**Estimate Second Moment of Inertia of each element from the Estimated Stiffness Matrix**

Element stiffness matrix of Euler beam

|  |  |
| --- | --- |
|  | (1) |

Stiffness matrix is composed with element stiffness matrix and compatibility matrix .

|  |  |
| --- | --- |
|  | (2) |

Minimization problem is defined as

|  |  |
| --- | --- |
|  | (3) |

The solution of Eq. (3) is obtained by solving following equation, which is the differentiation of Eq. (3) as

|  |  |
| --- | --- |
|  | (4) |

Substituting into Eq. (4) can be converted into the algebraic equation.

|  |  |
| --- | --- |
|  | (5) |

where,

can be implemented using Numpy as

|  |  |
| --- | --- |
|  | (6) |

**Estimate Damping Ratios from the Estimated Damping Matrix**

Damping matrix is composed with modal damping ratios.

|  |  |
| --- | --- |
|  | (7) |

where, is *i*th mass normalized mode shape and .

Minimization problem is defined as

|  |  |
| --- | --- |
|  | (8) |

The solution of Eq. (8) is obtained by solving following equation, which is the differentiation of Eq. (8) as

|  |  |
| --- | --- |
|  | (9) |

Substituting into Eq. (9) can be converted into the algebraic equation.

|  |  |
| --- | --- |
|  | (10) |

where,