École Polytechnique Fédérale de Lausanne
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Math 261 – Discrete Optimization (Spring 2022)

Assignment 2

Problem 1

Draw the feasible area of the following linear programs and indicate the direction of the objective function.

(a)

$$\begin{aligned} \min \quad & 3x - y \\ \text{s.t.} \quad & 3x + 2y \geq 5 \\ & 2x - 3y \leq 3 \\ & x + 2y \leq 6 \end{aligned}$$

(b)

$$\begin{aligned} \max \quad & 2x + y \\ \text{s.t.} \quad & x - y \geq 0 \\ & 3x - y \geq 2 \\ & - y \leq 2 \\ & 4x + 3y \leq 3 \end{aligned}$$

Problem 2

Consider the linear program

$$\begin{array}{rlrrrrcr} P = & \min & -x + & 3y & & & & \\ & \text{s.t.} & & y + & z & & = & 3 \\ & & -x & -y & & +w & = & -3 \\ & & 3x + & y & & & \leq & 15 \\ & & & & z, & w & \geq & 0 \end{array}$$

Write an equivalent linear program P' that uses only two variables. Show how the feasible solutions in P and P' correspond to each other.

Problem 3

Reformulate the (nonlinear) optimization problem

$$\begin{aligned} P = \quad \min \quad & 2x + 3|y - 10| \\ \text{s.t.} \quad & |x + 2| + y \leq 5, \end{aligned} \tag{1}$$

as a linear programming problem. That is, write a linear program P' which has an optimal solution that as the same objective value as an optimal solution to P (P and P' do not need to be equivalent).

Problem 4

Prove that the following are equivalent:

(a) $P = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{v}_i \cdot \mathbf{x} \leq b_i, i = 1, \ldots, m\}$ is nonempty and the $\mathbf{v}_i$ span $\mathbb{R}^n$.

(b) There exists a point $\mathbf{x} \in P$ which has n linearly independent active constraints (that is, their normal vectors are linearly independent).

Problem 5

Given a feasible region P and a point $\mathbf{x} \in P$, a *feasible direction at p* is any vector $\mathbf{v}$ for which $\mathbf{x} + \epsilon\mathbf{v} \in P$ for some $\epsilon > 0$.

(a) Let P be the polyhedron

$$P = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0\}.$$

Show that if $\mathbf{x} \in P$ and $\mathbf{v}$ is a feasible direction at $\mathbf{x}$ then $A\mathbf{v} = \mathbf{0}$.

(b) Let P be the polyhedron

$$P = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0\}.$$

Assume $\mathbf{x} \in P$ and $\mathbf{v}$ is a feasible direction at $\mathbf{x}$ and find a way to determine the set of ϵ for which $\mathbf{x} + \epsilon\mathbf{v}$ is feasible.

(c) Find the set of feasible directions of

$$P = \{\mathbf{x} \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 1, \mathbf{x} \geq \mathbf{0}\}$$

at the point $\mathbf{x} = (0, 0, 1)$ and for each feasible direction $\mathbf{v}$, find the set of ϵ for which $\mathbf{x} + \epsilon\mathbf{v}$ is feasible.