Proofs for final exam

Analysis IV, Spring Semester

EPFL, Mathematics section, Prof. Dr. Maria Colombo

- This is an exhaustive list of the *theorems* that you can be asked to **state** and **prove** during the final exam.
- For each proof, a reference in either Tao's book or Dacorogna's lecture notes is mentioned. They are just here in case you don't find in your notes the proof presented in class, you don't have to necessarily read them. In principle, the latters and the references are fairly similar, but there might be small differences in the way they are presented. Both versions are acceptable during the exam.
- The statement of any definition mentioned during the lectures can be asked.
- All other material treated in class can be useful for solving the exercises, but no proof on them will be asked.
- Partial proofs may be asked. For example, in the midterm you were asked to prove only part of Tonelli-Funbini's theorem. Another example could be to state the theorem on the approximation of L^p functions by C_c^∞ functions and prove only the approximation of C^0 functions by C_c^∞ functions.

Measure Theory

- Countable sub-additivity of the outer measure (Lemma 7.2.5, (x) in the book of Tao).
- For any open box B, $m^*(\bar{B}) = \text{vol}(B)$ (Proposition 7.2.6 in the book of Tao).
- Countable additivity of the Lebesgue measure (Lemma 7.4.8 in the book of Tao).
- Measurability of the sup, inf or pointwise limit of a sequence of measurable functions (Lemma 7.5.10 in the book of Tao).
- Linearity of the Lebesgue integral for simple functions (Proposition 8.1.10, (b) and (c) in the book of Tao).
- Fatou's lemma (Lemma 8.2.13 in the book of Tao).
- Lebesgue monotone convergence theorem (Theorem 8.2.9 in the book of Tao).
- Lebesgue dominated convergence theorem (Theorem 8.3.4 in the book of Tao).
- Fubini's theorem (Theorem 8.5.1 in the book of Tao).

- Hölder's inequality (Théorème 16.27 in Dacorogna's lecture notes).
- Minkowski's inequality (Théorème 16.28 in Dacorogna's lecture notes).
- Every convergent sequence in L^P has a pointwise convergent subsequence (Théorème 16.33 in Dacorogna's lecture notes).
- Density of C_c^∞ in L^P (Théorème 16.29 in Dacorogna's lecture notes).

Fourier series and Fourier transform

- Weierstrass Theorem (Theorem 5.4.1 in the book of Tao).
- Inversion formula and Parseval's identity for trigonometric polynomials (Page 115 and Corollary 5.3.6 in the book of Tao).
- Pointwise convergence of Fourier series (Théorème 17.16 in Dacorogna's lecture notes).
- Convergence in L^2 of Fourier series (Théorème 17.17 in Dacorogna's lecture notes).
- Parseval's identity for Fourier series (Théorème 17.17 in Dacorogna's lecture notes).
- Uniform convergence of Fourier series (Théorème 17.18 in Dacorogna's lecture notes).
- Fourier transform of derivative (Théorème 18.2, (iii) in Dacorogna's lecture notes).
- Fourier inversion formula (Théorème 18.5 in Dacorogna's lecture notes).
- Plancherel's identity for Fourier transform (Théorème 18.7 in Dacorogna's lecture notes).

PDEs

- Result on the heat kernel and the solution of the heat equation (section 19.2.2 and 19.2.3 in particular Theorem 19.9 in Dacorogna's lecture notes).