Series 3

David Wiedemann

15 mars 2021

1

We simply apply the expansion for the square of the geometric series, this result has been proven during the lectures :

$$\frac{1}{(1-2x)^2} = \sum_{i=0}^{\infty}(i+1)(2x)^i = \sum_{i=0}^{\infty}(i+1)2^i x^i$$

Hence the coefficient for the 5th power is given by

$$16 \cdot 2^5 = 512$$

2

First notice that we can factorize a x^8 in the expression :

$$\left(\sum_{i=2}^{\infty} x^i\right)^4 = x^8 \left(\sum_{i=}^{\infty} x^i\right)^4$$

Thus, we only have to find the coefficient of x^7 in the formal series expansion of

$$\left(\sum_{i=0}^{\infty} x^i\right)^4$$

This counting problem is equivalent to distributing 7 identical objects between 4 persons, thus, by a theorem proven in the first lecture, the result is

$$\binom{4+7-1}{4-1} = \binom{10}{3} = 120.$$