

**Instructions**

For each problem, create and run one or more Matlab programs to solve the problem.

To submit your solution, for each problem:

1. Create a Microsoft Word document named: **Matlab Exam Solution.docx**
2. Type your name on the first line of the file.
3. For each problem on the exam:
  - a) Place a line that says: Problem X solution (replace X with 1, 2, 3, etc)
  - b) Paste a copy of your complete MatLab code for the problem
  - c) Paste copies of the required MatLab results (text or plots)
  - d) You may also insert any comments you think are useful, into the word document (please type all such comments in RED).
4. You must save this *single* file (**Matlab Exam Solution.docx**) containing your solutions for the entire exam, to a thumb drive, and submit the thumb drive to the proctor.

Any other files (such as .m files) will be ignored. We will only grade the **Matlab Exam Solution.docx** file.

***Everyone was told to bring a thumb drive to the exam***, as a last resort means of submitting your exam, to be used only in the unlikely event that the classroom internet should fail during the exam. In the root directory of that drive, there should be a file whose file name includes your name ( mine would be " \_R D Delahoussaye.txt") Note that the leading "\_" usually makes the file show up at the top of the file list. During the exam you should SAVE YOUR WORK often to this thumb drive, in case your computer crashes during the exam!

The exam will be open book, open notes, open google searches, open thumb drive, etc.

You may not use any form of technology to communicate with, send to, or receive information from another person, during the time of the exam. Use of E-mail, chat, twitter, facebook, snapchat, instagram, etc are prohibited. You may not use your phone in any way during the exam.

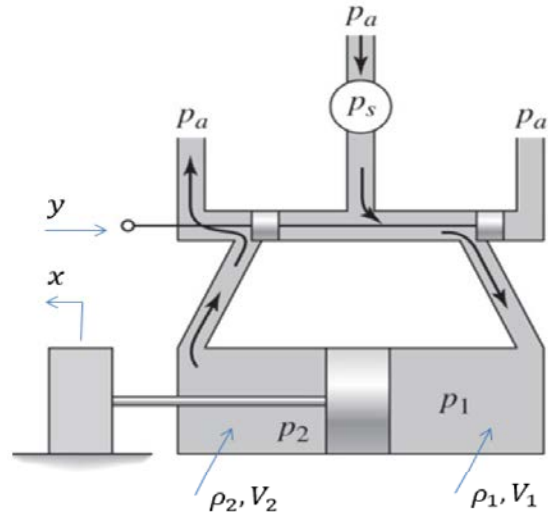
Because this is a computer based exam, no one will be allowed to leave the classroom early. If you think you might finish the exam early, bring something else to do during the remainder of the exam time. You may not leave the room during this 75 minute exam. Go the the restroom before the exam begins!!

1. (30 points) The following equations of motion describe the behavior of a hydraulic valve system. In these equations, the flow resistance terms have been linearized!

$$(p_1 - p_2) \cdot A = m \cdot \frac{d^2 x}{dt^2} \quad +$$

$$y \cdot K_{\text{valve}} \cdot (p_s - p_1) = \rho \cdot A \cdot \frac{dx}{dt} + V \cdot \frac{\rho}{\beta} \cdot \frac{dp_1}{dt}$$

$$y \cdot K_{\text{valve}} \cdot (p_2 - p_a) = \rho \cdot A \cdot \frac{dx}{dt} - V \frac{\rho}{\beta} \cdot \frac{dp_2}{dt}$$



The values for the constant parameters are:

$$A = 4.909 \times 10^{-4} \text{ m}^2$$

$$C_d = 0.6$$

$$p_s = 1.4 \times 10^7 \text{ Pa}$$

$$p_a = 1 \times 10^5 \text{ Pa}$$

$$V = 1.473 \times 10^{-4} \text{ m}^3$$

$$\beta = 2 \times 10^9 \text{ Pa}$$

$$\rho = 850 \frac{\text{kg}}{\text{m}^3}$$

$$K_{\text{valve}} = 2 \cdot 10^{-5}$$

$$m = 30 \cdot \text{kg}$$

**NOTE:** All units for the variables and the constants are **consistent** as given, and no unit conversions of any kind are necessary.

**Required:**

- a) Use Matlab and ode45() to solve the equations for the response to a step input of  $y = 0.002$

The initial conditions are:  $x=0$     $\dot{x}=0$     $p1=pa$     $p2=pa$

- b) Plot  $\dot{x}$  as a function of time, with nice title and labels.
- c) Plot  $p_1$  and  $p_2$  together as functions of time, on a new graph, with nice title and labels.

2. (25 points) For the system given in problem 1, perform a parameter study on the effect of  $K_{\text{valve}}$ , using values of:

$$K_{\text{valve}} = 1 \cdot 10^{-5}, \quad K_{\text{valve}} = 2 \cdot 10^{-5}, \quad K_{\text{valve}} = 10 \cdot 10^{-5}, \quad K_{\text{valve}} = 20 \cdot 10^{-5}$$

Plot ***p*<sub>1</sub>** vs time for the four different values of  $K_{\text{valve}}$ , all on one plot, nicely titled and labeled, with a legend entry to identify each curve.

3. (15 points) For the system given in problem 1, the analytical solution for displacement is claimed to be:

$$p1(t) = (7 - 14e^{-400t} \sin(800t + 1.55)) \cdot 10^6$$

Plot two curves on the same graph: The MatLab solution for  $p1(t)$  (as computed in problem 1) along with this claimed analytical solution, for the purpose of comparing the Matlab solution to this claimed analytical solution. Be sure to use meaningful plot labels. Use legends to identify each curve. Comment on how well the analytical solutions matches the Matlab solution.

4. (15 points) The valve terms in the equations given in problem 1 were linearized. The non-linear equations are:

$$\begin{aligned} (p1 - p2) \cdot A &= m \cdot \frac{d^2 x}{dt^2} \\ y \cdot K_{\text{valve}} \cdot \sqrt{p_s - p1} &= \rho \cdot A \cdot \frac{dx}{dt} + V \cdot \frac{\rho}{\beta} \cdot \frac{dp1}{dt} \\ y \cdot K_{\text{valve}} \cdot \sqrt{p2 - p_a} &= \rho \cdot A \cdot \frac{dx}{dt} - V \cdot \frac{\rho}{\beta} \cdot \frac{dp2}{dt} \end{aligned}$$

And changing to  $K_{\text{valve}} = 0.074$

Note that these equations are non-linear in  $p1$  and  $p2$  because of the square root term. To handle this non-linearity properly in Matlab, you must rewrite the square root terms as:

$$\sqrt{\text{abs}(p_s - p1)} \cdot \text{sign}(p_s - p1) \quad \text{and} \quad \sqrt{\text{abs}(p2 - p_a)} \cdot \text{sign}(p2 - p_a)$$

Plot  $p1$  vs time for the linear and nonlinear models, both on the same plot, nicely labeled, with a legend entry to identify each curve.

5. (15 points) The transfer function for an RLC circuit is:

$$T(s) = \frac{V_o(s)}{V_s(s)} = \frac{R_2}{R_2 C L s^2 + (R_1 R_2 C + L)s + (R_1 + R_2)} \quad \text{with } R_1 = R_2 = 8000, \quad L = 200 \quad \text{and} \quad C = 2.5 \cdot 10^{-6}$$

Plot the response of this system to a **step** input of magnitude 12 volts.