MAE 3723, System Analysis – Fall 2019

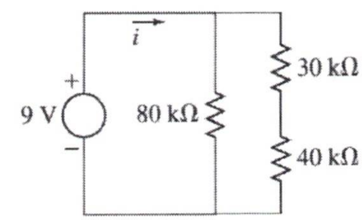
Homework Assignment #6

Due at 11:59 pm, Friday, October 4th, in the BrightSpace Homework 6 dropbox. Submit one and only one PDF file that includes any MatLab code and plot printouts. Start each problem on a new page.

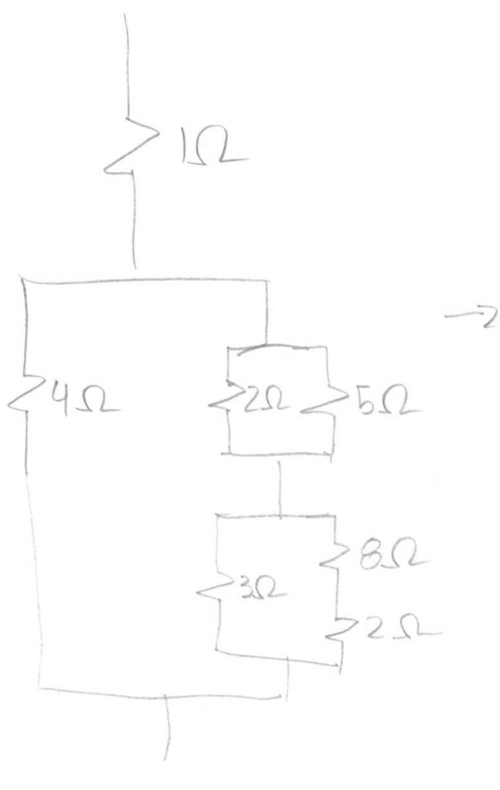
1) Determine the equivalent resistance for:

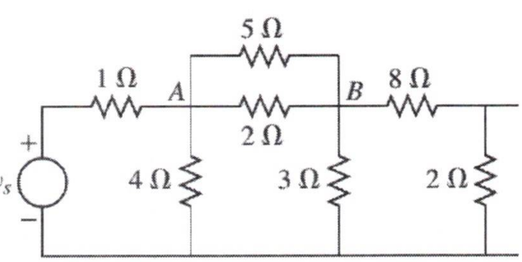
a)

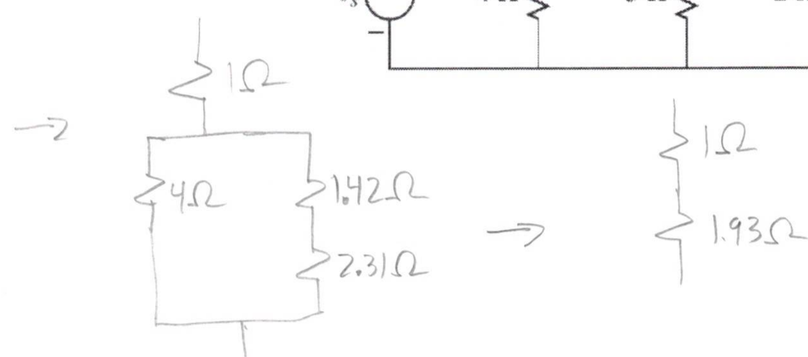
$$R_{eq} = \frac{1}{\frac{1}{80K\Omega} + \frac{1}{(30+40)\,K\Omega}} = 37.33\Omega$$



b)







$R_{eq} = 2.93\Omega$

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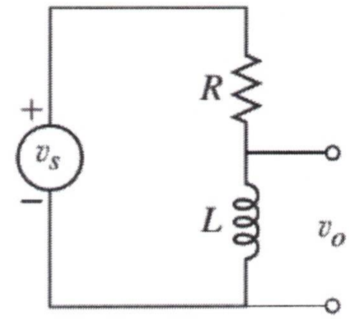
2) For the following circuit:



a) Obtain the model (ODE) of the voltage v_o, given the supply voltage v_s

$V_s - V_R - V_o = R$

$i = \frac{1}{R}(V_s - V_o)$

$\dot{i} = \frac{1}{R}(\dot{V}_s - \dot{V}_o)$

$V_o = L\dot{i}$

$V_o = \frac{L}{R}(\dot{V}_s - \dot{V}_o)$

$\frac{L}{R}\dot{V}_o + V_o = \frac{L}{R}\dot{V}_s$

$\dot{V}_o + \frac{R}{L}V_o = \dot{V}_s$

b) Determine the transfer function: $\frac{V_o(s)}{V_s(s)} = \frac{s}{s + \frac{R}{L}} = \frac{Ls}{Ls + R}$

c) Use the Laplace Transform method to determine $v_o(t)$

if: $v_s = 7.5u(t)$, $R = 8\,\Omega$ and $L = 5.3 \times 10^{-3}$ H.

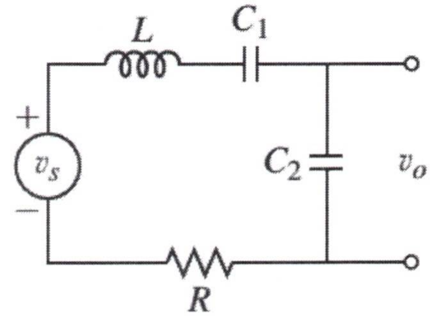
Use Matlab to plot your solution equation.

$V_o = \frac{Ls}{Ls+R}\left(\frac{A}{s}\right) = \frac{AL}{Ls+R} = \frac{A}{s + R/L}$

$V_o(t) = A\,e^{-\frac{R}{L}t}$

d) Use the your transfer function from part b), and Matlab tf() and step() functions to plot the response $v_o(t)$, and compare to part c).

3) For the following circuit:



a) Obtain the model (ODE) of the voltage v_o, given the supply voltage v_s

$V_s - V_L - V_{C_1} - V_O - V_R = 0$

$V_s - L\dot{i} - \frac{1}{C_1}\int i\,dt - v_o - iR = 0$

$\dot{V}_s - L\ddot{i} - \frac{1}{C_1} i - \dot{v}_o - \dot{i}R = 0$

$V_o = \frac{1}{C_2}\int i\,dt$

$\dot{V}_o = \frac{1}{C_2} i$

$\ddot{V}_o = \frac{1}{C_2}\dot{i}$

$\dddot{V}_o = \frac{1}{C_2}\ddot{i}$

$LC_2\dddot{V}_o + RC_2\ddot{V}_o + \left(1+\frac{C_1}{C_2}\right)\dot{V}_o = \dot{V}_s$

$LC_2\ddot{V}_o + RC_2\dot{V}_o + \left(1+\frac{C_1}{C_2}\right)V_o = V_s$

b) Determine the transfer function: $\dfrac{V_o(s)}{V_s(s)} = \dfrac{1}{LC_2 s^2 + RC_2 s + \left(1+\frac{C_1}{C_2}\right)}$

c) Use the Laplace Transform method to determine $v_o(t)$ for: $v_s = 3u(t)$, $R = 4\,\Omega$, $L = 6\times10^{-3}$ H and $C_1 = C_2 = 6.5\times10^{-4}$ N·m·s/rad

Then, use Matlab to plot your solution equation.

$V_o = \frac{A}{s}\left(\frac{1}{LC_2 s^2 + RC_2 s + \left(1+\frac{C_1}{C_2}\right)}\right) = \frac{A}{LC_2}\frac{1}{s\left(s^2 + \frac{R}{L}s + \frac{(1+C_1/C_2)}{LC_2}\right)}$

$V_o = \frac{A}{\left(1+\frac{C_1}{C_2}\right)}\left(\frac{\frac{\left(1+\frac{C_1}{C_2}\right)}{LC_2}}{s\left(s^2 + \frac{R}{L}s + \frac{\left(1+\frac{C_1}{C_2}\right)}{LC_2}\right)}\right)$

$V_o(t) = \frac{A}{\left(1+\frac{C_1}{C_2}\right)}\left[1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \phi\right)\right]$

$\omega_n^2 = \frac{\left(1+\frac{C_1}{C_2}\right)}{LC_2}$

$\zeta = \frac{R}{2L\omega_n}$

$\phi = \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

d) Use the Matlab ode45() function to plot the response $v_o(t)$, and compare to part c).

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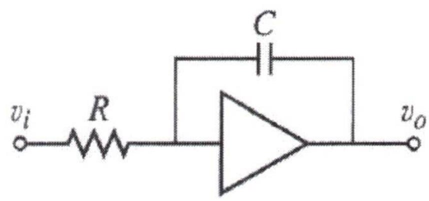
4) Determine the transfer function for the following op-amp circuits:

a)

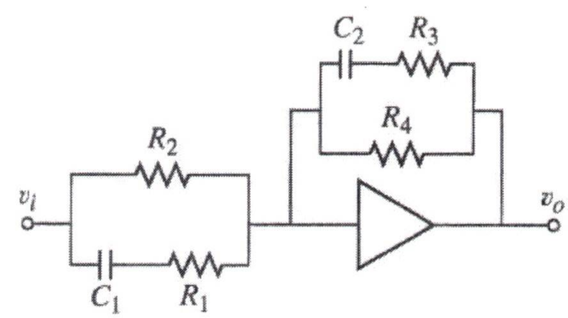


$$Z_f = \frac{1}{Cs}$$

$$Z_i = R$$

$$\frac{V_o(s)}{V_s(s)} = \frac{-Z_f}{Z_i} = -\frac{1}{RCs}$$

b)



$$Z_f = \frac{1}{\frac{1}{R_4} + \frac{1}{R_3 + C_2 s}}$$

$$Z_i = \frac{1}{\frac{1}{R_2} + \frac{1}{R_1 + C_1 s}}$$

$$\frac{V_o(s)}{V_s(s)} = \frac{-Z_f}{Z_i} = -\frac{\frac{1}{R_2} + \frac{1}{R_1 + C_1 s}}{\frac{1}{R_4} + \frac{1}{R_3 + C_2 s}}$$

```
clear all
clc
clf

%% Problem 2

A = 7.5;    R = 8;  L = 5.3e-3;

num = [L,0];
den = [L, R];
T = tf(num,den);
[vo_sim, t1] = step(T);

vo = A * exp(-R/L .* t1);

figure(1)
title('Simulation')
plot(t1,A*vo_sim)
xlabel('Time - [s]')
ylabel('Voltage - [V]')

figure(2)
title('Analytic Solution')
plot(t1,vo)
xlabel('Time - [s]')
ylabel('Voltage -[V]')

figure(3)
title('Comparison')
plot(t1,A*vo_sim,t1,vo)
xlabel('Time - [s]')
ylabel('Voltage -[V]')
legend('Simulation','Analytical')

%% Problem 3
global R;
global L;
global C1;
global C2;
global A;

R = 4; L = 6e-3; C1 = 6.5e-4; C2 = 6.5e-4; A = 3;

[t2,states] = ode45(@deriv,[0,0.1],[0,0]);

wn = sqrt(((1 + C1/C2))/(L*C2));
z = (R/L)/(2*wn);
sz = sqrt(1-z^2);
phi = atan(sz/z);
ex = -z.*wn.*t2;

C1 = A/(1 + C1/C2);

vl = C1.*(1-(exp(ex).*sin(wn.*sz.*t2+phi))/sz) ;

figure(4)
plot(t2,states(:,1))
title('Simulation')
xlabel('Time - [s]')
ylabel('Voltage - [V]')
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```
figure(5)
plot(t2,vl)
title('Analytical')
xlabel('Time - [s]')
ylabel('Voltage - [V]')

figure(6)
plot(t2,states(:,1),t2,vl)
title('Comparison')
xlabel('Time - [s]')
ylabel('Voltage - [V]')
legend('Simulation','Analytical')

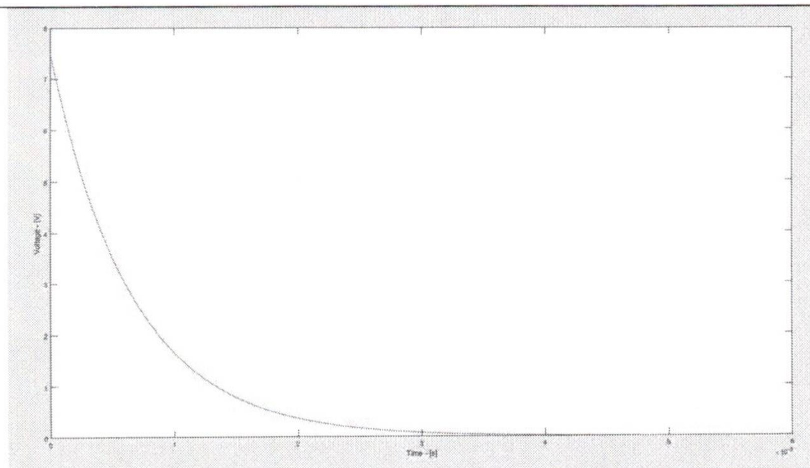
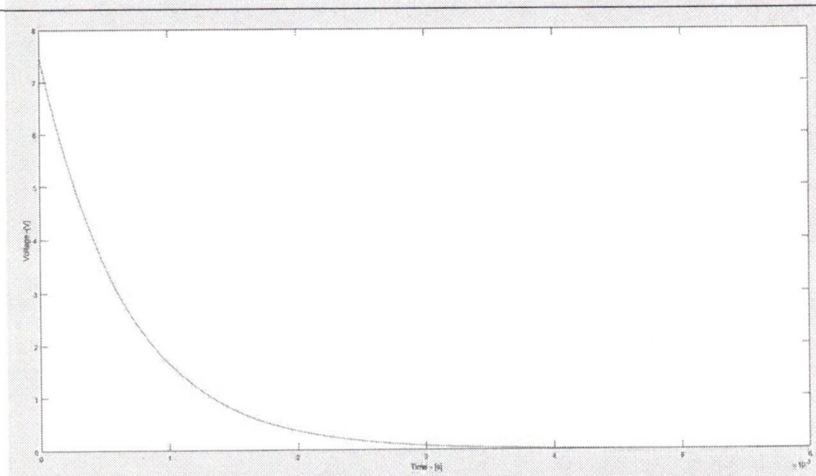
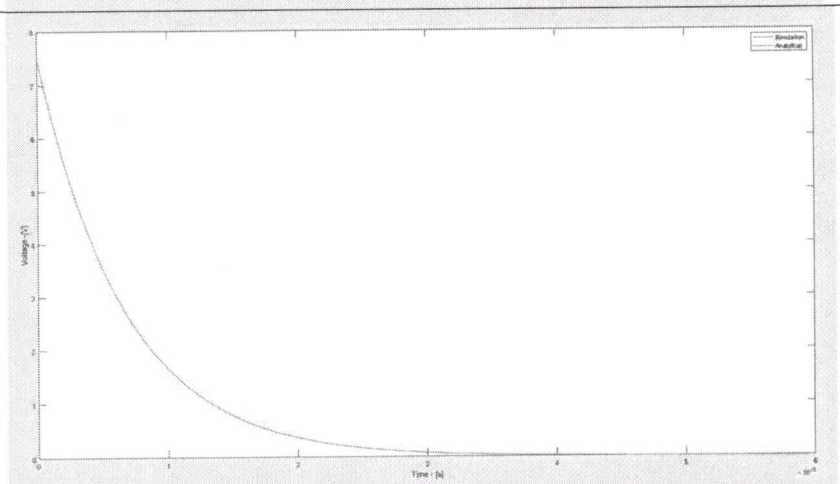
function XDOT = deriv(t,X)
    % System Parameters
    global R;
    global L;
    global C1;
    global C2;
    global A;

    % Rename states
    vo = X(1); vod = X(2);

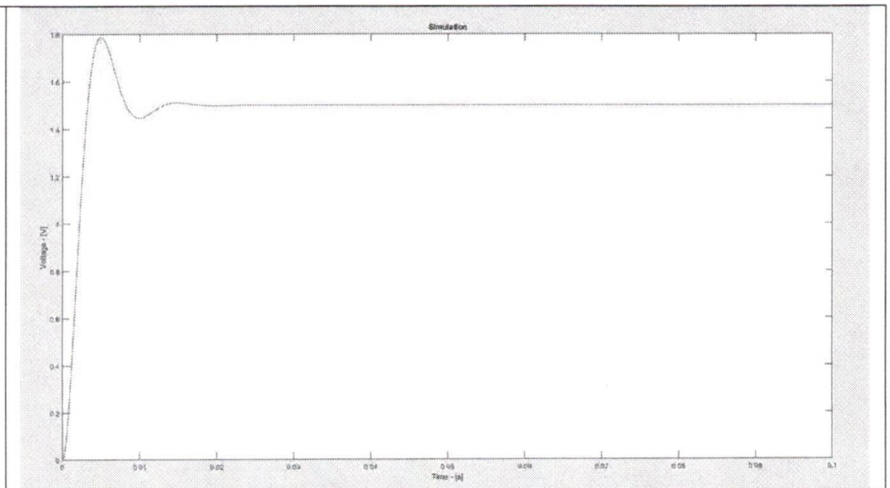
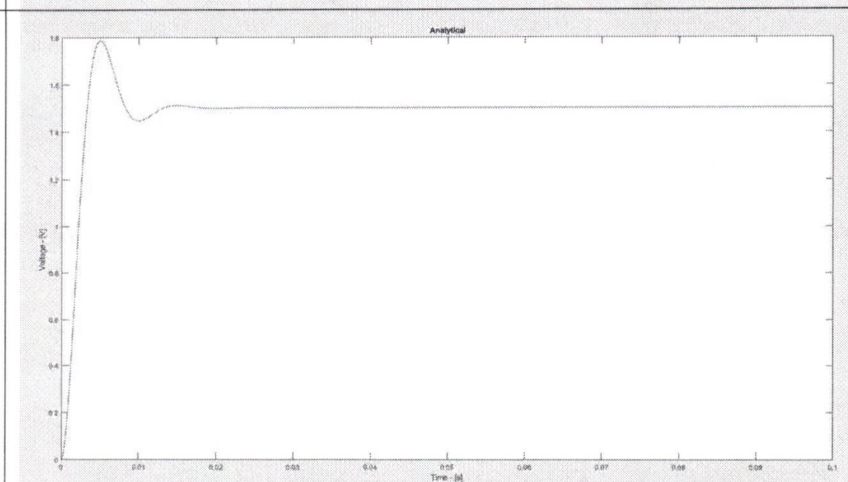
    % Initiate forcing function
    vs = A;
    % write the non-trivial equations using nice names
    vodd = (vs -R*C2*vod - (1 + C1/C2)*vo )/(L*C2);

    XDOT = [ vod; vodd] ;  %return the derivative values
end
```

Problem 2

Simulation	
Analytical	
Comparison	

Problem 3

Simulations	
Analytical	
Comparison