1)

$\ddot{x} + 2\dot{x} + 16x = 3u(t) \quad x(0) = 0 \ , \dot{x}(0) = 0$

$M_{\%} = 100 \exp\left(\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right)$

$= 44.43\%$

$2\omega_n \zeta = 2 \qquad \omega_n = \sqrt{k/m} = \sqrt{16/1}$

$\omega_n \zeta = 1 \qquad \omega_n = 4$

$\zeta = \frac{1}{\omega_n}$

$\zeta = \frac{1}{4}$

$M_p = \frac{1}{k} \exp\left(\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right)$

$= 27.77E{-3}$

$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

$= 811.15E{-3}\ s$

$t_r = \frac{2\pi - \phi}{\omega_n \sqrt{1-\zeta^2}} \ ; \ \phi = \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) + \pi$

$= 2.09\ s$

$t_d = \frac{1 + 0.7\zeta}{\omega_n}$

$= 293.75E{-3}\ s$

$t_s = \frac{4}{\zeta \omega_n}$

$= 4\ s$

2) $7\ddot{x}+c\dot{x}+6x=2u(t)$ $x(0)=0$; $\dot{x}(0)=0$

find c s.t

$\min(t_s)$ $t_s<8s$
$\min(M_{\%})$ $M_{\%}<20\%$
$\min(t_r)$ $t_r<1.5$

$7\ddot{x}+c\dot{x}+6x=2u(t) \rightarrow \ddot{x}+{}^{c}/_{7}\dot{x}+{}^{6}/_{7}=\frac{2}{7}u(t)$

$\omega_n^2=\frac{6}{7} \Rightarrow \omega_n=\sqrt{\frac{6}{7}}$

$2\zeta\omega_n=\frac{c}{7} \rightarrow \zeta=\frac{c}{14\omega_n}=\frac{c}{14\sqrt{\frac{6}{7}}}$

$t_s=\frac{4}{\zeta\omega_n}=\frac{4}{c/14}=\frac{56}{c} \rightarrow c=\frac{56}{t_s}$ $c_{max}=7$ $c_{min}=\infty$

$M_{\%}=100\,e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \rightarrow \frac{\ln(M_{\%})}{100}=\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} \rightarrow \sqrt{1-\zeta^2}\,\frac{\ln(M_{\%})}{100}=-\pi\zeta$

$(1-\zeta^2)\frac{\ln^2(M_{\%})}{100^2}=\pi^2\zeta^2 \rightarrow \frac{\ln^2(M_{\%})}{100^2}=\zeta^2\left(\pi^2+\frac{\ln^2(M_{\%})}{100}\right)$

$\left(\frac{c}{14\sqrt{6/7}}\right)^2=\frac{\frac{1}{100^2}\ln^2(M_{\%})}{\pi^2+\frac{1}{100^2}\ln^2(M_{\%})} \rightarrow c=\sqrt{\frac{\frac{1}{100^2}\ln^2(M_{\%})}{\pi^2+\frac{1}{100^2}\ln^2(M_{\%})}}\left(14\sqrt{\frac{6}{7}}\right)$

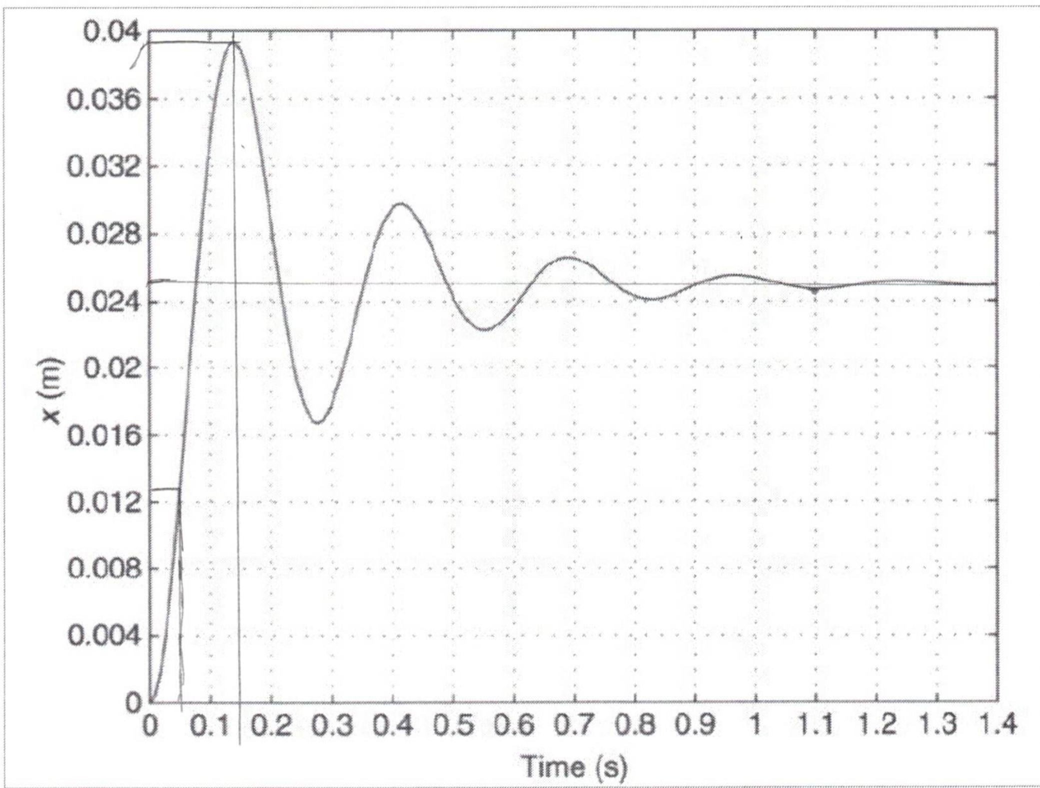
$c_{max}=0.12359$ $c_{min}=0.75869$

All three conditions cannot be met because $\min(t_r)=1.674$ for this system as a function of c. For this reason a c value of 9 was chosen. This gives a $t_s \doteq 6.22s$, $M_{\%}=4.825\%$; $t_r=3.51s$ & $M_p=0.00804$

Problem 3 – The figure below shows a measured response of a mechanical mass-spring-damper system subjected to a step input of 250 N. The assumed equation of motion is:

$$m\ddot{x} + c\ddot{x} + kx = 250u_s(t)$$

Estimate the values of m (Kg), c (N s/m), and k (N/m).



$t_p = 0.14\,s$ $\quad X_{ss} = 0.025\,m$ $\quad t_{ss} = 0.87\,s$

$M_p = 0.038\,m$ $\quad t_d = 0.5\,s$

$$M_{\%} = \left(\frac{0.038 - 0.025}{0.025}\right)100 = 52\%$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \rightarrow \omega_n = 3.2089$$

$$R = \ln\left(\frac{100}{M_{\%}}\right) = 0.65392$$

$$X_{ss} = \frac{f_{ss}}{K} \Longrightarrow K = 10E3\ \frac{N}{m}$$

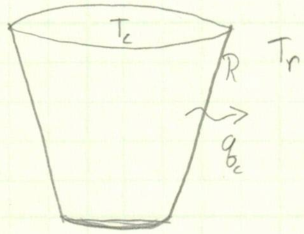
$$\zeta = \frac{R}{\sqrt{\pi^2 + R^2}} = 0.20378$$

$$\omega_n^2 = \frac{K}{m} \rightarrow m = 3.11E3\ Kg$$

$$\zeta = \frac{c}{2\sqrt{mK}} \rightarrow c = 2.27E3\ \frac{Ns}{m}$$

$$3.11E3\,\ddot{x} + 2.27E3\,\dot{x} + 10E3 = 250\,u_s(t)$$

4)



$$c\dot{T}_c = -\frac{(T_c - T_r)}{R} \quad ; \quad C = mc_p$$

$$mc_p\dot{T}_c + \frac{1}{R}(T_c - T_r) = 0 \quad ; \quad (T_c - T_r) = \Delta T$$

$$Rmc_p\Delta\dot{T} + \Delta T = 0$$

$$\Delta T(t) = \Delta T(0)\, e^{-\frac{t}{Rmc_p}}$$

$$\ln(\Delta T(t)) = \ln(\Delta T(0)) - \frac{1}{Rmc_p}$$

$$-\ln\left(\frac{\Delta T(t)}{\Delta T(0)}\right) = \frac{1}{Rmc_p}$$

$$R = \frac{-1}{\cancel{R}mc_p}\left[\ln\left(\frac{\Delta T(t)}{\Delta T(0)}\right)\right]^{-1}$$

$$R = \frac{-1}{mc_p}\left[\ln\left(\frac{T_c(t) - T_r}{T_c(0) - T_r}\right)\right]^{-1} \Bigg| \begin{array}{l} T_c(t) = 150°F \\ T_c(0) = 200°F \end{array}$$

$$R = \frac{-\left[\ln\left(\frac{65.55 - 21.66}{93.33 - 21.66}\right)\right]}{(2.957E\text{-}5\,m^3)(997\,kg/m^3)(4.4958\,KJ/kg°C)}$$

$$\boxed{R = 153.85E\text{-}3\ °C\,s/J}$$

$$\tau \Delta \dot{T} + \Delta T = 0 \rightarrow \Delta T(t) = \Delta T(0)\, e^{-t/\tau}$$

$$[T_c(t) - T_r] = [T_c(0) - T_r]\, e^{-t/\tau}$$

$$-\ln\left(\frac{T_c(t) - T_r}{T_c(0) - T_r}\right) = \frac{t}{\tau}$$

$$-\tau \ln\left(\frac{T_c(t) - T_r}{T_c(0) - T_r}\right) = t$$

$$t = -mc_p R \ln\left(\frac{T_c(t) - T_r}{T_c(0) - T_r}\right)$$

$$\boxed{t = 157.17\,s}$$