Solutions of 0. Exercises for the course
"Discrete Mathematics" (2021)

Exercise 1. How many symmetric $n \times n$ matrices are there with entries chosen from the numbers $[q]$?

Exercise 2. Let $n \in \mathbb{N}$ be even. How many permutations does the set $[n]$ have such that
(1) the sum of the first two elements is odd?
(2) the last two elements sum up to n.

Exercise 3. In how many different ways can the letters of the word MATHEMATICALLY be arranged such that
(1) the word starts always with MA?
(2) the three As are adjacent?
(3) the word MATH is always included?
(4) the word MAT is included twice?

Exercise 4. In how many different ways can we distribute 40 bottles of water to 10 kids and 5 adults, such that each kid gets at least one bottle?

Exercise 5. Let $n, r, k \in \mathbb{N}$ and $n \geq r \geq k$. Prove the following two equations:
(1)
$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$$
(2)
$$\binom{n}{k} = \sum_{j=k-1}^{n-1} \binom{j}{k-1}$$