Rigid Analytic Geometry

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List of Theorems

Lecture 1: Covering sieves

Wed 05 Apr

Theorem 1

Every sieve τ containing a covering sieve τ' of X is itself covering.
The intersection of two covering sieves is covering.

Proof

If $(v : V \to X)$ is a morphism in τ' then $v^*\tau = v\tau'^*$.
Let τ, τ' be covering sieves of X and $v : V \to X \in \tau$, then $v^*(\tau \cap \tau') = v^*\tau'$.
This covers V by GTTrans, by GTLoc, $\tau \cap \tau'$ covers X. $\square$

Remark

We are mostly interested in the case where the category C is the poset of open subsets of a topological space.
Then a sieve in V is just a set of open subsets of V such that $V \in \tau$, $W \subset V, W$ open implies $W \in \tau$.
The pullback along the (unique if it exists) morphism $V \to U$ are just the open subsets of V.
We write $\tau / = V$ if τ is a sieve oveoverV which covers V.
If several grothendieck topologies must be distinguished, I will write $\tau / =_\pi V$

Definition 1

We will write $[V_i | i \in I]$ for the sieve generated by the family V_i of open subsets of V. We have $[V_i] = \{V \in O_X | \exists i \in I \ st \ V \subset V_i\} = \bigcap_{\tau \text{ sieve in } X \text{ containing } V_i} \tau$.
A sieve is finitely generated if it can be written as $[V_i]$ for finitely many V_i

Remark

More generally, we consider Grothendieck topologies on B, a topology base for X, considered as posets.

Definition 2

Let $[\Omega_i]_B$ be the B-sieve generated by the Ω_i, ie.

$$\{\theta \in B | \theta \subset \Omega_i \text{ for at least one } i \in I\}$$

The subscript B will always be used when $B \subsetneq O_X$.

Proposition 4

Let X be a topological space and B a topology base for X.
Then we have a bijection between

— *Grothendieck topologices T_B on B*

— *Grothendieck topologies* T *on* O_X *st.* $[B_V]$ *covers* V.
If T_B *is given,* T *is defined by* $\tau / =_T V, \tau \cap B_\Omega / =_{T_B} \Omega$ *for all* $\Omega \in B_V$.
When T *is given,* T_B *is defined by*

$$\tau / =_{T_B} \Omega \text{ iff } [\tau] / =_T \Omega$$