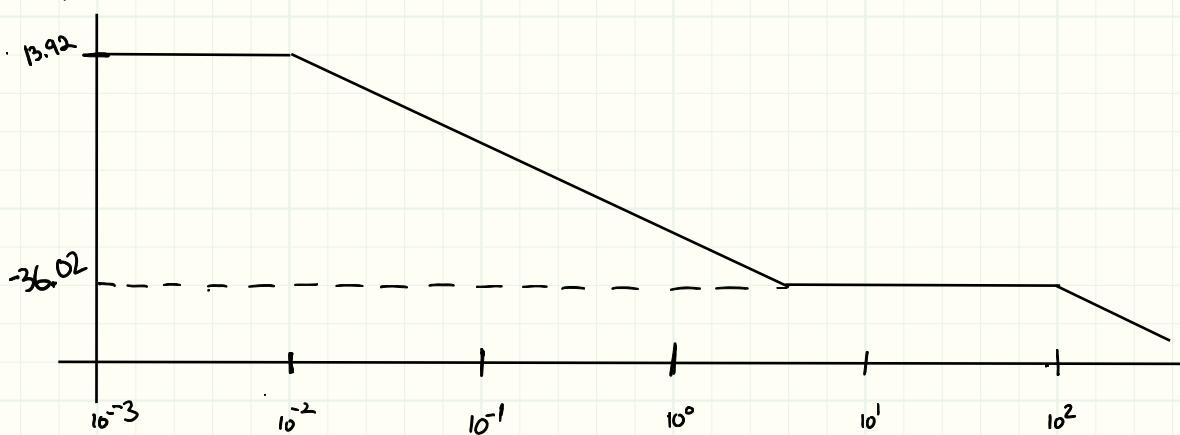
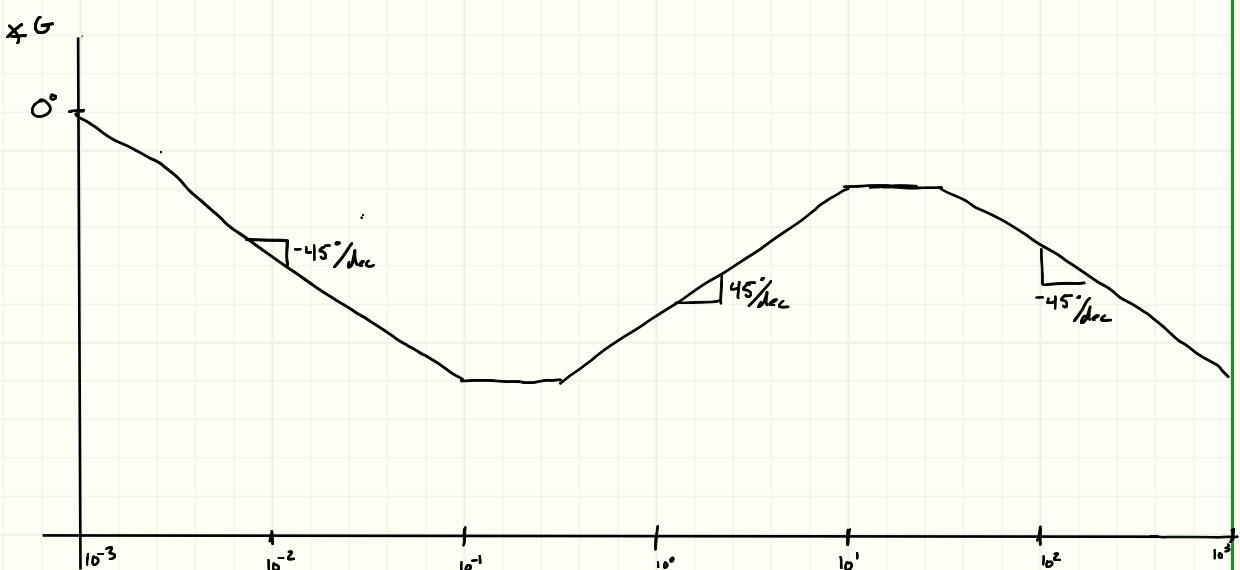
$$y(t) = |H_{yu}(0+0.1j)| \left| 0.5 \sin(0.1t + \angle H_{yu}(0+0.1j)) \right| + |H_{yu}(0+3j)| \left| \sin(3t + \frac{\pi}{2} + \angle H_{yu}(0+3j)) \right|$$

$$y(t) = 0.05 \sin(0.1t - 0.2) + 0.857 \sin(3t - 1.03)$$

3a)

$$G(s) = \frac{(s+5)}{(s+0.01)(s+100)} = \frac{5}{(s+0.01)} \cdot \frac{(s+5)}{(s+100)}$$

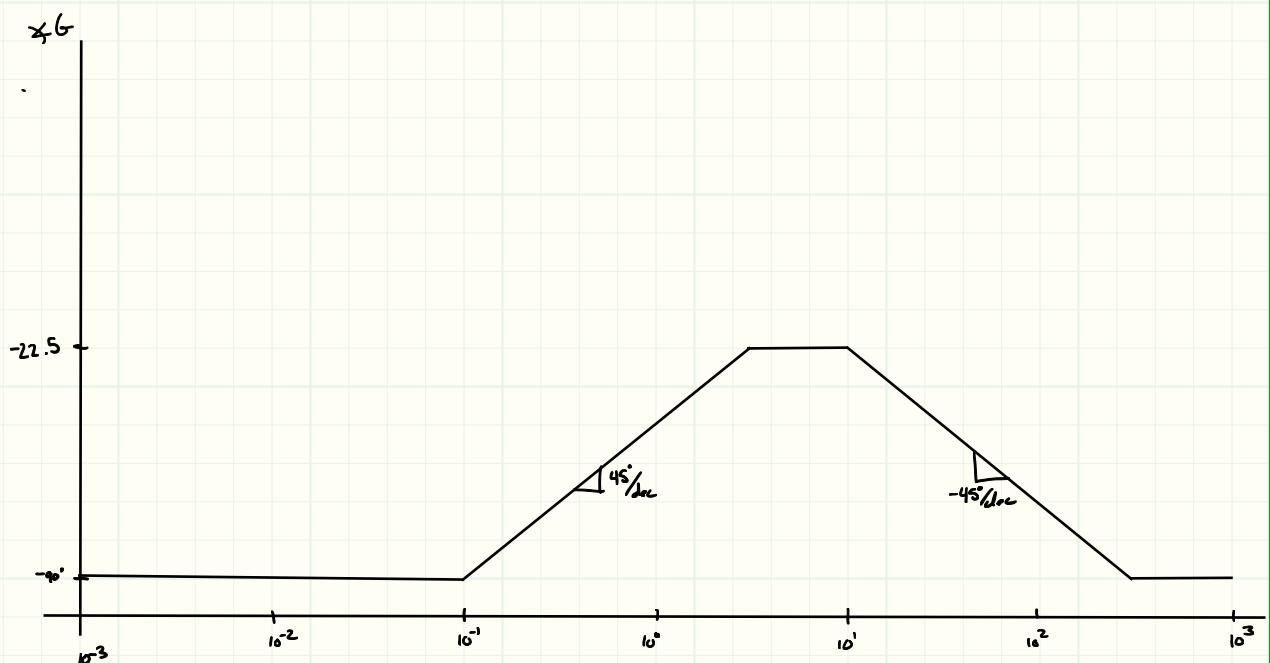
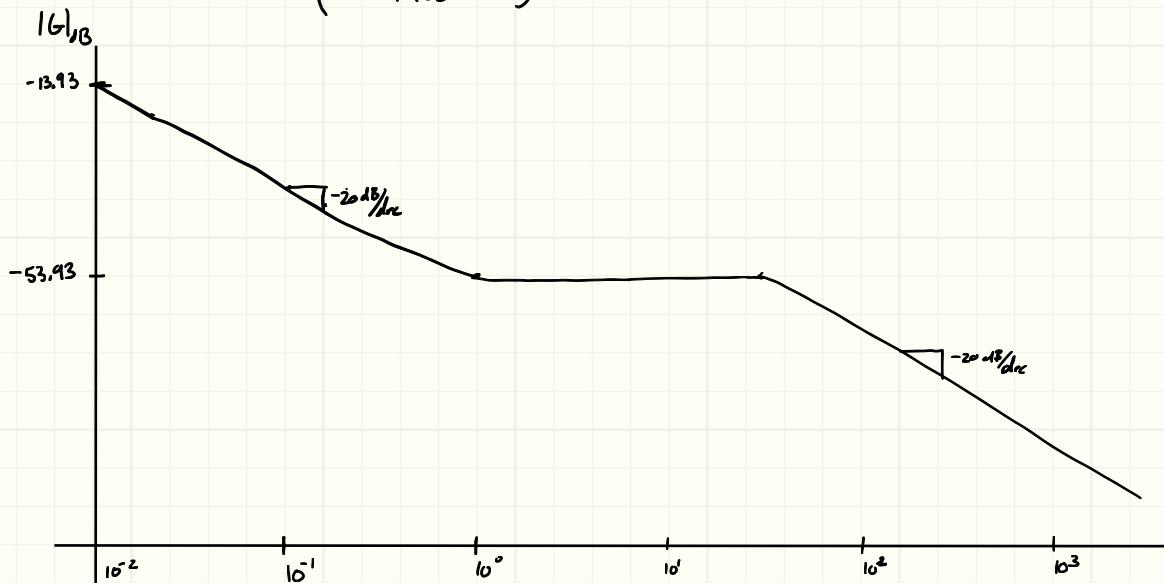
$$\omega_b = \{0.01, 5, 100\}$$

 $|G|_{dB}$  $\times G$ 

$$b) G(s) = \frac{(s+1)}{s(s+50)} \rightarrow G(j\omega) = \frac{j\omega + 1}{(j\omega)(j\omega + 50)} = \frac{49}{\omega^2 + 2500} - \frac{(\omega^2 + 50)}{\omega(\omega^2 + 2500)} j$$

$$|G(j\omega)|_{dB} = 10 \log (\omega^2 + 1) - 10 \log (\omega^4 + 2500\omega^2)$$

$$\angle G(j\omega) = \arctan \left(\frac{-(\omega^2 + 50)}{49\omega} \right)$$



$$c) \quad G(s) = \frac{s}{s^2 + 3s + 10} \quad G(j\omega) = \frac{j\omega}{(j\omega)^2 + 3j\omega + 10} = \frac{3\omega^2}{\omega^4 - 11\omega^2 + 100} - \frac{\omega(\omega^2 - 10)}{\omega^4 - 11\omega^2 + 100} j$$

$$|G(j\omega)|_{dB} = 20 \log(\omega) - 10 \log((10 - \omega^2)^2 + 9\omega^2)$$

$$\angle G(j\omega) = \arctan\left(\frac{-3\omega}{\omega^2 - 10}\right)$$