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Name: Last, First

MAE 3724 Systems Analysis
Fall 2019

Pre-Lab for Experiment 5
Electromechanical Systems / DC Motor

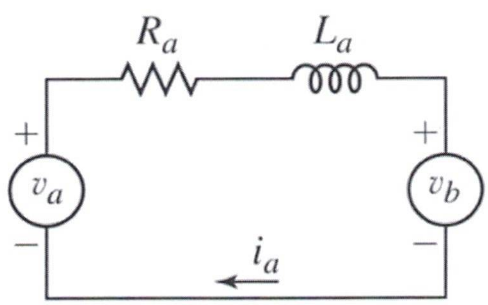
**Complete this Pre-Lab BEFORE you come to the laboratory.
The Lab Instructor will answer questions and provide feedback when you come to the laboratory to do the experiment.**

Show all your work and final answers on these pages. Attach the requested graphs.

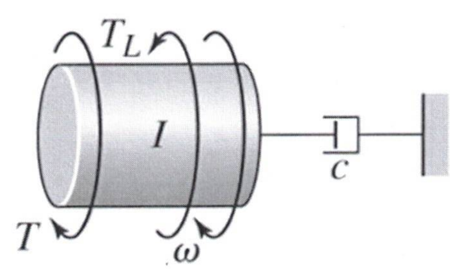
Background for Laboratory Experiment 5

In Experiment 5, you will study the dynamic behavior of an Armature-Controlled DC Motor. The background for this Experiment and Pre-Lab is in Sections 6.5.4 and 6.6 in the textbook.

A physical model of the Armature-Controlled DC motor is shown in the figure below.



Armature circuit



Pre-Lab for Experiment 5

1) *[3 pts]* Derive a transfer function for the DC motor that relates the Laplace Transform of the output rotational velocity $\Omega(s)$ to the Laplace Transform of the input voltage $V_a(s)$, assuming $T_L \approx 0$ and all initial conditions are equal to zero.

   Show your work on the following page starting with Kirchhoff's voltage law for the armature circuit and Newton's law for the inertia of the armature.

$$V_a - V_{R_a} - V_{L_a} - V_b = 0 \qquad I\ddot{\theta} = T - T_L - \dot{\theta}c$$

$$V_a - R_a i - L_a \dot{i} - v_b = 0$$

$$\dot{i} = \frac{1}{L_a}\left(V_a - R_a i - V_b\right) \qquad \dot{\omega} = \frac{1}{I}\left(T - T_L - \omega c\right) \quad ; \quad T = K_T i, \quad V_b = K_b \omega$$

$$\dot{i} = \frac{1}{L_a}\left(V_a - R_a i - K_b \omega\right) \qquad \dot{\omega} = \frac{1}{I}\left(K_T i - T_L - \omega c\right)$$

$$I_a s = \frac{1}{L_a}\left(V_a - R I_a - K_b \Omega\right) \qquad \Omega s = \frac{1}{I}\left(K_T I_a - T_L - \Omega c\right)$$

$$I_a\left(s + \frac{R}{L_a}\right) = \frac{1}{L_a}\left(V_a - K_b \Omega\right) \qquad \Omega\left(s - \frac{c}{I}\right) = \frac{1}{I}\left(K_T I_a - T_L\right)$$

$$I_a = \frac{1}{(L_a s + R)}\left[V_a - K_b \Omega\right] \qquad \Omega = \frac{1}{(I s - c)}\left[K_T I_a - T_L\right]$$

$$\Omega = \frac{1}{(Is - c)}\left[K_T\left(\frac{1}{L_a s + R}\left[V_a - K_b \Omega\right]\right) - T_L\right]$$

$$\Omega = \frac{K_T V_a}{L_a I s^2 + (R_a I + c L_a)s + (R_a + K_b K_T)} + \frac{-(L_a s + R_a) T_L}{L_a I s^2 + (R_a I + c L_a)s + (R_a + K_b K_T)}$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{L_a I s^2 + (R_a I + c L_a)s + (R_a + K_b K_T)}$$

2) *[1 pt]* Assuming that the inductance effect in the DC Motor is small ($L_a \approx 0$), how does that change the order of the system? (Rewrite the transfer function as the reduced model).

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{(R_a I + c L_a) s + (R_a + K_b K_T)}$$

3) *[1 pt]* Read the discussion in section 6.6.2 of the textbook about the assumption made in part b). In your own words, *BRIEFLY* explain why we must be careful with this assumption.

Assuming $L_a = 0$ removes the possibility of oscillations & can incorrectly predict settling time

4) *[1 pt]* Determine a numerical value for the time constant. The system parameters are as follows:

$$K_T = 0.0267\ N \cdot m/A\,, \qquad K_b = 0.0267\ N \cdot m/A\,,$$

$$c = 5X10^{-4}\ N \cdot m \cdot s/rad, \qquad I = 5X10^{-4}\ kg \cdot m^2,$$

$$R_a = 13.5\ \Omega, \qquad (T_L \approx 0), \quad (L_a \approx 0)$$

(Note that K_T and K_b have the same numerical values if expressed in the same units.)

$\tau = (R_a I + c L_a) = 6.75\ E\text{-}3\ s$

$\boxed{\tau = 6.75\ ms}$

5) [1 pt] Assume that the motor is driven by a step input in the applied voltage of $v_a = 5$ V. Determine an analytical expression for the response of ω vs. time using the Laplace Transform method (Pair 12). ***Show all of your work.***

$$\Omega = \frac{K_T}{(R_a I + cL_a)s + (R_a + K_b K_T)} \left(\frac{V}{s}\right)$$

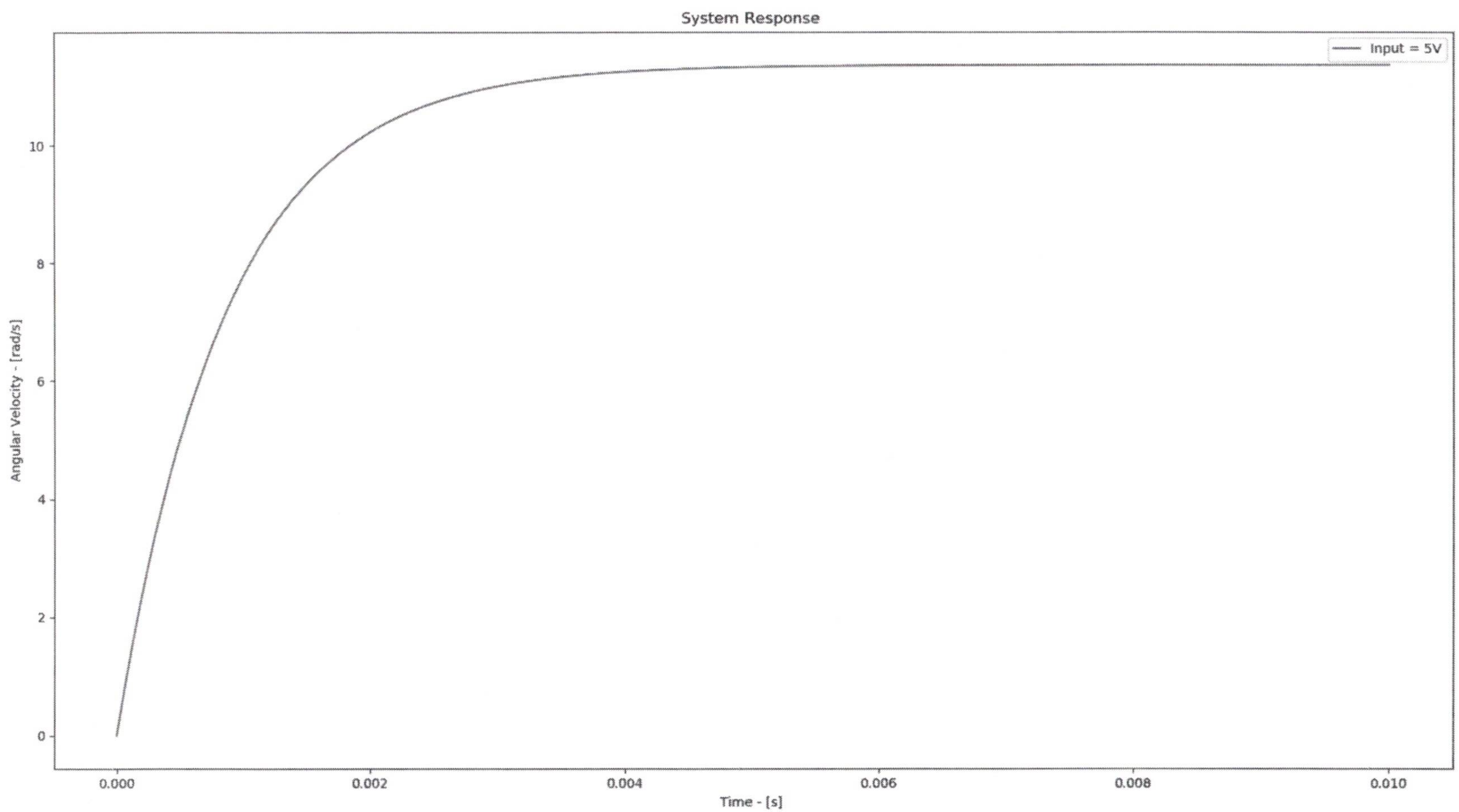
$$\Omega = \frac{K_T V}{(R_a I + cL_a)} \frac{1}{s\left(s + \frac{R_a + K_b K_T}{R_a I + cL_a}\right)}$$

$$\omega = \frac{V K_T}{R_a I + cL_a}\left(1 - e^{-\frac{R_a + K_b K_T}{R_a I + cL_a} t}\right)$$

$\omega(t) =$ ______________________________

6) *[3 pts]* Plot ω vs. time and attach your graph to this Pre-Lab.

(Include a title, labels on the axes, & units on the graph.)


System Response
Input = 5V
Angular Velocity - [rad/s]
0
2
4
6
8
10
0.000
0.002
0.004
0.006
0.008
0.010
Time - [s]

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