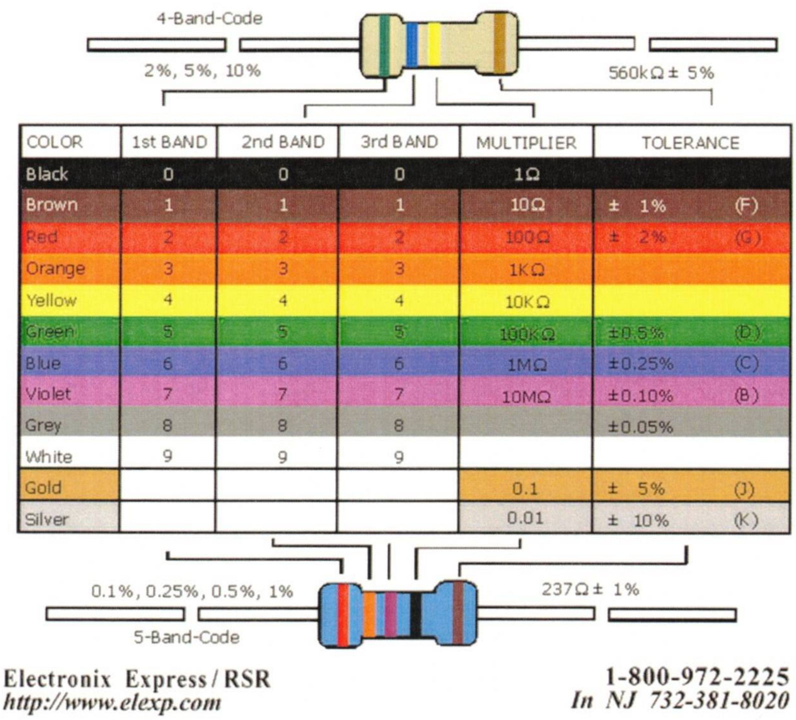
MAE 3724 Systems Analysis
Fall 2019

Laboratory Experiment 4
Step Response of First and Second Order Electrical Systems

Background

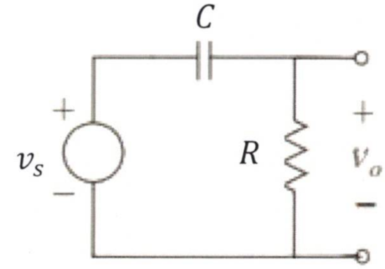
There can be a difference in the performance of electronic passive elements (resistors, capacitors and inductors) in the real case compared to ideal case because of the accuracy of the parameter values on commercial passive elements. The figure shows the color code used for commercial resistors.



COLOR	1st BAND	2nd BAND	3rd BAND	MULTIPLIER	TOLERANCE	
Black	0	0	0	1Ω		
Brown	1	1	1	10Ω	± 1%	(F)
Red	2	2	2	100Ω	± 2%	(G)
Orange	3	3	3	1KΩ		
Yellow	4	4	4	10KΩ		
Green	5	5	5	100KΩ	±0.5%	(D)
Blue	6	6	6	1MΩ	±0.25%	(C)
Violet	7	7	7	10MΩ	±0.10%	(B)
Grey	8	8	8		±0.05%	
White	9	9	9			
Gold				0.1	± 5%	(J)
Silver				0.01	± 10%	(K)

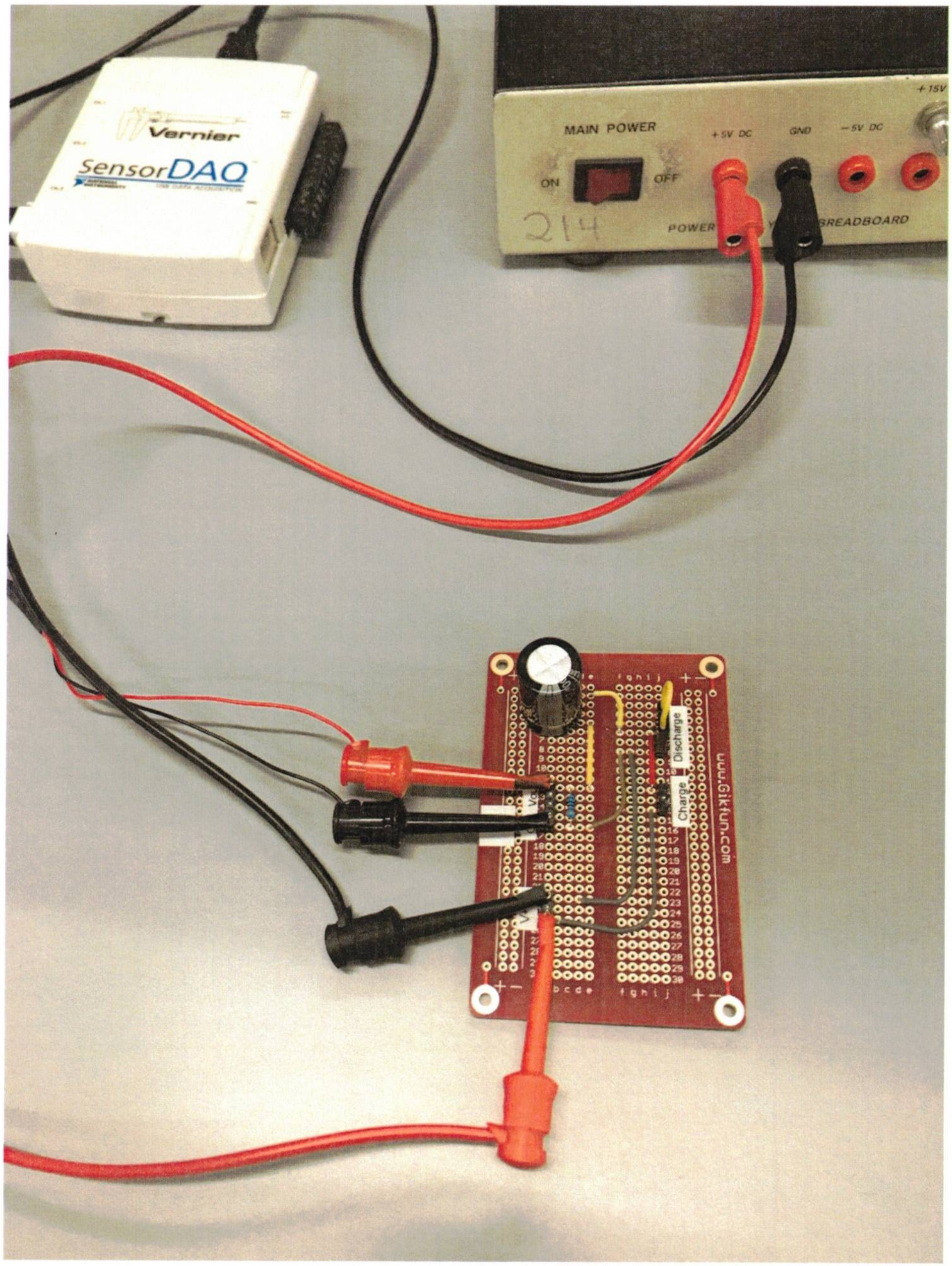
Part 1: RC Circuit

In this part you will do a step response experiment with the following RC circuit. You may use the model you derived in the Pre-Lab.



Setup

The circuit above is mounted on a breadboard. Nodes on the power supply (+5VDC) and (GND) should be clipped to the appropriate wires on the breadboard. This connection will provide a positive input voltage v_s. There is one 470 Ω (nominal) resistor for *R* and one 3300 μF (nominal) capacitor for C. The clips from SensorDAQ should be attached to both sides of the ***resistor***, and SensorDAQ should be plugged into the computer.



Procedure

Since the resistance properties of electronics components vary somewhat from their designated values, you will need to measure them directly. Use a multimeter to **measure the resistance R.** = 466

Open Logger Pro. Make sure the SensorDAQ is connected by selecting "Experiment" and looking at "Setup Sensors". Click "Data Collection" and make sure it is set to collect about 45 seconds of data at a rate of 100 Hz.

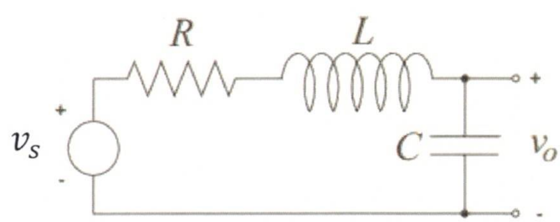
Set $v_s = 0\ V$ (see photo at the bottom of the previous page). Click "Collect Data" in Logger Pro. You are now ready to charge the capacitor. Set $v_s = 5\ V$.
Wait for v_0 to reach equilibrium ($v_o = 0\ V$). You are now ready to discharge the capacitor. Set $v_s = 0\ V$. Wait for v_0 to reach equilibrium ($v_o = 0\ V$).

Transfer the data from the Logger Pro window to an Excel spreadsheet. In your report, you will use these data to **find the experimental time constant τ** from the measured response. You should do this by determining the time it takes for the response during charging of the capacitor to be e^{-1} = 36.7% of its initial value of $v_0 = 5\ V$.

In your report for Part 1, use the measured values for the time constant and R to **compute C and compare it to the nominal value**. Use this computed value when you calculate the response using the mathematical model.

Part 2: RLC Circuit

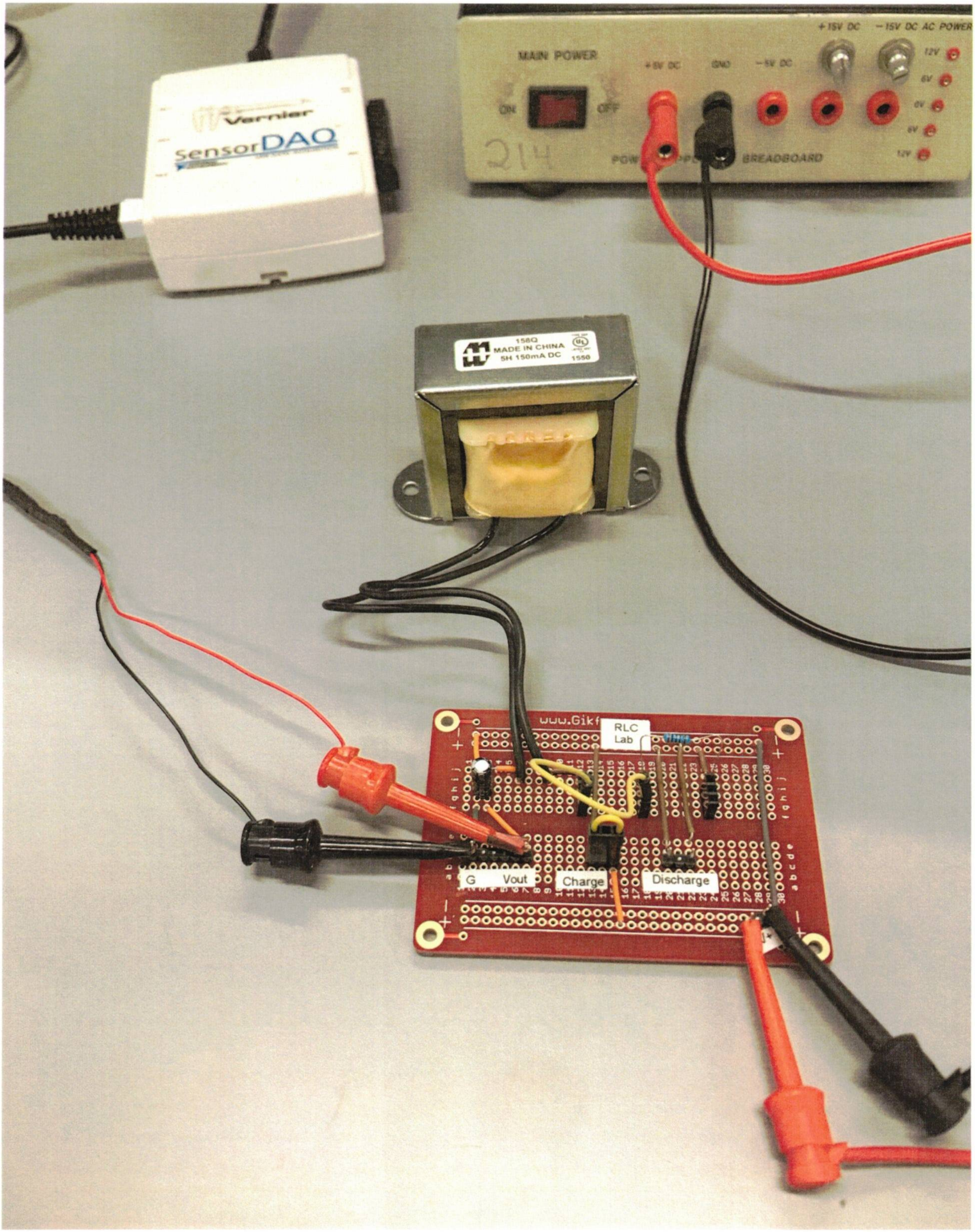
In this part, you will do a step response experiment with the following RLC circuit. There are two energy storage elements, so the system should be second order.

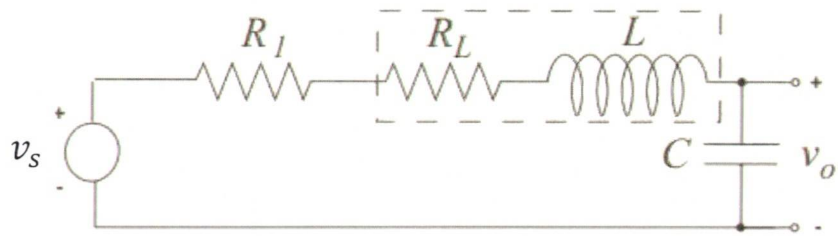


Setup

This circuit is assembled on a breadboard as shown below. Since the inductor (a long wire) actually has an internal resistance R_L of approximately 105 to 110 Ω, a better representation of what you will use is the circuit below showing an R_1 and an R_L. R_L = 109.75
Measure the value of R_L with a multimeter and record this for your final report. Assume L = 5 H, and C = 10 μF. **Determine R_1 such that the damping ratio ζ is approximately 0.15** (you calculated the best value for R_1 in the Pre-lab). Now **build an equivalent circuit for R_1** by using any combination of the available resistors, and **record the value of R_1**. Note that the resistance values given by the color code in the table on page 1 are only nominal.

Measure the actual resistances of the resistors in your circuit with the multimeter. Include a sketch of your circuit and these measured values in your report.





Procedure

Implement, measure and record the actual resistances of the resistors you used to make the desired R_1. Measure and record the inductor's actual resistance (R_L)

Open Logger Pro. Make sure the SensorDAQ is connected by selecting "Experiment", select "Set Up Sensors" and clicking on SensorDAQ. Ensure that the voltage drop across the capacitor is approximately zero (when the circuit is deenergized). Click "Data Collection" and set it to collect about 2 seconds of data at a rate of 500 Hz.

Set the Power Supply to OFF (this should correspond to $v_s = 0\ V$). Click "Collect Data" in Logger Pro. Set the Power Supply to ON (this should correspond to $v_s = 5\ V$). Wait for equilibrium. **Save the measured data on an Excel spreadsheet.**

$$R_1 = \left(\frac{1}{\frac{1}{220} + \frac{1}{220}}\right)_{ideal} \Omega \qquad R_1 = \left(\frac{1}{\frac{1}{219.2} + \frac{1}{220.8}}\right)_{act} \Omega$$

$$R_1 = 110\Omega \qquad R_1 = 104.998\ \Omega$$

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FINAL REPORT: (Each individual needs to submit a Final Report)

SHOW ALL YOUR WORK ON THIS AND THE FOLLOWING PAGES. Scan these pages and all requested plots and upload them into the Brightspace Dropbox for Lab 4

Final Report for Part 1

Your objective is to compare the measured response (from the lab) with your predicted response based on the mathematical model (from the PreLab). Include the following in your final report:

1) *[2 pts]* What is the time constant based on the measured data?

$V_1(6.57s) = 5.0767\ V \qquad 0.37V_1 = 1.878379 \qquad V_2(8.07s) = 1.8712\ V$

$\tau = t_2 - t_1 = 1.5s$

2) *[2 pts]* Calculate C from your experimental data, using τ (from Part 1) and the actual R you measured during the lab.

$\tau = \frac{m}{c}$ $\qquad \frac{V_o}{V_s} = \frac{RCs}{RCs+1} = \frac{m}{ms+c} \rightarrow \boxed{c = 1}$

$\frac{m}{ms+c}$

3) *[2 pts]* In the PreLab, the time-response for the output voltage was calculated to be:

$$v_o(t) = 5e^{-\frac{t}{RC}}$$

Plot this model response on top of the experimental data. (CHARGING ONLY). Clearly label the graph and distinguish between the measured response and the predicted response. Attach your graph to this report.

4) *[2 pts]* Discuss how the predictions compare with the actual measurements.

The prediction was very accurate with an Error squared of less than 1 for all points

5) *[1 pt]* Calculate the mean-squared error between the data you collected and your model. (CHARGING ONLY) *Show your work by providing the code you used or the formula you used.*

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - Y_{act})^2 = 0.0002$$

6) *[1 pt]* Is this a good way to compare measurements and predictions?

Yes

Final Report for Part 2

Your objective is to compare the measured response (from the Lab) with your predicted response based on the mathematical model (from the PreLab). Include the following in your final report:

1) *[2 pts]* Redraw the sketch of the circuit. Make sure to clearly show how you achieved a combination of resistors (and their values) to get the necessary R_1 to produce a damping ratio of approximately $\zeta = 0.15$.

$R = R_1 + R_L$; $s^2 + \frac{R}{L}s + \frac{1}{LC}$ $\quad \omega_n = \sqrt{\frac{1}{LC}}$

$2\zeta\omega_n = \frac{R_1 + R_L}{L}$

$R_1 = 2\zeta L\sqrt{\frac{1}{LC}} - R_L$ $\quad L = 5\,H$, $C = 10E\text{-}6\,F$, $\zeta = 0.15$, $R_L = 105\,\Omega$

$R_1 \approx 107.1\,\Omega$

2) *[2 pts]* What is the updated transfer function for the system? (using the true values you found for R_1 and R_L, not the ones given in the PreLab).

$$\frac{V_o(s)}{V_s(s)} = \frac{1}{(5E\text{-}6)s^2 + (219.75E\text{-}6)s + 1}$$

3) *[2 pts]* Using MATLAB's `step()` command or Excel (after hand-deriving the inverse-laplace), plot the updated model response on top of the experimental data, so they can be compared easily. Clearly label the graph and distinguish between the measured response and the predicted response.

Note: If you want overlay both plots in MATLAB, you will need to copy and paste the experimental data from Excel into the MATLAB workspace and make sure the vectors are the same length before plotting. An alternative is to simulate the model response in MATLAB and then copy the model data and paste it into EXCEL.

Attach your plot to this report.

4) *[2 pts]* Discuss how the predictions compare with the actual measurements.

The simulation follows the actual system closely

5) *1 pt]* Calculate the mean-squared error between the data you collected and your model. *Show your work by providing the code you used or the formula you used.*

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(V_{act,i} - V_{sim,i}\right)^2 = 0.0012$$

6) *[1 pt]* Is this a good way to compare measurements and predictions?

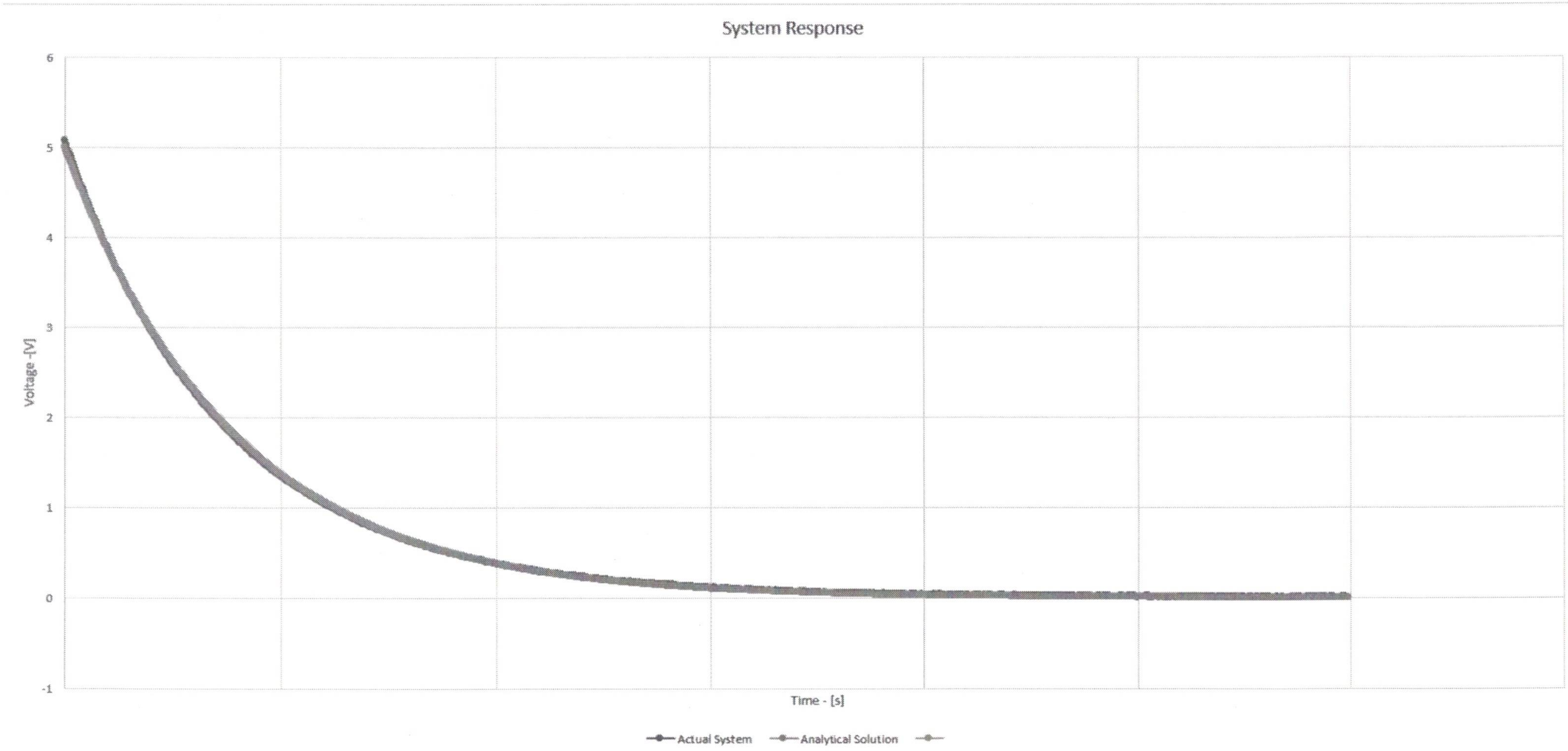
Yes.

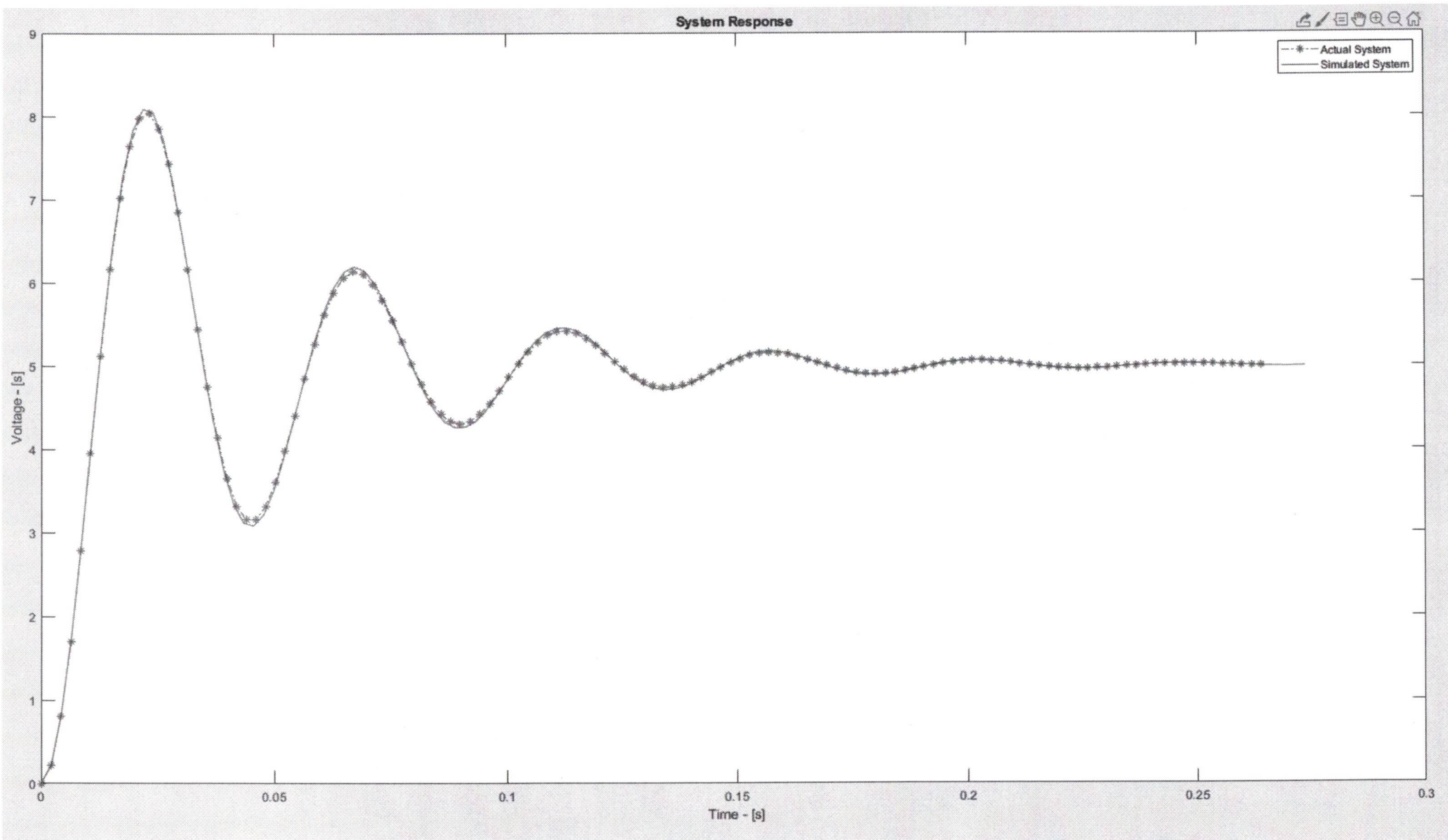
Final Evaluation **(for feedback purposes only, will not affect grade)**

a) What did you learn from these experiments?

b) What could we do to make the experience more beneficial?

c) Which parts of the instructions were confusing, if any? If so, why?


System Response
Voltage -[V]
6
5
4
3
2
1
0
-1
Time - [s]
Actual System
Analytical Solution


System Response
Actual System
Simulated System
Voltage - [s]
Time - [s]
0
1
2
3
4
5
6
7
8
9
0.05
0.1
0.15
0.2
0.25
0.3