1) $T(s) = \frac{6s}{7s+6} = \frac{s}{7/6 s+1} \rightarrow \frac{1}{7/6} \frac{7/6\, s}{7/6 s+1} = \frac{1}{\tau} \frac{\tau s}{\tau s+1}$

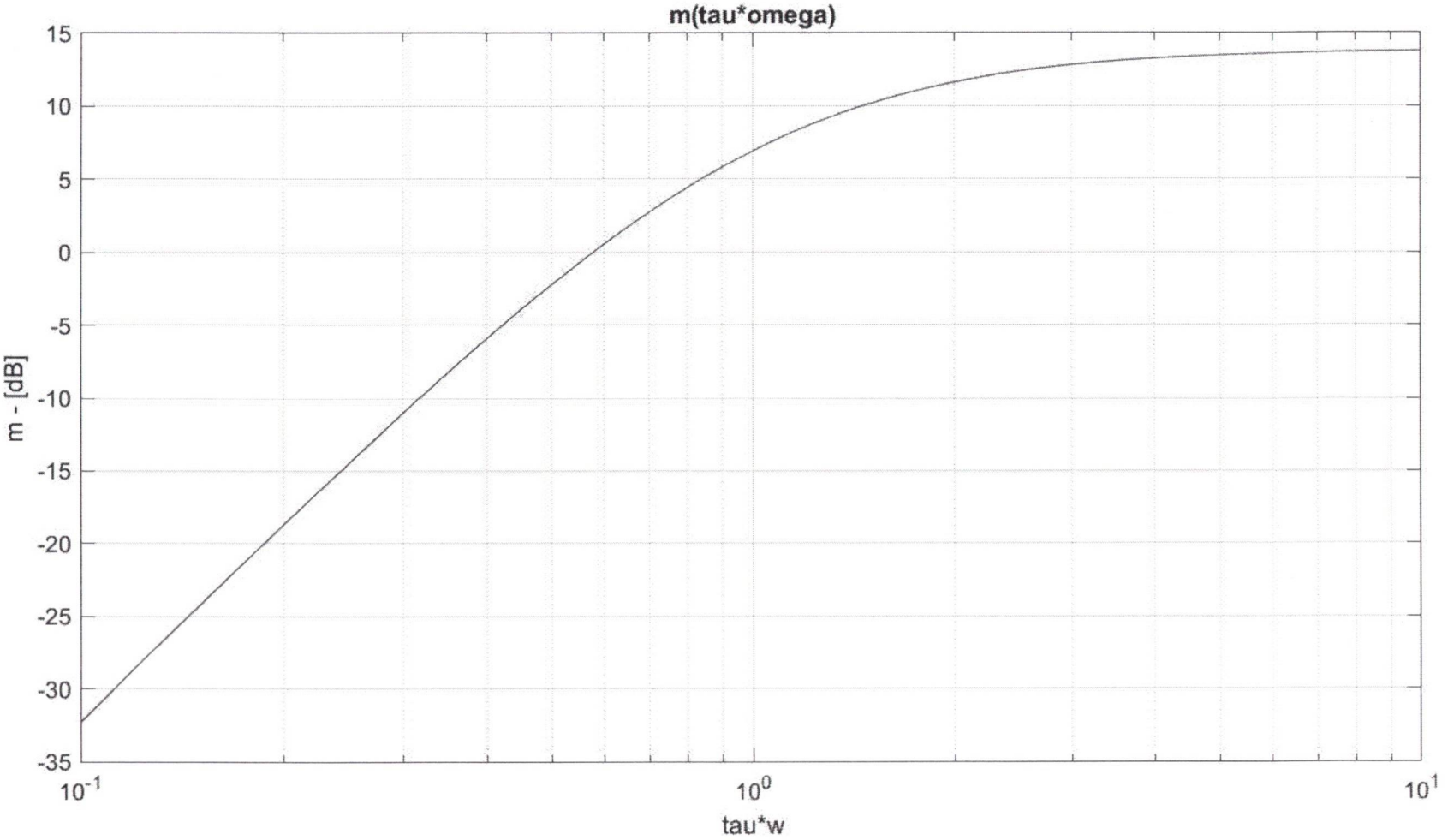
a) $\tau = 7/6$

b) $T(j\omega) = \frac{1}{\tau} \left(\frac{\tau \omega j}{\tau \omega j + 1} \right)$

c) $M = \frac{1}{\tau} \frac{\tau \omega}{\sqrt{1+(\tau\omega)^2}}$

b) $m = 20 \log \left(\frac{1}{\tau} \frac{\tau \omega}{\sqrt{1+(\tau\omega)^2}} \right)$

$m(\tau\omega) = 20 \log \frac{1}{\tau} + 20 \log (\tau\omega) - 10 \log (1+(\tau\omega)^2)$


m(tau*omega)
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
m - [dB]
10^{-1}
10^{0}
10^{1}
tau*w

2) $10\ddot{x} + c\dot{x} + 20x = f(x)$ Determine ω_r & M_r for a) $\zeta = 0.1$, b) $\zeta = 0.3$

$\omega_r = \omega_n\sqrt{1-2\zeta^2}$ $\quad M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$ $\quad \omega_n = \sqrt{\dfrac{K}{m}} = 1.414$

a) $\zeta = 0.1$

$\omega_r = 1.414\sqrt{1-2(0.1)^2} = 1.4 \text{ rad/s}$

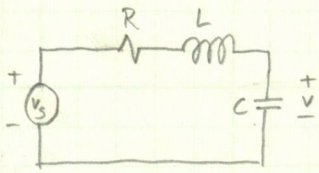
$M_r = \dfrac{1}{2(0.1)\sqrt{1-(0.1)^2}} = 5.025$

b) $\zeta = 0.3$

$\omega_r = 1.414\sqrt{1-2(0.3)^2} = 1.28 \text{ rad/s}$

$M_r = \dfrac{1}{2(0.3)\sqrt{1-(0.3)^2}} = 1.747$

3)



$LC\ddot{v} + RC\dot{v} + v = v_s$

$C = 1E\text{-}5\ F$

$L = 4E\text{-}3\ H$

a) $R|_{\zeta=0.5}$

$$\frac{V}{V_s} = \frac{1}{LCs^2 + RCs + 1} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$\omega_n = \sqrt{1/LC}$, $2\zeta\omega_n = R/L$

$R = 2L\zeta\sqrt{1/LC}$

$R = 20\Omega$

b) $\omega_n = \sqrt{\frac{1}{LC}} = 5E3\ rad/s$

c)

$$T(s) = \frac{1}{LC}\left(\frac{1}{s^2 + R/L\,s + 1/LC}\right)$$

$$T(j\omega) = \frac{1}{LC}\left(\frac{1}{(j\omega)^2 + R/L(j\omega) + 1/LC}\right) = \frac{1}{LC}\left(\frac{1}{-\omega^2 + \frac{R}{L}j\omega + 1/LC}\right) \quad ; \quad \omega_n^2 = \frac{1}{LC},\ 2\zeta\omega_n = R/L$$

$$T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2} = \frac{1}{(j\omega/\omega_n)^2 + (2\zeta\omega/\omega_n)j + 1} = \frac{1}{1 - (\omega/\omega_n)^2 + 2(\zeta\omega/\omega_n)j}$$

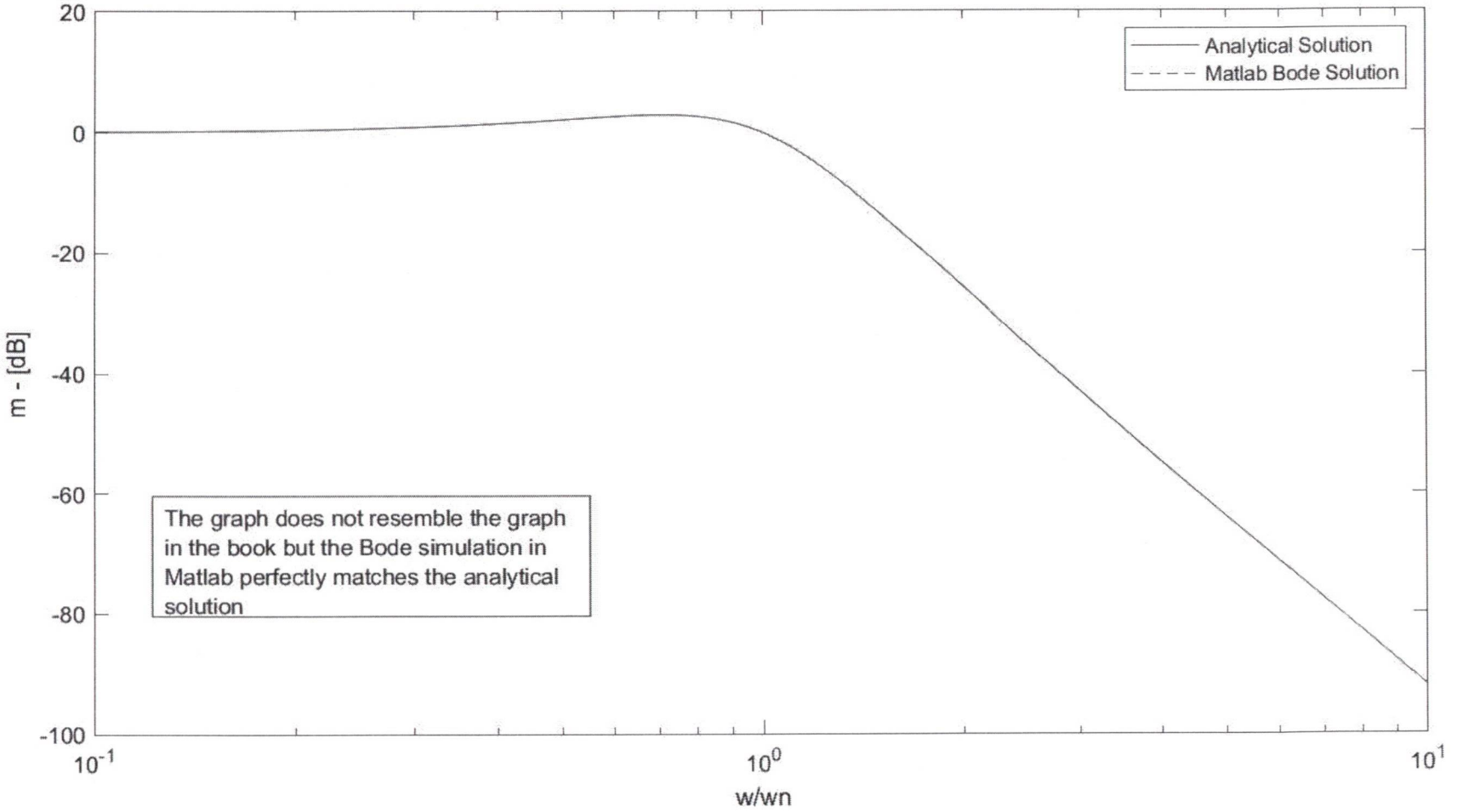
$r = \frac{\omega}{\omega_n}$

$$T(r) = \frac{1}{1 - r^2 + 2\zeta r j} \qquad |T(r)| = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$M(\omega/\omega_n) = \frac{1}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + 4\zeta^2(\omega/\omega_n)^2}}$$

d) $m(\omega/\omega_n) = 20 \log \left(\frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \right)$

$= -10 \log \left(\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2 \right)$


20
0
-20
-40
-60
-80
-100
m - [dB]
10^{-1}
10^{0}
10^{1}
w/wn
Analytical Solution
Matlab Bode Solution
The graph does not resemble the graph in the book but the Bode simulation in Matlab perfectly matches the analytical solution