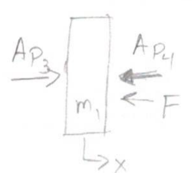
MAE 3723, System Analysis – Fall 2019

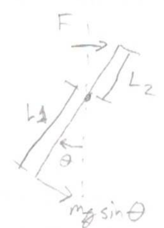
Homework Assignment #8

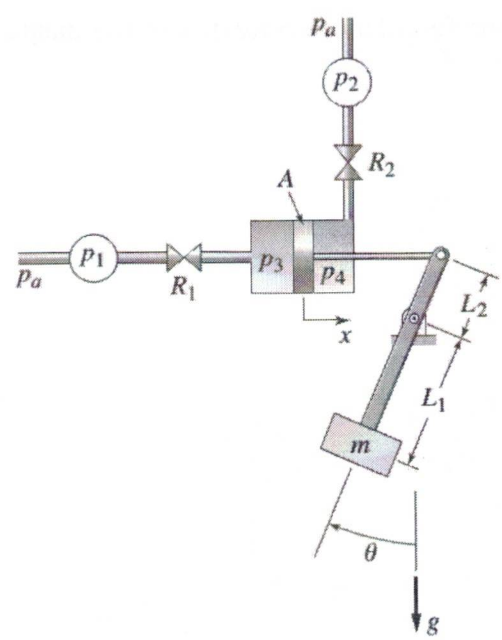
Due at 11:59 pm, Monday, Oct 21, in the Canvas Homework 8 dropbox.

1. The figure shows a pendulum driven by a hydraulic system. You will be deriving the equations of motion (ODEs) for the system in stages. Assume that the mass m lumped at the end of the pendulum is the only significant mass or inertia in the entire system. There is no friction between any of the mechanical parts. The fluid resistance is linear. You may assume small angles for the motion of the pendulum. You should treat the fluid as ***compressible***!

a) Draw freebody diagrams for the piston and the pendulum







b) Write one of Newton's equation for the piston

$$\sum F_x = AP_3 - AP_4 - F = m_1 \ddot{x}$$

c) Write one of Newton's equation for the pendulum

$$\sum \tau = -mg\sin\theta\, L_1 + F L_2 = I\ddot{\theta}$$

d) Write the kinematic equation relating the motion of the piston to the angle of the pendulum

$$\theta = L_2 x$$

e) Write the conservation of mass equation for each side of the cylinder

$$q_{m3} = \forall_3 \frac{\partial \rho_3}{\partial t} + \rho_3 \frac{\partial \forall_3}{\partial t} \qquad q_{m4} = \forall_4 \frac{\partial \rho_4}{\partial t} + \rho_4 \frac{\partial \forall_4}{\partial t}$$

f) Write the state equation for each side of the cylinder

$$\dot{\rho}_3 = \frac{(\rho_0)_3}{\beta}\dot{P}_3 \qquad \dot{\rho}_4 = \frac{(\rho_0)_4}{\beta}\dot{P}_4$$

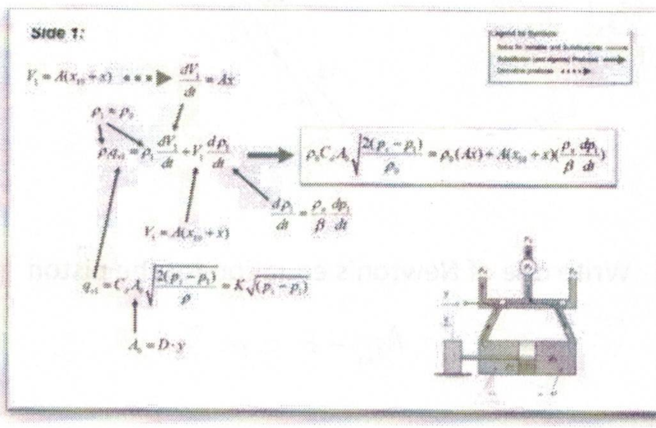
g) Write the flow resistance equation for each side of the cylinder

$$R_1 = \frac{(P_1-P_3)}{q_{m_3}} \qquad R_2 = \frac{(P_4-P_2)}{q_{m4}}$$

h) Write the equation relating chamber volume and piston position for each side of the cylinder

$$\forall_3 = A(x-x_0) \qquad \forall_4 = -A(x-x_0)$$

i) For the p_3 side of the cylinder, draw an equation solution diagram similar to the one shown here (and found in my notes). Use the diagram to derive an equation relating p_3 and x to p_1.



$$R_1 = \frac{(P_1-P_3)}{q_{m3}} \qquad \frac{(\rho_0)_3}{\beta}\dot{P}_3$$

$$q_{m3} = \forall_3 \frac{\partial \rho_3}{\partial t} + \rho_3 \frac{\partial \forall_3}{\partial t}$$

$$A(x-x_0) \qquad A(\dot{x})$$

$$\frac{P_1-P_3}{R_1} = A(x-x_0)\frac{(\rho_0)_3}{\beta}\dot{P}_3 + \rho_3 A\dot{x}$$

j) For the p_4 side of the cylinder, draw an equation solution diagram similar to the one shown above (and found in my notes). Use the diagram to derive an equation relating p_4 and x to p_2.

$$R_2 = \frac{(P_4 - P_2)}{q_{m4}}$$

$$\overset{\frac{(P_4-P_2)}{R_2}}{q_{m4}} = \underset{-A(x-x_0)}{V_4} \overset{\frac{(\rho_0)_4}{\beta}\dot{P}_4}{\frac{\partial \rho_4}{\partial t}} + \rho_4 \overset{-A\dot{x}}{\frac{\partial V_4}{\partial t}}$$

$$\frac{(P_4 - P_2)}{R_2} = -A(x-x_0)\frac{(\rho_0)_4}{\beta}\dot{P}_4 + \rho_4(\cancel{\rho} - A\dot{x})$$

k) For the two Newton's equations, draw an equation solution diagram similar to the one shown above (and found in my notes). Use the diagram to derive an equation relating x and θ to p_3 and p_4

$$F = AP_3 - AP_4 - m_p\ddot{x} \qquad -FL_2 - mgL_1\sin\theta = I\ddot{\theta}$$

$$(AP_3 - AP_4 - m_p\ddot{x})L_2 - mgL_1\sin\theta = I\ddot{\theta}$$

$$(AP_3 - AP_4 - m_p\ddot{x})L_2 - mgL_1\theta = mL_1^2\ddot{\theta}$$

l) For all 3 of the previous equations, use the kinematic equation to replace x with θ (so that x and its derivatives disappear from the equations. (If this weren't such a busy work week, I'd give you some parameter values and have you take these equations to Matlab to plot a few solutions.) Hmmmmmm . . . . this might make a nice Matlab Exam Problem.

$$X = \frac{\theta}{L_2} \qquad \dot{X} = \frac{\dot{\theta}}{L_2} \qquad \ddot{X} = \frac{\ddot{\theta}}{L_2}$$

$$\frac{P_1 - P_3}{R_1} = A\left(\frac{\theta}{L_2} - \frac{\theta_0}{L_2}\right)\frac{(\rho_0)_3}{\beta}\dot{P}_3 + \rho_3 A \frac{\dot{\theta}}{L_2}$$

$$\frac{P_4 - P_2}{R_2} = -A\left(\frac{\theta}{L_2} - \frac{\theta_0}{L_2}\right)\frac{(\rho_0)_4}{\beta}\dot{P}_4 - \rho_4 A \frac{\dot{\theta}}{L_2}$$

$$\left(AP_3 - AP_4 - m_p\frac{\ddot{\theta}}{L_2}\right)L_2 - mg\theta L_1 = mL_1^2\ddot{\theta}$$

$$m_p = 0, \quad \theta_0 = 0$$

$$\frac{P_1 - P_3}{R_1} = A\frac{\theta}{L_2}\frac{(\rho_0)_3}{\beta}\dot{P}_3 + \rho_3 A\frac{\dot{\theta}}{L_2} \qquad \frac{P_4 - P_2}{R_2} = -A\frac{\theta}{L_2}\frac{(\rho_0)_4}{\beta}\dot{P}_4 + \rho_4 A\frac{\dot{\theta}}{L_2}$$

$$mL_1^2\ddot{\theta} + mgL_1\theta = AL_1(P_3 - P_4$$

2. The non-linear equation relating pressure drop (ΔP) and flow rate (q_v) for an orfice is:

$$q_v = C_d A_0 \sqrt{\frac{2\Delta P}{\rho}}$$

a) Develop a substitute linear equation that we could use in the neighborhood of $\Delta P_o = 15000$.

$C_d = 0.75 \quad A_0 = 0.001 \qquad \rho = 675$ (all in compatible units)

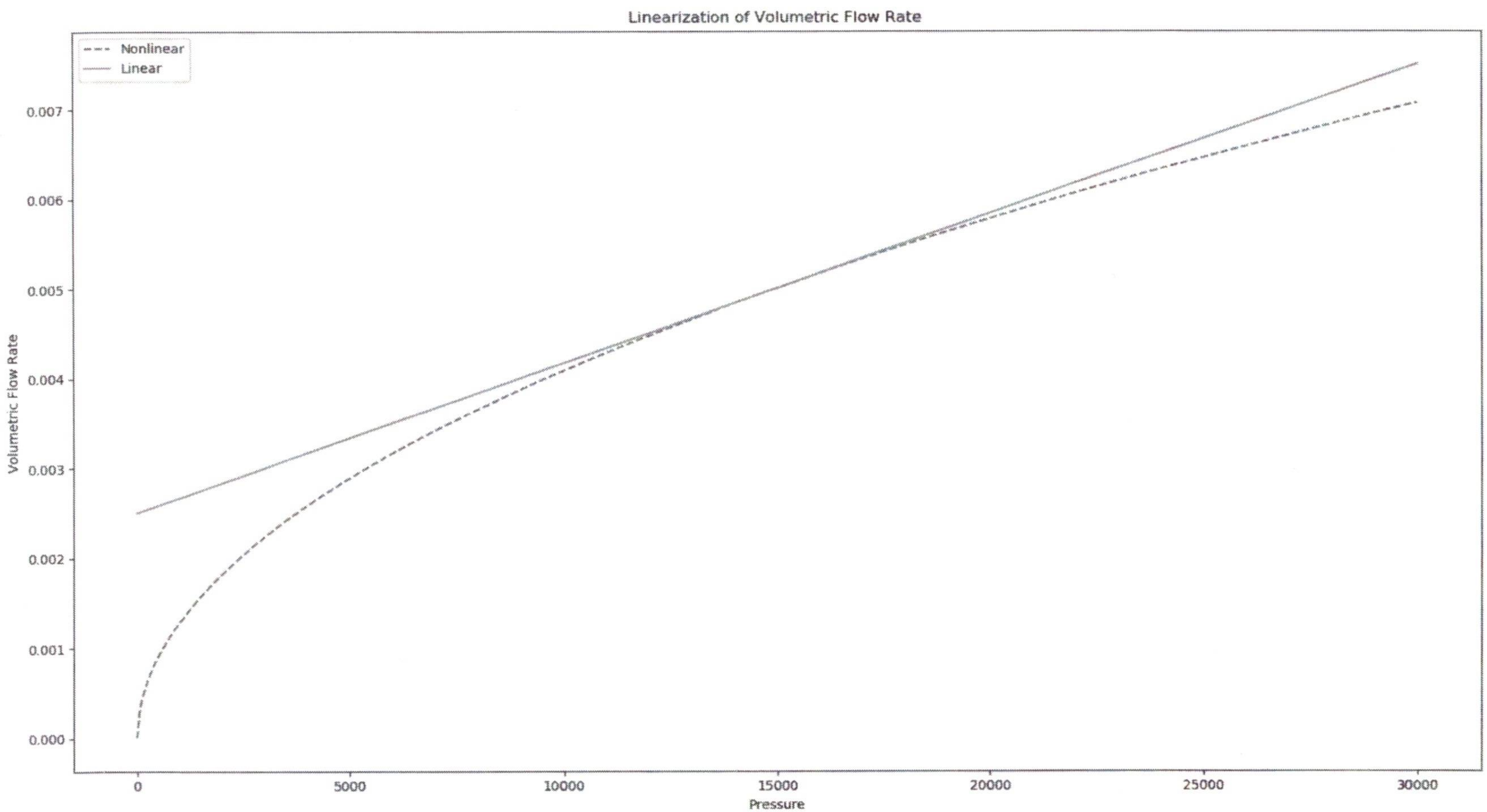
$$q_v = \frac{\partial q_v}{\partial \Delta P}(\Delta P - \Delta P_o) + q_v(\Delta P_o) \qquad q_v = 1\cancel{1}2.5E\text{-}6(\Delta P) + 5E\text{-}3$$

$$\frac{\partial q_v}{\partial \Delta P} = \frac{C_d A_o}{2}\sqrt{\frac{2}{\rho P_o}}$$

$$q_v = \frac{C_d A_o}{2}\sqrt{\frac{2}{\rho \Delta P_o}}(\Delta P - P_o) + C_d A_o \sqrt{\frac{2\Delta P_o}{\rho}}$$

b) Plot both equations (on the same graph) over the range $0 < \Delta P < 30000$. Is your linear equation tangent to the non-linear equation at $\Delta P = 15000$?

Yes


Linearization of Volumetric Flow Rate
Nonlinear
Linear
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
Volumetric Flow Rate
0
5000
10000
15000
20000
25000
30000
Pressure