HW 4 — Colón, Diego

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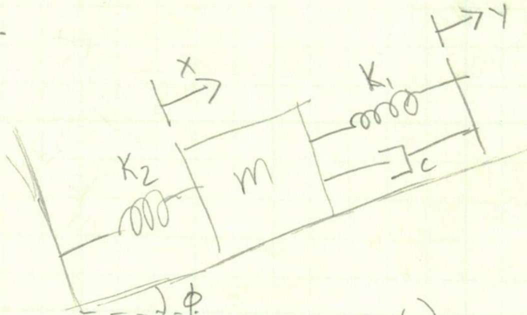

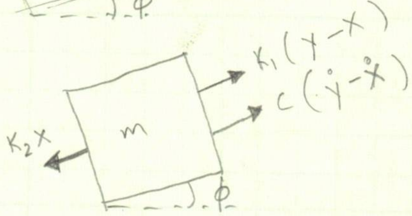

Assumptions:
- → ideal spring
- → ideal dampener
- → no friction
- → displacements measured from equilibrium.
- → $x(0) = 0$, $\dot{x}(0) = 0$

Pressumption
- → $x > 0$
- → $y > x$
- → $\dot{y} > \dot{x}$

$$\Sigma F = -K_2 x + K_1(y-x) + c(\dot{y}-\dot{x}) = m\ddot{x}$$

$$-K_2 x + K_1 y - K_1 x + c\dot{y} - c\dot{x} = m\ddot{x}$$

$$-(K_1+K_2)x + K_1 y + c\dot{y} - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + (K_1+K_2)x = K_1 y + c\dot{y}$$

$$\mathcal{L}\{m\ddot{x} + c\dot{x} + (K_1+K_2)x\} = \mathcal{L}\{K_1 y + c\dot{y}\}$$

$$(ms^2 + cs + (K_1+K_2))X(s) = (cs + K_1)Y(s)$$

$$\frac{X(s)}{Y(s)} = \frac{cs + K_1}{ms^2 + cs + (K_1+K_2)}$$

$$\frac{X(s)}{Y(s)} = \frac{cs + K_1}{ms^2 + cs + (K_1+K_2)} \quad ; \quad Y(s) = \frac{3}{s}$$

$$X(s) = \frac{3}{s}\left(\frac{cs + K_1}{ms^2 + cs + (K_1+K_2)}\right)$$

$$= 3\,\frac{c}{ms^2 + cs + (K_1+K_2)} + \frac{3}{s}\,\frac{K_1}{ms^2 + cs + (K_1+K_2)}$$

$$= 3\left(\frac{c/m}{s^2 + \frac{c}{m}s + (K_1+K_2)/m}\right) + 3K_1\left(\frac{1/m}{s(s^2 + c/m + (K_1+K_2)/m}\right)$$

$$= 3\frac{c}{K_1+K_2}\left(\frac{(K_1+K_2)/m}{s^2 + \frac{c}{m}s + (K_1+K_2)/m}\right) + 3\frac{K_1}{K_1+K_2}\left(\frac{(K_1+K_2)/m}{s(s^2 + c/m + (K_1+K_2)/m}\right)$$

$$= 3\frac{c}{K_1+K_2}\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) + 3\frac{K_1}{K_1+K_2}\left(\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2}\right)$$

$$x(t) = \frac{3c}{K_1+K_2}\left(\frac{\omega_n}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t)\right) + \frac{3K_1}{K_1+K_2}\left(1 - \frac{1}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \phi\right)\right)$$

$$\zeta = c/m \; ; \; \omega_n^2 = \frac{K_1+K_2}{m} \; ; \; \phi = \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

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clear all
clc
clf

% Analytical Solution

t = 0:0.01:10;

m = 2; c = 4; k1 =0; k2 = 50; A = 3;

ke = k1+k2;
z = c/(2*sqrt(ke*m))
wn = sqrt(ke/m)
sz = sqrt(1-z^2)
phi = atan(sz/z^2)

ex = exp(-z.*wn.*t);
s1 = sin(wn.*sz.*t);
s2 = sin(wn.*sz.*t + phi);

C1 = A*c/(k1+k2);
C2 = A*k1/(k1+k2);

x = C1.*((wn.*ex.*s1)/(sz)) + C2.*(1-(ex.*s2)/sz);

plot(t,x,'b')
xlabel('Time - [s]')
ylabel('Displacement - [s]')
title('Analytical Solution')
pause;

% Transfer Function Solution

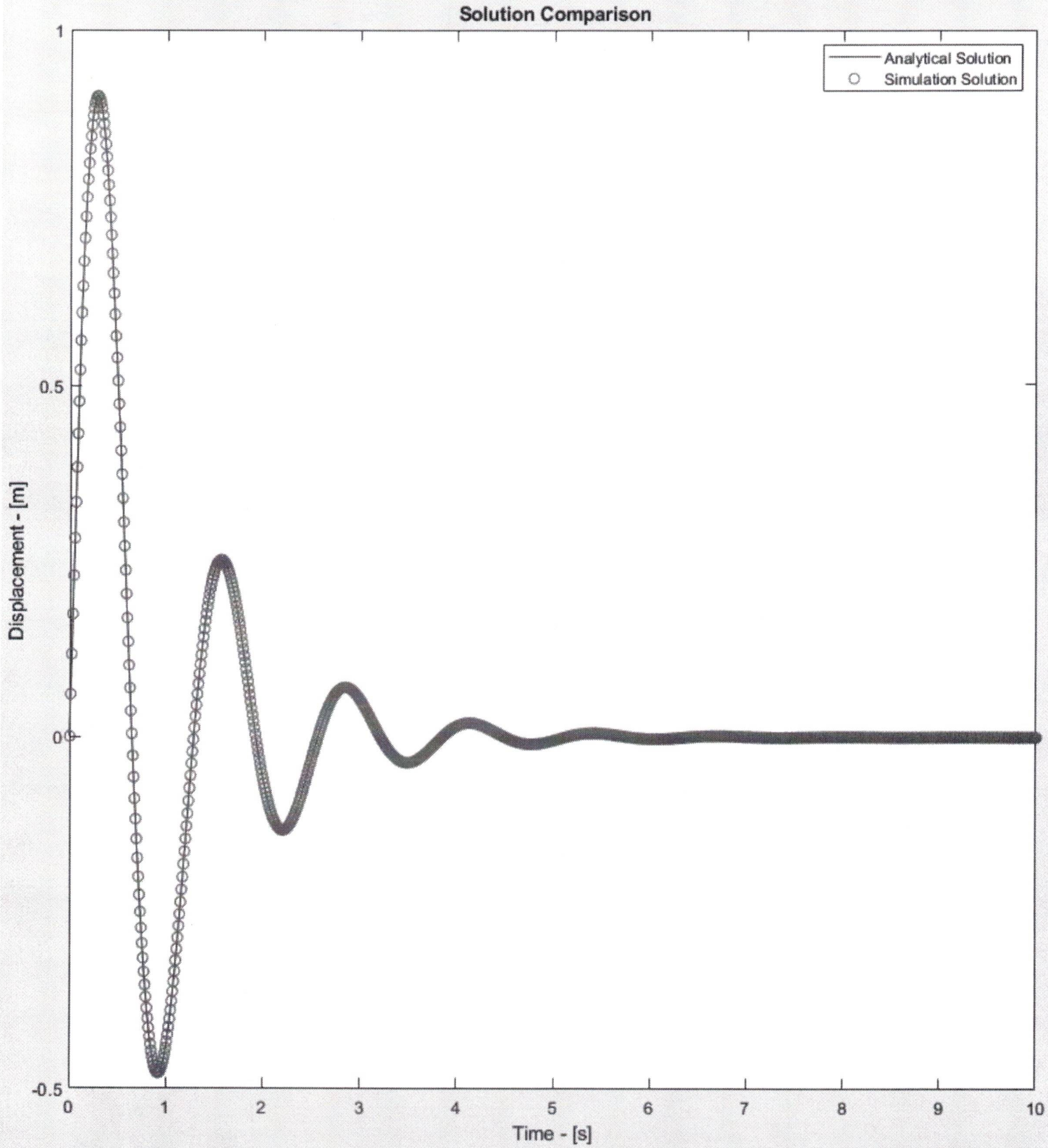
num = [c,k1];
den = [m,c,k1+k2];

T = tf(num,den);
opt = stepDataOptions('StepAmplitude',A);
[xsim, t] = step(T,t,opt);
plot(t,xsim,'k')
xlabel('Time - [s]')
ylabel('Displacement - [m]')
title('Simulation Solution')
pause;

% Solution Comparison

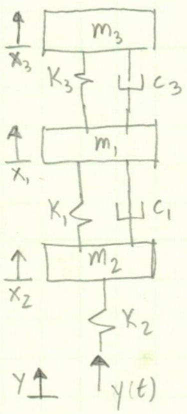
plot(t,x,'b',t,xsim,'k')
xlabel('Time - [s]')
ylabel('Displacement - [m]')
title('Solution Comparison')
legend('Analytical Solution','Simulation Solution')
pause;

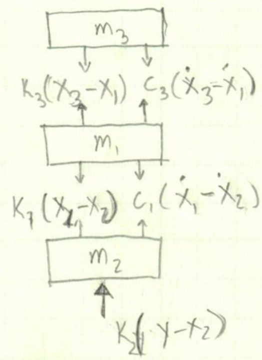
clear all
clc
clf
```



Solution Comparison
Analytical Solution
Simulation Solution
Displacement - [m]
Time - [s]
1
0.5
0
-0.5
0
1
2
3
4
5
6
7
8
9
10

2)





Assumptions:
→ ideal springs and dampeners
→ displacements measured from static equilibrium
→ $x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3 = 0 |_{t=0}$

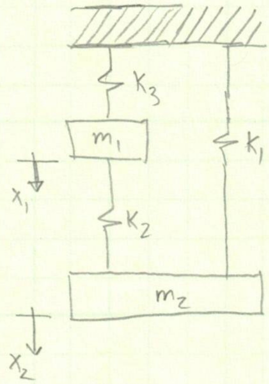
Pressumptions:
→ $\cancel{y}, x_1, x_2, x_3 > 0$
→ $y < x_2 < x_1 < x_3$

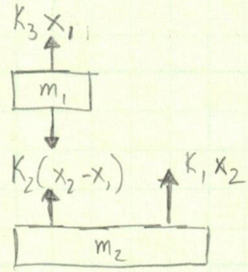
m_1: $m_1\ddot{x}_1 = k_3(x_3-x_1) + c_3(\dot{x}_3-\dot{x}_1) - K_1(x_1-x_2) - c_1(\dot{x}_1-\dot{x}_2)$

m_2: $m_2\ddot{x}_2 = K_1(x_1-x_2) + c_3(\dot{x}_1-\dot{x}_2) + K_2(y-x_2)$

m_3: $m_3\ddot{x} = -K_3(x_3-x_1) - c_3(\dot{x}_3-\dot{x}_1)$

3





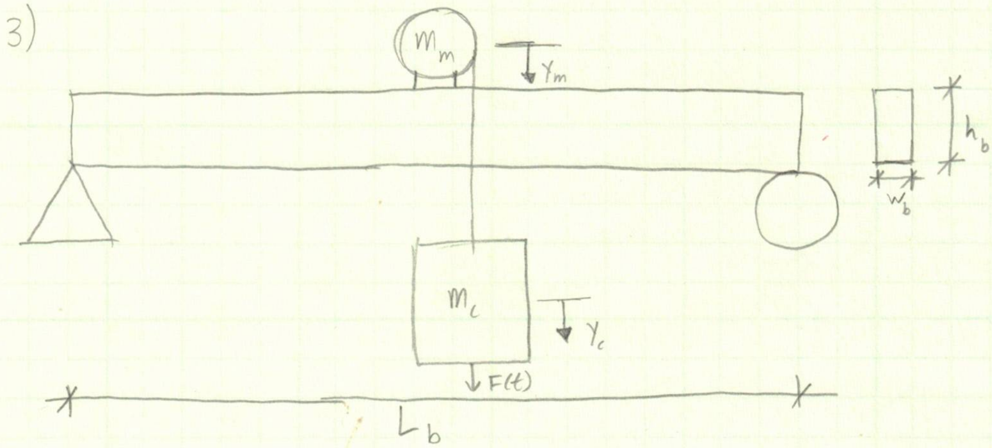
Assumptions:
→ ideal spring
→ displacements measured from static equilibrium
→ $\dot{x}_1 = \dot{x}_2 = 0$

Pressumptions
→ $x_2 > x_1$

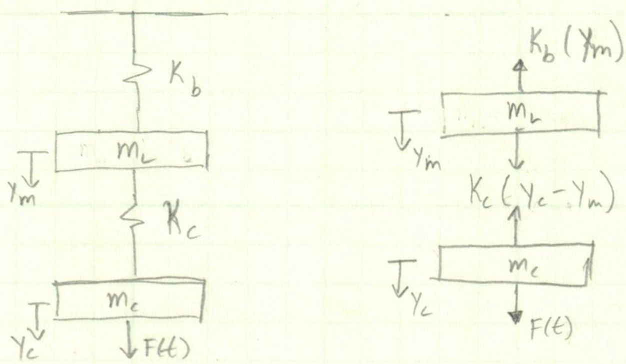
m_1: $m_1\ddot{x}_1 = k_3 x_1 - k_2(x_2 - x_1)$

m_2: $m_2\ddot{x}_2 = k_2(x_2 - x_1) + k_1 x_2$

3)



$\rho_b = 8000 \ kg/m^3$
$L_b = 3m$
$h_b = 0.10m$
$w_b = 0.075m$
$E_b = 207 \ GPa$
$m_m = 25 \ kg$
$m_c = 200 \ kg$
$L_c = 4m$
$d_c = 7E\text{-}3m$
$E_c = 207 \ GPa$



a) $K_b = \dfrac{4E w_b h_b^3}{L_b^3} = \dfrac{4(207E9 Pa)(75E\text{-}3m)(100E\text{-}3m)^3}{(3m)^3} = 2.3E6 \ N/m$

b) $K_c = \dfrac{\pi E d_c^2}{4L_c} = \dfrac{\pi(207E9 Pa)(7E\text{-}3m)^2}{4(4m)} = 1.99E6 \ N/m$

c) $m_L = m_m + \frac{1}{2} m_b = m_m + \frac{1}{2}\rho_b L_b w_b h_b = 25kg + \dfrac{(8000 \frac{kg}{m^3})(3m)(100E\text{-}3m)(75E\text{-}3)}{2}$

$= 115 \ kg$

d) $K_b(\delta_{y_m}) = (m_L + m_c) g \rightarrow \delta_{y_m} = \dfrac{g}{K_b}(m_L + m_c) = \dfrac{9.81 m/s^2}{2.3E6 N/m}\left(115kg + 200kg\right)$

$= 1.34E\text{-}3 \ m$

e) $m_L \ddot{y}_m = -K_b y_m + K_c(y_c - y_m)$

$m_c \ddot{y}_c = -K_c(y_c - y_m) + F(t)$