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Name: Last, First

MAE 3724 Systems Analysis
Fall 2019

Pre-Lab for Experiment 4
Electrical Circuits

Complete this Pre-Lab BEFORE you come to the laboratory.
The Lab Instructor will answer questions and provide feedback when you come to the laboratory to do the experiment.

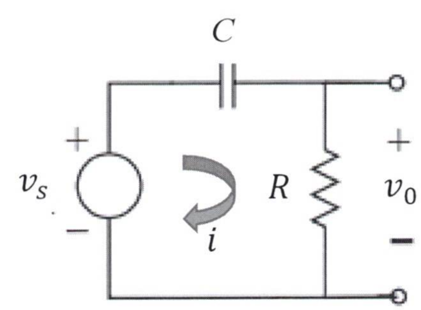
Show all your work and final answers on these pages. Attach the requested graphs.

Background for Laboratory Experiment 4

In Experiment 4 you will study two electrical circuits (1) an RC circuit (first order system) and (2) and RLC circuit (second order system). In the previous experiments you have studied the free responses of systems with initial conditions only. In this experiment, you will study the responses of the two systems to step inputs (forced inputs) where the initial conditions are zero.

Part 1 – RC Circuit

In this part, you will analyze the step response of the following RC circuit.



1. *[1 pt]* Determine the governing differential equation which relates $v_0(t)$ (the dependent variable or output) to $v_s(t)$ (the independent variable or input). *(Hint: refer to sections 6.1.6 for important capacitor properties).*

$V_s - V_c - V_0 = 0$ $\qquad$ $V_0 = iR \rightarrow \frac{V_0}{R} = i$

$V_s - \frac{1}{C}\int i\,dt - V_0 = 0$

$V_s - \frac{1}{C}\int \frac{V_0}{R}\,dt - V_0 = 0$ $\qquad$ $\dot{V}_0 + \frac{1}{CR}V_0 = \dot{V}_s$

$\dot{V}_s - \frac{1}{C}\frac{V_0}{R} - \dot{V}_0 = 0$

$$\frac{dv_s(t)}{dt} = \dot{V}_0 + \frac{1}{RC}V_0$$

2. *[1 pt]* Determine the transfer function for the system. *(Assume zero initial conditions)*

$\dot{V}_o + \frac{1}{RC} V_o = \dot{V}_s$

$V_o \left(s + \frac{1}{RC}\right) = s V_s$

$\frac{V_o}{V_s} = \frac{s}{s + \frac{1}{RC}}$

$$\frac{V_o(s)}{V_s(s)} = \frac{s}{s + \frac{1}{RC}}$$

3. *[1 pt]* Solve the differential equation assuming that $v_s = 5u_s(t)$.

$V_o = \left(\frac{5}{s}\right)\left(\frac{s}{s + \frac{1}{RC}}\right)$

$V_o = \frac{5}{s + \frac{1}{RC}}$

$V_o(t) = 5e^{-\frac{t}{RC}}$

$$v_o(t) = 5e^{-\frac{t}{RC}}$$

4. *[1 pt]* Use Excel or MATLAB to plot $v_0(t)$ as a function of time. Use the following parameters: $R = 470\ \Omega$ and $C = 3300\ \mu\text{F}$.

(Print and attach the graph)

Note that $v_0(t)$ is discontinuous from $t = 0^-$ to $t = 0^+$. Consider the Initial Value Theorem (Sect. 2.2.4) and the Final Value Theorem (Sect. 2.2.5).

5. *[1 pt]* Calculate the time constant for the system.

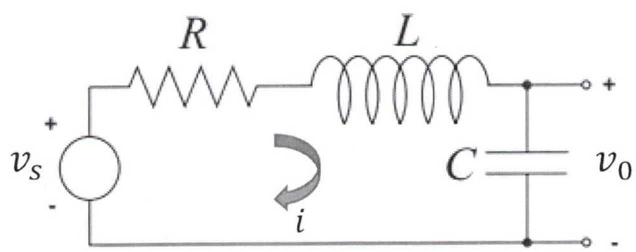
$\dot{V}_o + \frac{1}{RC} V_o = V_s$

$\dot{V}_o + \frac{1}{\tau} V_o = V_s \ ; \ \tau = RC$

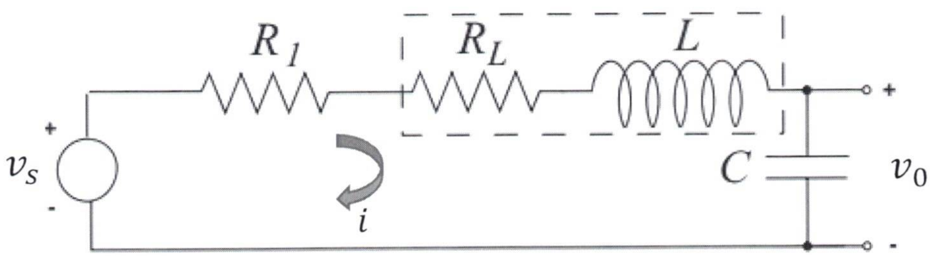
$$\tau = 155.1E-3$$

Part 2 – RLC Circuit

In this part, you will analyze the step response of the following RLC circuit.



Since the inductor actually has an internal resistance R_L of 105 Ω, a better representation of the circuit we will build in the lab is the following:



where $R = R_1 + R_L$. R_L is the built-in inductor resistance and R_1 is the resistor you will pick to solve parts 4a) and 4b) below.

1. *[1 pt]* Derive the governing differential equation relating the output voltage v_0 to the input voltage v_S. *(Hint: refer to sections 6.1.7 for important inductor properties)*

$V_S - V_R - V_L - V_o = 0$ $\quad V_R = iR \quad V_L = L\dot{i} \quad V_o = \frac{1}{C}\int i\,dt$

$V_S - iR - L\dot{i} - V_o = 0$ $\quad \dot{V}_o = \frac{1}{C} i \rightarrow \dot{V}_o C = i$

$V_S - RC\dot{V}_o - LC\ddot{V}_o - V_o = 0$ $\quad \ddot{V}_o = \frac{1}{C}\dot{i} \rightarrow \ddot{V}_o C = \dot{i}$

$\dddot{V}_o =$

$V_S = LC\ddot{V}_o + RC\dot{V}_o + V_o$

$v_S(t) =$ $LC\ddot{V}_o + RC\dot{V}_o + V_o$

2. *[1 pt]* Develop the transfer function for the system. *(Assume zero initial conditions)*

$$\frac{V_o(s)}{V_s(s)} = \frac{1}{LCs^2 + RCs + 1}$$

3. *[1 pt]* Write the characteristic equation for the system.

$$0 = LCs^2 + RCs + 1$$

4. Determine the values of R_1 which are needed to meet the specifications in parts a) and b) below. The known parameters are R_L = 105 Ω, L = 5 H, C = 10 μF.

   a) *[0.5 pts]* The system is underdamped (with ζ = 0.15).

   $LCs^2 + RCs + 1 \rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC}$

   $\omega_n = \sqrt{\frac{1}{LC}}$

   $R_1 = 2\zeta\omega_n L - R_L$

   $2\zeta\omega_n = \frac{R_1 + R_L}{L}$

   $R_1 = 2\zeta L\sqrt{\frac{1}{LC}} - R_L$

   $R_{1a} =$ 107.1 Ω

   b) *[0.5 pts]* The system is critically damped ($\zeta = 1.0$).

   $R_1 = 2\zeta L\sqrt{\frac{1}{LC}} - R_L$

   $R_{1b} =$ 1.309 KΩ

5. *[1 pt]* For each case in part 4), using the MATLAB `step` command, plot the response of v_o as a function of time assuming that v_s is equal to 5 $u_s(t)$.

   (Print and attach the 2 graphs)

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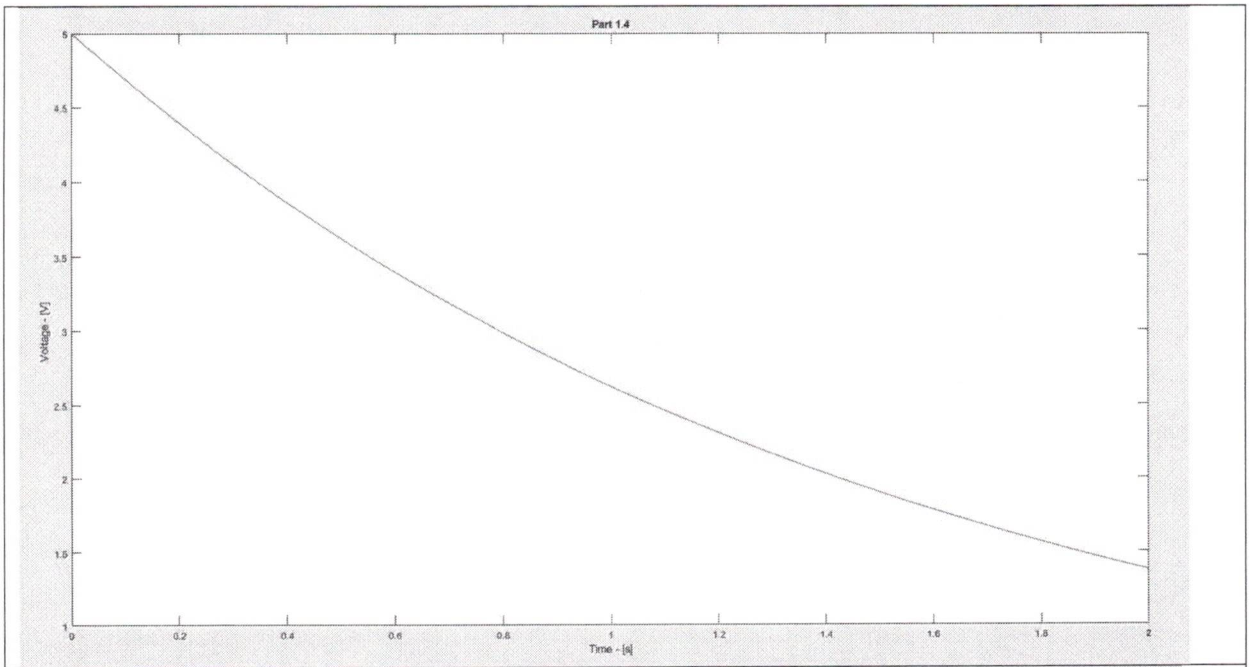
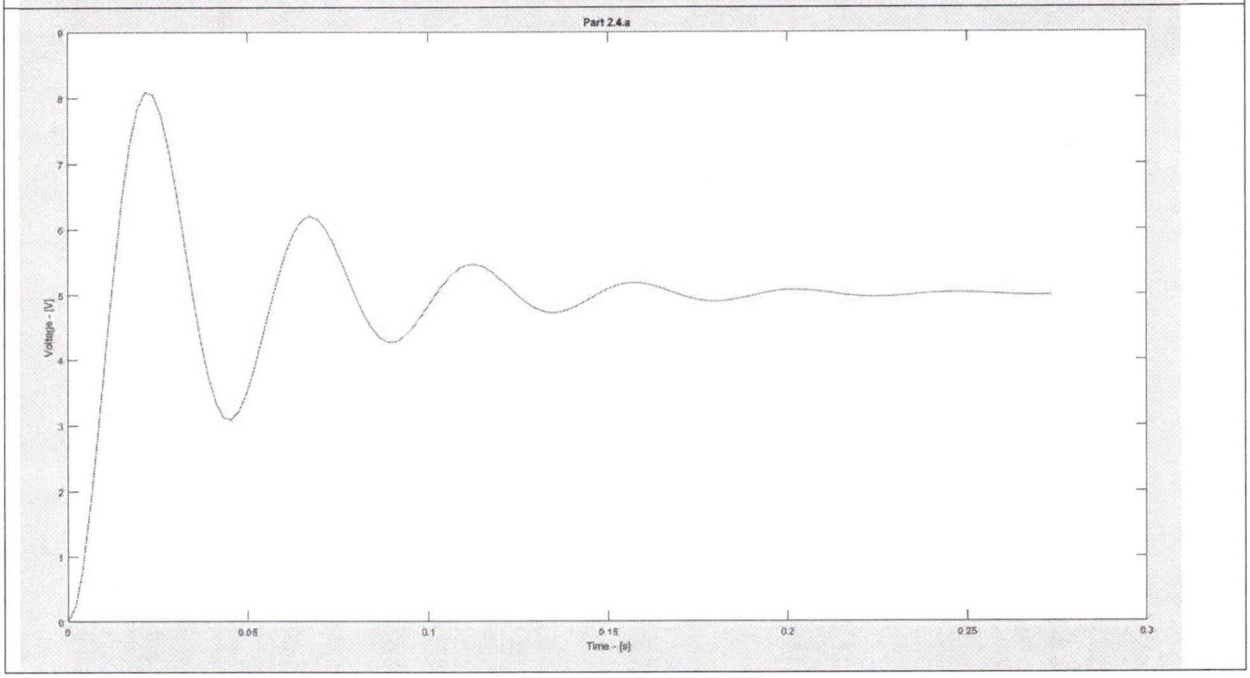
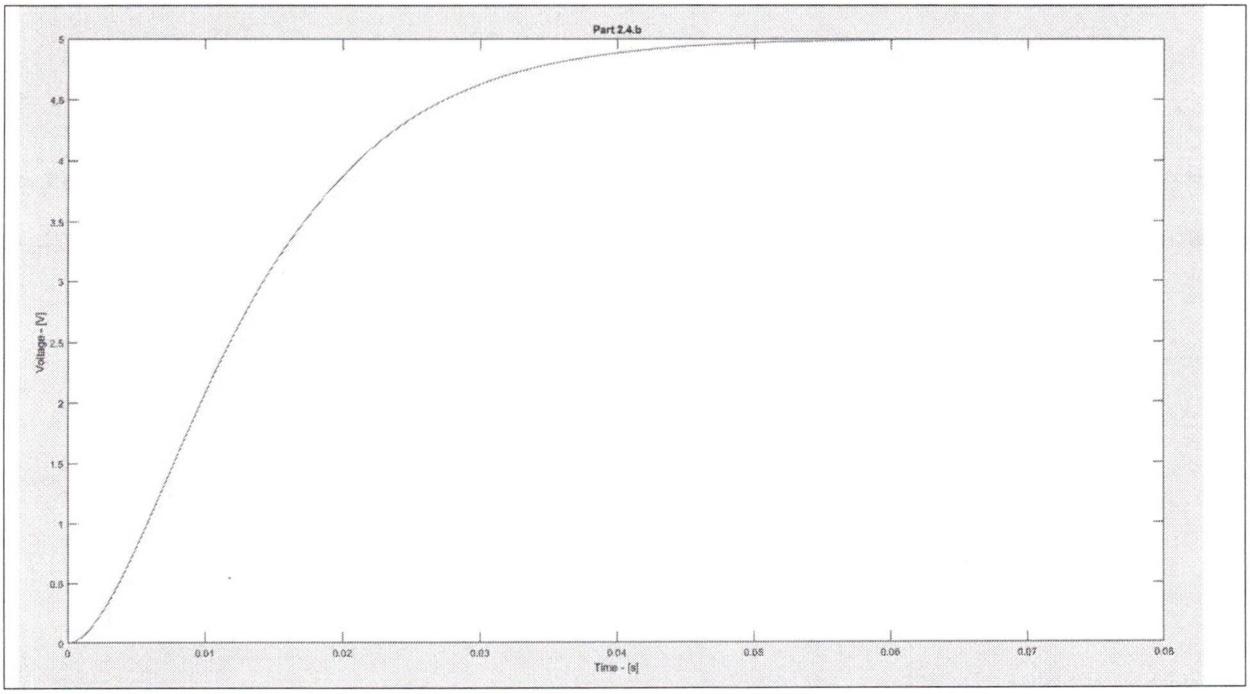


Part 1.4
Voltage - [V]
Time - [s]
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



Part 2.4.a
Voltage - [V]
Time - [s]
9
8
7
6
5
4
3
2
1
0
0.05
0.1
0.15
0.2
0.25
0.3



Part 2.4.b
Voltage - [V]
Time - [s]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08