Assignment 2

David Wiedemann

30 mars 2022

Problem Set 4, Exercise 1.a, 1.b

1.a)

By definition of the reduced cost, we get that

$$\overline{c}_i = c_i - c_B^T B^{-1} \operatorname{col}_j(A)$$

Hence, the reduced cost vector in the basis B is of the form

$$\overline{c}_B^T = c_B^T - c_B^T B^{-1} \operatorname{col}_B(A) = c_B^T - c_B^T B^{-1} B = 0$$

Hence, if x_i is in the basis, then $\overline{c}_i x_i = 0$.
If x_i is not in the basis, then by definition, $x_i = 0$, which implies $\overline{c}_i x_i = 0 \forall i$

1.b)

Since d is a feasible direction, in particular, we get that $A \cdot d = 0$.
Indeed, for some $\theta > 0$ we have that

$$A(x + \theta d) = b \implies Ax + \theta Ad = b \implies \theta Ad = 0 \implies Ad = 0$$

Hence

$$\overline{c} \cdot d = (c^T - c_B^T B^{-1} A) \cdot d = c^T \cdot d - c_B^T B^{-1} Ad = c \cdot d$$

Problem Set 5, Exercise 1.a, 1,b

1.a)

The data of the program is

$$c = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

and

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & -1 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$

Hence we first turn it into a minimization problem by multiplying the cost by -1

$$\begin{aligned} P' = \min -3x_1 - x_2 - 4x_3 \\ \text{s.t. } 2x_1 + x_2 + x_3 \leq 1 \\ x_1 - x_2 + 2x^3 \leq 1 \\ -x_1 - x_2 + x_3 \leq 1 \\ -2x_1 + x_2 - x_3 \leq 1 \end{aligned}$$

Hence our dual program will be

$$\begin{aligned} P' = \min \quad b \cdot \lambda \\ \text{s.t. } \lambda^t A = -c^T \\ \lambda \geq 0 \end{aligned}$$

which when written out is

$$\begin{aligned} P' = \min \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\ \text{s.t. } \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & -1 & 1 \\ -2 & 1 & -1 \end{pmatrix} = \begin{pmatrix} -3 & -1 & -4 \end{pmatrix} \\ \lambda_i \geq 0 \forall i \end{aligned}$$

1.b)

We use the procedure described in the course, our starting program is

$$\begin{aligned} \max \quad x_1 + 4x_2 - 2x_3 - x_4 \\ 3x_1 + x_3 + 2x_4 = 6 \\ -x_1 + 2x_2 - x_3 - x_4 = 2 \\ x_i \geq 0 \end{aligned}$$

Setting $c = \begin{pmatrix} 1 & 4 & -2 & -1 \end{pmatrix}$, $b = \begin{pmatrix} 6 & 2 \end{pmatrix}$ and

$$A = \begin{pmatrix} 3 & 0 & 1 & 2 \\ -1 & 2 & -1 & -1 \end{pmatrix}$$

This gives us a dual program

$$\begin{aligned} \min \quad & b \cdot \lambda \\ \text{s.t. } & A^T \lambda \leq c \end{aligned}$$

Which, when written out yields

$$\begin{aligned} \max \quad 6\lambda_1 + 2\lambda_2 & \\ \text{s.t. } 3\lambda_1 - \lambda_2 &\geq 1 \\ 2\lambda_2 &\geq 4 \\ \lambda_1 - \lambda_2 &\geq -2 \\ 2\lambda_1 - \lambda_2 &\geq -1 \end{aligned}$$

as the dual program.