This project entails the representation of the Helmholtz Equation in two dimensions using numerical methods in MATLAB. Boundary conditions, domain limits, and constant parameters are prescribed in the assignment. The Gauss-Seidel/Liebmann method with and without relaxation is used to represent the equation.

MATHEMATICAL FORMULATION

The Helmholtz equation is an elliptic partial differential equation of the second order with respect to the spatial variables. It is similar to the Poisson equation but includes an additional output term that is not differentiated. The equation is

The parameter is given as 1. The domain is a rectangle, given as

,

Where

The boundary conditions for y are given as

,

The boundary conditions for x are given as

DISCRETIZATION

For the discretization of this PDE, the centered difference formula is used, where

The spatial terms are set such that where is the area of the squares used to discretize the domain. Substituting the approximate terms into the original equation yields

Solving for ,

This is the equation used to implement the Gauss-Seidel method. The insulated/Neumann boundary condition on the right side of the rectangle is accounted for through the use of a ghost node. The ghost node uses the centered difference formula as such:

Exploiting this fact for the right side of the domain results in

allowing for the use of the discretized formula at the edge of the domain.

PSEUDOCODE

Helmholtz

while error > 0.01

uprev = u

for i = 2:length(x)- 1

for j = 2:length(y) - 1

u(i,j) = [u(i+1,j) + u(i-1,j) + u(i,j+1) + u(i,j-1)

* Δ^2 \* F(i,j)] / (4 - Δ^2 \* Λ)

end

u(i,end) = [2 \* u(i,end-1) + u(i+1,end) + u(i-1,end)

- Δ^2 \* F(i,end)] / (4 - Δ^2 \* Λ)

end

error =

end

MACHINE TECHNICAL SPECIFICATIONS

OS: Windows 10 Home 64-bit

CPU:

* Intel Celeron N2830 2.16 GHz (Dual-core)
* L1 Cache: 2 x 24 KB
* L2 Cache 1024 KB

Memory: 4 GB DDR3 800 MHz

GPU: Intel HD Graphics 2GB VRAM

Disk: WD Blue 500GB 5400 RPM

RESULTS