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P.2-1

Sol. a) $a_A = \dfrac{\vec{a}_x + \vec{a}_y 2 - \vec{a}_z 3}{\sqrt{14}}$

b) $|\vec{A}-\vec{B}| = \left|\vec{a}_x + 6\vec{a}_y - 4\vec{a}_z\right| = \sqrt{36+16+1} = \sqrt{53}$

c) $\vec{A}\cdot\vec{B} = (\vec{a}_x + 2\vec{a}_y - 3\vec{a}_z)\cdot(-4\vec{a}_y + \vec{a}_z) = -8-3 = -11$

d) $\cos\theta_{AB} = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{-11}{\sqrt{14}\cdot\sqrt{17}} = \dfrac{-11}{\sqrt{238}}$

$\Rightarrow \theta_{AB} = \arccos\left(\dfrac{-11}{\sqrt{238}}\right) = 135.48^\circ$

e) $\vec{A}\cdot\vec{C} = (\vec{a}_x + 2\vec{a}_y - 3\vec{a}_z)\cdot(5\vec{a}_x - 2\vec{a}_z) = 5+6 = 11$

the component of A in direction of C $\Rightarrow A\cdot\cos\theta = \dfrac{\vec{A}\cdot\vec{C}}{C} = \dfrac{11}{\sqrt{29}}$.

f) $\vec{A}\times\vec{C} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & 2 & -3 \\ 5 & 0 & -2 \end{vmatrix} = -4\vec{a}_x - 13\vec{a}_y - 10\vec{a}_z$

g) $(\vec{B}\times\vec{C}) = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & -4 & 1 \\ 5 & 0 & -2 \end{vmatrix} = 8\vec{a}_x + 5\vec{a}_y + 20\vec{a}_z$

$\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{a}_x + 2\vec{a}_y - 3\vec{a}_z)\cdot(8\vec{a}_x + 5\vec{a}_y + 20\vec{a}_z)$

$= 8+10-60 = -42$

$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & 2 & -3 \\ 0 & -4 & 1 \end{vmatrix} = -10\vec{a}_x - \vec{a}_y - 4\vec{a}_z$

$(\vec{A}\times\vec{B})\cdot\vec{C} = (-10\vec{a}_x - \vec{a}_y - 4\vec{a}_z)\cdot(5\vec{a}_x - 2\vec{a}_z) = -50+8 = -42$

h) $\vec{A}\times\vec{B} = -10\vec{a}_x - \vec{a}_y - 4\vec{a}_z$; $(\vec{A}\times\vec{B})\times\vec{C} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ -10 & -1 & -4 \\ 5 & 0 & -2 \end{vmatrix} = 2\vec{a}_x - 40\vec{a}_y + 5\vec{a}_z$

$\vec{B}\times\vec{C} = 8\vec{a}_x + 5\vec{a}_y + 20\vec{a}_z$; $\vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & 2 & -3 \\ 8 & 5 & 20 \end{vmatrix} = 55\vec{a}_x - 44\vec{a}_y - 11\vec{a}_z$

P2-2

Sol. assume that $\vec{C} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$ and $\sqrt{x^2+y^2+z^2} = 1$

$\vec{A}\cdot\vec{C} = \vec{B}\cdot\vec{C} = 0 \Rightarrow \begin{cases} x-2y+3z=0 \\ x+y-2z=0 \\ x^2+y^2+z^2=1 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{\sqrt{35}} \\ y = \frac{5}{\sqrt{35}} \\ z = \frac{3}{\sqrt{35}} \end{cases}$ or $\begin{cases} x = -\frac{1}{\sqrt{35}} \\ y = -\frac{5}{\sqrt{35}} \\ z = -\frac{3}{\sqrt{35}} \end{cases}$.

$\Rightarrow \vec{C} = -\frac{1}{\sqrt{35}}\vec{a}_x - \frac{5}{\sqrt{35}}\vec{a}_y - \frac{3}{\sqrt{35}}\vec{a}_z$ or $\frac{1}{\sqrt{35}}\vec{a}_x + \frac{5}{\sqrt{35}}\vec{a}_y + \frac{3}{\sqrt{35}}\vec{a}_z$.

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P2-20.

Sol. $\vec{F} = \vec{a_x}\,xy + \vec{a_y}(3x-y^2) = \langle xy,\ 3x-y^2\rangle$

a) $\overrightarrow{P_1P_2} = \langle -2,-3\rangle \qquad d\vec{\ell} = \langle dx, dy\rangle$ along path ①. $y = \frac{3}{2}x - \frac{3}{2}$

$$\int_{\overrightarrow{P_1P_2}} \vec{F}\cdot d\vec{\ell} = \int \langle xy, 3x-y^2\rangle\cdot\langle dx, dy\rangle = \int_{\overrightarrow{P_1P_2}} xy\cdot dx + \int_{\overrightarrow{P_1P_2}} 3x\,dy - \int_{\overrightarrow{P_1P_2}} y^2\cdot dy$$

$$= \int_5^3 x\cdot\left(\tfrac{3}{2}x-\tfrac{3}{2}\right)\cdot dx + \int_6^3 3\cdot\left(y+\tfrac{3}{2}\right)\times\tfrac{2}{3}\cdot dy - \int_6^3 y^2\cdot dy$$

$$= \int_3^5 -\tfrac{3}{2}x^2+\tfrac{3}{2}x\cdot dx - \int_3^6 2y+3\cdot dy + \int_3^6 y^2\cdot dy$$

$$= \left(-\tfrac{1}{2}x^3+\tfrac{3}{4}x^2\right)\Big|_3^5 - (y^2+3y)\Big|_3^6 + \tfrac{1}{3}y^3\Big|_3^6$$

$$= \left(-\tfrac{125}{2}+\tfrac{75}{4}\right) - \left(-\tfrac{27}{2}+\tfrac{27}{4}\right) + 18 + 9 = -37 + 27 = -10.$$

b) $$\int_{\overrightarrow{P_1A}+\overrightarrow{AP_2}} \vec{F}\cdot d\vec{\ell} = \int_6^3 15\,dy - \int_6^3 y^2\cdot dy + \int_5^3 3x\cdot dx$$

$$= 15y\Big|_6^3 + \tfrac{1}{3}y^3\Big|_3^6 + \tfrac{3}{2}x^2\Big|_5^3$$

$$= 18 + \left(\tfrac{27}{2} - \tfrac{75}{2}\right) = -6.$$

P2-23

Sol. $V = (\sin\frac{\pi}{2}x)\cdot(\sin\frac{\pi}{3}y)\,e^{-z}$.

a) $$\nabla V = \left(\vec{a_x}\frac{\partial}{\partial x} + \vec{a_y}\frac{\partial}{\partial y} + \vec{a_z}\frac{\partial}{\partial z}\right)\cdot V = \frac{\pi}{2}\cos\frac{\pi}{2}x\cdot\sin\frac{\pi}{3}y\cdot e^{-z}\cdot\vec{a_x} + \frac{\pi}{3}\cos\frac{\pi}{3}y\cdot\sin\frac{\pi}{2}x\cdot e^{-z}\cdot\vec{a_y} - \sin\frac{\pi}{2}x\cdot\sin\frac{\pi}{3}y\cdot e^{-z}\cdot\vec{a_z}$$

$$\nabla V\Big|_P = \left(-\frac{\pi}{6}\vec{a_y} - \frac{\sqrt{3}}{2}\vec{a_z}\right)e^{-3}$$

b) $\overrightarrow{PO} = \langle -1, -2, -3\rangle \qquad |\overrightarrow{PO}| = \sqrt{14}.$

$$(\nabla V)_P\cdot\frac{\overrightarrow{PO}}{|\overrightarrow{PO}|} = \frac{\left(\frac{\pi}{3}+\frac{3\sqrt{3}}{2}\right)e^{-3}}{\sqrt{14}}$$

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P2-29.

Sol. $\vec{A} = r^2\vec{a_r} + 2z\vec{a_z}$ and divergence thoerom: $\oint_s \vec{F}\cdot d\vec{s} = \int_v \nabla\cdot\vec{F}\cdot dv$

$\nabla\cdot\vec{A} = \frac{\partial(r\cdot r^3)}{r\partial r} + \frac{\partial\cdot(2z)}{\partial z} = 3r+2$

$\int_v \nabla\cdot\vec{A}\cdot dv = \int_0^4\int_0^{2\pi}\int_0^5 \nabla\cdot\vec{A}\ r dr d\theta dz = \int_0^4\int_0^{2\pi}\int_0^5 (3r^2+2r)\cdot dr\cdot d\theta\cdot dz = 1200\pi.$

for top surface ($z=4$) $\vec{A} = r^2\vec{a_r} + 8\vec{a_z}$ $\int_{top\ surface}\vec{A}\cdot d\vec{s} = \int_{top\ surface} 8\cdot ds = 8\times(5)^2\pi = 200\pi$

for bottom surface ($z=0$) $\vec{A} = r^2\vec{a_r}$ $\int_{bottom}\vec{A}\cdot d\vec{s} = 0$

for side surface ($r=5$) $\vec{A} = 25\vec{a_r} + 2z\vec{a_z}$ $\int_{side}\vec{A}\cdot d\vec{s} = \int_{side} 25\cdot ds = 25\times 40\pi = 1000\pi$

$\Rightarrow \oint_s \vec{A}\cdot d\vec{s} = 1200\pi = \int_v \nabla\cdot\vec{A}\cdot dv$

P2-32

Sol. $\vec{D} = \cos^2\phi\ \vec{a_R}/R^3.$ $ds = R^2\cdot\sin\theta\cdot d\theta\cdot d\phi$

a) $\oint\vec{D}\cdot d\vec{s} = \int_0^{2\pi}\int_0^{\pi}(\frac{1}{2}-1)\sin\theta\cdot d\theta\cos^2\phi\cdot d\phi = -\frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\sin\theta\cdot d\theta\cos^2\phi\, d\phi = -\int_0^{2\pi}\cos^2\phi\cdot d\phi = -\pi$

b) $\nabla\cdot\vec{D} = \frac{1}{R^2}\cdot\frac{\partial}{\partial R}\cdot(\frac{\cos^2\phi}{R^3}\cdot R^2) = -\frac{\cos^2\phi}{R^4}$

$dV = R^2\sin\theta\cdot dR d\theta\cdot d\phi.$

$\int_V\nabla\cdot\vec{D}\cdot dV = \int_0^{2\pi}\int_0^{\pi}\int_1^2 R^2(-\frac{\cos^2\phi}{R^4})\cdot\sin\theta dR\, d\theta d\phi = -\pi.$

P2-34

Sol. $\vec{A} = 3x^2y^3\vec{a_x} - x^3y^2\vec{a_y}$

$d\vec{l} = \vec{a_x}\cdot dx + \vec{a_y}\cdot dy$

from $(1,1)$ to $(2,2)$ directly, denoted as path ① : $y=x$.

$\int_① \vec{A}\cdot d\vec{l} = \int_1^2 3x^5\cdot dx - \int_1^2 y^5\cdot dy = \int_1^2 2x^5\cdot dx = 21$

from $(2,2)$ to $(2,1)$ denoted as path ② $x=2$, $dx=0$

$\int_② \vec{A}\cdot d\vec{l} = \int_2^1 -8y^2\cdot dy = \frac{56}{3}$

from $(2,1)$ back to $(1,1)$ denoted as path ③ $y=1$, $dy=0$

$\int_③ \vec{A}\cdot d\vec{l} = \int_2^1 3x^2\cdot dx = -7$

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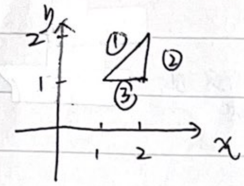
$$\Rightarrow \oint \vec{A}\cdot d\vec{l} = 21 + \frac{56}{3} - 7 = \frac{98}{3}$$

b)

$$\nabla\times\vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3 & -x^3y^2 & 0 \end{vmatrix} = -\vec{a_z}\,(3x^2y^2 + 9x^2y^2) = -12x^2y^2\vec{a_z}$$

$$d\vec{S} = -\vec{a_z}\,dx\,dy$$

$$\int_S \nabla\times\vec{A}\cdot d\vec{S} = \int_S 12x^2y^2\,dx\,dy = 12\int_1^2 y^2\int_1^y x^2\,dx\,dy = \frac{98}{3}$$




c) ?

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