École Polytechnique Fédérale de Lausanne
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Math 261 – Discrete Optimization (Spring 2022)

Assignment 5

Problem 1

Determine the dual program for each of the following linear programs:

(a)

$$\begin{array}{lrcrcrcl} \max & 3x_1 & + & x_2 & + & 4x_3 & & \\ & 2x_1 & + & x_2 & + & x_3 & \leq & 1 \\ & x_1 & - & x_2 & + & 2x_3 & \leq & 1 \\ & -x_1 & - & x_2 & + & x_3 & \leq & 1 \\ & -2x_1 & + & x_2 & - & x_3 & \leq & 1 \end{array}$$

(b)

$$\begin{array}{lrcrcrcrcl} \max & x_1 & + & 4x_2 & - & 2x_3 & - & x_4 & & \\ & 3x_1 & & & + & x_3 & + & 2x_4 & = & 6 \\ & -x_1 & + & 2x_2 & - & x_3 & - & x_4 & = & 2 \\ & & & & & & & x_i & \geq & 0 \end{array}$$

(c)

$$\begin{array}{lrcrcrcr} \min & 7x_1 & + & x_2 & - & 5x_3 & & \\ & x_1 & - & x_2 & + & 3x_3 & \geq & 10 \\ & 5x_1 & - & 2x_2 & - & x_3 & \geq & 6 \\ & & & & & \mathbf{x} & \geq & 0 \end{array}$$

(d)

$$\begin{array}{lrcrcrcrcr} \min & 3x_1 & + & 2x_2 & - & 3x_3 & + & 4x_4 & & \\ & 2x_1 & - & 2x_2 & + & 3x_3 & + & 4x_4 & \leq & 3 \\ & & & x_2 & + & 3x_3 & + & 4x_4 & \geq & -5 \\ & 2x_1 & - & 3x_2 & - & 7x_3 & - & 4x_4 & = & 2 \\ & & & & & & & x_1 & \geq & 0 \\ & & & & & & & x_4 & \leq & 0 \end{array}$$

Problem 2

Find a linear program $\mathcal{P}$ such that $\mathcal{P}$ is infeasible and its dual $\mathcal{D}$ is infeasible as well.

Problem 3

Consider the linear program from Problem 1 (b):

$$\begin{array}{lrcrcrcrcl} \max & x_1 & + & 4x_2 & - & 2x_3 & - & x_4 & & \\ & 3x_1 & & & + & x_3 & + & 2x_4 & = & 6 \\ & -x_1 & + & 2x_2 & - & x_3 & - & x_4 & = & 2 \\ & & & & & & & \mathbf{x} & \geq & 0 \end{array}$$

and the solution $\mathbf{x}^* = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$.

Show that $\mathbf{x}^*$ is an optimal solution by finding a feasible solution $\boldsymbol{\lambda}^*$ to the dual problem that satisfies complementary slackness.

Problem 4

Recall the linear program from Problem 4 in Problem Set 4:

$$\begin{array}{rrcrcrcrcr} \max & 6a & + & 9b & + & 2c & + & 3d & & \\ \text{s.t.} & a & + & 3b & + & c & + & 2d & = & -4 \\ & & & b & + & c & - & d & \leq & -1 \\ & 3a & + & 3b & - & c & & & \leq & 1 \\ & a & & & & & & & \leq & 0 \\ & & & b & & & & & \leq & 0 \\ & & & & & c & & & \leq & 0 \end{array} .$$

Prove that the solution you obtained is optimal by providing matching certificates. Note that your certificates should be checkable on this linear program — not the standard form program that you formed in Phase 0.