Design of Gravity Balanced Systems with Workspace in 3D

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***Abstract*— A controlled gravity balancing device is designed and demonstrated in this paper to replace a standard step process to help people reach the place high and reduce or eliminate the force on the knee. This can be used to assist people with knee injuries or people with disability in their knees. We used the Sarrus Linkage mechanism [1] to keep the human body to be raised along the vertical axis without rotation. And we used a 4-cable system to help people move in the horizontal plane. We used one vertical cable to drive the device to rise, and then use the incomplete gravity balance to slow the device down. The Kinect system is used in detecting the movement intention of the person as part of the control system. This device can move in 3D space inside of the Sarrus Linkage mechanism. And the concept of this device can be further developed to assist any person with weakness in their legs to jump or reach.**

I. Introduction

A gravity balanced machine is defined that the joint forces are not needed to keep the system in equilibrium. A gravity balanced system behaves as if it is moving in gravity-less environment. Previous works on a gravity-balanced machines include designs with counterweights, zero free length spring method and auxiliary parallelograms. [3,4,5,6]

To achieve gravity balancing, the potential energy of the system should remain constant in every configuration. Several springs are added to the system at appropriate places to keep the potential energy of the system remains constant.

Based on the previous, a method of gravity balancing planar mechanism with springs, along with the Sarrus Linkage mechanism to keep the object to move vertically is used in this design. This device relates to a simple harness and not interfere with the range of motion. This device can move vertically in a certain range to help people step up and reach a higher position without being affected by gravity. It can also move in the horizontal plane with the four-cable system to reach every workspace inside of the Sarrus Linkage mechanism.

The gravity balanced machine has many aspects of applications. Stepping up to reach high objects can be a chore for people with knee injury or workers who must constantly go up and down, and these people may be more likely to fall and hurt themselves. These people might need this design to make their frequent step and reach process easier in their daily life. The control-based approach can allow the user to move where they want (even an object out of reach by using the ladder) with minimal effort and strain on limbs.

Another application is for rehabilitation. Cerebral palsy (CP) is a group of disorders that affect a person's ability to move and maintain balance and posture.[7] These group of people cannot experience the feeling of a jump, especially the children.

Using this design can be a relief of the pressure and the pain of those disabilities and bring children a sense of jumping without pain and efforts, which is meaningful during the rehabilitation process.

II. Solution Approach

A. Gravity Balancing

The focus of this design is its proposed ability to counter the effect of gravity on the human body, thus allowing the user to move up and down with minimal strain on the knee. This is a technique referred to as gravity balancing.

One of the simplest examples of gravity balancing is the single link and spring system shown in Fig. 1. In this system, the change in potential energy of both the spring and the link is a function of sin(θ). Therefore, when the sin is factored out the remaining terms which make up the sin coefficient can be set to zero, thus eliminating the effect of variable θ on the potential energy. We based our design heavily on this simple solution so that we could achieve a similar effect.

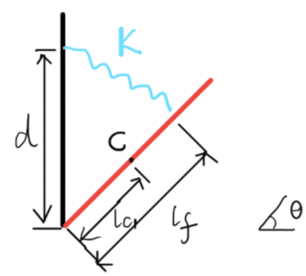

Figure 1: Simple gravity balancing system.

At first, we want the harness to have the ability to move along a single axis without rotation, the attachment point to the harness needs to adhere to this restriction. However, using the simple gravity balancing technique has the restriction to move under the trajectory of the curve due to a variable angle input. In order to achieve both goals, we applied the Peucellier straight line mechanism to our design, shown in Fig. 2. This linkage design can translate the endpoint across a linear path when the angle of the smallest base link is changed.

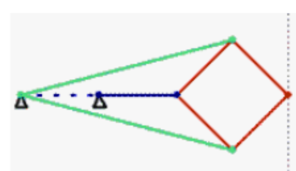

Figure 2: Example of a Peaucellier mechanism. [1]

However, the force transmitted to the link varies as the crank angle changes continuedly, which means the device cannot fully balance the gravity of the object. What’s more, the Peaucellier mechanism limits the degree of freedom to only move up/down. This is obviously not a design we want that can move in 3D space. So, we chose another mechanism called the Sarrus linkage mechanism.

B. Degrees of Freedom and design

The first challenge in designing the mechanism was to determine the ideal number of degrees of freedom (DoFs) for the motion of the harness and therefore the user as well. It is easy to imagine that a full 6-DoF motion would be very unstable, unsafe, and unusable for a majority of users of the device. It was best for both simplicity and safety that the DoFs are as limited as possible.

Roll and pitch rotations would be very uncomfortable due to shifting the user's center of mass away from the stable location, and would also unnecessarily tilt the head. The yaw rotation could have some benefits in special applications, so we set a connection between the human body and the Sarrus Linkage mechanism, which is a mechanism using the combination of inner and outer rings with rotational freedom.

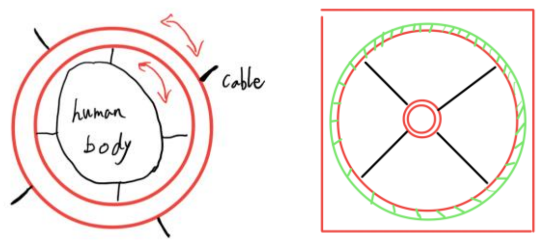

Figure 3a: The mechanism of the yaw rotation. Figure 3b: The connection between the yaw rotation mechanism and the Sarrus linkage mechanism.

The small red circle is the yaw rotation mechanism which is connected to the human body and the black line is the four-cable system. The green part is using a soft material to prevent people from getting injured.

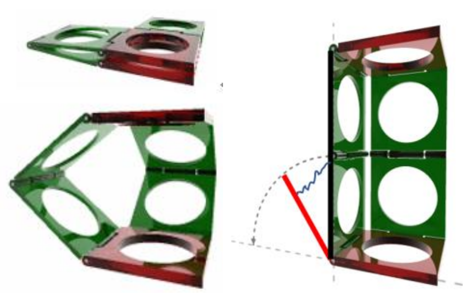

Figure 4. Design of the gravity balancing system using the Sarrus linkage mechanism. [2]

Then we used the gravity balancing system to provide the DoF along the vertical (superior/inferior) axis. There is one black bar connect with the red bar using a spring, it's the same as the simplest gravity balancing mechanism. And we used one vertical cable to drive the device to rise, and then use the incomplete gravity balance to slow the device down. And we used the four-cable system to provide the DoF of moving in a horizontal plane.

For the control part, we placed the Kinect device to detect the moving intention of the human and decide which direction should the device move the human body. The red point is a control point set in the Kinect software and after the human moved to that point, the device will stop.

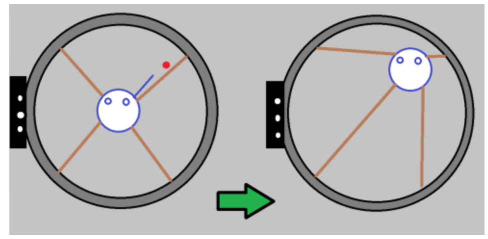

Figure 5a. the Lateral movement controls

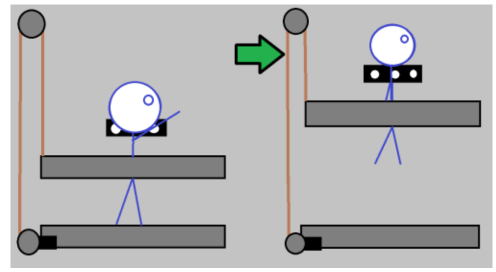

Figure 5b. the Vertical movement controls

C. Draft of the design

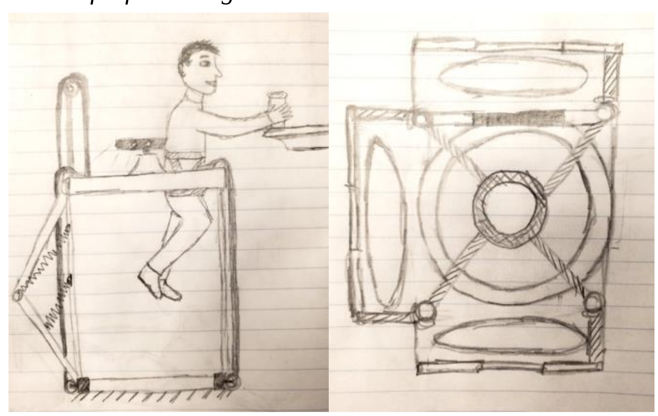

Figure 6. Design of the whole system.

III. Calculations and Results

A. Sarrus Gravity Balancing

Assuming a point mass equal to the user's weight is fixed on the top of the Sarrus mechanism, the potential energy equation is expressed as:

$$P = mgh + \frac{N}{2}kx^2 \tag{1}$$

Where m is the mass of the user, g is the gravitational constant, h is the height of the user relative to the fixed base of the mechanism, N is the number of attached springs used for the balancing, k is the spring constant, and x is the spring length. In order to rewrite this equation using only one variable, h and x need to be written in terms of the crank angle as follows:

$$h = 2l sin(\theta) \tag{2}$$

$$x^2 = h_s^2 + l_c^2 - 2h_s l_c \sin(\theta) \tag{3}$$

Where l is the length of the links h_s is the fixed height of the point at which the spring is connected to the device, l_c is the distance to the center of the link where the spring is connected, and θ is the angle of the crank link.

Plugging these terms back into the original equation we get:

$$P = 2mglsin(\theta) + \frac{N}{2}k(h_s^2 + l_c^2 - 2h_s l_c \sin(\theta)) \quad (4)$$

To gravity balance the system, the above equation needs to be constant, i.e. all the terms that include variable θ must cancel out:

$$2mgl - Nkh_s l_c = 0 \quad (5)$$

Therefore, the spring constant required to gravity balance this system is calculated as:

$$k = \frac{2mgl}{Nh_S l_c} \quad (6)$$

Assuming $m = 70kg$, $g = \frac{9.81m}{s^2}$, $N = 6$, $l = 1m$, $l_c = 0.5m$, $h_s = 2.5m$, we get $k = \frac{183.12N}{m}$ for each of the springs which is very reasonable.

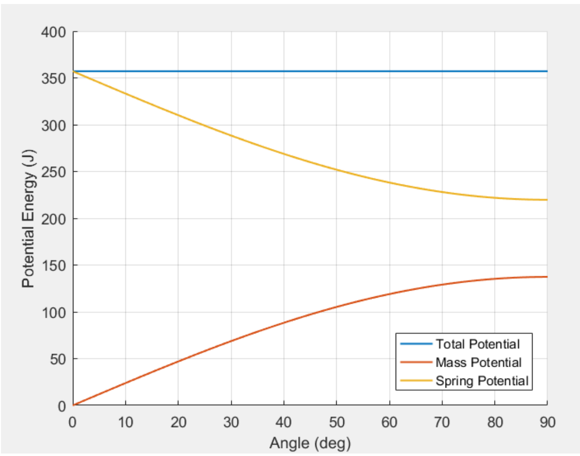

Figure 7: Potential energy curves for all possible values of the crank angle. Note that while mass and spring potentials change, the total potential remains constant.

B. *Vertical PD-Controller*

The vertical dynamic equation of motion is written as:

$$T - mg(1 - r) = ma \quad (7)$$

Where T is the tension of the cable on the device, r is the percentage of reduction of the user's weight as controlled by the gravity balancing mechanism, and a is the net acceleration as a result of both tension and gravity. In order to control this device, the Microsoft Kinect will recognize a vertical arm motion and set a one-dimensional control point relating to desired height based on the direction of the gesture. The relationship between the control point and the angle of the crank link is:

$$h_{des} = \arcsin\left(\frac{\theta_{des}}{2l}\right) \quad (8)$$

Rearranging the equation of motion to solve for cable tension yields:

$$T = ma + mg(1 - r) \quad (9)$$

To apply this equation for the PD controller, the net acceleration is taken to be:

$$a = \ddot{z} - k_p(z_{des} - z) - k_d(\dot{z}_{des} - \dot{z}) \quad (10)$$

where z represents the current vertical position of the point mass. Therefore, by substituting Eq. 10 into Eq. 9, for each time step the controller will obey the following law:

$$T = m(\ddot{z} - k_p(z_{des} - z) - k_d(\dot{z}_{des} - \dot{z}) + g(1 - r) \quad (11)$$

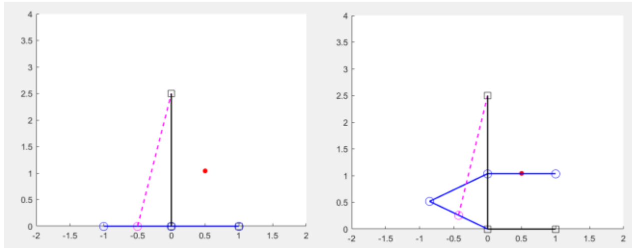

Figure 8: Start and end frames of the simulation for vertical control. Sarrus mechanism is composed of rigid (black) and dynamic (blue) components and springs (magenta). The red point indicates the control point set by the computer.

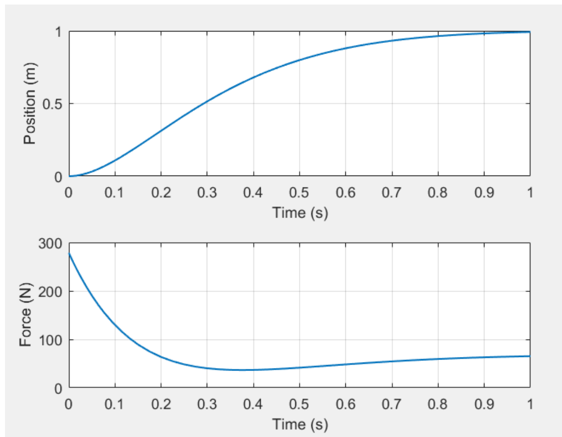

Figure 9: Position and force curves for vertical control when $kp = 30$ and $kd = 10$. Mass was reduced by 90% ($r = 0.9$)

B. *Horizontal Controller*

For the horizontal controller, the user point mass is supported by four cables instead of one. This adds complexity to the control as it is a redundant form of control. In order to simplify this problem, we introduce some constraints. First, the range of motion is limited to the horizontal X-Y plane which differentiates it from the SkyCam control that this design took inspiration from [9]. In other words, we neglect the vertical Z-axis for the control calculations. Second, we limit the freedom of the control point to just direction while keeping distance constant. This constraint paired with a desired time of completion makes it convenient to use a velocity profile to control the motion of the point mass.

To achieve a linear acceleration, the equations for position and velocity can be expressed as:

$$x(t) = at^3 + bt^2 + ct + d \quad (12)$$

$$\dot{x(t)} = 3at + 2bt + c \quad (13)$$

Providing the initial conditions $x(0) = 0$, $x(1) = \frac{l}{4}$, $\dot{x}(0) = \dot{x}(1) = 0$, the constants can be solved for and the equation is simplified to:

$$x = -\frac{1}{2}t^3 + \frac{3}{4}t^2 \tag{14}$$

$$\dot{x} = -\frac{3}{2}t^2 + \frac{3}{2}t \tag{15}$$

Using this profile, the vectors representing each cable can be found for each time step using the following equation:

$$\boldsymbol{q_i} = [x(t),0] * \boldsymbol{u} - \boldsymbol{O_i} \tag{16}$$

Where q_i is the vector representing the ith cable, u is the unit vector pointing from the center of the mechanism to the desired location, and O_i is the origin coordinate of the ith cable. The lengths of each cable can be found taking the magnitude of each vector:

$$\|q_i\| = \sqrt{(q_{ix})^2 + \left(q_{iy}\right)^2} \tag{17}$$

Given the velocity profile, the net force acting on the point mass is determined to be:

$$F = m([\ddot{x}(t), 0] * \boldsymbol{u}) \tag{18}$$

The tension on each active cable is then defined as a percentage of the net force based on relative dot product.

A consequence of this control method is that it does not implicitly prevent cables from going slack by trying to produce a negative tension. However, due to the symmetry of this design, it is guaranteed that at least one cable will always be in tension . Therefore, instead of asking a cable to produce negative tension, at each time step, the controller will request additional tension from all cables that is proportionally the same to eliminate the negative forces while maintaining the desired velocity.

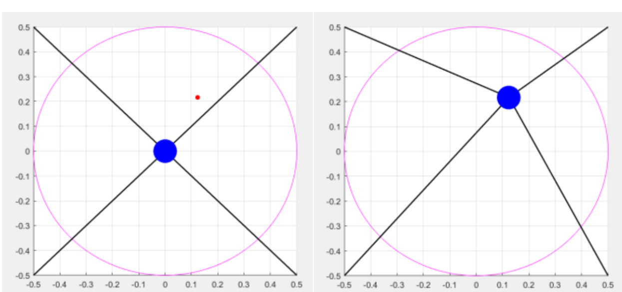

Figure 10: Start and end frames of the simulation for horizontal control. Four cables (black) are attached to point mass (blue). Movement is bounded by a circle (magenta). The red point indicates the control point set by the computer.

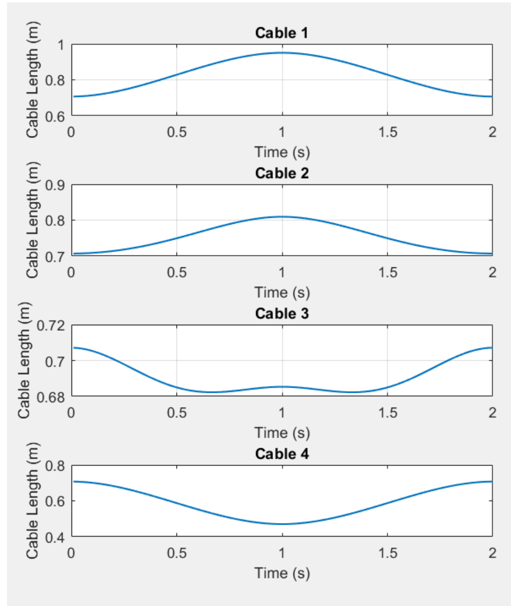

Figure 11: Cable lengths for the four cables for horizontal control. Cables are defined as the bottom left (1), bottom right (2), top left (3), top right (4). Symmetry indicates moving to control point and then returning to start position.

IV. Conclusion

A. Summary

From our original design, we added a vertical control to the gravity balancing system. The user can adjust their height without applying force. Using a four-cable control system connected to the top of the Sarrus linkage [8,9], we can add limited horizontal translational degrees of freedom so that the user is not restricted to the vertical axis.

The Kinect can detect the user's intent and set a control point (desired end effector position) accordingly for the control system to solve.

Overall this design is much more flexible than the original and gives the user more range of motion while removing some limitations.

B. Future Extension

For the former iteration, we focused on the gravity balancing equations and worked through the problems that arose. For this iteration, we focused on how can design this device to move freely in 3D space. For the future extension, the problems of the gravity balancing mechanism consist of two parts: one is what if the gravity of the object changed, the second is that does the mechanical structure's self-weight effect the function?

For the first problem, we can adjust it by changing the connecting position of the spring. According to the equations, it can always balance the gravity of the object in a normal range. And under the condition that we set the device to not fully balance the gravity of the object, we used one vertical cable to drive the device to rise, and then use the incomplete gravity balance to slow the device down. So, the spring length might not be changed frequently since the weight of humans is similar.

For the second problem, many modifications need to be made for stability in order to make the device mobile as originally intended. Right now, the overall frame is very large so it would be best if we could reduce the size. We will have a wheel mechanism to move and self-locked while working. The wheel locking mechanism can stop the wheels from rolling when the step is being used. This can be achieved by lowering a stopper on the wheel when the step is pressed.

Finally, through testing the position of the Kinect should be optimized for the best readability of the human's motion.

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