Diseño SISI

Daniel Czarnievicz

2017

Estrategia de selección

El diseño $SISI$ consiste en la siguiente estrategia de selección:

- **Primer etapa**: bajo un diseño SI se selecciona una muestra s_I de tamaño n_I de los N_I clusters.
- **Segunda etapa**: $\forall i \in s_I$ se toma una muestra bajo diseño SI de tamaño n_i de los N_i elementos en el cluster i.

Probabilidades de inclusión

Probabilidades de inclusión para la primera etapa

$$\star\ \pi_{I_i} = \frac{n_I}{N_I}\ \forall i \in U \qquad \star\ \pi_{I_{ij}} = \frac{n_I}{N_I}\frac{(n_I-1)}{(N_I-1)}\ \forall i \neq j \in U$$

$$\star\ \Delta_{I_{ij}} = \begin{cases} -\dfrac{f_I(1-f_I)}{N_I-1} & \text{si} \quad i \neq j \\ \\ f_I(1-f_I) & \text{si} \quad i = j \end{cases} \qquad \star\ \Delta^{\checkmark}_{I_{ij}} = \frac{\Delta_{I_{ij}}}{\pi_{I_{ij}}}$$

Probabilidades de inclusión para la segunda etapa

$$\star\ \pi_{k|i} = \frac{n_i}{N_i}\ \forall k \in i = 1;\ldots;N_I \qquad \star\ \pi_{kl|i} = \frac{n_i}{N_i}\frac{(n_i-1)}{(N_i-1)}\ \forall k \neq l \in i = 1;\ldots;N_I$$

$$\star\ \Delta_{kl|i} = \begin{cases} -\dfrac{f_i(1-f_i)}{N_i-1} & \text{si} \quad k \neq l \\ \\ f_i(1-f_i) & \text{si} \quad k = l \end{cases} \qquad \star\ \Delta^{\checkmark}_{kl|i} = \frac{\Delta_{kl|i}}{\pi_{kl|i}}$$

Probabilidades de inclusión de los elementos

$$\star\ \pi_k = \pi_{I_i}\,\pi_{k|i}\ \forall k \in U_i \qquad \star\ \pi_{kl} = \begin{cases} \pi_{I_i}\,\pi_{k|i} & \text{si} \quad k = l \in U_i \\ \pi_{I_i}\,\pi_{kl|i} & \text{si} \quad k;l \in U_i \\ \pi_{I_{ij}}\,\pi_{k|i}\,\pi_{l|j} & \text{si} \quad k \in U_i \text{ y } l \in U_j \end{cases}$$

El estimador $\hat{t}_\pi$

$$\star\ \hat{t}_\pi = \sum\nolimits_s y^{\checkmark}_k = \sum\nolimits_{s_I}\sum\nolimits_{s_i} y^{\checkmark}_k = \sum\nolimits_{s_I}\sum\nolimits_{s_i}\frac{y_k}{\pi_k} = \sum\nolimits_{s_I}\sum\nolimits_{s_i}\frac{y_k}{\pi_{I_i}\,\pi_{k|i}} =$$

$$= \sum\nolimits_{s_I}\frac{1}{\pi_{I_i}}\sum\nolimits_{s_i}\frac{y_{k|i}}{\pi_{k|i}} = \frac{N_I}{n_I}\sum\nolimits_{s_I} N_i\,\bar{y}_{s_i} = \frac{N_I}{n_I}\sum\nolimits_{s_I}\hat{t}_{\pi_i}$$

$$\star\ V_{SISI}(\hat{t}_\pi) = \frac{N_I^2}{n_I}(1-f_I)S^2_{t_{U_I}} + \frac{N_I}{n_I}\sum\nolimits_{U_i}\frac{N_i^2}{n_i}(1-f_i)S^2_{y_{U_i}}$$

donde:

- $S^2_{t_{U_I}} = \dfrac{1}{N_I - 1} \sum_{U_I} \left(t_{y_i} - \bar{t}_{U_I} \right)^2$ con $\bar{t}_{U_I} = \dfrac{1}{N_I} \sum_{U_I} t_{y_i}$
- $S^2_{y_{U_i}} = \dfrac{1}{N_i - 1} \sum_{U_i} \left(y_{k|i} - \bar{y}_{U_i} \right)^2$ con $\bar{y}_{U_i} = \dfrac{1}{N_i} \sum_{U_i} y_{k|i}$

$$\star\ \hat{V}_{SISI}(\hat{t}_\pi) = \frac{N_I^2}{n_I}(1 - f_I) S^2_{\hat{t}_{s_I}} + \frac{N_I}{n_I} \sum_{s_i} \frac{N_i^2}{n_i}(1 - f_i) S^2_{y_{s_i}}$$

donde:

- $S^2_{\hat{t}_{s_I}} = \dfrac{1}{n_I - 1} \sum_{s_I} \left(\hat{t}_{\pi_i} - \dfrac{1}{n_I} \sum_{s_I} \hat{t}_{\pi_i} \right)^2$
- $S^2_{y_{s_i}} = \dfrac{1}{n_i - 1} \sum_{s_i} \left(y_{k|i} - \bar{y}_{s_i} \right)^2$ con $\bar{y}_{s_i} = \dfrac{1}{n_i} \sum_{s_i} y_{k|i}$