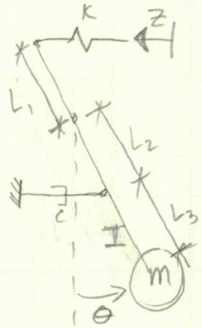
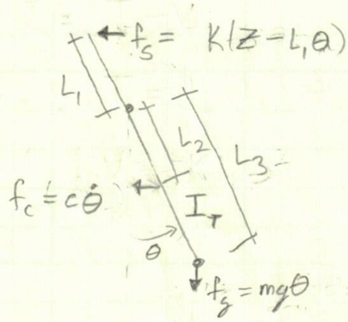
HW #5 Diego Colón

1)



Assumptions
- → $\theta(0) = z(0) = 0$
- → frictionless pivot
- → $\sin\theta = \theta$
- → ideal springs and dampeners



$$I_T\ddot{\theta} = L_1 f_s - L_2 f_c - L_3 f_g$$

$$I_T\ddot{\theta} = L_1(K(z - L_1\theta)) - L_2 c\dot{\theta} - mgL_3\theta$$

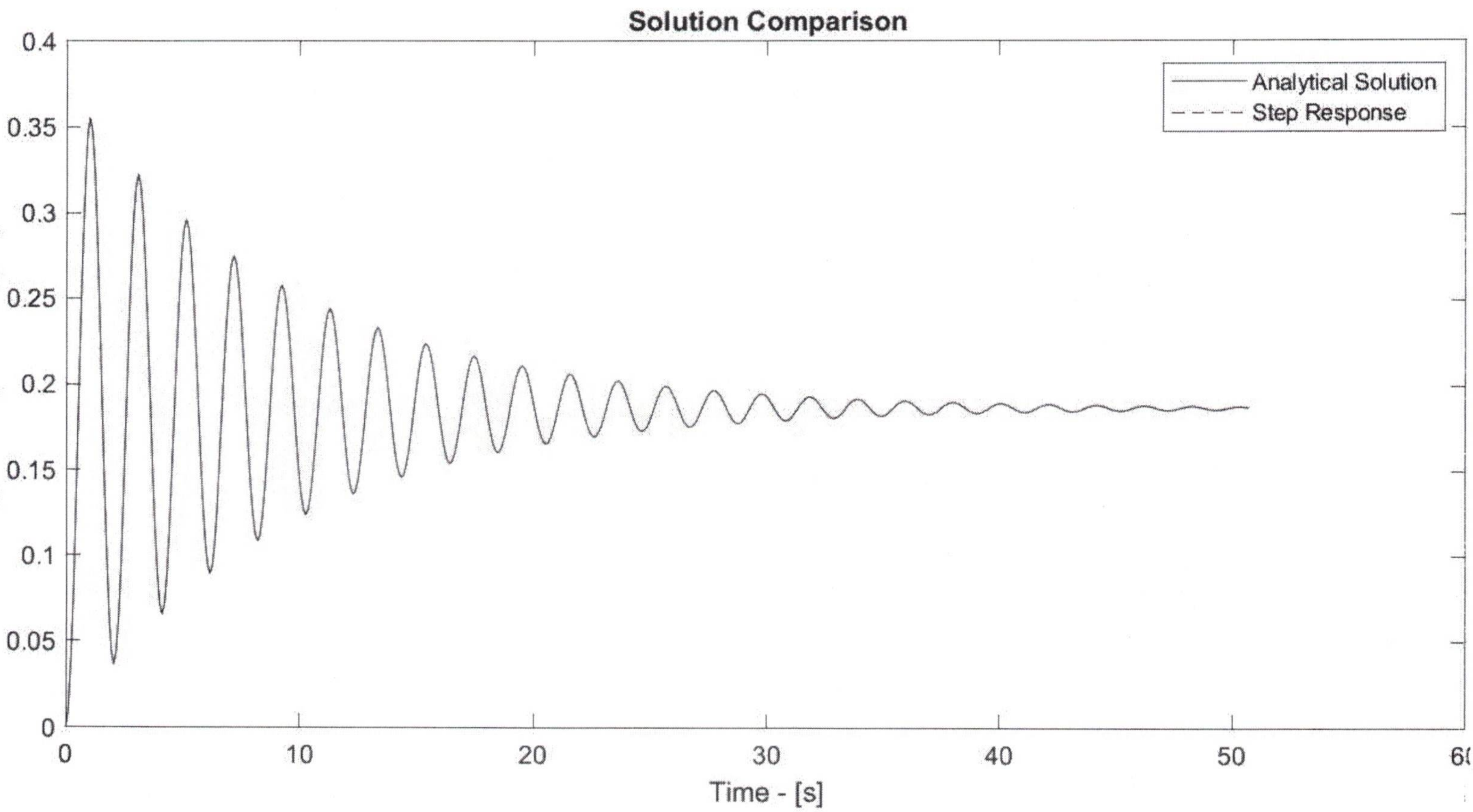
$$I_T\ddot{\theta} = L_1 K z - L_1^2 K\theta - L_2 c\dot{\theta} - mgL_3\theta$$

$$I_T\ddot{\theta} + L_2 c\dot{\theta} + (L_1^2 K + mgL_3)\theta = L_1 K z$$

$$I_T\ddot{\theta} + L_2 c\dot{\theta} + (L_1^2 K + mgL_3)\theta = L_1 K z$$

$$[s^2 I_T + L_2 c s + (L_1^2 K + mgL_3)]\Theta(s) = L_1 K Z(s)$$

$$\frac{\Theta(s)}{Z(s)} = \frac{L_1 K}{I_T s^2 + L_2 c s + (L_1^2 K + mgL_3)}$$



The two solutions overlay ∴ the analytical solution and the step response solution are close to, if not, identical.

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clear all
clc

I = 1.8;     L1 = 0.6;   L2 = 0.6;
L3 = 1.5;    m = 3;      c = 5;
k = 100;     g = 9.81;   A = 0.25;
It = I + m*L3^2

num = [L1*k];
den = [It, L2^2*c, (m*g*L3+L1^2*k)];
THETA = tf(num,den)
[theta2, t] = step(THETA);

wn = sqrt((m*g*L3+k*L1^2)/It)
z = (c*L2^2)/(2*wn*It)
sz = sqrt(1-z^2)
phi = atan(sz/z)
C1 = (A*L1*k)/(It*wn^2)
ex = -z.*wn.*t;
theta1 = C1.*(1-(exp(ex).*sin(wn.*sz.*t+phi))/sz) ;


figure(1)
plot(t,theta1)
title('Analytical Solution')
xlabel('Time - [s]')
ylabel('Angular Displacent - [rad]')


figure(2);
plot(t,A.*theta2)
title('Step Response')
xlabel('Time - [s]')
ylabel('Angular Displacent - [rad]')

figure(3)
plot(t,theta1,'b',t,0.25.*theta2,'k--')
title('Solution Comparison')
xlabel('Time - [s]')
ylabel('Angular Displacent - [rad]')
legend('Analytical Solution','Step Response')
pause

clc
clf(1)
clf(2)
clf(3)

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$$\Theta(s) = \frac{L_1 K}{I_T s^2 + L_2 c s + (L_1^2 K + mgL_3)} \left(\frac{A}{s}\right)$$

$$= \frac{A L_1 K}{I_T} \left(\frac{1}{s(s^2 + {}^{L_2 c}/_{I_T}\, s + (L_1^2 K + mgL_3)/I_T)} \right)$$

$$= \frac{A L_1 K}{I_T \omega_n^2} \left(\frac{\omega_n^2}{s(s^2 + {}^{L_2 c}/_{I_T}\, s + \omega_n^2)} \right) \quad ; \quad \omega_n^2 = \frac{L_1^2 K + mgL_3}{I_T}$$

$$2\zeta\omega_n = \frac{L_2 c}{I_T} \rightarrow \zeta = \frac{L_2 c}{2\omega_n I_T}$$

$$= \frac{A L_1 K}{I_T \omega_n^2} \left(\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n + \omega_n^2)} \right)$$

$$\Theta(t) = \frac{A L_1 K}{I_T \omega_n^2} \left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \phi\right) \right)$$

$$\omega_n = \frac{L_1^2 K + mgL_3}{I_T} \;,\; \zeta = \frac{L_2 c}{2\omega_n I_T} \;,\; \phi = \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$\Theta(t) = 0.1872 \left(1 - \frac{1}{0.9994} e^{-0.10531904t} \sin(3.05976304t + 1.5364) \right)$$

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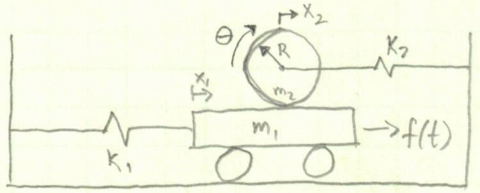
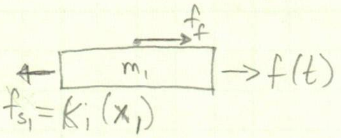
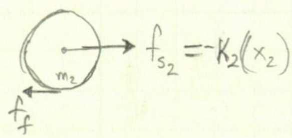
2)



Assumptions
→ measured from equilibrium
→ ideal springs
→ sufficient friction between bodies to avoid slipping





$$x_2 - x_1 = R\theta$$
$$\dot{x}_2 - \dot{x}_1 = R\dot{\theta}$$
$$\ddot{x}_2 - \ddot{x}_1 = R\ddot{\theta}$$

$$m_1\ddot{x}_1 = -K_1 x_1 + f_f + f(t)$$

$$m_1\ddot{x}_1 = -K_1 x_1 + \frac{I}{R^2}(\ddot{x}_2 - \ddot{x}_1) + f(t)$$

$$m_2\ddot{x}_2 = -K_2 x_2 - f_f$$

$$m_2\ddot{x}_2 = -K_2 x_2 - \frac{I}{R^2}(\ddot{x}_2 - \ddot{x}_1)$$

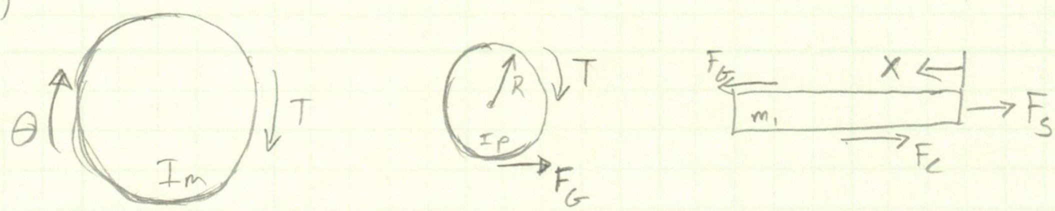
$$I\ddot{\theta} = f_f R$$

$$I\left(\frac{\ddot{x}_2 - \ddot{x}_1}{R}\right) = f_f R$$

$$\frac{I}{R^2}(\ddot{x}_2 - \ddot{x}_1) = f_f$$

$$\boxed{\begin{aligned} \left(m_1 + \frac{I}{R^2}\right)\ddot{x}_1 &= -K_1 x_1 + \frac{I}{R^2}\ddot{x}_2 + f(t) \\ \left(m_2 + \frac{I}{R^2}\right)\ddot{x}_2 &= -K_2 x_2 + \frac{I}{R^2}\ddot{x}_1 \end{aligned}}$$

3)



$R\theta = x$

$R\dot{\theta} = \dot{x}$

$R\ddot{\theta} = \ddot{x}$

$\sum F_x = F_G - F_S - F_C = m\ddot{x}$

$\sum T = T - F_G R = (I_m + I_p)\ddot{\theta}$

$m\ddot{x} = F_G - F_S - F_C$

$F_G = \frac{1}{R}(T - I_T\ddot{\theta})$

$m\ddot{x} = T/R - \frac{I_T}{R}\ddot{\theta} - F_S - F_C$

$I_T = (I_m + I_P)$

$m\ddot{x} = T/R - I_T/R^2\,\ddot{x} - F_S - F_C$

$$\boxed{(m + \tfrac{1}{R^2} I_T)\ddot{x} = -c\dot{x} - Kx + T/R}$$

$\sum (T_i + V_i) = const.$

$T = \frac{1}{2} I_T \dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}^2 \quad ; \quad V = \frac{1}{2} K x^2$

$\frac{1}{2} I_T \dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} K x^2 = 0$

$I_T \dot{\theta}\ddot{\theta} + m_1 \dot{x}\ddot{x} + Kx\dot{x} = 0$

$I_T (\frac{\dot{x}}{R})(\frac{\ddot{x}}{R}) + m_1 (\dot{x})(\ddot{x}) + Kx\dot{x} = 0$

$\frac{1}{R^2} I_T \ddot{x} + m_1 \ddot{x} + Kx = 0$

$$\boxed{(\tfrac{1}{R^2} I_T + m_1)\ddot{x} = -Kx}$$

$$\boxed{m_{eq} = \frac{I_T}{R_2} + m_1 \; ; \; I_T = I_m + I_p}$$