```
(* Finding  an Initial  BFS *)
(* MIN    3x - y
     s.t.  3x + 2y - r = 5
           2x - 3y + s = 3
            x + 2y + t = 6
            x, y, r, s, t ≥ 0
*)
(* Observation : life  is easier  if x = 0, y = 0 *)
(* Observation  2: r = -5, s = 3, t = 6 *)


(* add  w -- cheating  variable  *)
(* Auxiliary  Linear  Program  *)
(* MIN                    w
     s.t.  3x + 2y - r + w = 5
           2x - 3y + s = 3
            x + 2y + t = 6
            x, y, r, s, t, w ≥ 0
*)
(* Observation  3: x=0, y=0, r=0, w = 5, s = 3, t = 6 *)


(* { x, y, r, s, t, w } *)
c = {0, 0, 0, 0, 0, 1};
A = {{3, 2, -1, 0, 0, 1}, {2, -3, 0, 1, 0, 0}, {1, 2, 0, 0, 1, 0}};
b = {5, 3, 6};


In[6]:= basis  = {4, 5, 6};(* s, t, w *)
B = A[[ ;; , basis ]];
MatrixForm [B]
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

```
In[10]:= xB = Inverse [B].b
xB . c[[basis ]]
```

Out[10]= {3, 6, 5}

Out[11]= 5

In[13]:=
```
(* vector of reduced costs *)
(* c^T - c_B^T B^{-1} A *)
c - c[[basis]].Inverse[B].A
```

Out[13]=
```
{-3, -2, 1, 0, 0, 0}
```

In[18]:=
```
(* Bland's Rule: add x ( = 1 ) to the basis *)
uB = Inverse[B].A[[;; , 1]];
xB / uB // N (* Pick one with smallest nonnegative number *)
(* 4, 5, 6 *)
```

Out[19]=
```
{1.5, 6., 1.66667}
```

```
(* Do the wrong thing -- swap x1 with x5 *)
basis = {1, 4, 6};
B = A[[;; , basis]];
xB = Inverse[B].b
```

Out[22]=
```
{6, -9, -13}
```

In[26]:=
```
(* Do the right thing -- swap x1 with x4 *)
basis = {1, 5, 6};
B = A[[;; , basis]];
xB = Inverse[B].b
xB . c[[basis]]
```

Out[28]=
$$\left\{\frac{3}{2}, \frac{9}{2}, \frac{1}{2}\right\}$$

Out[29]=
$$\frac{1}{2}$$

In[30]:=
```
c - c[[basis]].Inverse[B].A
```

Out[30]=
$$\left\{0, -\frac{13}{2}, 1, \frac{3}{2}, 0, 0\right\}$$

In[33]:=
```
(* Bland's Rule: add y (= 2) to the basis *)
uB = Inverse[B].A[[;; , 2]];
xB / uB // N (* Pick one with smallest nonnegative number *)
(* 1, 5, 6 *)
```

Out[34]=
```
{-1., 1.28571, 0.0769231}
```

In[35]:=
```
(* Do the right thing -- swap x2 with x6 *)
basis = {1, 2, 5};
B = A[[ ;; , basis ]];
xB = Inverse[B].b
xB . c[[basis]]
```

Out[37]= $\left\{\frac{21}{13}, \frac{1}{13}, \frac{55}{13}\right\}$

Out[38]= 0

In[39]:=
```
c - c[[basis]].Inverse[B].A
```

Out[39]= {0, 0, 0, 0, 0, 1}