

Exercise for Week 4

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1

Let us first count the case $n = 3$.

We denote each point by its index, here are all the possibilities :

$$\{(1, 2), (3, 4), (5, 6)\}$$

$$\{(1, 2), (3, 6), (4, 5)\}$$

$$\{(1, 4), (2, 3), (5, 6)\}$$

$$\{(1, 6), (2, 3), (4, 5)\}$$

$$\{(1, 6), (2, 5), (3, 4)\}$$

Hence, there are 5 possibilities for the case $n = 3$.

2

Let us search a recursive formula.

We will denote by $a(n)$ the number of ways to do this for a circle with $2n$ labelled points.

We define $a(0) = 1$.

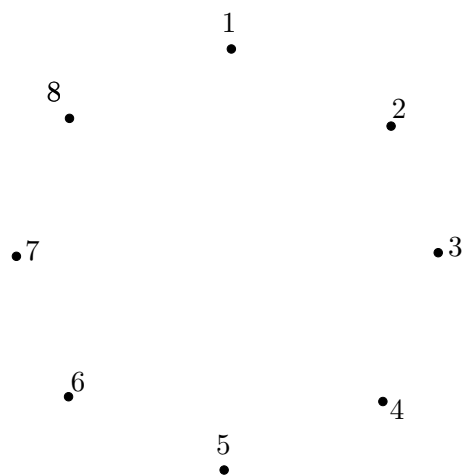
We first label each point on the circle in the following way :

First we choose what point to connect with point 1, there are exactly n points to choose, since choosing a point with an uneven index will clearly force 2 lines to intersect.

Hence, the coordinates of the chosen point have to be even, suppose we choose the point with coordinates $2k$.

Now we are left with two subset of points, one contains $2n - 2k$ points, the other one contains $2k - 2$ points. Hence, the formula of arrangements for a fixed k is given by

$$a(n - k) \cdot a(k - 1)$$



Summing over all possibilities for k yields

$$a(n) = \sum_{k=1}^n a(n-k) \cdot a(k-1)$$