Exercise sheet 1

1. Show that $\pi_n(X, A, x)$ has natural group structure for $n \geq 2$, and it is abelian for $n \geq 3$.

   Hint:

2. Finish the proof of the long exact sequence of homotopy groups by showing the exactness at the term $\pi_n(X, x)$.

3. Let $f : X \to Y$ satisfy the Homotopy Lifting Property for the space T. Show that

   (a) If $g : Y \to Z$ and $e : W \to X$ are homeomorphisms, then $g \circ f \circ e$ satisfies HLP for T;

   (b) For any map $k : V \to Y$ the pullback of f, i.e. the canonical map $V \times_Y X \to V$, satisfies HLP for T.

4. Construct the Serre fibration

$$S^3 \hookrightarrow S^7 \to S^4$$

   Hint: