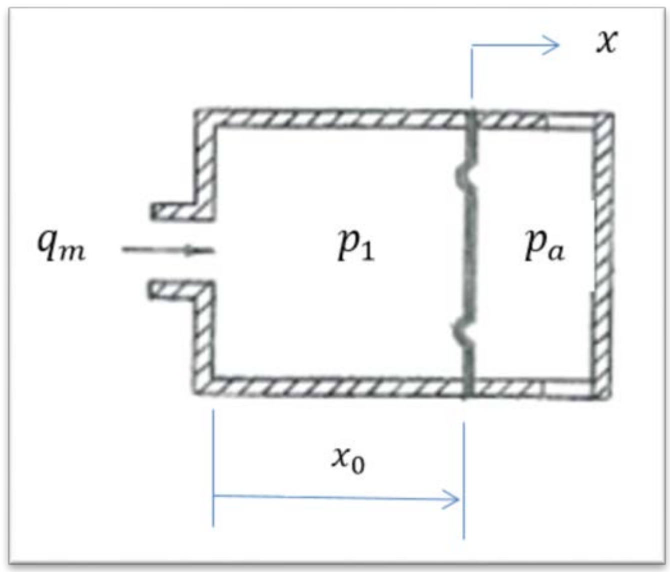
MAE 3723, System Analysis – Fall 2019

Homework Assignment #9

Due at 11:59 pm, Friday, October 25th, in the Canvas Homework 9 dropbox.

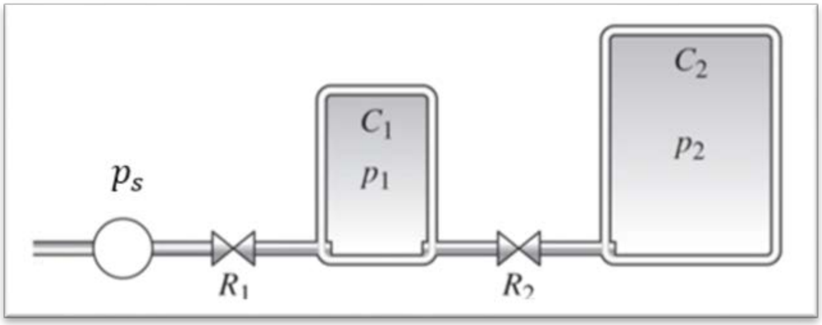
<u>Problem 1</u> – Consider the tank with a diaphragm of area A and a spring stiffness k. Both sides of the tank are filled with air. The pressure on the right side of the diaphragm is atmospheric pressure (constant). The mass flow rate q_m of air into the cylinder moves the diaphragm in the x direction. Assume that the diaphragm is massless. Assume that x is small compared to x_0. Assume the process of charging the chamber is sufficiently slow that the process is effectively isothermal. Derive an expression for the capacitance of the system



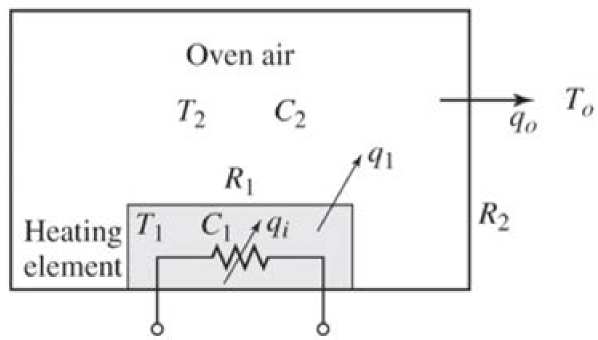
Problem 2 - The figure below shows two rigid tanks whose pneumatic capacitances are C_1 and C_2. The pneumatic lines have linear resistors R_1 and R_2. The pressure source is p_s. Assume that the processes in both tanks are isothermal.

a) Draw the equivalent electrical circuit diagram for the system.

b) Derive equations of motion which relate p_1, p_2 and p_s.



Problem 3 – A simplified representation of the temperature dynamics of two adjacent masses is shown in the figure. The mass with capacitance C_2 is perfectly insulated on all sides except the right side. That side has a convective resistance R_2. The thermal capacitances of the masses are C_1 and C_2, and their representative uniform temperatures are T_1 and T_2. The thermal capacitance of the surroundings is very large and the temperature of the surroundings is T_0.



a) Develop a model of the dynamic behavior of T_1 and T_2.

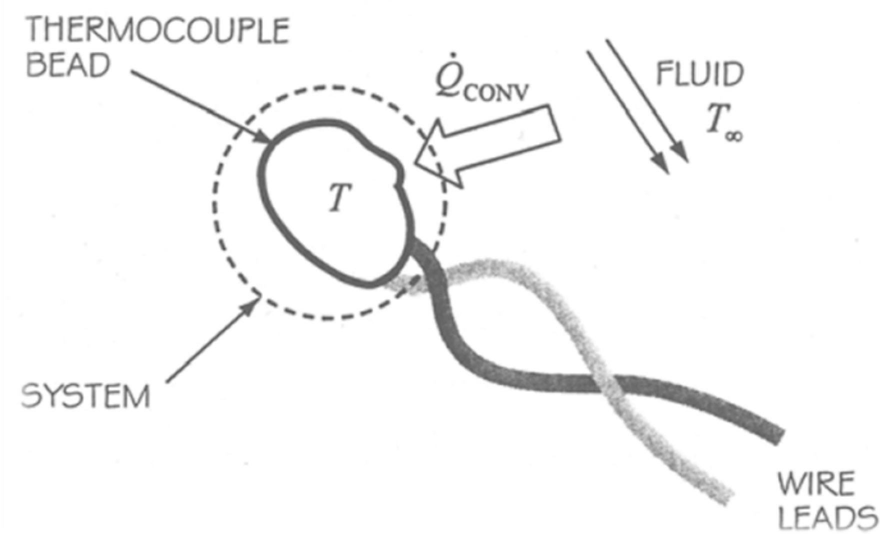
b) Discuss what happens if the thermal capacitance C_2 is very small.

Problem 4 – Thermocouple (this problem serves as a Pre-Lab for Lab 8)

The figure below shows a simple thermocouple. The "system" is the bead. The temperature throughout the bead is T (lumped parameter approach) and the temperature of the surrounding fluid is T_∞. Heat flows into the bead from the surrounding fluid. The bead properties are mass density ρ, volume V, specific heat c_p, and surface area A.

a) Starting with the equation for the conservation of energy, derive a dynamic model (ODE) for the thermocouple which relates the temperature of the bead to the temperature of the surrounding fluid. Assume that the temperature of the surrounding fluid is constant, that conduction through the wire leads and radiation heat transfer are negligible, and that heat transfer into the bead is primarily due to convection. Assume the convection coefficient is h.

b) Write an equation for the time constant.



Problem 5 – Problem 7.56 in the text – part a. only