

# Feature Decorrelation

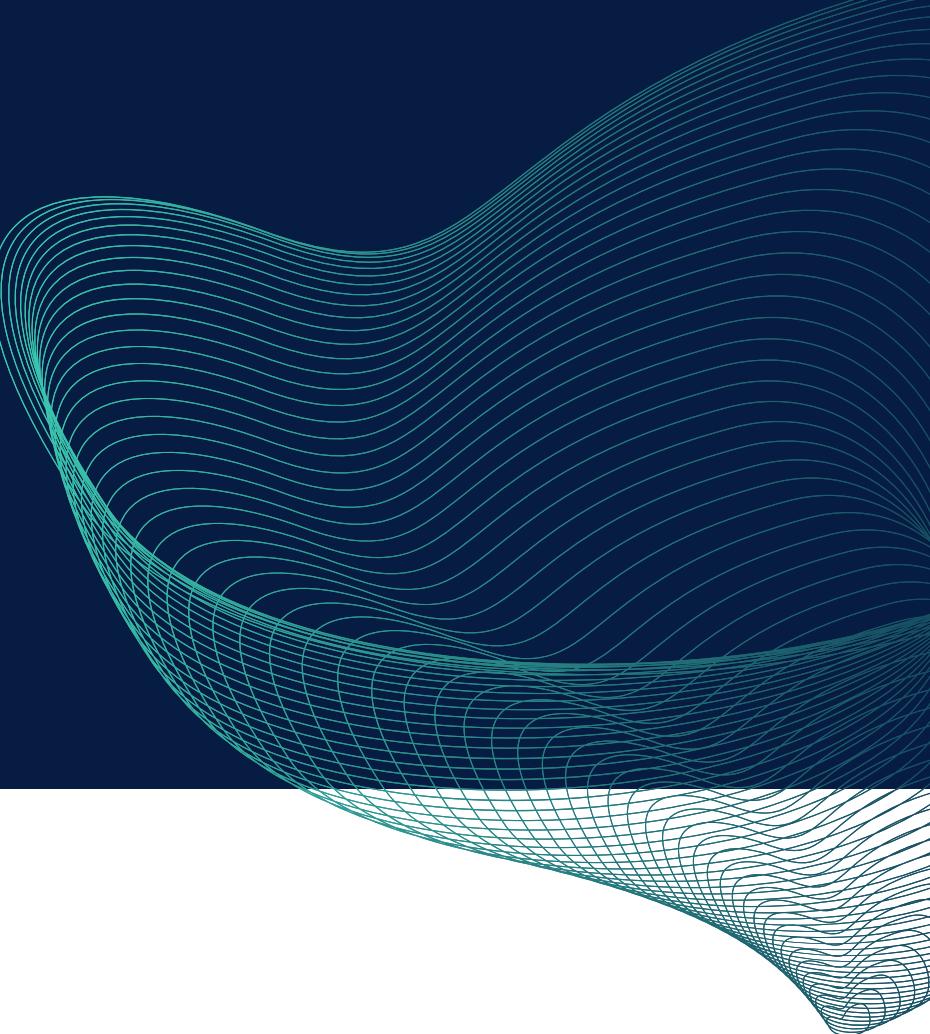
Presented by Davide Perozzi and Fabian Vincenzi

# Introduction

## Problem overview

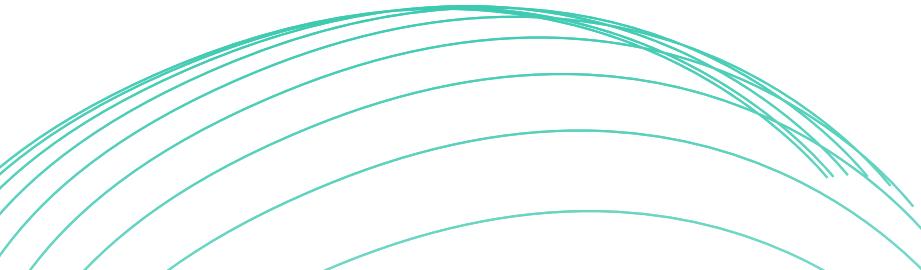
The task is to investigate the effect of feature decorrelation in model training, specifically the stability of the training.

To achieve this, we built a dataset and its decorrelated version with three different standard data distribution graphs and two with noise and observed the difference in training with two different model architectures.



# Graphs

Using the Python library Datagen we created three different graphs:



## Graph 1

```
A = dg . normal (mu=0, sigma =1)
```

```
B = dg . normal (mu=0, sigma =1)
```

```
C = A + B
```

```
D = A + B - 3 * C
```

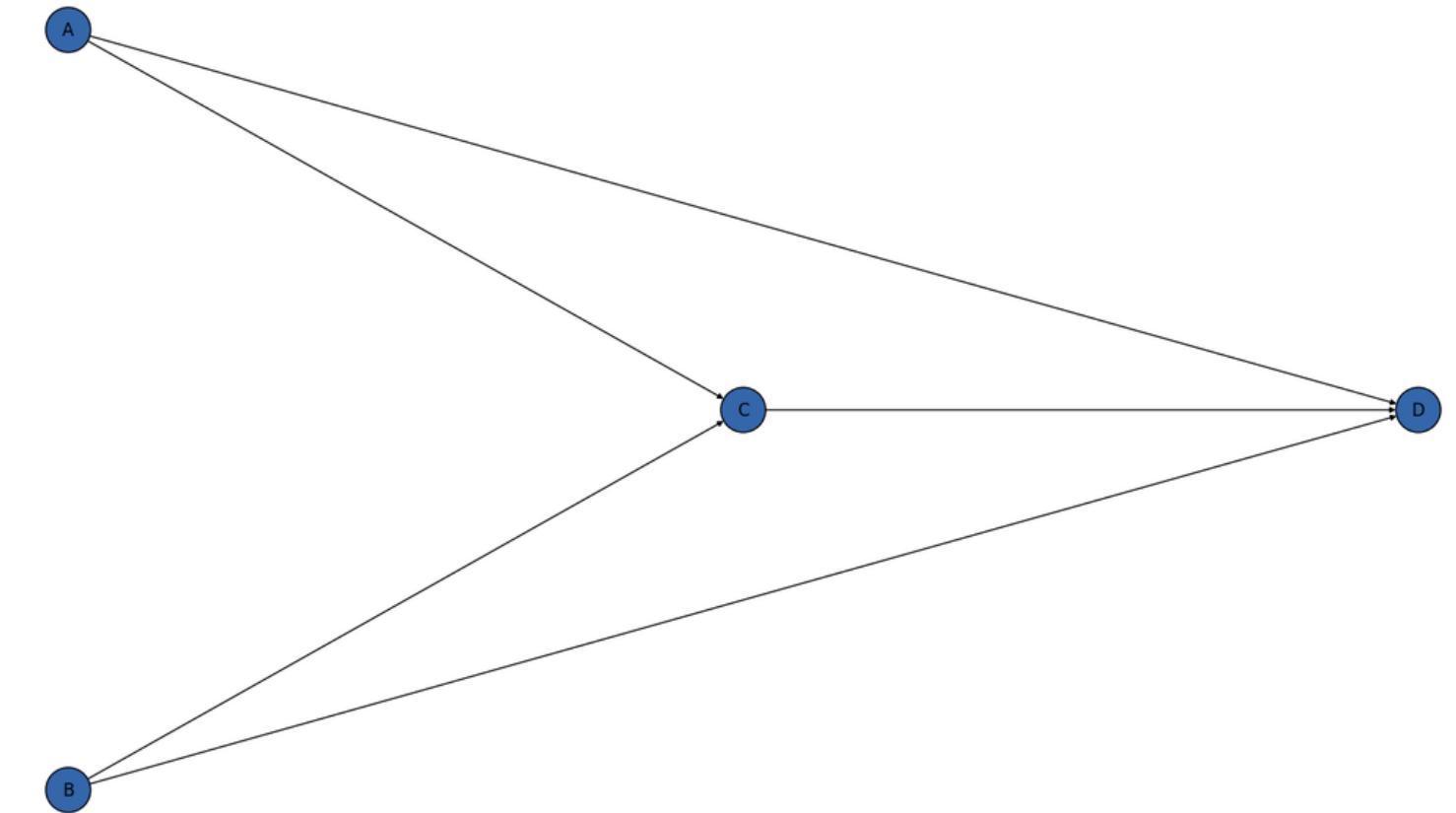
## Graph 2

```
A = dg . normal (mu=0.5 , sigma =0.5)
```

```
B = dg . normal (mu=0.5 , sigma =0.5)
```

```
C = A + B
```

```
D = A + B - 3 * C
```



## Graph 3

```
A = dg . normal (mu=1.0 , sigma =0.2)
```

```
B = dg . normal (mu=0.8 , sigma =0.4)
```

```
C = A + B
```

```
D = A + B - 3 * C
```

# Graphs with noise

The first 2 graphs have also  
a noisy version defined so:



## Graph with noise #1

```
A = dg . normal (mu=0, sigma=1)
```

```
B = dg . normal (mu=0, sigma=1)
```

```
C = A + B
```

```
D = A + B - 3 * C + dg . n o i s e ( )
```

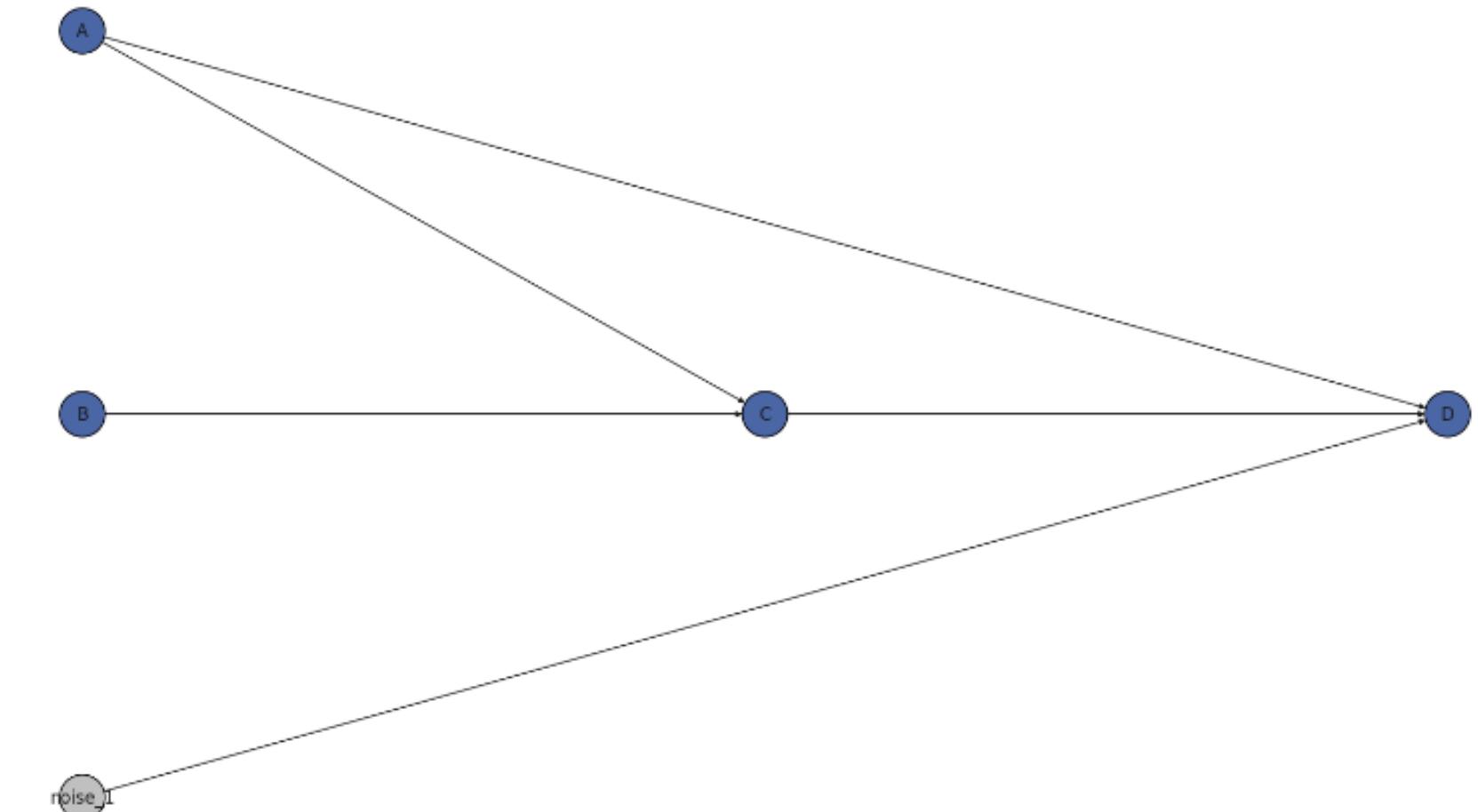
## Graph with noise #2

```
A = dg . normal (mu=0.5 , sigma=0.5)
```

```
B = dg . normal (mu=0.5 , sigma=0.5)
```

```
C = A + B
```

```
D = A + B - 3 * C + dg . n o i s e ( )
```



# Dataset

We generated a total of 10,000 samples.

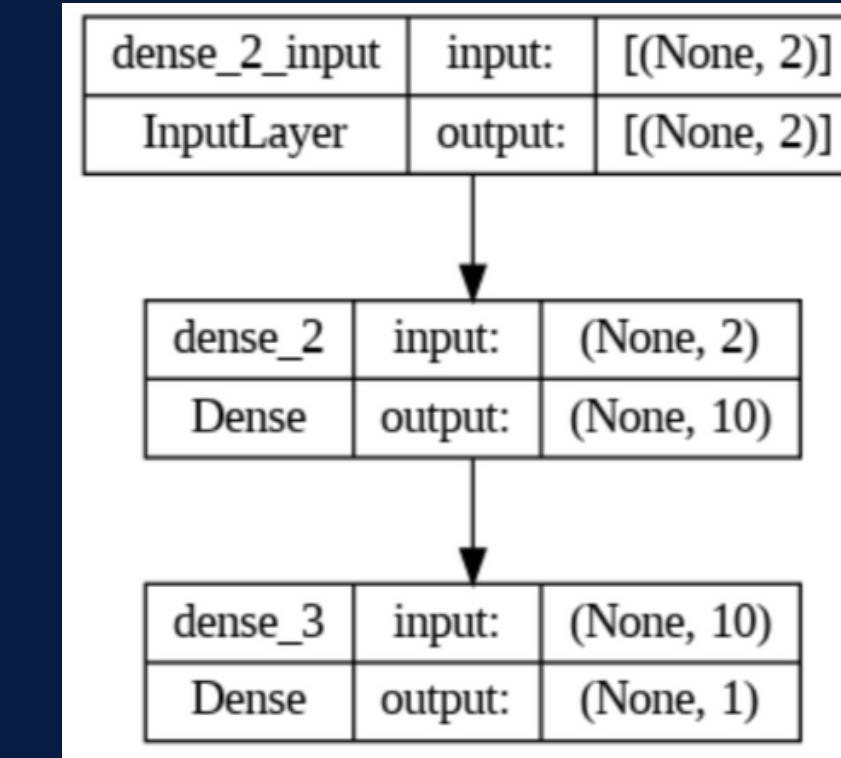
Using this data we compute the decorrelated feature C\_prime.

C\_prime is defined as the features C minus the correlation with A and B.

$$C_{\text{prime}} = C - M(A, B)$$

To do so we trained a NN Model using Keras library. The Model captured the correlation between features A,B and predict C, giving as output M(A,B).

Next, we can compute C\_prime. This allowed us to decorrelate feature C from features A and B in the dataset, and thus investigate the impact of feature decorrelation on training stability.



	A	B	C	D	C_new	C_prime
0	0.875226	0.380171	1.255397	-1.255397	1.255379	0.000017
1	0.970282	-0.045318	0.924964	-0.924964	0.924950	0.000014
2	-0.475518	1.931827	1.456309	-1.456309	1.456267	0.000042
3	-0.151090	-0.001890	-0.152980	0.152980	-0.152998	0.000019
4	0.563920	-0.248622	0.315298	-0.315298	0.315284	0.000014

# Training

We implemented two stochastic models from Keras and SciKit-Learn libraries to evaluate the impact of feature decorrelation.

We trained both models with 25 different seeds for 2 epochs.

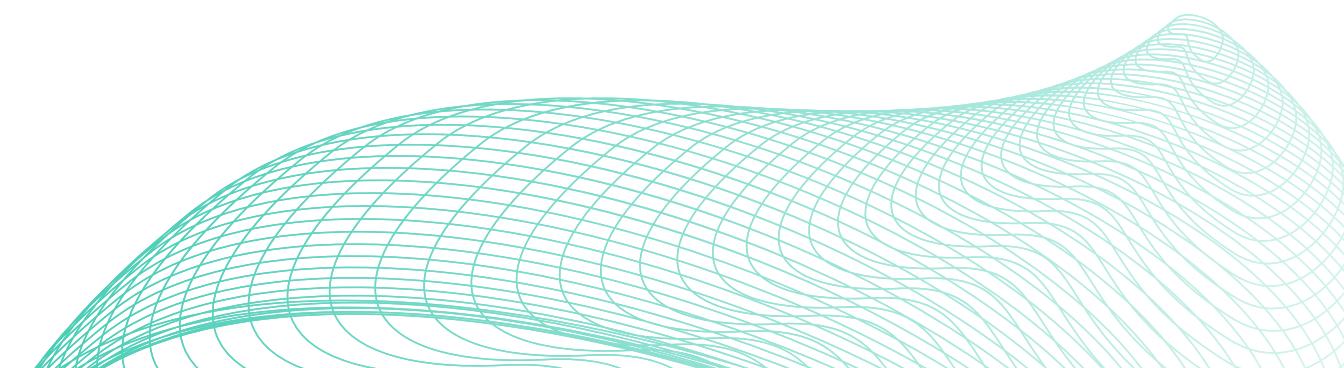
We collected the coefficients (feature importances) at the end of each training and for each seed, then we computed the distance of the coefficients vector from the mean coefficients vector using the L1 and L2 distance metrics.

## Stochastic Linear Regression

```
keras.Sequential([  
    Dense (1, inputdim=3, activation='linear')  
])
```

## Stochastic Decision Tree

```
sklearn.tree.DecisionTreeRegressor(  
    splitter="random",  
    randomstate=seed  
)
```



# Results

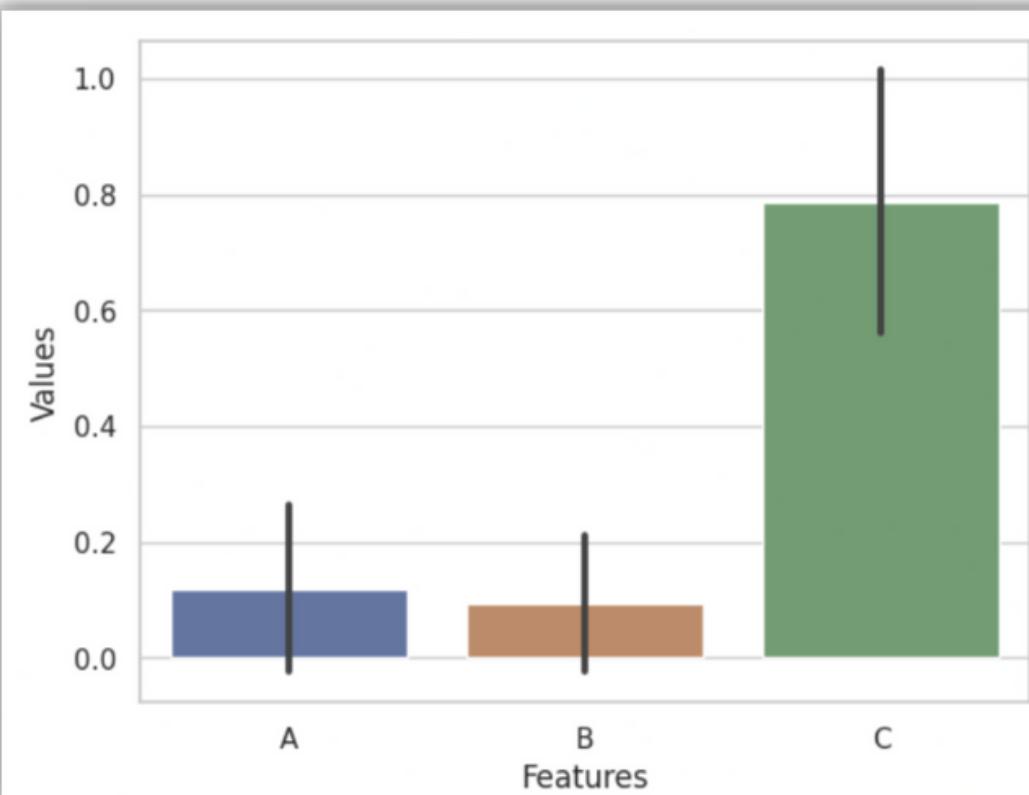
We conducted a comparison between the models, trained on C\_prime and C, L1s and L2s metrics, specifically analyzing the ratio of them.

This comparison aimed to detect an improvement in the variability of the final coefficients.

Our results in the table demonstrate that the ratios of the average L1 and L2 are consistently below 0.35 for SDT and below 0.95 for SLR. This indicates that feature decorrelation significantly improves training stability.

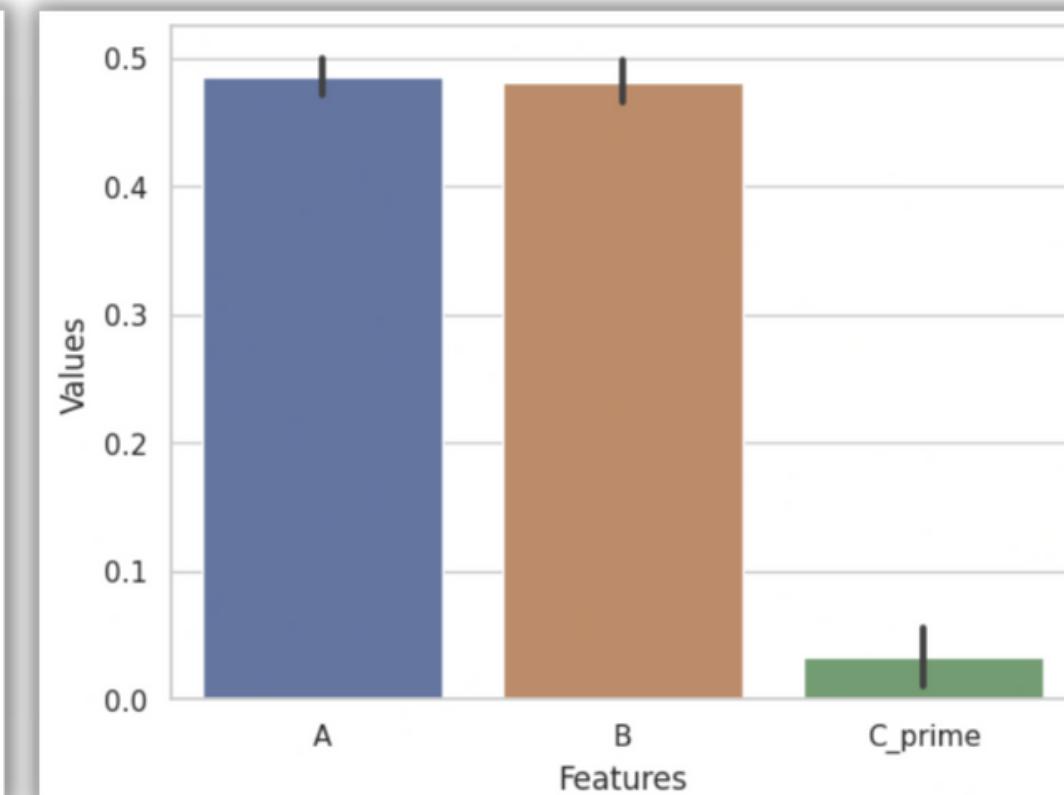
Notably, this improvement is particularly pronounced when using SDT (Stochastic Decision Tree) model.

This behaviour is also confirmed in presence of noise.



Train with C

Graph 1, SLR



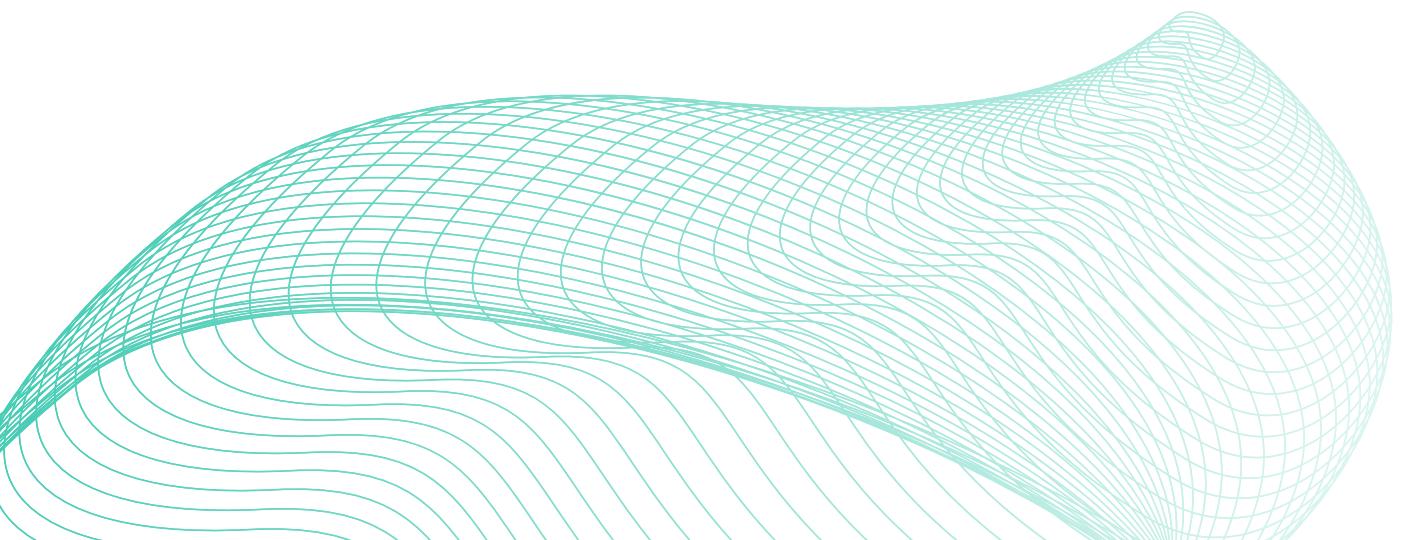
Train with C\_prime

Graph	Model	Ratio L1 C'/C	Ratio L2 C'/C
Graph 1	SLR	0.499974	0.864705
Graph 1	SDT	0.096018	0.097270
Graph 2	SLR	0.568582	0.864672
Graph 2	SDT	0.174069	0.170915
Graph 3	SLR	0.785187	0.931922
Graph 3	SDT	0.355453	0.339534
Graph w/ noise	SLR	0.498876	0.863772
Graph w/ noise	SDT	0.089377	0.091166
Graph w/ noise 2	SLR	0.568750	0.863956
Graph w/ noise 2	SDT	0.162088	0.165554

# Error Analysis

We wanted to investigate a deteriorating of other performance indicators.

We collected the R2 score of the predictions for each seed and calculated the average for each model.



Our values indicate that for SLR the feature decorrelation increase the loss because the C\_prime R2 score is consistently slightly lower than the C R2 score.

This aspect should be carefully considered during a cost-benefit analysis.

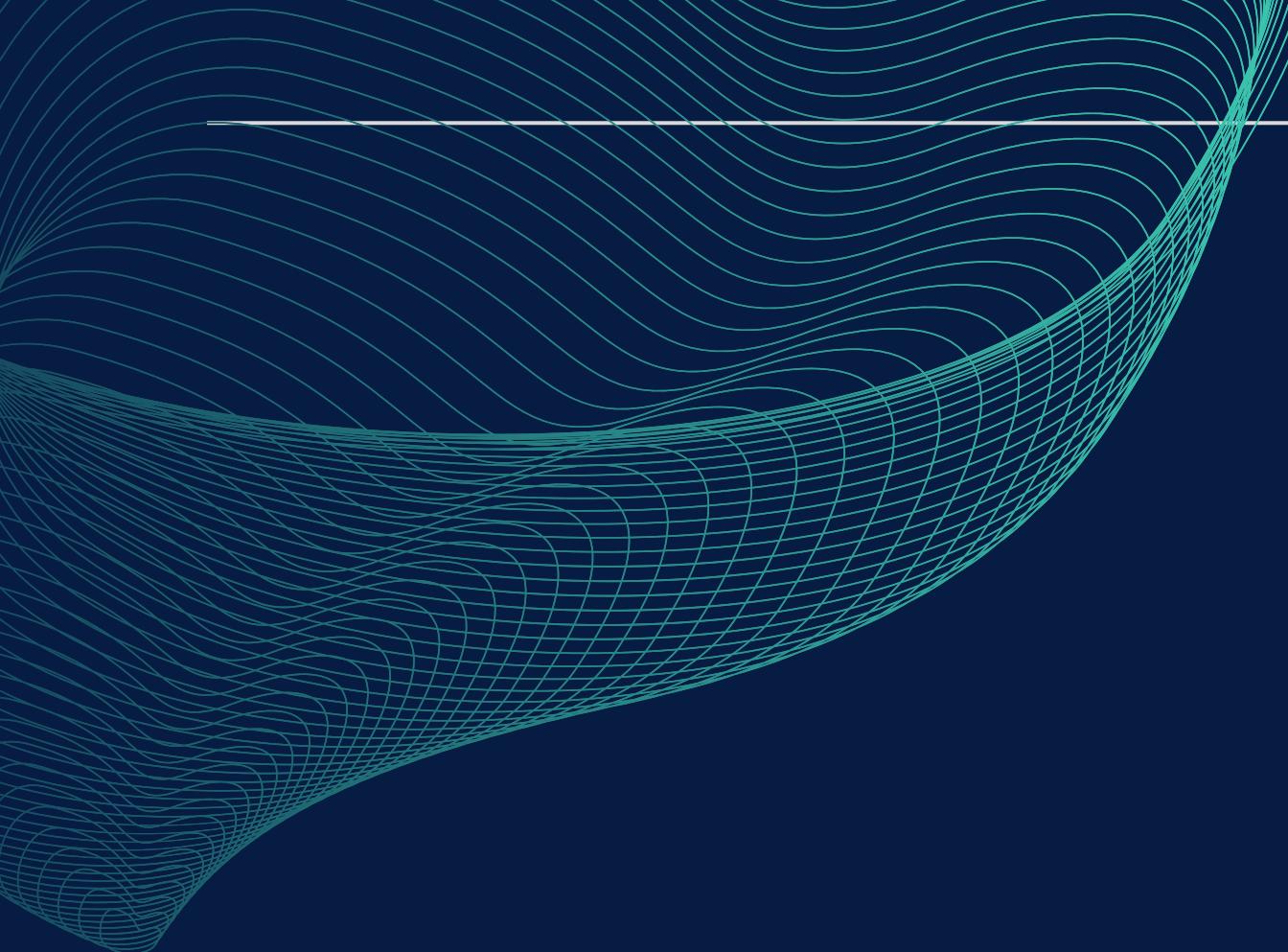
Graph	Model	R2 score C	R2 score C'
Graph 1	SLR	0.999999	0.999969
	SDT	1.0	1.0
Graph 2	SLR	0.999696	0.991859
	SDT	1.0	1.0
Graph 3	SLR	0.989710	0.956236
	SDT	1.0	1.0
Graph w/ noise	SLR	0.888754	0.888872
Graph w/ noise	SDT	1.0	1.0
Graph w/ noise 2	SLR	0.668298	0.663281
	SDT	1.0	1.0

# Conclusion

Findings:

- Feature decorrelation significantly improves training stability,
- Improvements are greater using SDT model,
- The improvement is achieved also in presence of noise,
- Loss is slightly increased.

In conclusion we can say that feature decorrelation offers benefits in terms of improving training stability but it is important to acknowledge that it can result in a higher loss.



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**THANK YOU FOR YOUR ATTENTION**

