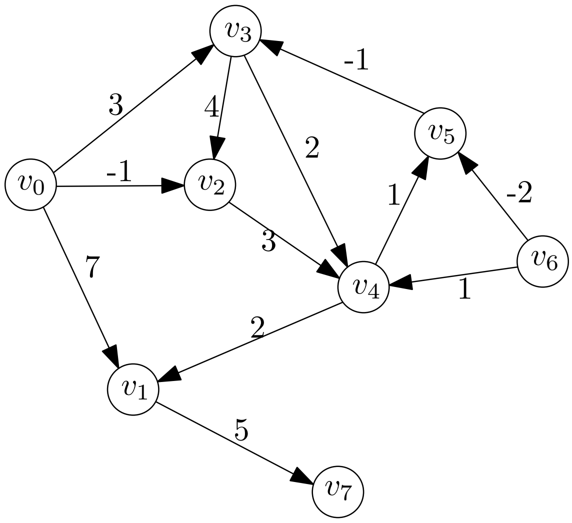
École Polytechnique Fédérale de Lausanne
Prof. Marcus May 19, 2022

Math 261 – Discrete Optimization (Spring 2022)

Assignment 11

Problem 1

In the following network, compute the minimum cost path from v_0 to all other vertices using the Bellman-Ford algorithm. Note the vector of shortest lengths for each iteration t.



Problem 2

Let D be a network with no negative cost cycles and let $\mathbf{B} \in \mathbb{R}^V$ be a vector with the values of the minimum cost walks from v_i to v_n. Show how you can use $\mathbf{B}$ to construct the actual minimum costs walks.

Problem 3

Given a directed graph $D = (V, W)$, the Bellman-Ford algorithm defines a collection of vectors $\mathbf{B}(t)$ for $t = 0, \ldots, n = |V|$ using

$$B_i(t) = \{\text{the smallest cost walk that one can have from } v_i \text{ to } v_n \text{ using at most } t \text{ edges }\}$$

with $B_i(t) = \infty$ if no walk of length at most t exists. Assume all edge costs are finite.

1. Show that $B_i(n-1) < \infty$ if and only if there exists a directed walk from v_i to v_n in D.
2. Assume that $B_i(n-1) < \infty$ for all i. Show that $\mathbf{B}(n) = \mathbf{B}(n-1)$ if and only if there is no negative cost cycle.

Problem 4

Due to the decentralized nature of the global currency market, it might be the case that an individual or an organized group makes a large profit without risk. Arbitrage is a phenomenon that refers to cases when it is possible to convert one unit of a currency into more than one unit of the same currency by using discrepancy in exchange rates. For example, consider the case that 1 CHF buys 60 RUB, 1 RUB buys 0.019 USD and 1 USD buys 0.93 CHF. This means that a trader can transform 1 CHF into $60 \cdot 0.019 \cdot 0.93 = 1.0602$ CHF gaining a profit of 6.02%.

1. Given a list of currencies $r_1, \ldots, r_n$ and a matrix $E \in \mathbb{R}_{>0}^{n \times n}$ where $E_{i,j}$ denotes the amount of currency r_j that one can buy for 1 unit of r_i (the exchange rate between currencies r_i and r_j), design an algorithm to test if there is a possibility of arbitrage.

2. Show how you can use your algorithm to find the arbitrage (not just show it exists).

   Hint: First find the length of the shortest walk (in terms of number of edges) that ends at v_n and contains a negative cost cycle.