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1) Design Requirements:

$V_A = 50V$, $P = 600W$ @ $1300 rpm$ $(136.13 rad/s)$, $T_{stall} = 1.55\, T_{Dop}$

$T_{fric} = 0.07\, T_{Dop}$

a) $T_{Dop} = \frac{P_{Dop}}{\omega_{Dop}} = \frac{0.600KW}{1300 rpm}\left(9.5488\right) = 4.40E\text{-}3\, Nm$

$T_{stall} = 1.55\, T_{Dop} = 1.55\,(4.40E\text{-}3\, Nm) = 6.82E\text{-}3\, Nm$

$T_{fric} = 0.07\, T_{Dop} = 0.07\,(4.40E\text{-}3 Nm) = 308E\text{-}6\, Nm$

b) $T_{fric} = c\,\omega_{Dop} \rightarrow \quad c = \frac{T_{fric}}{\omega_{Dop}} = \frac{308E\text{-}6\, Nm}{136.13\, rad/s} = 2.26E\text{-}6\, \frac{Nms}{rad}$

c) $\omega_{Dop} = \frac{K V_a - R_a T_{Dop}}{c R_a + K^2}$ $\qquad \omega_{stall} = \frac{K V_a - R_a T_{stall}}{c R_a + K^2} = 0$

$\omega_{Dop} c R_a + \omega_{Dop} K^2 = K V_a - R_a T_{Dop}$ $\qquad K V_a - R_a T_{stall} = 0$

$\omega_{Dop} K^2 - K V_a + \omega_{Dop} c R_a + R_a T_{Dop} = 0$ $\qquad K V_a - R_a T_{stall} = 0$

$R_a = \frac{K V_a}{T_{stall}}$

$\omega_{Dop} K^2 - K V_a + \left(\omega_{Dop} c + T_{Dop}\right)\frac{K V_a}{T_{stall}} = 0$

$\omega_{Dop} K^2 + K V_a \left(\frac{\omega_{Dop} c + T_{Dop}}{T_{stall}} - 1\right) = 0$

$\frac{K}{\omega_{Dop}}\left(K + \frac{V_a}{\omega_{Dop}}\left(\frac{\omega_{Dop} c + T_{Dop}}{T_{stall}} - 1\right)\right) = 0$

$K = 0, \; -\frac{V_a}{\omega_{Dop}}\left(\frac{\omega_{Dop} c + T_{Dop}}{T_{stall}} - 1\right)$

$K_T, K_b = \frac{-V_a}{\omega_{Dop}}\left(\frac{\omega_{Dop} c - T_{Dop}}{T_{stall}} - 1\right)$

$R_a = \frac{-V_a^2}{T_{stall}\,\omega_{Dop}}\left(\frac{\omega_{Dop} c - T_{Dop}}{T_{stall}} - 1\right)$

$$K_T = K_b = \frac{-V_a}{\omega_{oop}}\left(\frac{\omega_{oop} C - T_{oop}}{T_{stall}} - 1\right) = -\frac{50V}{136.13\frac{rad}{s}}\left(\frac{(136.13\frac{rad}{s})(2.26E\text{-}6\frac{Nms}{rad}) - (4.4E\text{-}3Nm)}{6.82E\text{-}3\,Nm} - 1\right)$$

$$= 0.5893\,\frac{Nm}{A}$$

$$R_a = \frac{-V_a^2}{T_{stall}\,\omega_{oop}}\left(\frac{\omega_{oop} C - T_{oop}}{T_{stall}} - 1\right)$$

$$= 4.32E3\,\Omega$$

d) $$i_{stall} = \frac{V_a}{R_a} = \frac{50V}{4.32E3\Omega} = 11.57\,mA$$

$$i_{oop} = \frac{c V_a + K_b T_{oop}}{c R_a + K_b K_T} = \frac{(50V)(2.26E\text{-}6\frac{Nms}{rad}) + (0.5893\frac{Nm}{A})(4.4E\text{-}3Nm)}{(2.26E\text{-}6\frac{Nms}{rad})(4.32E3\Omega) + (0.5893\frac{Nm}{A})^2} = 7.58\,mA$$

e) $$\omega_{nL} = \frac{K_T V_a}{c R_a + K_b K_T} = \frac{(0.5893\frac{Nm}{A})(50V)}{(2.26E\text{-}6\frac{Nms}{rad})(4.32E3\Omega) + (0.5893\frac{Nm}{A})^2} = 82.5\,rad/s$$

f) $$\eta = \frac{P_{out}}{P_{in}} = \frac{600W}{(50V)(7.58E\text{-}3A)} = 158\,;$$ (known to be wrong but cannot find computational mistakes)

g) $$\Omega(s) = \frac{V_a(s) K_T}{(R_a I + c L_a)s + c R_a + K_b K_T} \rightarrow \dot{\omega}(R_a I + c L_a) + \omega(c R_a + K_b K_T) = V_a K_T$$

$$\dot{\omega} + \frac{(c R_a + K_b K_T)}{R_a I + c L_a}\omega = V_a K_T \rightarrow \tau = \frac{R_a I + c L_a}{c R_a + K_b K_T}\,;\ L_a \approx 0$$

$$\tau = \frac{R_a I}{c R_a + K_b K_T} \rightarrow I = \frac{\tau}{R_a}\left(c R_a + K_b K_T\right)$$

$$I = \frac{2.475}{4.32E3\Omega}\left((7.26E\text{-}6\,Nms)(4.32E3\Omega) + (0.5893\frac{Nm}{A})^2\right)$$

$$I = 204.5E\text{-}6\,kg\,m^2$$

2)

a) $L_a I s^2 + (R_a I + c L_a) s + c R_a + K_b K_T = 0$

$240E{-}9\, s^2 + (64E{-}6 + 3E{-}3c)\, s + 0.8c + 25E{-}6 = 0$

b) $c_1 = 0 \quad s_1 = -0.391, \; s_2 = -266.27$

$c_2 = 0.01 \quad s_1 = -125.73, \; s_2 = -265.9$

$c_3 = 0.1 \quad s_1 = -266.7, \; s_2 = -1249.9$

e)

$$\tau = \frac{R_a I + c L_a}{c R_a + K_b K_T}$$

$c_1 = 0 \quad \tau = 2.56\,ms \quad t_{ss} = 12.8\,ms$

$c_2 = 0.01 \quad \tau = 11.7\,ms \quad t_{ss} = 58.5\,ms$

$c_3 = 0.1 \quad \tau = 4.54\,ms \quad t_{ss} = 22.7\,ms$

f)

The simulated models do not fully agree with calculations given that the larger c becomes, the lower t_{ss} becomes in the simulation but not the calculations.