MAE 3724 Systems Analysis
Spring 2019

Laboratory Experiment 2
Free Response of a Second-Order Rotational System

Objectives of the Experiment:

To have a hands-on experience with a real dynamic system in the laboratory, and to compare the dynamic behavior of the actual system to that predicted from dynamic modeling and analysis. To become familiar with measurement methods and techniques for determining the experimental behavior of a dynamic system, and for estimating values of parameters that are difficult to determine from first principles modeling.

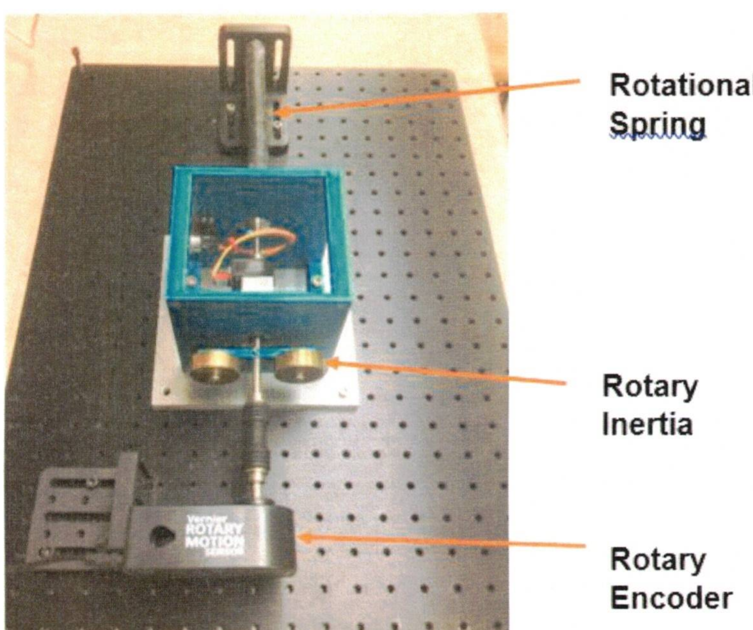
Background for Laboratory Experiment 2

In Experiment 2, you will study a rotational inertia-spring-damper system consisting of two masses attached to a shaft that travels through bearings and connects to a torsional spring (one end fixed). This is the rotary equivalent of Lab 1. A rotary encoder will be used to measure the angular displacement. The displacement is measured from equilibrium. As in Lab 1, you will study the free response of the system, i.e., the response due to initial conditions only.

In the Pre Lab, the rotational inertia of this system was calculated to be 0.000239 $kg\ m^2$.

Setup:

The equipment you will use for the experiment is shown in the photo below.



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- Double check that all parts are secured to the breadboard adequately. Ask the Lab TA for assistance if necessary.
- Connect the Vernier Rotary Motion Sensor to the first digital channel "DIG/SONIC 1" on the LabPro (blue-green device).
- Plug the LabPro to the power outlet using the power cable, and connect it to the computer using the USB cable.
- Launch the Logger Pro program on the computer.

Procedures:

- On the menu bar in Logger Pro, under "Experiment", go to "Setup sensor" and choose the "LabPro 1." In the new window, go to the digital channels and select the first channel ("DIG/SONIC 1"). Under "Choose sensor" select "Rotary Motion." Then click on the channel again and click the "Zero" button. **The sensors have to be zeroed each time you make a measurement.** (Make sure that you are tracking "*Angle*", "*Velocity*", and "*a*")

- On Logger Pro, under "Experiment", go to "Data Collection." Change the "Duration" to 20 seconds and double check that the "Sampling Rate" is 20 samples/second. Then click "Done."

Experiment:

Click "Collect" (wait until you see the data being collected) and then turn the shaft approximately 90 degrees and release it. Copy the data into an excel spreadsheet. Repeat this step three times and use the "best" result.

Before you leave the laboratory, make sure your data have been saved. These data will be needed for the plots required in the Final Report.

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Fall 2019

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FINAL REPORT: (Each individual must submit a Final Report)

SHOW ALL YOUR WORK ON THIS AND THE FOLLOWING PAGES. Scan these pages and all requested plots and upload them into the Brightspace Dropbox for Lab 2

1) *[2 pts]* Choose the best set of data from your 3 trials. Explain why you chose that set.

Trial 3 was chosen because it provided the smothest data

2) *[2 pts]* Using the first peak, last peak, and number of peaks from Excel, use the Logarithmic decrement method detailed in the online document titled "*Logarithmic Decrement*" to estimate the damping ratio (ζ). Note that at the end of the data, the value of the peaks may be in error due to the resolution of the sensor. It is OK to cut off the data before the values become too small in order to avoid this issue, but you must still use an appropriate number of peaks.

$B_1 = 1.6929$ $\delta = 0.191955$

$B_{n+1} = 0.1396$ $\zeta = 0.55518$

$n = 13$

3) *[1 pt]* Is the system critically damped, overdamped, or underdamped? (Hint: look at the discussion in section 2.5.7 of your book.)

Underdamped

4) *[3 pts]* Calculate the damped natural frequency (ω_d) using the period and corresponding number of cycles measured in your data. (Look at Example 8.4.4 in the textbook for an example.)

$$\cancel{\omega_d = \frac{8}{(6.13 - 2.83)s}} \quad \cancel{2.02\ Hz}$$

$$P = \frac{(6.95 - 3)s}{8} = 0.49375 \qquad \omega_d = \frac{2\pi}{P} = 12.72$$

5) *[1 pt]* Determine the natural frequency (ω_n) of the system using the damped natural frequency (ω_d) and the damping ratio (ζ). (Equation 2.5.17)

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 15.25$$

6) *[2 pts]* In general, the characteristic equation (denominator) for a 2nd order system can be written in the form:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Looking at the coefficients in your transfer function from the Pre-Lab, determine the relationship between k_T, ω_n, and I_e. Then back-calculate k_T using that relationship. (Remember $I_e = 0.000239\ kg \cdot m^2$)

$$\omega_n^2 = \frac{K_T}{I_e} \rightarrow K_T = (0.000239\ kg\,m^2)(15.25)^2$$

$$K_T = \cancel{0.003645}\ 0.05559$$

7) *[1 pt]* How does the k_T determined from your data compare to the k_T you calculated in the Pre Lab? (In the Pre Lab your model for the ideal rubber tube should have resulted in a $k_{TPreLab} = 0.0263\ Nm/rad$.) If there is a difference, give a reasonable explanation for it.

% diff = 71%

There is a significant difference & I would think it is due to propagated error in the calculations to get that value

8) *[1 pt]* (Similar to Part 6) Determine the relationship between c_T, ζ, ω_n and I_e. Then calculate c_T using that relationship.

$$2\zeta\omega_n = \frac{C_T}{I_e} \rightarrow C_T = 2(0.55518)(15.25)(0.000239)$$

$$C_T = 0.0040$$

9) *[1 pt]* What in the real-life system contributes to this damping?

Friction in joints, encoder, air-resistance

10) *[4 pts]* Using the plotting tool in Excel or MATLAB, plot the data for the angular displacement on one graph, the angular velocity on another, and the angular acceleration on a third. *(3 graphs total). Attach these graphs to your report.*

11) *[2 pts]* Briefly discuss the relationship between $\theta, \dot{\theta}$, and $\ddot{\theta}$, both graphically and mathematically.

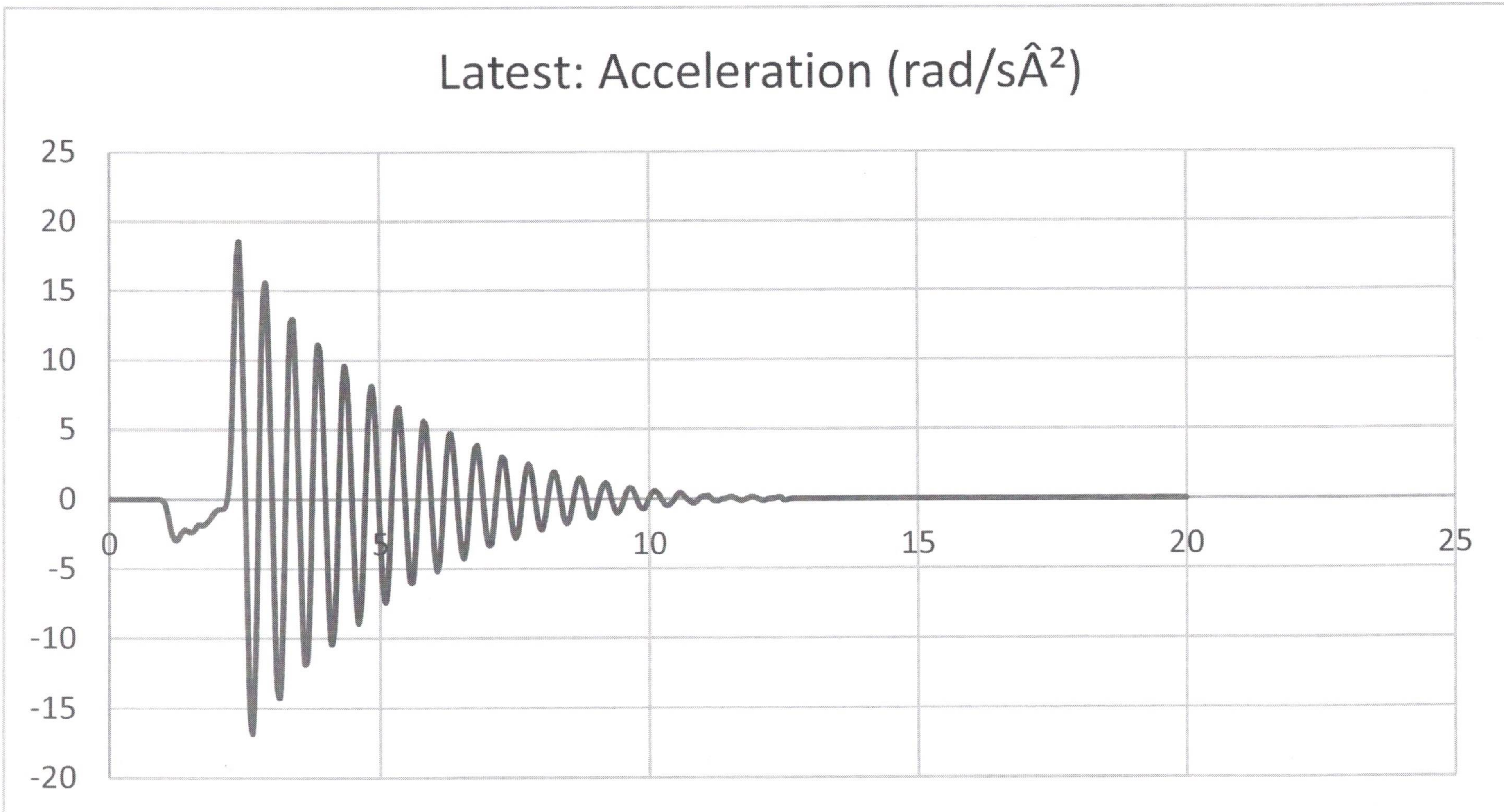
Assuming θ is a ~~sin~~ sin function, $\dot{\theta}$ would be a cos function & $\ddot{\theta}$ be a sin function because they are derivative of each other. This can be seen in the starting position & phase shifts between the graphs

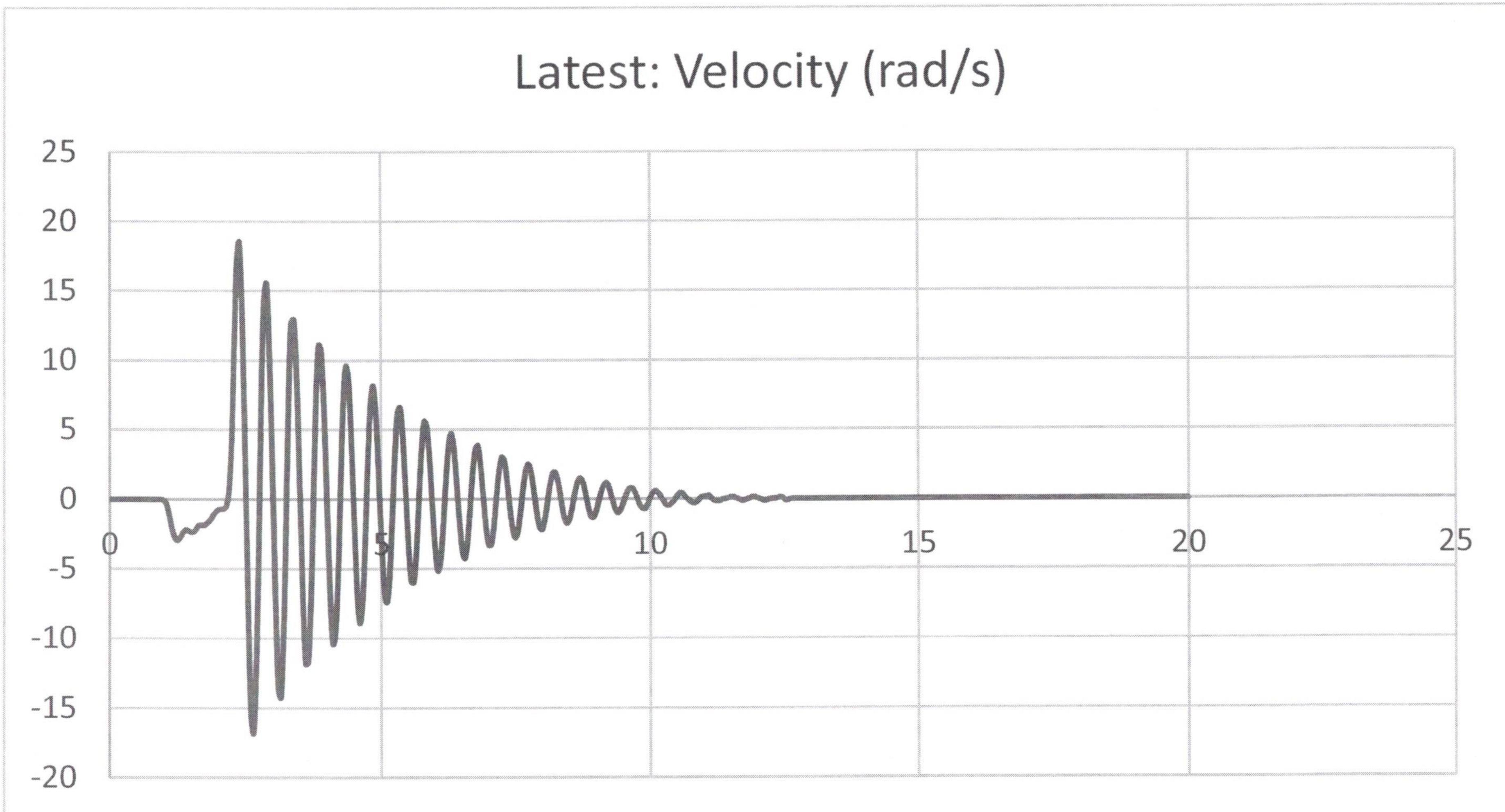
Final Evaluation (for feedback purposes, will not affect grade)

12) What did you learn from this experiment?

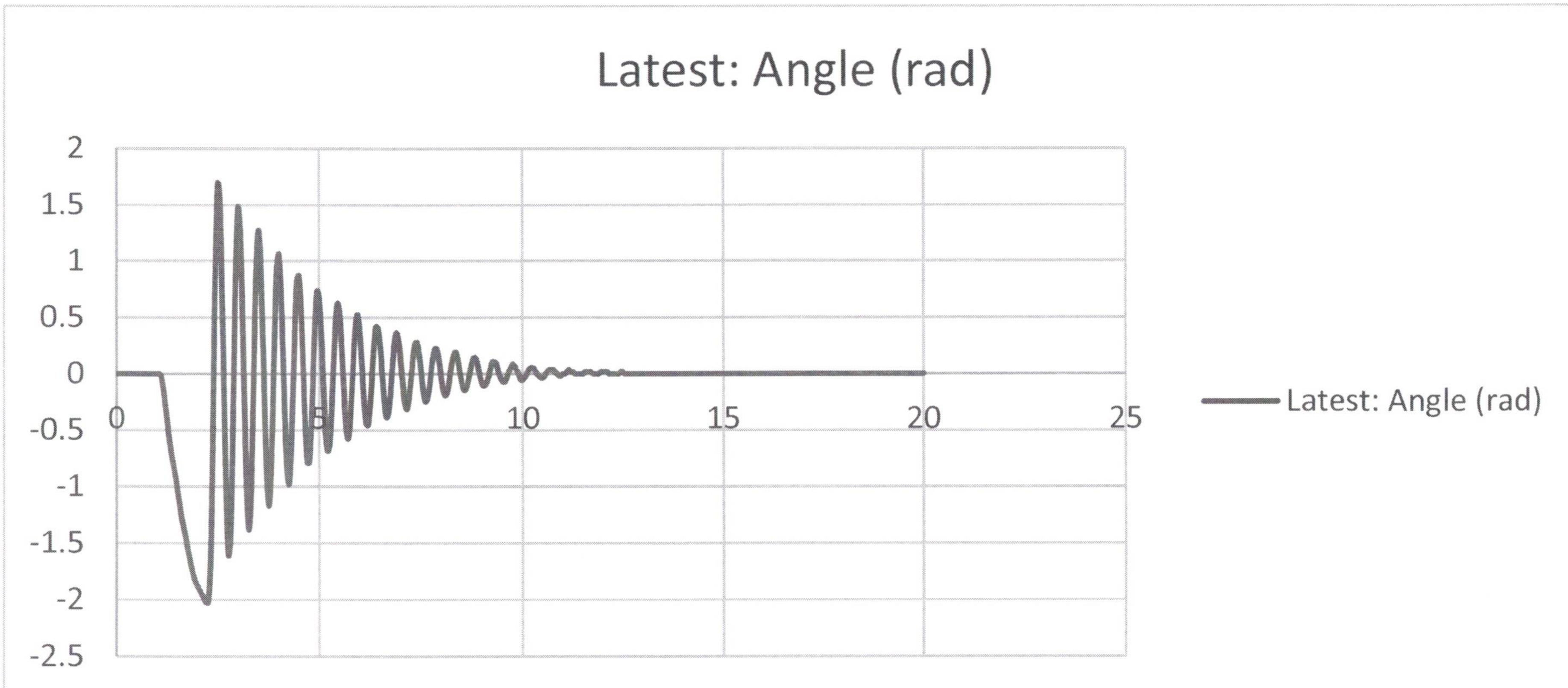
13) What could we do to make the experience more beneficial?

14) Which parts of the Pre-Lab, Lab, and Post-Lab were confusing, if any. Explain.


Latest: Acceleration (rad/sÂ²)
25
20
15
10
5
0
-5
-10
-15
-20
0
5
10
15
20
25



Latest: Velocity (rad/s)
25
20
15
10
5
0
-5
-10
-15
-20
0
5
10
15
20
25



Latest: Angle (rad)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
5
10
15
20
25
Latest: Angle (rad)