Fun. de Respuesta: Koyck

FRI

$$y_t = \alpha^* + \beta x_t + \lambda y_{t-1} + u_t^*$$

$$\beta;\ \lambda\beta;\ \lambda^2\beta; \ldots$$

$$y_{t+1} = \alpha^* + \beta x_{t+1} + \lambda y_t + u^*_{t+1} =$$

$$= \alpha^* + \beta x_{t+1} + \lambda\alpha^* + \lambda\beta x_t + \lambda^2 y_{t-1} + \lambda u_t^* + u^*_{t+1}$$

$$y_{t+2} = \alpha^* + \beta x_{t+2} + \lambda y_{t+1} + u^*_{t+2} =$$

$$= \alpha^* + \beta x_{t+2} + \lambda\alpha^* + \lambda\beta x_{t+1} + \lambda^2 y_t + \lambda u^*_{t+1} + u^*_{t+2} =$$

$$= \alpha^*(1+\lambda) + \beta(x_{t+2} + \lambda x_{t+2}) + \lambda^2\alpha^* + \lambda^2\beta x_t + \lambda^3 y_{t-1} + \lambda^2 u_t^* + \lambda u^*_{t+1} + u^*_{t+2}$$

⊛ $FRI = \beta;\ \lambda\beta;\ \lambda^2\beta;\ \lambda^3\beta; \ldots = \{\beta\lambda^j\}_{j=0}^{j=+\infty}$

⊛ $FRE = \beta;\ \beta+\lambda\beta;\ \beta+\lambda\beta+\lambda^2\beta; \ldots = \sum_i \{\beta\lambda^j\}_{j=0}^{j=i}$

Otra forma

$$y_t = \alpha^* + \beta x_t + \lambda y_{t-1} + u_t^* \Rightarrow \frac{\partial y_t}{\partial x_t} = \beta$$

$$y_{t+1} = \alpha^* + \beta x_{t+1} + \lambda y_t + u^*_{t+1} \Rightarrow \frac{\partial y_{t+1}}{\partial x_t} = \frac{\partial y_{t+1}}{\partial y_t} \cdot \frac{\partial y_t}{\partial x_t} = \lambda\beta$$

$$y_{t+2} = \alpha^* + \beta x_{t+2} + \lambda y_{t+1} + u^*_{t+2} \Rightarrow \frac{\partial y_{t+2}}{\partial x_t} = \frac{\partial y_{t+2}}{\partial y_{t+1}} \cdot \frac{\partial y_{t+1}}{\partial y_t} \cdot \frac{\partial y_t}{\partial x_t} =$$

$$= \lambda \cdot \lambda \cdot \beta = \lambda^2\beta$$

Por lo tanto: $FRI = \beta;\ \lambda\beta;\ \lambda^2\beta; \ldots = \{\beta\lambda^j\}_{j=0}^{j=+\infty}$