Colón, Diego

Name: Last, First

MAE 3724 Systems Analysis
Fall 2019

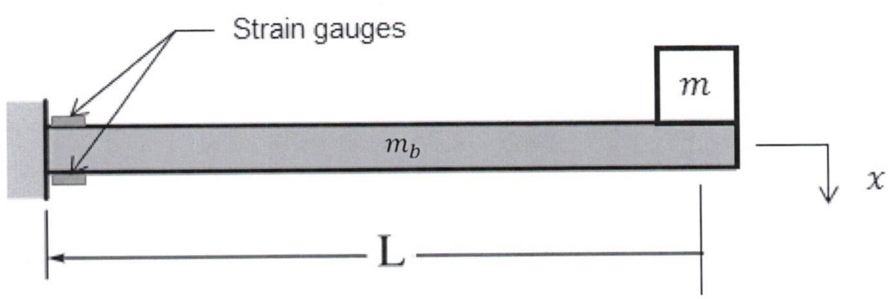
Pre-Lab for Experiment 3
Cantilever Beam Experiment

**Complete this Pre-Lab BEFORE you come to the laboratory.
The Lab Instructor will answer questions and provide feedback when you come to the laboratory to do the experiment.**

Show all your work and final answers on these pages.

Background for Laboratory Experiment 3

In Experiment 3 you will study a mass-spring-damper system comprising a cantilever beam with a mass on the end as shown in the schematic below. A two-arm strain gage bridge will be used to measure the free response of the system, i.e., the motion $x(t)$ following an initial condition x_0 imposed on the system. This pre-lab will prepare you for the experiment.



Pre-Lab for Experiment 3

You may model the cantilever beam system shown above with an equivalent mass concentrated (lumped) at the end of the beam, and with an equivalent spring dependent on the properties of the beam. Clearly state any additional assumptions.

For this Pre-Lab you may use the following parameters:

- Length from fixed support to the **center of the mass** at the end: $L = 0.740\ m$
- Modulus of elasticity of the material making up the beam: $E = 69.0\ GPa$
- Width of beam: w = 0.051 m
- Thickness of beam: h = 0.005 m
- Mass of the beam without the mass at the end: $m_b = 0.402$ kg
- Added mass at the end of the beam: $m = 0.680$ kg (assume point mass at the end and then L is the full length of the beam)

While there is not an obvious damper, some energy is dissipated during the bending of the beam (internal damping because of material characteristics). For this Pre-Lab, assume a damping coefficient of $c = 0.1\ N \cdot s/m$. You will determine in the laboratory experiment whether this is a good assumption.

Calculations

a) *[1 pts]* Calculate the effective spring constant of the system using the appropriate equation on p.173 of the text (Table 4.1.1).

$$K = \frac{E w h^3}{4L^3} = \frac{(69E9\ Pa)(51E\text{-}3m)(5E\text{-}3m)}{4\ (740E\text{-}3m)} = 5.94E6\ N/m$$

$k =$ 5.94 E6 N/m

b) *[1 pts]* Calculate the effective mass of the system using the appropriate equation on p.197 of the text (Table 4.3.1).

$$m_e = m + 0.23\, m_b = 680g + 402g$$

$m_e =$ 1.082 kg

c) *[1 pt]* Determine the undamped natural frequency of the system (p. 183, Eq. 4.2.2).

$$\omega_n = \sqrt{\frac{5.94E6\ N/m}{1.082\ kg}}$$

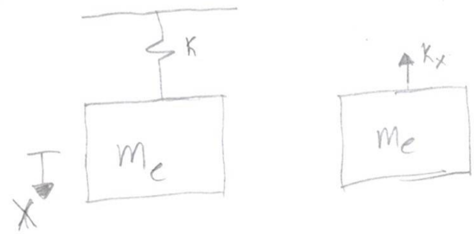
$\omega_n =$ 2.34 E3 H~~z~~

d) *[1 pt]* Determine the theoretical damping ratio (using $c = 0.1\ N \cdot s/m$). (p. 76, Eq. 2.5.15).

$$\zeta = \frac{c}{2\sqrt{mK}} = \frac{0.1\ \frac{Ns}{m}}{2\sqrt{(1.082\ kg)(5.94E6\ N/m)}} =$$

$\zeta =$ 19.722 E-6

e) *[2 pts]* Develop a dynamic model for the system.



$-Kx = m\ddot{x}$

$m\ddot{x} + Kx = 0$

f) *[2 pts]* Develop an expression for $X(s)$, the Laplace Transform of $x(t)$. Assume the initial conditions are $x_0 = 1, \dot{x} = 0$.

$m_e\left(s^2 X - s\,x(0) - \dot{x}(0)\right) + K X = 0$

$m_e s^2 X - m_e s\,x(0) - m_e\dot{x}(0) + KX = 0$

$X = \dfrac{m_e\left(s\,x(0) - \dot{x}(0)\right)}{m_e s^2 + K}$

$$X(s) = \frac{m_e s}{m_e s^2 + K}$$

g) *[1 pts]* Write the "characteristic equation" for the system.

$$m_e s^2 + K = 0$$

h) *[1 pts]* Use the Laplace Transform method to determine $x(t)$ as a function of time.

$X(s) = \dfrac{m_e s}{m_e s^2 + K} \rightarrow \dfrac{s}{s^2 + \frac{K}{m_e}}$

$\mathcal{L}^{-1}\left\{\dfrac{s}{s^2 + \frac{K}{m_e}}\right\} = \cos\left(\frac{K}{m_e} t\right)$

$x(t) = \cos\left(\frac{K}{m_e} t\right)$