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Name: Last, First

MAE 3724 Systems Analysis
Fall 2019

Pre-Lab for Experiment 2
Rotational Inertia-Spring-Damper System Experiment

**Complete this Pre-Lab BEFORE you come to the laboratory.
The Lab Instructor will answer questions and provide feedback when you come to the laboratory to do the experiment.**

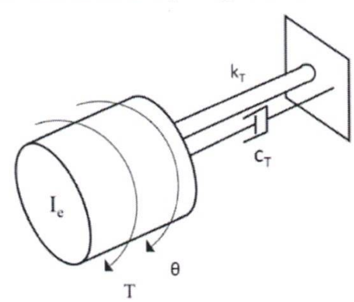
Show all your work and final answers on these pages.

Background for Laboratory Experiment 2

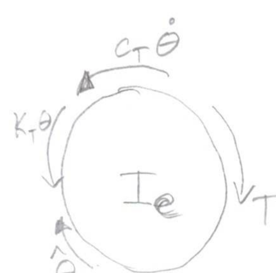
In Experiment 2, you will study a rotational inertia-spring-damper system consisting of two masses attached to a shaft that travels through bearings and connects to a torsional spring (one end fixed). This is the rotary equivalent of Lab 1. A rotary encoder will be used to measure the angular displacement. The displacement is measured from equilibrium. As in Lab 1, you will study the free response of the system, i.e., the response due to initial conditions only.

Pre-Lab for Experiment 2

A simplified physical model of the rotational system is shown in the figure below. The angular displacement θ is measured from an equilibrium position (when there is no torque in the torsional spring). Damping occurs during the rotational movement of the shaft in the bearings. The damping coefficient is c_T. The torsional spring is a tube made of rubber. It is assumed that the torsional spring constant is k_T.



1) *[2 pts]* Draw a free body diagram for the system. Clearly list all assumptions below.



2) *[2 pts]* Develop an equation of motion for the system.

$$T - K_T\theta - C_T\dot{\theta} = I_e\ddot{\theta}$$

3) *[2 pts]* Assume that the external torque is $T = 0$ and that the initial conditions are: $\theta(0) = \pi/2 \ rad$ and $\dot{\theta}(0) = 0 \, rad/s$. Using the Laplace Transform method develop an expression for $\theta(s)$, the Laplace Transform of $\theta(t)$. Arrange your answer into a form such that the highest order term in the denominator has a coefficient of 1 (i.e., divide all terms by the rotary inertia).

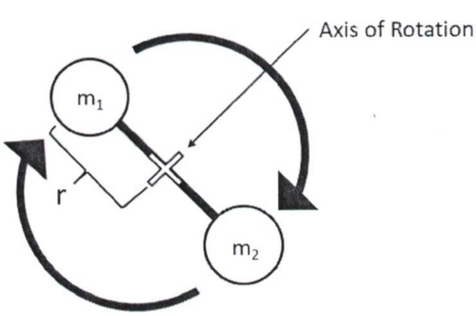
$$I_e\ddot{\theta} + C_T\dot{\theta} + K_T\theta = 0$$

$$I_e\left(s^2\Theta - s\theta(0) - \dot{\theta}(0)\right) + C_T\left(s\Theta - \theta(0)\right) + K_T\Theta = 0$$

$$\left(I_e s^2 + C_T s + K_T\right)\Theta - sI_e\theta(0) - C_T\theta(0) = 0$$

$$\Theta(s) = \frac{\theta(0)\left(sI_e + C_T\right)}{s^2 + \frac{C_T}{I_e}s + \frac{K_T}{I_e}}$$

4) *[2 pts]* Calculate the equivalent rotational moment of inertia (I_e) for the actual body shown schematically below. Assume that the masses can be modeled as point masses equidistant from the axis of rotation on a massless rigid rod.



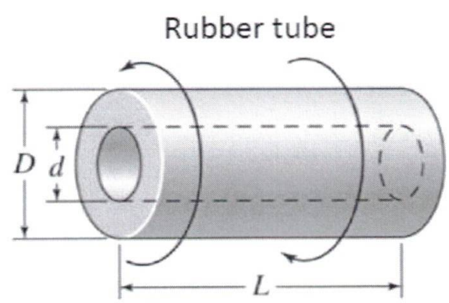
$$I_i = m_i r_i^2$$

$$I = \sum_i m_i r_i^2$$

$$I = 3.838E\text{-}3 \ kg\,m^2$$

Let $m_1 = m_2 = 123 \, g$
and $r = 0.0312 \, m$.

5) *[2 pts]* Using the equation for a "Hollow shaft" in Table 4.1.2 of the Text, compute the equivalent torsional spring constant (k_T) of the rubber tube. The dimensions of the tube are D = 0.0188 m, d = 0.0041 m, and L = 0.1397 m. You will need the modulus of rigidity for your calculation, that is G for the rubber (*Hint: use www.engineeringtoolbox.com to find Modulus of Rigidity of common materials*).



$K_T = \dfrac{\pi G (D^4 - d^4)}{32L}$

$G = 0.0003E9\,Pa$
$D = 0.0188\,m$
$d = 0.0041\,m$
$L = 0.1397\,m$

$\dfrac{\frac{N}{m}(m^4)}{m} = Nm^2$

$K_T = 26.28E{-3}\ Nm^2$