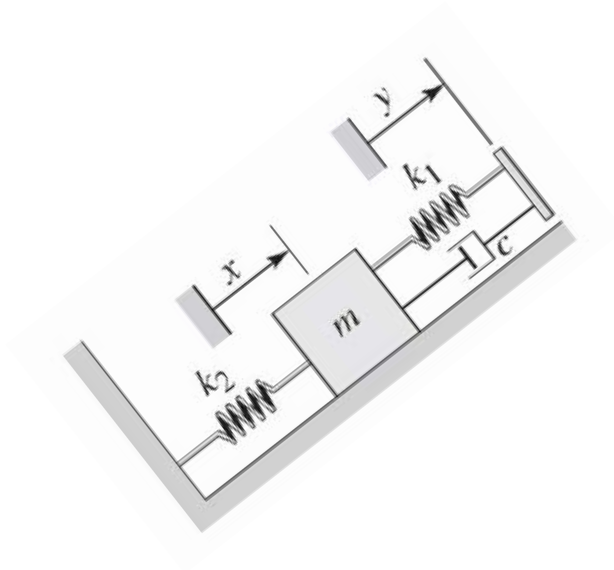
MAE 3723/4, System Analysis - Homework Assignment #4

Due at 11:59 pm, Friday, Sept. 13, in the Canvas Homework 4 dropbox.

Upload a single PDF file. Start each problem on a new page.

1) For the system shown in the figure below, the input is the displacement *y(t)* and the output is the displacement *x(t)* of the mass m. The equilibrium position corresponds to *x = y = 0*. Neglect any friction between the mass and the surface.



a) Clearly state all assumptions to be used for modeling this system, including any given above.
b) Draw the freebody diagram. State your "presumptions" on the configuration of the states of the system. Be sure to label the magnitude and direction of the forces consistent with those presumptions.
c) Use the freebody diagram to derive the equation of motion (ODE) for the system.
d) Determine the Transfer Function for the system.

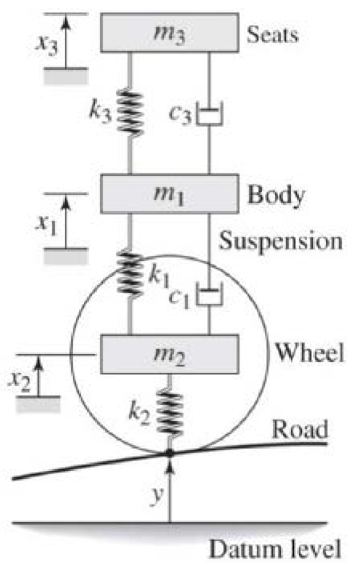
e) If the displacement input is a setp function - ***y(t) =3 u(t)***, Determine the analytical solution for the time response of the system - *x(t)*
Hint: look at table entries 21,22 and 23.

f) The model parameters are: m = 2 kg, c = 4 N-s/m, k1 = 100 N/m, k2 = 50 N/m. Use Matlab to plot your solution equation from part e). Inlcude a printout of your code and that plot. Hint: your solution equation is probably long, containing many terms and many constants. It will be much easier to avoid or eliminate errors if you develop the equation over several lines. Example:
Create variable names and assign values for the parameters.
Create variables containing things like ω_n and ζ
Create variables containing things like $\sqrt{1-\zeta^2}$
Create variables containing small to medium size pieces of the final equation
Combine those pieces into the full equation

g) Use the MatLab tf() and step() functions along with your transfer function from part d) and the model parameters given above, to plot the response of the system to a unit step input. Compare this plot with the plot from part f). Include a printout of your code and the plot.

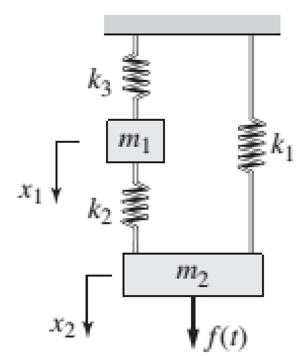
2) The figure below shows a quarter car model that includes the mass of the seats and passengers. The input is the road displacement *y(t)*.

a) Clearly state all assumptions to be used for modeling this system.
b) Draw the freebody diagrams. State your "presumptions" on the configuration of the states of the system. Be sure to label the magnitude and direction of the forces consistent with those presumptions.
c) Use the freebody diagrams to derive the equations of motion (ODEs) for the system. Arrange the equations so that the highest derivative term appears on the left side of the equations and all other terms are on the right side of the equation.
For example: $m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1) - c_1\dot{x}_1 + c_2(\dot{x}_2 - \dot{x}_1)$

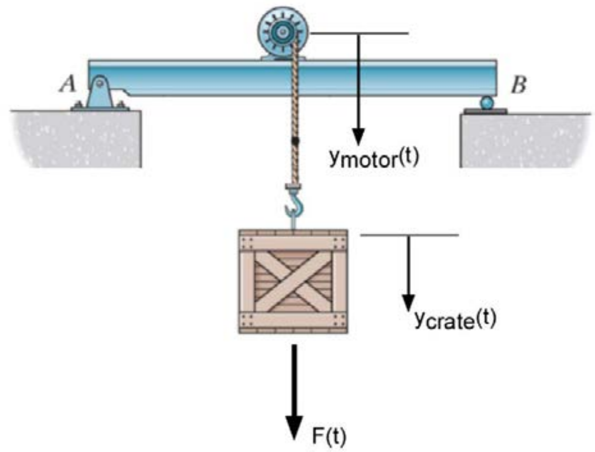


3) The figure below shows a model including two masses and three springs. The input is an external force *f(t)*. Important: assume that mass2 will always remain horizontal - it cannot rotate!

a) Clearly state all assumptions to be used for modeling this system.
b) Draw the freebody diagrams. State your "presumptions" on the configuration of the states of the system. Be sure to label the magnitude and direction of the forces consistent with those presumptions.
c) Use the freebody diagrams to derive the equations of motion (ODEs) for the system.



4) Consider the system shown in the figure below, comprised of a motor, a simply supported steel beam, a steel cable and a crate. Assume that the motor is rigidly attached at the center of the beam. The beam has a rectangular cross-section. For this problem, assume that the cable is flexible and behaves like a massless spring. The beam is neither rigid nor massless. Also, assume that the cable will always remain in tension, and never go slack.



Beam properties:

length = 3 m, width = 0.075 m, height = 0.10 m,
E = 207 GPa, density = 8000 kg/m3
Motor: mass = 25 kg,
Crate: mass = 200 kg

Cable properties:

length = 4 m, diameter = 0.007 m, E = 207 GPa,

a) Calculate the equivalent stiffness of the beam, in units of N/m. (See table 4.1.1 in your textbook)
a) Calculate the equivalent stiffness of the cable, in units of N/m.
b) Calculate the equivalent-mass of the combined beam and motor
c) Draw an equivalent system diagram where the beam looks like typical coiled spring and the motor and beam equivalent mass are lumped onto one mass. (See example 4.2.1)
d) Calculate the static deflection of the motor/beam, in meters.
e) Write the equations of motion (ODEs) for the displacement of the motor/beam and the crate, measured from the static equilibrium position. The applied force F(t) is the system input.