K

$$S = \{f_1, \ldots\}$$

$$V(S) = \{(x_1, \ldots) \in K^n \mid f_i(x_1, \ldots) = 0 \, \forall i\}$$

$a_1, \ldots \in K[x_1, \ldots]$

$$\sum_i a_i(x) f_i(x) = 0$$

s

$I(S)$

V

R

R

$\forall a, b \in R$

$$a.b = 0 \implies a = 0 \text{ or } b = 0$$

I

I

$$\forall a, b \in R, ab \in I \implies a \in I \text{ or } b \in I$$

I

$$a^n \in I \implies a \in I$$

$I \subset R$

$\sqrt{I}$

I

$I \subset R$

$$\Longleftrightarrow$$

R/I

$I \subset R$

$$\Longleftrightarrow$$

R/I

$\iff R/I$

$S \subset R$

S

$$I(S) = \left\{ \sum_i a_i s_i \right\}$$

I

$$I = I(S)$$

s

R

刀

R

$R[x]$

$K[x_1, \ldots, x_n]$

I

$a \in R$

a

$$a = b.c$$

b

c

$(a) \subset R$

$\Longleftrightarrow$

a

R

R

R

R

$$\implies R[x]$$

R

R

R

$a \in R[X]$

$a \in Q(R)[X]$

R

$S \subset R$

R

S

$$S^{-1}R = \left\{ x \in Q(R) \mid x = \frac{a}{b}, b \in S \right\}$$

M

R–

$$S^{-1}M = \left\{ \frac{m}{s} \middle| m \in M, s \in M \right\} \Big/ \left\{ \frac{m}{s} = \frac{m'}{s'} \iff ms' = sm' \right\}$$

$p \subset R$

$$R_p = (R \setminus p)^{-1} R$$

$p \subset R$

R_p

R_p

$I \subset p$

$$o \rightarrow I \rightarrow R \rightarrow R/I \rightarrow 0$$

$$0 \to I_p \to R_p \to (R/I)_p \to 0 \tag{1}$$

$a \in \mathbb{N}^n$

$$X^a = X_1^{a_1} \ldots \in k[X_1, \ldots]$$

$F \in k[X_1, \ldots, X_n]$

$$F = \sum_{a \in \mathbb{N}^n} \lambda_a X^a$$

F

d

$\lambda_a = 0$

$$a_1 + \ldots + a_n = d$$

F

$$F = F_0 + \ldots + F_d$$

F_i

i

$$F = \sum_{a \in \mathbb{N}^n} \lambda_a X^a$$

X_i

$$F_{X_i} = \frac{\partial F}{\partial X_i}$$

F

d

$$\sum_{i=1}^{n} \frac{\partial F}{\partial X_i} X_i = dF$$

$$\sum_{i=1}^{n} \frac{\partial F}{\partial X_i} X_i = dF$$