Conformación de matrices en MC2E

Caso de 1 VI con p Zs

* Supongamos $Y = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k + \varepsilon$

con $E(X_i ; \varepsilon) \neq 0$ e instru: $Z_1 ; \ldots ; Z_p$

$$\mathbb{X}_{n\times(k+1)} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \qquad \mathbb{Y}_{n\times 1} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$W_{n\times p} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1p} \\ z_{21} & z_{22} & \cdots & z_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{np} \end{bmatrix}$$

$$W'_{p\times n} = \begin{bmatrix} z_{11} & z_{21} & \cdots & z_{n1} \\ z_{12} & z_{22} & \cdots & z_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ z_{1p} & z_{2p} & \cdots & z_{pn} \end{bmatrix}$$

* $$\hat{\delta}_{p\times(k+1)} = \big(\underbrace{W'_{p\times n} W_{n\times p}}_{p\times p}\big)^{-1} \underbrace{W'_{p\times n} \mathbb{X}_{n\times(k+1)}}_{p\times(k+1)}$$

$$\hat{\delta}_{p\times(k+1)} = \begin{bmatrix} \hat{\delta}_{11} & \hat{\delta}_{12} & \cdots & \hat{\delta}_{1(k+1)} \\ \hat{\delta}_{21} & \hat{\delta}_{22} & \cdots & \hat{\delta}_{2(k+1)} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\delta}_{n1} & \hat{\delta}_{n2} & \cdots & \hat{\delta}_{n(k+1)} \end{bmatrix}$$

$$\hat{X}_{n\times k+1} = \underbrace{W_{n\times p}\,\hat{S}_{p\times k+1}}_{n\times k+1} \Rightarrow \hat{X}'_{(k+1)\times n}$$

* Luego: $\hat{X} = Z$

$$\hat{\beta}_{VI\,(k+1)\times 1} = (Z'X)^{-1}(Z'Y) = \underbrace{\underbrace{\left[\hat{X}'_{(k+1)\times n}\, X_{n\times(k+1)}\right]^{-1}}_{(k+1)\times(k+1)} \underbrace{\left[\hat{X}'_{(k+1)\times n}\, Y_{n\times 1}\right]}_{(k+1)\times 1}}_{(k+1)\times 1}$$

$$\hat{\beta}_{VI\,(k+1)\times 1} = \underbrace{\begin{bmatrix} \hat{\beta}_{0;VI} \\ \hat{\beta}_{1;VI} \\ \vdots \\ \hat{\beta}_{k;VI} \end{bmatrix}}_{1} \Big\} k+1$$

* Varianza: $$V(\hat{\beta}_{MC2E}) = \sigma^2_{\varepsilon}\left[\underbrace{\underbrace{\underbrace{X'_{(k+1)\times n} W_{n\times p}}_{(k+1)\times p} \underbrace{(W'_{p\times n} W_{n\times p})^{-1}}_{p\times p}}_{(k+1)\times p} \underbrace{W'_{p\times n} X_{n\times(k+1)}}_{p\times(k+1)}}_{(k+1)\times(k+1)}\right]^{-1}$$

$$V(\hat{\beta}_{MC2E}) = \begin{bmatrix} V(\hat{\beta}_0) & COV(\hat{\beta}_1;\hat{\beta}_0) & COV(\hat{\beta}_2;\hat{\beta}_0) & \cdots & COV(\hat{\beta}_{k+1};\hat{\beta}_0) \\ COV(\hat{\beta}_0;\hat{\beta}_1) & V(\hat{\beta}_1) & COV(\hat{\beta}_2;\hat{\beta}_1) & \cdots & COV(\hat{\beta}_{k+1};\hat{\beta}_1) \\ COV(\hat{\beta}_0;\hat{\beta}_2) & COV(\hat{\beta}_1;\hat{\beta}_2) & V(\hat{\beta}_2) & \cdots & COV(\hat{\beta}_{k+1};\hat{\beta}_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ COV(\hat{\beta}_0;\hat{\beta}_{k+1}) & COV(\hat{\beta}_1;\hat{\beta}_{k+1}) & COV(\hat{\beta}_2 & \cdots & V(\hat{\beta}_{k+1}) \end{bmatrix}$$

$$\boxed{\begin{array}{l}\text{Caso de 2VI} \\ \text{con } p \text{ y } q \text{ Zs}\end{array}}$$

Supongamos: $Y = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k + \varepsilon$

con $E(X_i; \varepsilon) \neq 0 \Rightarrow$ Instrums: $z_1^{(i)}; \dots ; z_p^{(i)}$

con $E(X_j; \varepsilon) \neq 0 \Rightarrow$ Instrums: $z_1^{(j)}; \dots ; z_q^{(j)}$

$\boxed{i \neq j}$

$$\mathbb{X}_{n\times(k+1)} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \qquad \mathbb{Y}_{n\times 1} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$W_{n\times(p+q)} = \left.\begin{bmatrix} \underbrace{\begin{matrix} z_{11}^{(i)} & z_{12}^{(i)} & \cdots & z_{1p}^{(i)} \\ z_{21}^{(i)} & z_{22}^{(i)} & \cdots & z_{2p}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}^{(i)} & z_{n2}^{(i)} & \cdots & z_{np}^{(i)} \end{matrix}}_{p} & \underbrace{\begin{matrix} z_{11}^{(j)} & z_{12}^{(j)} & \cdots & z_{1q}^{(j)} \\ z_{21}^{(j)} & z_{22}^{(j)} & \cdots & z_{2q}^{(j)} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}^{(j)} & z_{n2}^{(j)} & \cdots & z_{nq}^{(j)} \end{matrix}}_{q} \end{bmatrix}\right\} n$$

$\Downarrow$

$W'_{(p+q)\times n}$

$$\hat{\delta}_{(p+q)\times(k+1)} = \underbrace{\underbrace{\left[W'_{(p+q)\times n} W_{n\times(p+q)}\right]^{-1}}_{(p+q)\times(p+q)} \cdot \underbrace{\left[W'_{(p+q)\times n} \mathbb{X}_{n\times(k+1)}\right]}_{(p+q)\times(k+1)}}_{(p+q)\times(k+1)}$$

$$\hat{\delta}_{(p+q)\times(k+1)} = \begin{array}{cc} \begin{bmatrix} \hat{\delta}_{11} & \hat{\delta}_{12} & \cdots & \hat{\delta}_{1(k+1)} \\ \hat{\delta}_{21} & \hat{\delta}_{22} & \cdots & \hat{\delta}_{2(k+1)} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\delta}_{p1} & \hat{\delta}_{p2} & \cdots & \hat{\delta}_{p(k+1)} \\ \hat{\delta}_{(p+1)1} & \hat{\delta}_{(p+1)2} & \cdots & \hat{\delta}_{(p+1)(k+1)} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\delta}_{(p+q)1} & \hat{\delta}_{(p+q)2} & \cdots & \hat{\delta}_{(p+q)(k+1)} \end{bmatrix} & \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} p \\ \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} q \end{matrix} \\ \underbrace{\hphantom{\hat{\delta}_{11} \quad \hat{\delta}_{12} \quad \cdots \quad \hat{\delta}_{1(k+1)}}}_{k+1} & \end{array}$$

$$\hat{\mathbb{X}}_{n\times(k+1)} = \underbrace{W_{n\times(p+q)}\ \hat{\delta}_{(p+q)\times(k+1)}}_{n\times(k+1)} \Rightarrow \hat{\mathbb{X}}'_{(k+1)\times n}$$

Luego: $\hat{\mathbb{X}} = Z$

$$\hat{\beta}_{VI\,(k+1)\times 1} = (Z'\mathbb{X})^{-1}(Z'\mathbb{Y}) = \underbrace{\left[\underbrace{\hat{\mathbb{X}}'_{(k+1)\times n}\ \mathbb{X}_{n\times(k+1)}}_{(k+1)\times(k+1)}\right]^{-1}\left[\underbrace{\hat{\mathbb{X}}'_{(k+1)\times n}\ \mathbb{Y}_{n\times 1}}_{(k+1)\times 1}\right]}_{(k+1)\times 1}$$

$$\hat{\beta}_{VI} = \begin{array}{c} \begin{bmatrix} \hat{\beta}_{0;VI} \\ \hat{\beta}_{1;VI} \\ \vdots \\ \hat{\beta}_{k;VI} \end{bmatrix} \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} k+1 \\ \underbrace{\hphantom{\hat{\beta}_{k;VI}}}_{1} \end{array}$$