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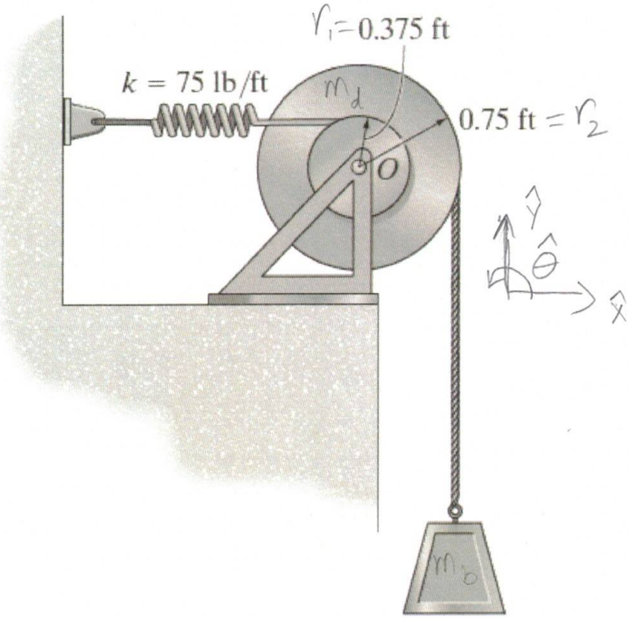
MAE 3723, Systems Analysis – Fall 2019
Prerequisite Exam

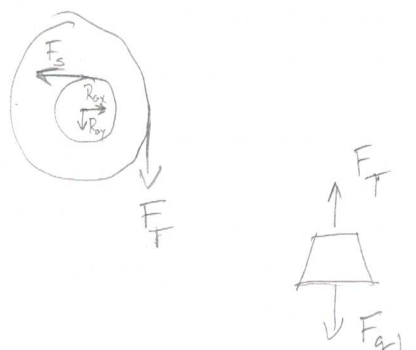
1) (30 points) The following is a dynamics problem given on my ENSC 2123 Final Exam this summer.

If the 250-lb block is released from rest when the spring is unstretched, determine the velocity of the block after it has descended 5 ft. The drum has a weight of 50 lb and a radius of gyration of $k_O = 0.5$ ft about its center of mass O.

Do not solve this problem as presented. Instead, use the information in the problem to do the following:



a) Draw complete freebody diagrams for the block and the pulley. Hint: show the forces in the rope and spring.



b) Write an equation relating the force in the spring to the change in angle of the pulley.

$F_s = -K\Delta x = -K r_1 \theta$

$\Delta x = r_1 \theta$

$\boxed{F_s(\theta) = -K r_1 \theta}$

c) Use the freebody diagrams to write the rotational form of Newton's Second Law ($\Sigma M = I\alpha$) for the pulley.

$I = mK_o^2$

$\sum M_o = I\alpha \rightarrow F_s r_1 - F_b r_2 = I\ddot{\theta}$

$I\ddot{\theta} - (-Kr_1\theta) = F_T r_2$

$I\ddot{\theta} + Kr_1\theta = F_T r_2$

$\boxed{m_d K_o^2 \ddot{\theta} + Kr_1\theta = F_T r_2}$

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d) Use the freebody diagrams to write Newton's Second Law (ΣF = ma) for the block.

$$\Sigma F = ma \rightarrow \boxed{-F_{gb} + F_T = m_b\ddot{y}}$$

e) Write an equation relating the acceleration of the block to the angular acceleration of the pulley.

$$F_T = m_b\ddot{y} + F_{gb}$$

$$m_d K_o^2\ddot{\theta} + k r_1\theta = F_T r_2$$

$$\frac{1}{r_2}(m_d K_o^2\ddot{\theta} + K r_1\theta) = F_T$$

$$y = r_2\theta$$

$$\dot{y} = \cancel{\dot{r}_2\theta} + r_2\dot{\theta}$$

$$\ddot{y} = \cancel{\ddot{r}_2\theta} + r_2\ddot{\theta}$$

$$\boxed{\frac{1}{r_2}(m_d K_o^2\ddot{\theta} + K r_1\theta) = m_b\ddot{y} + F_{gb}} \qquad \boxed{\ddot{y} = r_2\ddot{\theta}}$$

f) Solve those equations to determine the acceleration of the block at the instant it is released. Remember that the moment of inertia of the pulley is $I = m\,k_o^2$

$$\frac{1}{r_2}(m_d K_o^2\ddot{\theta} + K r_1\theta) = m_b\ddot{y} + F_{gb}$$ * spring unstretched @ $\theta = 0$

$$\frac{1}{r_2}(m_d K_o^2\ddot{\theta}) = m_b\ddot{y} + F_{gb}$$

$$\ddot{y} = \frac{1}{m_b}\left(\frac{1}{r_2} m_d K_o^2\ddot{\theta} - F_{gb}\right) \; ; \ddot{\theta} = \frac{\ddot{y}}{r_2}$$

$$\ddot{y} = \frac{m_d}{m_b}\frac{K_o^2}{r_2}\left(\frac{\ddot{y}}{r_2}\right) - \frac{F_{gb}}{m_b}$$

$$\ddot{y}\left(1 - \frac{m_d}{m_b}\left(\frac{K_o}{r_2}\right)^2\right) = -\frac{F_{gb}}{m_b}$$

$$\ddot{y} = \frac{-F_{gb}}{m_b\left(1 - \frac{m_d}{m_b}\left(\frac{K_o}{r_2}\right)^2\right)} = \frac{-(250\text{ lbf})}{\frac{250\text{ lbf}}{32.2\text{ ft/s}^2}\left(1 - \frac{56}{250}\left(\frac{0.5}{0.75}\right)^2\right)}$$

$$\boxed{\ddot{y} = -35.34\text{ ft/s}^2}$$

g) Briefly explain the process you would use to solve the problem as originally presented

Given that time is not stated in the problem, I would use the Work energy principle to solve the problem.

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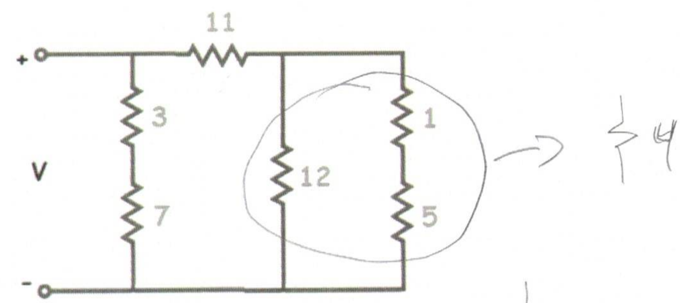
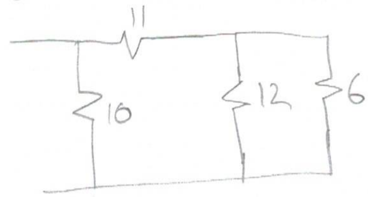
2) (20 points) From your knowledge of Physics and Electrical Circuits, for the parallel and series resistor circuit shown:

a) Compute the equivalent resistance

$$R_{eq} = \cfrac{1}{\cfrac{1}{10} + \cfrac{1}{11 + \cfrac{1}{\frac{1}{12} + \frac{1}{6}}}} \ \Omega$$





$$\frac{1}{\frac{1}{12} + \frac{1}{6}}$$

$$\boxed{R_{eq} = 6\,\Omega}$$

b) The input voltage is 14V. Compute the current i produced by that 14-volt power supply

$$V_T = I_T R_{eq} \rightarrow I_T = \frac{V_T}{R_{eq}} = \frac{14V}{6\Omega} = \frac{7}{3} A = 2.33A \qquad \boxed{I = 2.33A}$$

c) Compute the power delivered by the 14 V source.

$$P = IV = (14V)(2.33A) = 32.67W \qquad \boxed{P = 32.67W}$$

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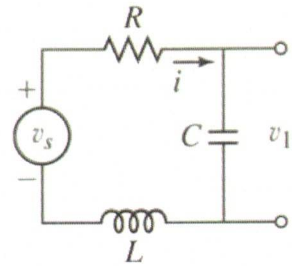
3) (25 points) From your knowledge of Physics and Electrical Circuits: for the following RLC circuit, derive an ordinary differential equation relating the ***current i*** (and it's derivatives) to the supply voltage $v_s(t)$ (and it's derivatives).



$$-V_s + V_R + V_C + V_L = 0$$

$$V_R + V_C + V_L = V_s$$

$$iR + \frac{1}{C}\int i\,dt + \dot{i}L = V_s$$

$$\frac{d}{dt}\left(iR + \frac{1}{C}\int i\,dt + \dot{i}L\right) = \frac{d}{dt}(V_s)$$

$$\boxed{\ddot{i}L + \dot{i}R + \frac{1}{C}i = \dot{V}_s}$$

Electrical Elements - Basic

Resistance:	$v = iR$
Capacitance:	$i = C\frac{dv}{dt}$
Inductance:	$v = L\frac{di}{dt}$

$V_R = iR$

$V_C = \frac{1}{C}\int i\,dt$

$V_L = L\dot{i}$

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4) (25 points) From your knowledge Differential Equations and Engineering Analysis:

a) Write the Laplace transform of the following differential equation:

$2\,\dot{x} + 6\,x = 4t$ with initial condition $x(0) = 8$

$$2\left(sX - x(0)\right) + 6X = \frac{4}{s^2}$$

$$\boxed{2sX - 16 + 6X = \frac{4}{s^2}}$$

b) Solve your transformed equation for X(s), ***not x(t)***.

$$2sX - 16 + 6X = \frac{4}{s^2}$$

$$X\left(2s+6\right) = \frac{4}{s^2} - 16$$

$$\boxed{X(s) = \left(\frac{1}{2s+6}\right)\left(\frac{4}{s^2} - 16\right)}$$

$$\frac{1}{(\quad)} = \frac{A}{s} + \frac{D}{2s+6}$$

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c) Determine the inverse Laplace transform for X(s) to determine ***x(t)***.

$$\boxed{x(t) = -8e^{-3t} - \tfrac{2}{9}u(t) + \tfrac{2}{3}t + \tfrac{2}{9}e^{-3t}}$$

$$X(s) = \left(\frac{1}{2s+6}\right)\left(\frac{4}{s^2} - 16\right)$$

$$= \frac{4}{s^2(2s+6)} - \frac{16}{2s+6}$$

$$= \frac{2}{s^2(s+3)} - \frac{8}{s+3}$$

$$\frac{-8}{s+3} : \mathcal{L}^{-1}\left\{8\left(\frac{1}{s+3}\right)\right\} = -8e^{-3t}$$

$$\frac{2}{s^2(s+3)} : \frac{2}{s^2(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3}$$

$$2 = A(s)(s+3) + B(s+3) + Cs^2$$

$$2 = As^2 + 3As + Bs + 3B + Cs^2$$

$$As^2 + Cs^2 = 0$$

$$3As + Bs = 0$$

$$3B = 2$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 0 \\ 0 & 3 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} -\frac{2}{9} \\ 2/3 \\ 2/9 \end{bmatrix}$$

$$= \frac{-2}{9s} + \frac{2}{3s^2} + \frac{2}{9(s+3)}$$

$$-\frac{2}{9}u(t) + \frac{2}{3}t + \frac{2}{9}e^{-3t}$$

$X(s)$		$x(t), t \geq 0$
1.	1	$\delta(t)$, unit impulse
2.	$\frac{1}{s}$	$u_s(t)$, unit step
3.	$\frac{c}{s}$	constant, c
4.	$\frac{e^{-sD}}{s}$	$u_s(t-D)$, shifted unit step
5.	$\frac{n!}{s^{n+1}}$	t^n
6.	$\frac{1}{s+a}$	e^{-at}

2.	$\frac{dx}{dt}$	$sX(s) - x(0)$
3.	$\frac{d^2x}{dt^2}$	$s^2X(s) - sx(0) - \dot{x}(0)$