PRÁCTICO 11

Ejercicio 1

1 ⊛ Si $E(u|X)=0 \Rightarrow E(\hat{\beta}_{MCO}|X) = \beta + E\left[(X'X)^{-1}X'u|X\right] =$

$$= \beta + (X'X)^{-1}\underbrace{E(u|X)}_{=0} = \beta$$

⊛ $E(u'u|X) = E\left[(Y-X\hat{\beta}_{MCO})(Y-X\hat{\beta}_{MCO})\right] = \sigma_u^2 I_n$

⊛ $\Sigma_{(\hat{\beta}|X)} = E\left[(X'X)^{-1}X'uu'X(X'X)^{-1}|X\right] =$

$$= (X'X)^{-1}X'\underbrace{E(uu'|X)}_{=\sigma_u^2 I_n}X(X'X)^{-1} = \sigma_u^2(X'X)^{-1}$$

⊛ $\hat{\sigma}_u^2 = \dfrac{u'u}{n-k} \Rightarrow E(\hat{\sigma}_u^2|X) = \sigma_u^2$

⊛ $\hat{\Sigma}_{(\beta|X)} = \hat{\sigma}_u^2 (X'X)^{-1}$

⊛ $\lim\limits_{n\to+\infty} E(\hat{\beta}_{MCO}) = \beta \quad (1)$

$$\lim_{n\to+\infty} V(\hat{\beta}_{MCO}) = \lim_{n\to+\infty}\sigma_u^2(X'X)^{-1} = \lim \frac{\sigma_u^2}{n}\left(\frac{X'X}{n}\right)^{-1} =$$

$$= 0 \cdot Q^{-1} = 0 \quad (2)$$

$\left.\begin{matrix}(1)\\(2)\end{matrix}\right\} \Rightarrow \hat{\beta}_{MCO} \xrightarrow{p} \beta \Rightarrow \text{plim}\,\hat{\beta}_{MCO} = \beta \Rightarrow \hat{\beta}_{MCO}$

es consistente de β

$\boxed{2}$ $E(u|X)=0$ y $E(uu'|X)=\sigma_u^2 I$

$\Sigma_{(\hat\beta|X)} = \sigma_u^2 (X'X)^{-1}$

$\boxed{3}$ Si $E(u|X)\neq 0 \Rightarrow \hat\beta_{MCO}$ es sesgado de β y no es consistente

Ejercicio 2

$Y = X\beta+\varepsilon$ $\quad E(\varepsilon|X)=0$

$E(\varepsilon\varepsilon'|X)=\sigma_\varepsilon^2 I_n$ $\quad \text{plim}\,\frac{X'X}{n}=Q$

$\boxed{1}$ $E(\hat\beta^*) = E(\hat\beta_{MCO}+c) = E(\hat\beta_{MCO})+E(c) = \beta+c$

$V(\hat\beta^*) = V(\hat\beta_{MCO}+c) = V(\hat\beta_{MCO}) = \sigma_\varepsilon^2 (X'X)^{-1}$

$\boxed{2}$ $\lim\limits_{n\to+\infty} E(\hat\beta^*) = \lim\limits_{n\to+\infty} E(\hat\beta_{MCO}+c) =$

$= \lim\limits_{n\to+\infty} E(\hat\beta_{MCO}) + \lim\limits_{n\to+\infty} E(c) =$

$= \beta + \lim\limits_{n\to+\infty}\left(\frac{c_1}{n}, \frac{c_2}{n}, \ldots, \frac{c_k}{n}\right) = \beta \quad (1)$

$\lim\limits_{n\to+\infty} V(\hat\beta^*) = \lim\limits_{n\to+\infty} V(\hat\beta_{MCO}+c) = \lim\limits_{n\to+\infty} V(\hat\beta_{MCO}) =$

$= \lim\limits_{n\to+\infty} \frac{\sigma_\varepsilon^2}{n}\left(\frac{X'X}{n}\right)^{-1} = 0\cdot Q^{-1} = 0 \quad (2)$

$\left.\begin{matrix}(1)\\(2)\end{matrix}\right\} \Rightarrow \hat\beta^* \xrightarrow{p} \beta \Rightarrow \text{plim}\,\hat\beta^* = \beta \Rightarrow \hat\beta^*$ es consistente de β

$\boxed{3}$ $\frac{X'\varepsilon}{\sqrt{n}} \xrightarrow{D} N(0; \sigma_\varepsilon^2 Q)$

$\beta_{MCO} = \beta + (X'X)^{-1} X'\varepsilon \Rightarrow \beta_{MCO} - \beta = (X'X)^{-1} X'\varepsilon$

$\Rightarrow \beta_{MCO} - \beta = \left(\frac{X'X}{n}\right)^{-1} \frac{X'\varepsilon}{\sqrt{n}\sqrt{n}} \Rightarrow$

$\Rightarrow \sqrt{n}(\beta_{MCO} - \beta) = \underbrace{\left(\frac{X'X}{n}\right)^{-1}}_{= Q^{-1}} \underbrace{\frac{X'\varepsilon}{\sqrt{n}}}_{\xrightarrow{D} N} \Rightarrow$

$\Rightarrow \sqrt{n}(\beta_{MCO} - \beta) \xrightarrow{D} N(0; \sigma_\varepsilon^2 Q^{-1})$

luego dado que $\hat{\beta}_{MCO} \xrightarrow{P} \beta$ y

$\hat{\beta}^* \xrightarrow{P} \beta \Rightarrow (\hat{\beta}^* - \beta)\sqrt{n} \xrightarrow{D} N(0; \sigma_\varepsilon^2 Q^{-1})$

(*) $\lim_{n \to +\infty} \hat{\beta}^* = \lim_{n \to +\infty} \hat{\beta}_{MCO} + \left(\frac{c_1}{n}; \frac{c_2}{n}; \dots; \frac{c_K}{n}\right) =$

$= \lim_{n \to +\infty} \hat{\beta}_{MCO} = \beta$

Ejercicio 3

$Y = X\beta + \varepsilon \quad E(\varepsilon|X) = 0 \quad E(Y|X) = X\beta + E(\varepsilon|X) = X\beta$

$E(\varepsilon'\varepsilon|X) = \sigma_\varepsilon^2 I_n \quad \text{plim}\ \frac{\varepsilon'\varepsilon}{n} = \sigma_\varepsilon^2$

$\text{plim}\left(\frac{X'X}{n}\right) = Q \quad \text{plim}\left(\frac{X'\varepsilon}{n}\right) = 0$

$$\hat{\sigma}_{\hat{\varepsilon}}^2 = \frac{\hat{\varepsilon}'\hat{\varepsilon}}{n-k} = \frac{(Y - X\hat{\beta})'(Y - X\hat{\beta})}{n-k} = \frac{Y'Y}{n-k} - \frac{Y'X\hat{\beta}}{n-k} - \frac{\hat{\beta}'X'Y}{n-k} + \frac{\hat{\beta}'X'X\beta}{n-k}$$

$Y = X\beta + \varepsilon \Rightarrow \varepsilon =$