Assignment 2

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Problem Set 4, Exercise 1.a, 1.b

1.a)

First, suppose that $x_i \neq 0$, then note that the column basis contains the i'th vector.
We get that for β our current basis and $B = \operatorname{col}_\beta(A)$ the associated matrix

$$B^{-1} \operatorname{col}_i(A) = e_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \end{pmatrix}$$

where the 1 appears in the i-th position.
Hence

$$\overline{c}_i = -c_\beta^T B^{-1} \operatorname{col}_i(A) + c_i = -c_\beta^T e_i + c_i = -c_i + c_i = 0.$$

Now suppose that $\overline{c}_i \neq 0$, and suppose $x_i \neq 0$ also, then x_i is in the column basis which, by the computation above implies that $\overline{c}_i = 0$, hence $x_i = 0$, concluding the proof.

1.b)

Problem Set 5, Exercise 1.a, 1,b

1.a)

The data of the program is

$$c = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

and

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & -1 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$

Hence we first turn it into a minimization problem by multiplying the cost by -1

$$\begin{aligned} P' = \min -3x_1 - x_2 - 4x_3 \\ \text{s.t. } 2x_1 + x_2 + x_3 \leq 1 \\ x_1 - x_2 + 2x^3 \leq 1 \\ -x_1 - x_2 + x_3 \leq 1 \\ -2x_1 + x_2 - x_3 \leq 1 \end{aligned}$$

Hence our dual program will be

$$\begin{aligned} P' = \min \quad b \cdot \lambda \\ \text{s.t. } \lambda^t A = -c^T \\ \lambda \geq 0 \end{aligned}$$

which when written out is

$$\begin{aligned} P' = \min \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\ \text{s.t. } \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & -1 & 1 \\ -2 & 1 & -1 \end{pmatrix} = \begin{pmatrix} -3 & -1 & -4 \end{pmatrix} \\ \lambda_i \geq 0 \forall i \end{aligned}$$

1.b)

We use the fact that the duality map is an involution.
We first turn the LP into a minimization problem

$$\begin{aligned} \min \quad -x_1 - 4x_2 + 2x_3 + x_4 \\ 3x_1 + x_3 + 2x_4 = 6 \\ -x_1 + 2x_2 - x_3 - x_4 = 2 \\ x_i \geq 0 \end{aligned}$$

Setting $c = \begin{pmatrix} -1 & -4 & 2 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 6 & 2 \end{pmatrix}$ and

$$A = \begin{pmatrix} 3 & 0 & 1 & 2 \\ -1 & 2 & -1 & -1 \end{pmatrix}$$

This gives us a dual program

$$\begin{aligned} \max \quad & b \cdot \lambda \\ \text{s.t. } & A^T \lambda \leq c \end{aligned}$$

Which, when written out yields

$$\begin{aligned} \max \quad 6\lambda_1 + 2\lambda_2 & \\ \text{s.t. } 3\lambda_1 - \lambda_2 &\leq -1 \\ 2\lambda_2 &\leq -4 \\ \lambda_1 - \lambda_2 &\leq 2 \\ 2\lambda_1 - \lambda_2 &\leq 1 \end{aligned}$$

as the dual program.