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Name: Last, First

MAE 3724 Systems Analysis
Fall 2019

Pre-Lab for Experiment 6
Electroacoustic System / Speaker

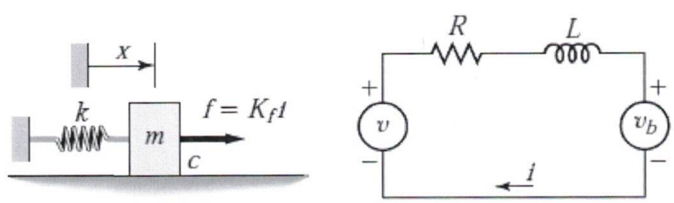
Complete this Pre-Lab BEFORE you come to the laboratory.
The Lab Instructor will answer questions and provide feedback when you come to the laboratory to do the experiment.

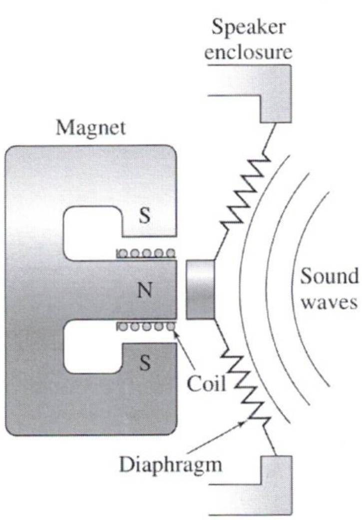
Show all your work and final answers on these pages. Attach the requested graphs.

Background for Laboratory Experiment 6

In Experiment 6, you will study the dynamic behavior of an electroacoustic system (or speaker). The background for this Pre-Lab is in Section 6.7.5 in the textbook.

A physical model of the electroacoustic system is shown in the figure below.





The motion of the center of the speaker cone is x. The motion is reference to an equilibrium position.

The coil is connected to the center of the speaker cone so that it moves a distance x when the center of the cone moves a distance x.

1) *[3 pts]* Derive a transfer function model for the speaker that relates the Laplace Transform of the output displacement $X(s)$ to the Laplace Transform of input voltage $V(s)$, assuming all initial conditions are equal to zero.

   Show all your work on the following page, starting with Newton's Law for the diaphragm and connected inertia, and Kirchhoff's voltage law for the circuit.

$$-c\dot{x} - Kx + K_f i = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + Kx = K_f i$$

$$X(ms^2 + cs + K) = K_f I$$

$$V - V_R - V_L - V_b = 0$$

$$V - Ri - L\dot{i} - K_b\dot{x} = 0$$

$$V - Ri - L\dot{i} = K_b\dot{x}$$

$$V - I(Ls + R) = K_b X s$$

$$I = [V - K_b X s]\frac{1}{(Ls + R)}$$

$$X(ms^2 + cs + K) = K_f[V - K_b X s]\frac{1}{Ls + R}$$

$$X(ms^2 + cs + K)(Ls + R) = K_f[V - K_b s X]$$

$$X(mLs^3 + cLs^2 + KLs + mRs^2 + cRs + KR) = K_f V - K_f K_b s X$$

$$X(mLs^3 + (cL + mR)s^2 + (KL + cR)s + KR) = K_f V - K_f K_b s X$$

$$X(mLs^3 + (cL + mR)s^2 + (KL + cR + K_f K_b)s + KR) = K_f V$$

$$\frac{X(s)}{V(s)} = \frac{K_f}{mLs^3 + (cL + mR)s^2 + (KL + cR + K_b K_f)s + KR}$$

2) *[1 pt]* Set $L = 0$. Rewrite the transfer function as the reduced model below.

$$\frac{X(s)}{V(s)} = \frac{K_f}{mRs^2 + (cR + K_b K_f)s + KR}$$

3) *[1 pt]* Write the transfer function in the form below. (Keep in terms of $m, k, K_f, K_b, R, \& c$) Show your work.

$$T.F. = A \cdot \frac{{\omega_n}^2}{s^2 + 2\omega_n \zeta s + {\omega_n}^2}$$

$$T.F = \frac{mK_f}{K}\left[\frac{K/m}{s^2 + \frac{cR + K_b K_f}{mR}s + K/m}\right]$$

$$\frac{X(s)}{V(s)} = \rule{8cm}{0.4pt}$$

4) *[1 pt]* Assume that the speaker parameters are as follows:

$m = 0.002\ kg$	$k = 4 \times 10^5\ N/m$
$K_f = 16\ N/A$	$K_b = 13\ V \cdot \frac{s}{m}$
$R = 5.91\ \Omega$	$c = 4.4\ N \cdot s/m$
$(L \approx 0\ H)$	

Determine numerical values for the undamped natural frequency ω_n and the damping ratio ζ.

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{4E5\ N/m}{2E\text{-}3\ Kg}} = 14.14E3\ \frac{rad}{s}$$

$$\zeta = \frac{1}{2\omega_n}\left[\frac{cR + K_b K_f}{mR}\right] = 0.699$$

5) *[1 pt]* The speaker is driven by a step function in the input voltage of $v(t) = 5$ V.

Determine an analytical expression for the response of x vs. time using the Laplace Transform method (Pair 23). ***Show all your work***.

$$X = \frac{mK_f}{K}\left[\frac{K/m}{s^2 + \frac{cR+K_bK_f}{mR}s + K/m}\right]\left[\frac{V_A}{s}\right]$$

$$X = \frac{V_A m K_f}{K}\left[\frac{K/m}{s\left(s^2 + \frac{cR+K_bK_f}{mR}s + K/m\right)}\right]$$

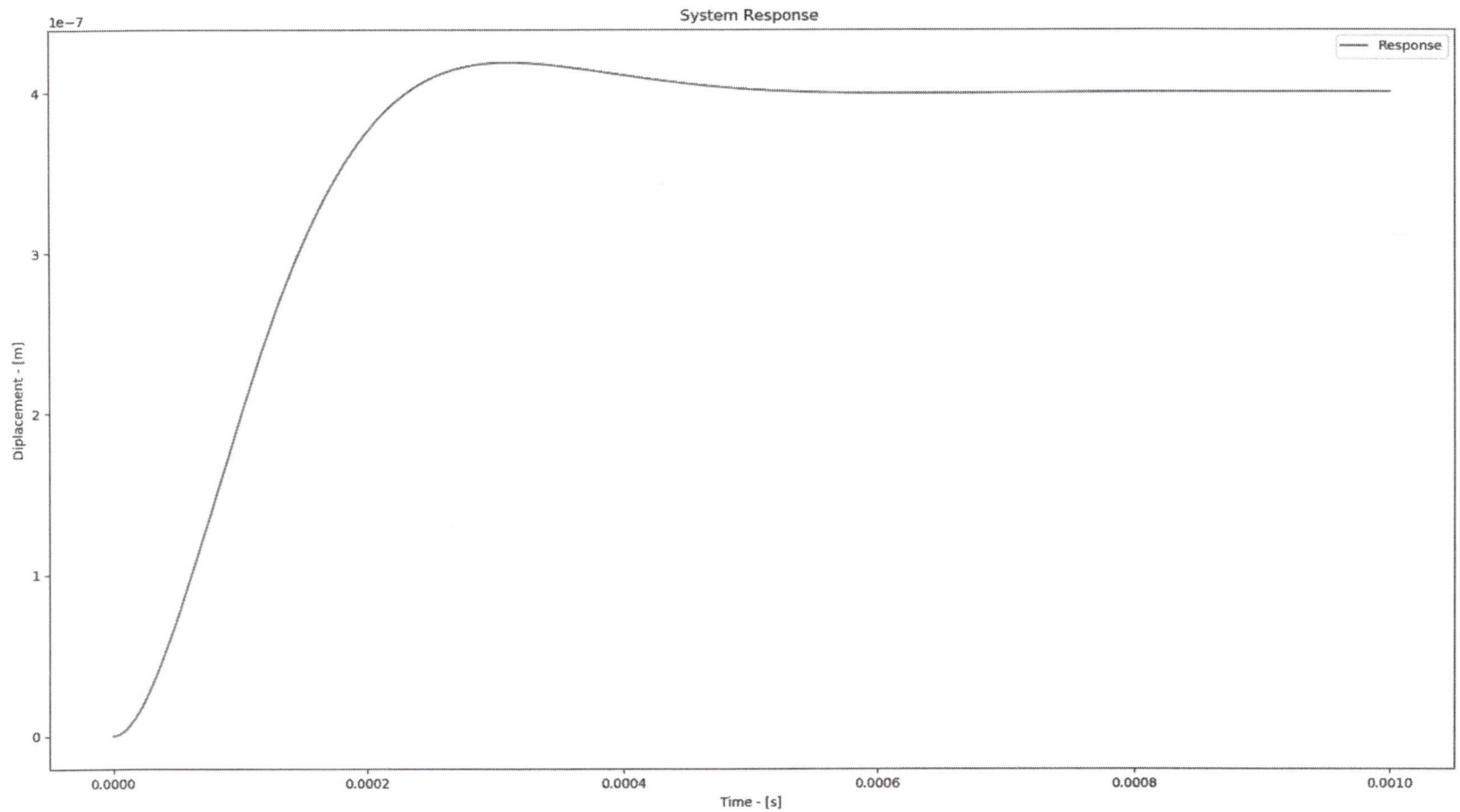
$$x(t) = \frac{V_A m K_f}{K}\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)\right)$$

$$\omega_n = \sqrt{K/m}$$

$$\zeta = \frac{1}{2\omega_n}\left[\frac{cR+K_bK_f}{mR}\right]$$

$x(t) =$

6) *[3 pts]* Plot x vs. time and attach your graph to the end of this Pre Lab. ***(Include a title, labels on the axes, & units on the graph)***


System Response
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Response
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