Exercises Week 1

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Part 1

We describe the process which leads to the construction of such a word. Since the order of the letters matter, we proceed letter by letter.
The first letter gives m choices.
For the $n-1$ remaining letters, there are $m-1$ choices per letter. Thus, the answer is

$$m \cdot (m-1)^{n-1}$$

Where we have used the formula for a $n-1$ letter word in an alphabet of $m-1$ letters.
In the specific case where $m = n = 1$, the above expression is undefined, however the result is clearly 1.

Part 2

In order to solve the problem, we differentiate between the different amount of a's the word could contain.
Indeed, by differentiating the number of a's we will be able to consider a pair of a's as one letter, hence simplifying the computations.

— If the word contains 0 a's, we simply form a 10 letter word in an alphabet of size 2 :

$$2^{10}$$

— If the word contains 2 a's, it is as if the word contains 9 letters, one of which is replaced by a "double a". Once the spot for the double a is chosen, we build a 8 letter word in a 2 letter alphabet, hence we have

$$\binom{9}{1} \cdot 2^8$$

— The same reasoning for 4 a's yields

$$\binom{8}{2} \cdot 2^6$$

— For 6 a's we have

$$\binom{7}{3} \cdot 2^4$$

— For 8 a's, the result is

$$\binom{6}{4} \cdot 2^2$$

— And finally, for 10 a's we simply have

$$1$$

We can now sum up these possibilities to get the desired result

$$2^{10} + 9 \cdot 2^8 + 28 \cdot 2^6 + 35 \cdot 2^4 + 15 \cdot 2^2 + 1 = 5741$$