

$$1) \quad 6\dot{x} + 4x = 12 \quad x(0) = 3$$

$$6(sX - x(0)) + 4(X) = \frac{12}{s}$$

$$X(6s+4) - 18 = \frac{12}{s}$$

$$X = \left( \frac{1}{6s+4} \right) \left( \frac{12}{s} - 18 \right) = \frac{12}{6} \left( \frac{1}{s(s+\frac{4}{6})} \right) + \frac{18}{6} \left( \frac{1}{s+\frac{4}{6}} \right)$$

$$X(t) = \frac{12}{6} \left( \frac{6}{4} \left( e^{-0t} - e^{-\frac{4}{6}t} \right) \right) + \frac{18}{6} \left( e^{-\frac{4}{6}t} \right)$$

$$\boxed{x(t) = 3}$$

$$2) \quad F(s) = \frac{6s+2}{(s)(s+4)(s+1)}$$

$$= 6 \left( \frac{s+\frac{1}{3}}{s(s+4)(s+1)} \right) \quad a=0, b=4, c=1$$

$$f(t) = 6 \left( \frac{(\frac{1}{3}-0)}{(1-0)(1-0)} e^{-0t} + \frac{(\frac{1}{3}-4)}{(1-4)(0-4)} e^{-4t} + \frac{(\frac{1}{3}-1)}{(0-1)(4-1)} e^{-t} \right)$$

$$\boxed{f(t) = \frac{1}{2} - \frac{11}{6} e^{-4t} + \frac{2}{9} e^{-t}}$$

$$3) \quad F(s) = \frac{6s+2}{s(s+4)^2} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{(s+4)^2}$$

$$s=0:$$

$$A = \frac{2}{(4)^2} = \frac{1}{8}$$

$$s=-4:$$

$$C = \frac{6(-4)+2}{-4} = \frac{11}{2}$$

$$s=1:$$

$$\frac{6+2}{5^2} = \frac{1}{8} + \frac{B}{5} + \frac{C}{5^2}$$

$$B = -\frac{1}{8}$$

$$F(s) = \frac{1}{8s} - \frac{1}{8(s+4)} + \frac{11}{2} \left( \frac{1}{(s+4)^2} \right)$$

$$\boxed{f(t) = \frac{1}{8} - \frac{1}{8} e^{-4t} + \frac{11}{2} t e^{-4t}}$$

$$4) \quad F(s) = \frac{4s+2}{2s^2+20s+48}$$

$$\frac{2s+1}{s^2+10s+24} \rightarrow 2 \left( \frac{s+\frac{1}{2}}{(s+6)(s+4)} \right)$$

$$F(s) = 2 \left( \frac{s+\frac{1}{2}}{(s+6)(s+4)} \right) \quad a=6, b=4, p=\frac{1}{2}$$

$$f(t) = 2 \left( \frac{1}{4-6} \left( \left(\frac{1}{2}-6\right)e^{-6t} - \left(\frac{1}{2}-4\right)e^{-4t} \right) \right)$$

$$\boxed{f(t) = \frac{1}{2} (11e^{-6t} + 7e^{-4t})}$$

$$5) \quad F(s) = \frac{3s+1}{2s^2+8s+26} \rightarrow \frac{1}{2} \left( \frac{3s+1}{(s+2)^2-3^2} \right)$$

$$\frac{1}{2} \left( \frac{3s+1}{(s+2)^2-3^2} \right) = \frac{A(s+2)}{(s+2)^2-3^2} + \frac{B(3)}{(s+2)^2-3^2}$$

$$3s+1 = A(s+2) + 3B$$

$$3s+1 = As + 2A + 3B$$

$$A = \frac{3}{2} ; \quad 1 = 3 + 3B$$

$$B = -\frac{2}{3}$$

$$F(s) = \frac{3}{2} \left( \frac{(s+2)}{(s+2)^2-3^2} \right) - \frac{2}{3} \left( \frac{3}{(s+2)^2-3^2} \right)$$

$$\boxed{f(t) = \frac{3}{2} e^{-2t} \cos(3t) - \frac{2}{3} e^{-2t} \sin(3t)}$$



6)  $\ddot{x} + 6\dot{x} + 34x = 0$  ;  $x(0) = 4$  ;  $\dot{x}(0) = -3$

$$[s^2 X - s x(0) - \dot{x}(0)] + 6[sX - x(0)] + 34X = 0$$

$$X(s^2 + 6s + 34) = 4s + 21$$

$$X(s) = \frac{4s + 21}{s^2 + 6s + 34} \rightarrow \frac{4s + 21}{(s+3)^2 + 5^2} = \frac{A(s+3)}{(s+3)^2 + 5^2} + \frac{B(5)}{(s+3)^2 + 5^2}$$

$$4s + 21 = As + 3A + 5B$$

$$A = 4 ; 21 = 3A + 5B$$

$$B = \frac{9}{5}$$

$$X(s) = 4 \left( \frac{s+3}{(s+3)^2 + 5^2} \right) + \frac{9}{5} \left( \frac{5}{(s+3)^2 + 5^2} \right)$$

$$x(t) = 4e^{-3t} \cos(5t) + \frac{9}{5} e^{-3t} \sin(5t)$$

7)  $\ddot{x} + 10\dot{x} + 21x = f(t)$

a)  $\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 21}$

b)  $f(t) = 68u(t)$

$$X(s) = \frac{68 \left( \frac{1}{s} \right)}{s^2 + 10s + 21} = 68 \left( \frac{1}{s(s+7)(s+3)} \right)$$

$$x(t) = 68 \left[ \frac{e^{-0t}}{(7-0)(3-0)} + \frac{e^{-7t}}{(0-7)(3-7)} + \frac{e^{-3t}}{(0-3)(7-3)} \right]$$

$$x(t) = \frac{68}{21} + \frac{17}{7} e^{-7t} - \frac{17}{3} e^{-3t}$$

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syms s;  
  
F = (4*s+2)/(2*s^2+20*s+48);  
disp("Inverse Laplace of 4")  
ilaplace(F)  
  
F = (3*s+1)/(2*s^2+8*s+26);  
disp("Inverse Laplace of 5")  
ilaplace(F)
```

Inverse Laplace of 4)

ans =

$(11\exp(-6t))/2 - (7\exp(-4t))/2$

Inverse Laplace of 5)

ans =

$(3\exp(-2t))(\cos(3t) - (5\sin(3t))/9))/2$

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