CIVE586 Assignment #3

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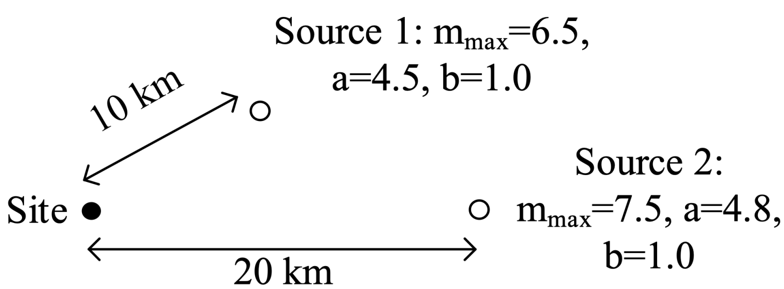
Question:

Given the information of earthquake sizes and location, compute the seismic hazard curves [i.e., $\lambda(PGA > x)$ versus x curves] for individual sources and total hazard. The GMPE of Cornell et al. (1979) is used to compute the predicted ground motion. The minimum moment magnitude for both earthquake sources is 4.0. Identify

1) The mean annual rate of exceedance of PGA,
2) The PGAs that will be exceeded at 2% and 10% in 50 years, respectively?
3) Deaggregation for PGA>0.2g using mean annual rate.



Q1:

Step1: The source-to-site distance characterization:

For source 1 and source 2, since they are both point sources, the distance is fixed to 10km and 20km respectively. Thus, the source-to-site probability distribution is 1.

Step2: Earthquake magnitude uncertainty characterization:

The distribution of earthquake recurrence can be characterized using the recurrence relationships provided in the problem statement. The minimum moment magnitude for both earthquake sources is 4.0, the mean rates of exceedance of magnitude 4.0 events from each of the source zones are:

$$\begin{cases} Source1: \quad \lambda(M > m_{min}) = 10^{4.5-1.0\cdot(4.0)} = 3.162, \quad \alpha = 10.362, \beta = 2.303 \\ Source2: \quad \lambda(M > m_{min}) = 10^{4.8-1.0\cdot(4.0)} = 6.310, \quad \alpha = 11.052, \beta = 2.303 \end{cases}$$

Probability of discrete set of magnitudes:

$$P(M = m_j) = f_M(m_j)\Delta m = \frac{\beta e^{-\beta(m-m_{min})}}{1 - e^{-\beta(m_{max}-m_{min})}}\Delta m$$

Setting the magnitude interval $\Delta m = 0.25$, for source 1, the lowest magnitude interval will be from $M = 4 \sim 4.25$. The middle value of 1^{st} bin is $m_1 = \frac{4+4.25}{2} = 4.125$. The probability that the magnitude would fall within that interval would be:

$$P[4.0 < M < 4.25] \approx \frac{2.303e^{-2.303(4.125-4.0)}}{1 - e^{-2.303(6.5-4.0)}} \times 0.25 = 0.433$$

The probabilities of various magnitudes for each source zone are as shown in Figure 1.

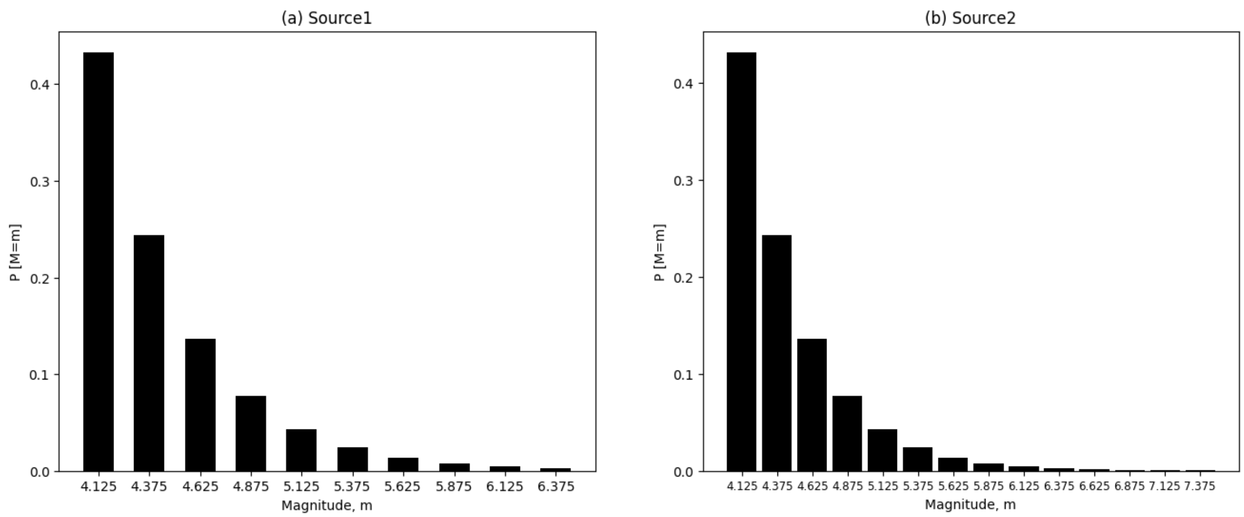


Figure 1 Approximations to magnitude probability distributions for source zones

Step3: Identify the ground motion intensity:

Using the GMPE of Cornell:

$$\overline{\ln PGA}\,(\mathrm{g}) = -0.152 + 0.859M - 1.803 \ln (R + 25)$$

The standard deviation $\sigma_{\ln y} = 0.57$.

Step4: Combine step 1, 2, and 3.

For zone1 and the lowest magnitude interval $j = 1$,

$$P[M = m_1] = P[M = 4.125] = 0.433$$

as computed in step2. For the fixed distance,

$$P[R = r_1] = P[R = 10\text{km}] = 1$$

as computed in step1. This combination of magnitude and distance indicates an expected value of $\ln PGA$ of

$$\overline{\ln PGA} = -0.152 + 0.859 \times 4.125 - 1.803 \times \ln(10 + 25) = -3.019$$

Calculate the probabilities that various target peak acceleration levels will be exceeded. For $x = 0.1g$, the corresponding standard normal variable is

$$z = \frac{\ln x - \overline{\ln PGA}}{\sigma_{\ln PGA}} = \frac{\ln(0.1) - (-3.019)}{0.57} \approx 1.26$$

According to the CDF of the standard normal distribution, the probability that the peak acceleration is greater than $0.1g$

$P[PGA > 0.1g|M = 4.125, R = 10km] = P[z > 1.26] = 1 - F_z(1.26) = 1 - 0.8956 = 0.1044.$

Annual rate of exceedance of a peak acceleration of $0.1g$ by an earthquake of magnitude 4.125 at a distance of 10km on source zone 1 will be:

$$\lambda_{0.1g} = \lambda(M > m_{min}) \times P[PGA > 0.1g|M = 4.125, R = 10\text{km}] \times P[M = 4.125] \\ \times P[R = 10\text{km}] = 3.162 \times 0.1044 \times 0.433 \times 1 = 0.1429$$

If the preceding calculations are repeated for the 9 other possible combinations of magnitude and distance for source zone 1, the contributions of each will be:

Table 1 Annual rate of exceedance for source 1

Distance (km) = 10km	
Magnitude	Annual rate of exceedance
4.125	0.1429
4.375	0.1459
4.625	0.1331
4.875	0.1095
5.125	0.0820
5.375	0.0566
5.625	0.0365
5.875	0.0223
6.125	0.0132
6.375	0.0076

Summing all of these contributions indicates that the mean annual rate at which an acceleration of $0.1g$ will be exceeded bn an earthquake on source 1 will be 0.7496. Repeating all of these calculations for the source 2 yield equivalent exceedance rate of 0.5828. Consequently, the probability that a target acceleration of $0.1g$ will be exceeded by an earthquake of $M > m_0$ on any of these two source zones will be 1.3324. By repeating this process for different target accelerations, the seismic hazard curves of Figure 2 can be developed.

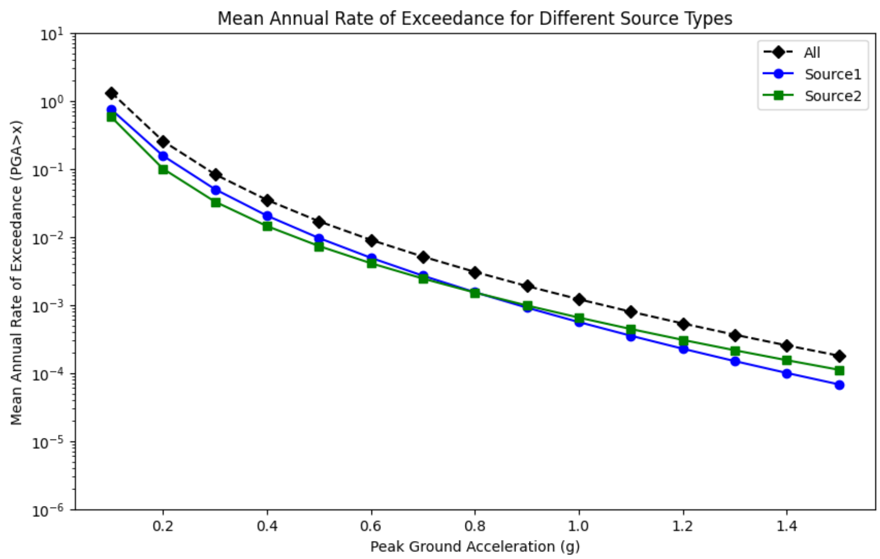


Figure 2 Seismic hazard curves for source zones 1, 2 and total seismic hazard curve for all sources

Table 2 Contributions to the mean annual rate of exceedance

	Point Source 1	Point Source 2	Sum
PGA (g)	λ [PGA>x]	λ [PGA>x]	λ [PGA>x]
0.1	0.74960381	0.58205229	1.33165609
0.2	0.15549511	0.10025397	0.25574907
0.3	0.05012531	0.03291126	0.08303656
0.4	0.02049792	0.01439283	0.03489076
0.5	0.00961257	0.00733272	0.01694529
0.6	0.00492461	0.00410063	0.00902523
0.7	0.00268624	0.00244172	0.00512796
0.8	0.00153679	0.00152163	0.00305842
0.9	0.00091327	0.00098191	0.00189519
1.0	0.00056006	0.00065156	0.00121162
1.1	0.00035273	0.00044243	0.00079516
1.2	0.00022733	0.00030634	0.00053368
1.3	0.00014951	0.00021573	0.00036523
1.4	0.00010011	0.00015418	0.00025429
1.5	0.00006812	0.00011165	0.00017977

Q2:

Using the Possion model and linear interpolation ($\log_{10}\lambda$),

$$P(N>1) = 1 - e^{-\lambda t}$$

When $P = 2\%$, $t = 50$, $\lambda = \frac{1}{2475} \approx 0.000404054$, $\log_{10}\lambda = -3.3936$, $PGA = 1.273g$.

When $P = 10\%$, $t = 50$, $\lambda = \frac{1}{475} \approx 0.00210721$, $\log_{10}\lambda = -2.6763$, $PGA = 0.878g$.

Q3:

The process of deaggregation requires that the mean annual rate of exceedance be expressed as a function of magnitude and/or distance.

The mean annual rate of exceedance can be expressed as a function of magnitude by:

$$\lambda(IM > x, M = m) \approx P\left[M_i = m_j\right] \sum_{i=1}^{N_S} \sum_{k=1}^{N_R} \lambda_i(M > m_{min}) \cdot P\left[IM > x \middle| m_j, r_k\right] P[R_i = r_k]$$
$$= \lambda(IM > x) \cdot P(M = m | IM > x)$$

Similarly, the mean annual rate of exceedance can be expressed as a function of source-site distance by:

$$\lambda(IM > x, R = r) \approx P[R_i = r_k] \sum_{i=1}^{N_S} \sum_{j=1}^{N_M} \lambda_i(M > m_{min}) \cdot P\left[IM > x \middle| m_j, r_k\right] P\left[M_i = m_j\right]$$
$$= \lambda(IM > x) \cdot P(R = r | IM > x)$$

The result of deaggregation is shown below:

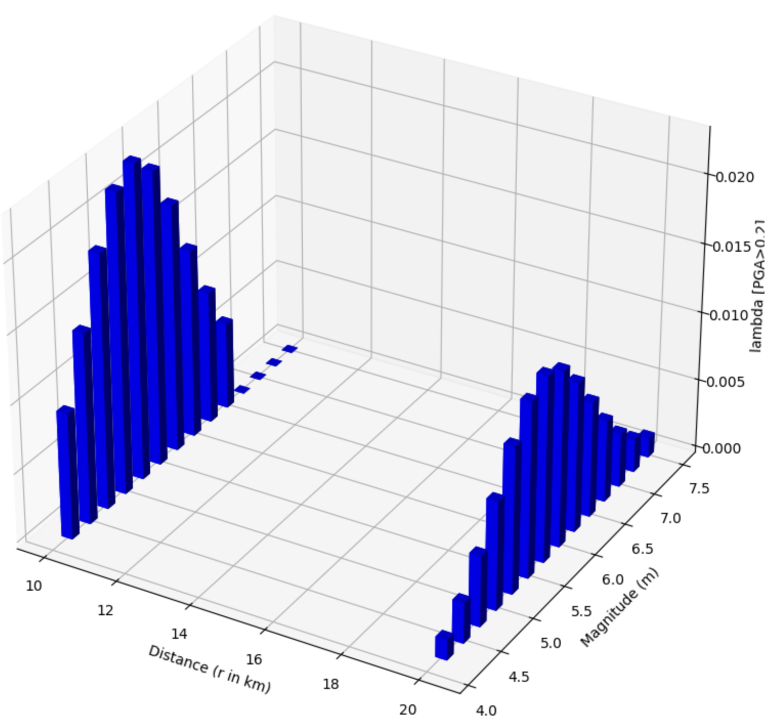


Figure 3 Deaggregation by PGA>0.2g |R=r, M=m