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Name: Last, First

MAE 3724 Systems Analysis
Fall 2019

Pre-Lab for Experiment 9
Frequency Response of a Speaker

Complete this Pre-Lab BEFORE you come to the laboratory.
The Lab Instructor will answer questions and provide feedback when you come to the laboratory to do the experiment.

Show all your work and final answers on these pages. Attach the specified graphs and MATLAB Codes.

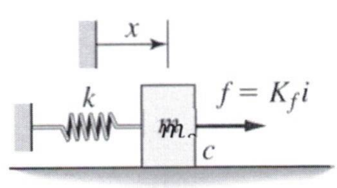
Background for Laboratory Experiment 9

In Lab 6, you measured the **step response** of a speaker of the type shown schematically in the figure below. That was a **Time Domain Experiment**. You measured the displacement of the speaker cone x as a function of time when the input voltage v was a step with a magnitude of 2 volts.

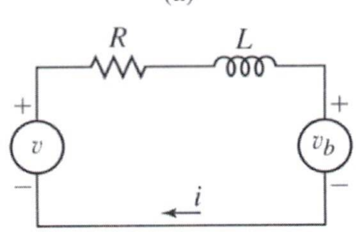
In this experiment, you will measure the **frequency response** of the same speaker. This will be a **Frequency Domain Experiment**. In Frequency Domain analysis, we subject the system to a "steady sinusoidal input" of constant amplitude and varying frequency.

In Frequency Domain Analysis, we determine the magnitude (and phase) and of the output as a function of frequency ω of the input, while holding the amplitude of the input sinusoid constant. That is, if $v = A \cdot \sin(\omega t)$, we vary ω while holding A constant.

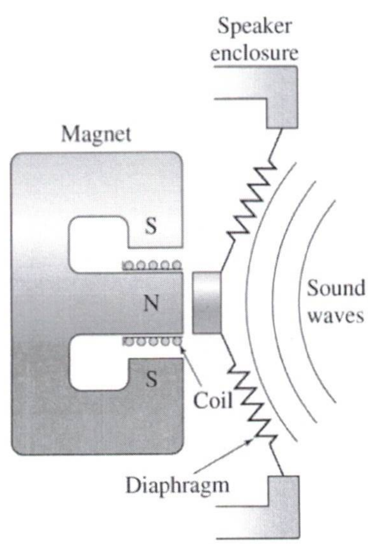
A physical model of the Speaker is shown in the figure below. We found in Lab 6 that a reasonably good assumption is that the effect of the inductance L is of minimal importance compared to other effects. We set $L = 0$. **But in this Pre-Lab, we are going to explore the effects of coil inductance by using the manufacturer's value for L.**



(a)



(b)



The motion of the center of the speaker cone is x. The motion is reference to an equilibrium position.

The coil is connected to the center of the speaker cone so that it moves a distance x when the center of the cone moves a distance x.

The governing differential equations are derived in Example 6.7.1 in the textbook. It is a third order, linear differential equation. The transfer function is as shown below.

$$\frac{X(s)}{V(s)} = \frac{K_f}{m_s L s^3 + (cL + m_s R)s^2 + \left(kL + cR + K_f K_b\right)s + kR}$$

1. *[1 pts]* Demonstrate that the expression for the magnitude $M(\omega) = |X(j\omega)/V(j\omega)|$ is as shown below.

$a = m_s L$
$b = cL + m_s R$
$c = KL + cR + K_f K_b$
$d = KR$

$$T(j\omega) = \frac{K_f}{a(j\omega)^3 + b(j\omega)^2 + c(j\omega) + d}$$

$$= \frac{K_f}{-a\omega^3 j + b\omega^2 + cj\omega + d}$$

$$= \frac{K_f}{(d - b\omega^2) + (c\omega - a\omega^3)j}$$

$$M = \frac{K_f}{\sqrt{(d-b\omega^2)^2 + (c\omega - a\omega^3)^2}}$$

$$M = \frac{K_f}{\sqrt{(KR - (cL + m_s R)\omega^2)^2 + ((KL + cR + K_f K_b)\omega - m_s L\omega^3)^2}}$$

$$\boldsymbol{M(\omega) = \frac{K_f}{\sqrt{[kR - (cL + m_s R)\omega^2]^2 + \left[\left(kL + cR + K_f K_b\right)\omega - m_s L\omega^3\right]^2}}}$$

2. *[1 pts]* Write an expression for the log magnitude $m = 20\, log_{10} M(\omega)$.

$$m = 20\log(K_f) - 10\log\left((KR - (cL + m_s R)\omega^2)^2 + ((KL + cR + K_f K_b)\omega - m_s L\omega^3)^2\right)$$

You could now predict what the log magnitude of the speaker's responses would be for any specific input frequencies. That is, a "Bode plot" can be obtained by plotting m for various ω values on semi-log graph paper. There is also a way to use Excel to make the plot. But we will use MATLAB.

For Part 3 use the following values for the parameters:

$$m_s = 0.03\,kg,\ c = 1.532\,N\cdot\frac{s}{m},\ k = 885\,\frac{N}{m},\ K_b = 5.71\ T\cdot m,$$
$$K_f = 9.62\ T\cdot m,\ R = 5.91\,\Omega,\ \text{and}\ L = 8.17X10^{-4}\,H$$

3. *[2 pts]* As an alternate approach, use MATLAB's function "`bode`" to plot the log magnitude m (in dB) versus frequency ω (in rad/s) on a logarithmic axis (i.e., use the MATLAB "***SpeakerBode***" code posted on Brightspace under "*MATLAB Resources*" to create the transfer function X(s)/V(s) and plot the frequency response using the "`bode()`" function). **Label your plots. Print and attach a copies of the Bode plot (both magnitude and phase) from MATLAB and the MATLAB Code you used to obtain the graph.**

4. *[1 pts]* What is the high-frequency slope of the Bode magnitude plot from Part 3?

$slope =$ -60 $dB/decade$

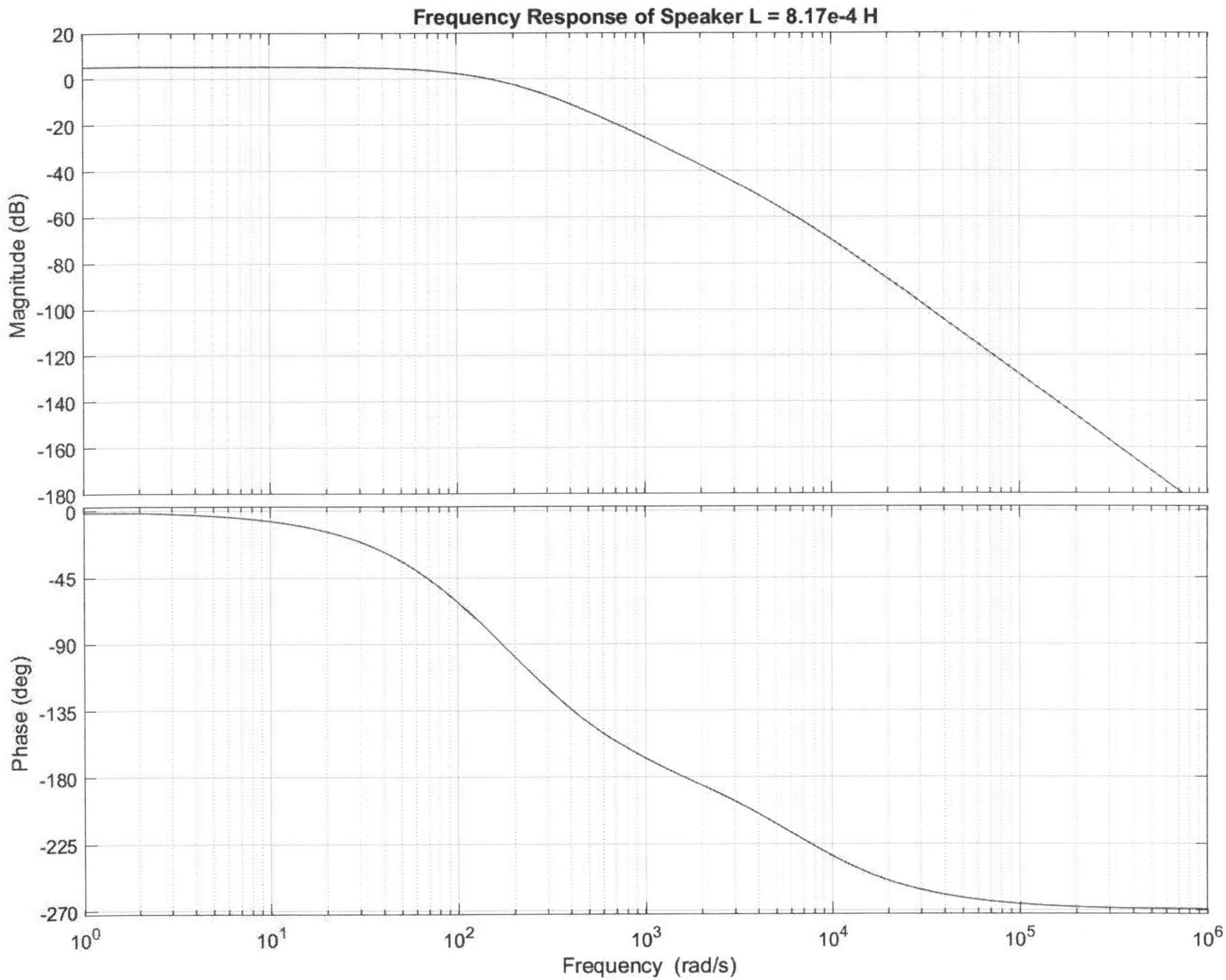
5. *[2 pts]* Explore the potential of "Model Reduction". Repeat Part 3 for the case when the speaker coil inductance is equal to zero ($L = 0$). **Print and attach a copies of the the Bode plot of the reduced order model from MATLAB and the MATLAB Code you used to obtain the graph.**

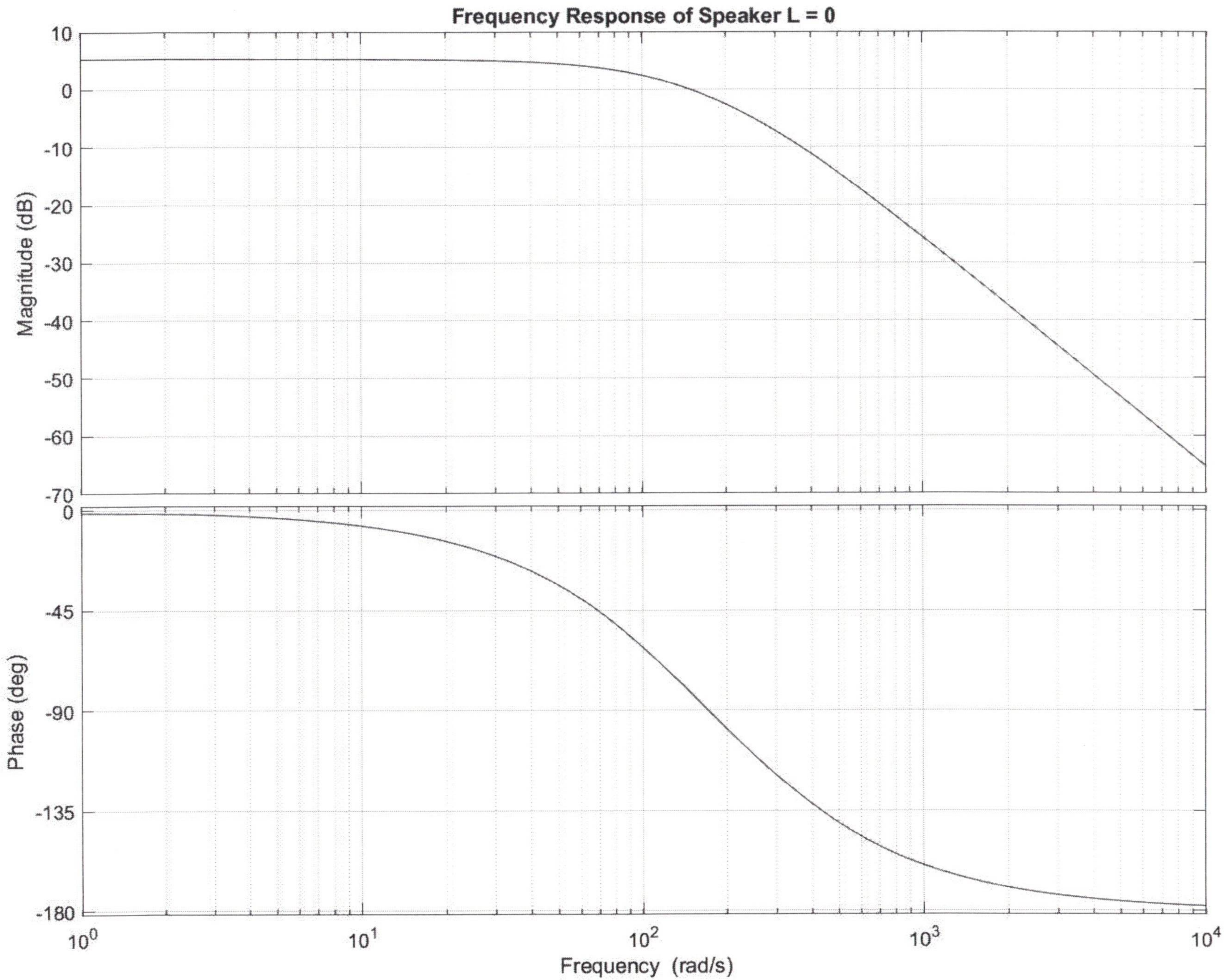
6. *[1 pts]* What is the high-frequency slope of the Bode magnitude plot for the reduced model (Part 5)?

$slope =$ -40 $dB/decade$

7. *[2 pts]* After comparing your answers for Part 4 & 6, develop an equation that predicts the high-frequency slope of a Bode magnitude plot, based on the relative order of the system (n).

$$m = -20n\left[\frac{dB}{decade}\right]$$


Frequency Response of Speaker L = 8.17e-4 H
Magnitude (dB)
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
Phase (deg)
0
-45
-90
-135
-180
-225
-270
10^0
10^1
10^2
10^3
10^4
10^5
10^6
Frequency (rad/s)


Frequency Response of Speaker L = 0
Magnitude (dB)
10
0
-10
-20
-30
-40
-50
-60
-70
Phase (deg)
0
-45
-90
-135
-180
10^0
10^1
10^2
10^3
10^4
Frequency (rad/s)

```
%Frequency Response of a Speaker

%define parameters
m = 0.03;        % [kg]
c = 1.532;       % [Ns/m]
k = 885;         % [N/m]
Kb = 5.71;       % [T.m]
Kf = 9.62;       % [T.m]
R = 5.91;        % [ohms]
L = 8.17*10^-4;    % [H]
%L=0;

%create transfer function
num=[Kf*1000];      %multipled by 1000 to convert from m to mm
den=[(m*L), (c*L+m*R), (k*L+c*R+Kf*Kb), (k*R)];
sys1=tf(num,den);

L = 0;
num=[Kf*1000];      %multipled by 1000 to convert from m to mm
den=[(m*L), (c*L+m*R), (k*L+c*R+Kf*Kb), (k*R)];
sys2=tf(num,den);

%plot bode plot of TF
figure (1)
bode(sys1);
title('Frequency Response of Speaker L = 8.17e-4 H')
grid on

figure (2)
bode(sys2);
title('Frequency Response of Speaker L = 0')
grid on
```