MAE 3724 | Homework 1 | Diego Colón

1) $6\dot{x} + 4x = 12 \quad x(0) = 3$

$6(sX - x(0)) + 4(X) = \frac{12}{s}$

$X(6s+4) - 18 = \frac{12}{s}$

$X = \left(\frac{1}{6s+4}\right)\left(\frac{12}{s} - 18\right) = \frac{12}{6}\left(\frac{1}{s(s+4/6)}\right) + \frac{18}{6}\left(\frac{1}{s+4/6}\right)$

$X(t) = \frac{12}{6}\left(\frac{6}{4}\left(e^{-0t} - e^{-4/6 t}\right)\right) + \frac{18}{6}\left(e^{-4/6 t}\right)$

$\boxed{X(t) = 3}$

2) $F(s) = \frac{6s+2}{(s)(s+4)(s+1)}$

$= 6\left(\frac{s+\frac{1}{3}}{s(s+4)(s+1)}\right) \quad a=0, b=4, c=1$

$f(t) = 6\left(\frac{(\frac{1}{3}-0)}{(4-0)(1-0)}e^{-0t} + \frac{(\frac{1}{3}-4)}{(1-4)(0-4)}e^{-4t} + \frac{(\frac{1}{3}-1)}{(0-1)(4-1)}e^{-t}\right)$

$\boxed{f(t) = \frac{1}{2} - \frac{11}{6}e^{-4t} + \frac{2}{9}e^{-t}}$

3) $F(s) = \frac{6s+2}{s(s+4)^2} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{(s+4)^2}$

$s=0$:

$A = \frac{2}{(4)^2} = \frac{1}{8}$

$s=-4$:

$C = \frac{6(-4)+2}{-4} = \frac{11}{2}$

$s=1$:

$\frac{6+2}{5^2} = \frac{1}{8} + \frac{B}{5} + \frac{C}{5^2}$

$B = -\frac{1}{8}$

$F(s) = \frac{1}{8s} - \frac{1}{8(s+4)} + \frac{11}{2}\left(\frac{1}{(s+4)^2}\right)$

$\boxed{f(t) = \frac{1}{8} - \frac{1}{8}e^{-4t} + \frac{11}{2}te^{-4t}}$

4) $F(s)=\dfrac{4s+2}{2s^2+20s+48}$

$$\frac{2s+1}{s^2+10s+24} \rightarrow 2\left(\frac{s+\frac{1}{2}}{(s+6)(s+4)}\right)$$

$$F(s) = 2\left(\frac{s+\frac{1}{2}}{(s+6)(s+4)}\right) \qquad a=6,\ b=4,\ p=1/2$$

$$f(t) = 2\left(\frac{1}{4-6}\left(\left(\tfrac{1}{2}-6\right)e^{-6t}-\left(\tfrac{1}{2}-4\right)e^{-4t}\right)\right)$$

$$\boxed{f(t) = \tfrac{1}{2}\left(11e^{-6t}+7e^{-4t}\right)}$$

5) $F(s) = \dfrac{3s+1}{2s^2+8s+26} \rightarrow \dfrac{1}{2}\left(\dfrac{3s+1}{(s+2)^2-3^2}\right)$

$$\frac{1}{2}\left(\frac{3s+1}{(s+2)^2-3^2}\right) = \frac{A(s+2)}{(s+2)^2-3^2} + \frac{B(3)}{(s+2)^2-3^2}$$

$$3s+1 = A(s+2)+3B$$

$$3s+1 = As+2A+3B$$

$$A=\tfrac{3}{2} \quad ; \quad 1 = 3+3B$$

$$B = \tfrac{-2}{3}$$

$$F(s) = \tfrac{3}{2}\left(\frac{(s+2)}{(s+2)^2-3^2}\right) - \tfrac{2}{3}\left(\frac{3}{(s+2)^2-3^2}\right)$$

$$\boxed{f(t) = \tfrac{3}{2}e^{-2t}\cos(3t) - \tfrac{2}{3}e^{-2t}\sin(3t)}$$

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6) $\ddot{x} + 6\dot{x} + 34x = 0 \; ; \; x(0) = 4 \; ; \; \dot{x}(0) = -3$

$$\left[s^2 X - s\,x(0) - \dot{x}(0)\right] + 6\left[sX - x(0)\right] + 34X = 0$$

$$X(s^2 + 6s + 34) = 4s + 21$$

$$X(s) = \frac{4s+21}{s^2+6s+34} \rightarrow \frac{4s+21}{(s+3)^2 - 5^2} = \frac{A(s+3)}{(s+3)^2+5^2} + \frac{B(5)}{(s+3)^2+5^2}$$

$$4s + 21 = As + 3A + 5B$$

$$A = 4 \; ; \; 21 = 3A + 5B$$

$$B = \frac{9}{5}$$

$$X(s) = 4\left(\frac{s+3}{(s+3)^2+5^2}\right) + \frac{9}{5}\left(\frac{5}{(s+3)^2+5^2}\right)$$

$$\boxed{x(t) = 4e^{-3t}\cos(5t) + \tfrac{9}{5}e^{-3t}\sin(5t)}$$

7) $\ddot{x} + 10\dot{x} + 21x = f(t)$

a) $$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 21}$$

b) $f(t) = 68\,u(t)$

$$X(s) = \frac{68\left(\frac{1}{s}\right)}{s^2+10s+21} = 68\left(\frac{1}{s(s+7)(s+3)}\right)$$

$$x(t) = 68\left[\frac{e^{-0t}}{(7-0)(3-0)} + \frac{e^{-7t}}{(0-7)(3-7)} + \frac{e^{-3t}}{(0-3)(7-3)}\right]$$

$$\boxed{x(t) = \frac{68}{21} + \frac{17}{7}e^{-7t} - \frac{17}{3}e^{-3t}}$$