Colón, Diego

Name: Last, First

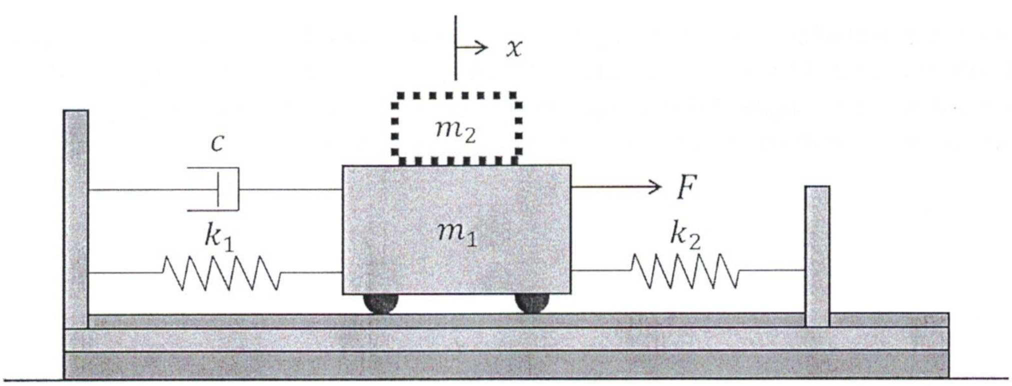
Pre-Lab for Experiment 1
Mass-Spring-Damper System Experiment

Complete this Pre-Lab BEFORE you come to the laboratory.
The Lab Instructor will answer questions and provide feedback when you come to the laboratory to do the experiment.

Show all your work and final answers on these pages. Attach the requested graphs.

Background for Laboratory Experiment 1

In Experiment 1, you will study a mechanical mass-spring-damper system, consisting of a low friction cart rolling on a track. A physical model of the system is shown in the figure below. An optical encoder (digital) is used to measure the position x of the cart on the track. The displacement x is measured from equilibrium. Force sensors (analog) are used to measure the forces in the springs. Although the cart is termed "low friction" by the manufacturer, damping still occurs during the movement of the cart on the track. You will study the "free response" of the system, i.e., the response of the system to initial conditions only.



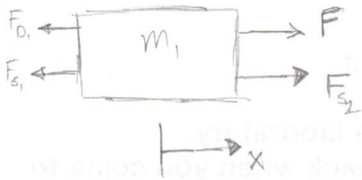
Pre-Lab for Experiment 1

1) *[2 points]* Derive an ordinary differential equation (equation of motion) for the simplified mass-spring-damper system shown in the figure above. Assume the body is rigid and that the springs are massless and linearly elastic. Use the following parameters for the masses and springs:

Spring 1, k_1: 70 N/m	Mass of the cart, m_1: 0.61 Kg
Spring 2, k_2: 80 N/m	Additional mass, m_2: 0.52 kg

Assume that the damping can be modeled as an ideal damper where the damping force $f_d = c\dot{x}$. For this Pre-Lab, assume that the damping coefficient is $c = 0.20\ N \cdot s/m$. You will evaluate this assumption when you do the experiment.

Part 1) continued. Show your work below (including the Free Body Diagram and assumptions).



$$\sum F_x = m\ddot{x}$$

$$-F_{D_1} + F_{S_1} + F_{S_2} + F = m\ddot{x}$$

$$m\ddot{x} + F_{D_1} - F_{S_1} - F_{S_2} = F \; ; \; F_{D_1} = c\dot{x} \; , \; F_{S_i} = -K_i x$$

$$m\ddot{x} - c\dot{x} + K_1 x + K_2 x = F$$

$$m\ddot{x} - c\dot{x} + (K_1 + K_2)x = F$$

$m = 0.61$ kg
$c = 0.2$ Ns/m
$K_1 = 70$ N/m
$K_2 = 80$ N/m

Ordinary Differential Equation $(0.61 k_g)\ddot{x} + (0.2 \frac{Ns}{m})\dot{x} + (150 \text{ N/m})x = F$

2) *[2 points]* Develop the transformed equation of motion (i.e., take the Laplace Transform of the equation of motion) for the case when the mass is only the mass of the cart m_1 and no input force is applied (free response). Use the initial conditions $x_0 = 1$, $\dot{x}_0 = 0$. Develop an equation for $X(s)$. Show your work below.

$$m\ddot{x} + c\dot{x} + (K_1 + K_2)x = F$$

$$m(s^2 X - s\,x(0) - \dot{x}(0)) + c(sX - x(0)) + (K_1 + K_2)X = F$$

$$ms^2 X + csX + (K_1 + K_2)X = F$$

$$X = \frac{F}{ms^2 + cs + (K_1 + K_2)}$$

$$X(s) = \frac{F}{0.61s^2 + 0.2s + 150}$$

3) *[0 points]* Using the following form:

$$X(s) = \frac{c_1 s + c_2}{c_3 s^2 + c_4 s + c_5}$$

You may enter the following code into MATLAB to obtain the output x as a function of time. You will cover how to generate model data in later MATLAB exercises.

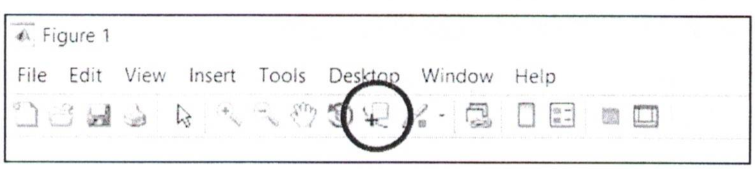
(Note: You need to input the coefficients of the numerator (c_1, c_2) in the first vector brackets, and the coefficients of the denominator (c_3, c_4, c_5) in the second vector brackets. Entries must start with the coefficients of the highest order, and must be separated by a comma or space.)

```
sys=tf([c1,c2],[c3,c4,c5])
sys1=ss(sys);
x0=[1,0];
t=0:0.02:20;
F=0*t;
lsim(sys1,F,t,x0)
x=lsim(sys1,F,t,x0);
```

4) *[1 point]* A graph of the simulated response should have popped-up. Make the plot full screen (and adjust the window range if necessary) so that the peaks are more clearly seen.

From the graph, determine the number of peaks and the height (amplitude) of both the first and last peak. **(You do not need to use the entire data set, but you must use a reasonable amount. You can pick 10-15 peaks and find the initial and final height of that set).**

Note: To select a point on the graph and read its coordinates, click on the 'Data Cursor' icon in the tool bar (see image below). Then select the peak (or any other data point) for which you want the coordinates. You can click on as many data points as you like or you can toggle through all of the data points with the left and right arrow keys.



Part 4) continued

After obtaining the first peak, the last peak and the number of peaks, use the Logarithmic Decrement method to find the damping ratio (ζ). The Logarithmic Decrement may be calculated using Equations (8.4.14) and (8.4.13) in the text. Show your work below. **Attach a copy of the graph you used.**

$x(0.1) = 0.411$

$t(15.7) = 0.0316$

$n = 40$

$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$, $\delta = \frac{1}{n} \ln\left(\frac{B_1}{B_{n+1}}\right)$

$\zeta = \frac{0.0641}{\sqrt{4\pi^2 + (0.0641)^2}}$ $\delta = \frac{1}{40} \ln\left(\frac{0.411}{0.0316}\right)$

$\delta = 0.0641$

$\zeta = 0.010207$

$\delta =$ 0.064

$\zeta =$ 0.01

5) *[1 point]* Using Equation 2.5.15, determine the value of the damping coefficient (c) from the results in Part 4). Show your work below.

$\zeta = \frac{c}{2\sqrt{mk}} \rightarrow c = 2\zeta\sqrt{mk} = 2(0.10)\sqrt{(0.61)(150)} = 0.191$

$c =$ 0.19

6) *[0.5 points]* How does this value compare with your assumed value ($c = 0.20\ N \cdot s/m$)? If different, why is it different? Explain below.

$\%\,diff = \frac{abs(0.2 - 0.19)}{\frac{0.2+0.19}{2}} \cdot 100 = 5.12\%$ difference

The difference in the value could be different due to the discrete nature of computer systems.

7) *[1 point]* Develop the transformed equation of motion for the case when the total mass is the mass of the cart plus the additional mass (same initial conditions). Develop an equation for $X(s)$. *(Hint: Only one parameter was changed…)*

$$X(s) = \frac{F}{1.13s^2 + 0.2s + 150}$$

8) [2 points] Repeat procedures in Parts 3), 4), and 5) for the transformed model in Part 7). Show your work below. **Attach a copy of your graph.**

$x(0.14) = 0.303$
$x(17.6) = 0.0646$
$n = 33$

$\delta = \frac{1}{n} \ln\left(\frac{B_1}{B_{n+1}}\right)$

$\delta = \frac{1}{33} \ln\left(\frac{0.303}{0.0646}\right)$

$\delta = 0.04683$

$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

$\zeta = \frac{0.04683}{\sqrt{4\pi^2 + (0.04683)^2}}$

$\zeta = 0.007453$

$c = 2\zeta\sqrt{mk}$

$c = 2(0.007453)\sqrt{(1.13)(150)}$

$c = 0.1940$

$\delta =$ 0.047

$\zeta =$ 0.0075

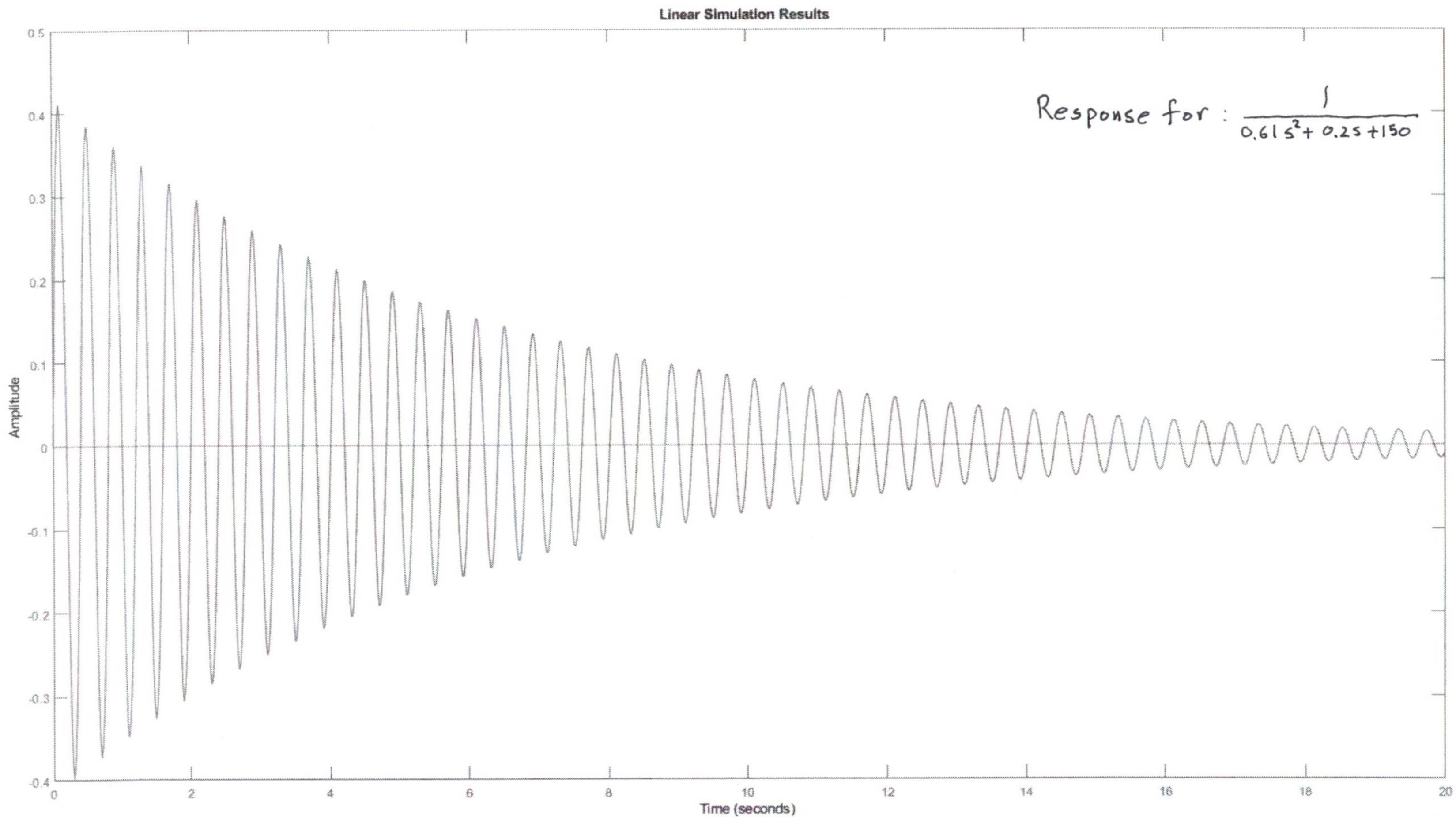
$c =$ 0.19

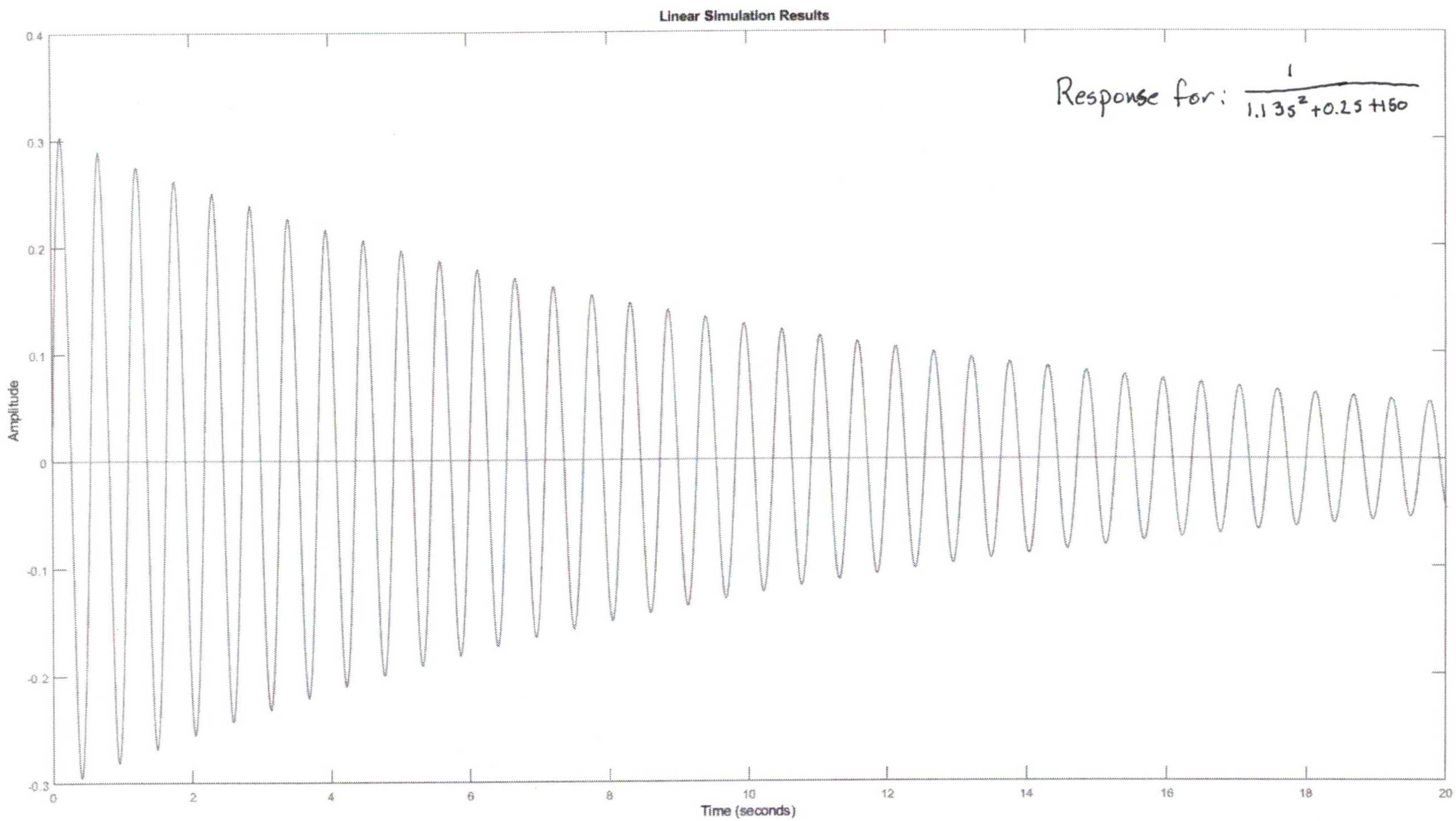
9) [0.5 points] Is there a difference between this value of c and the one you obtained in Part 5)? Explain below.

There is no difference between this value and the one obtained in Part 5 out to two decimal places.

10) [0.5 points extra credit] One of the objectives we have for the laboratory experiments is for you to learn about different measurement systems (sensors and data acquisition). A linear encoder will be used as one sensor in Lab 1. What is an "encoder" and how does it work? Write your response below.

An encoder is a data acquisition device that rotates along with a shaft in the system and sends two electrical signals, A & B. Based on which signal comes first and the number of pulses produced, one can measure direction and displacement/position.


Linear Simulation Results
Response for : $\frac{1}{0.61s^2 + 0.2s + 150}$
Amplitude
Time (seconds)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14
16
18
20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-0.3
0
2
4
6
8
10
12
14
16
18
20

Response for