Linear Algebra and SVD

Part 1: Linear Algebra Concepts

1.1 Vector Norms and Properties

- Exercise 1: Compute the 2-norm, 1-norm, and infinity norm of the following vector $(1, -2, 3, -4)$
- Exercise 2: Verify the triangle inequality holds for the vector norms.

1.2 Matrix operations

- Exercise 3: Perform basic matrix operations: matrix multiplication, transpose, and inverse.
- Exercise 4: Compute the determinant and rank of a matrix.

1.3 Vector Projection and Orthogonality

- Exercise 5: Implement a function to compute the projection of vector (b}) onto (a).
- Exercise 6: Check if two vectors are orthogonal.

Part 2: Singular Value Decomposition (SVD)

2.1 SVD Basics

- Exercise 1: Perform SVD on a matrix ($A \in \mathbb{R}^{4x4}$).
- Exercise 2: Reconstruct matrix (A) from its SVD components.
- Exercise 3: Compare the original matrix (A) with the reconstructed matrix to check reconstruction accuracy.

2.2 Image Compression with SVD

- Exercise 4: Load and display a grayscale image.
- Exercise 5:
    - o Perform SVD on the image matrix. o Reconstruct the image using only the top (k) singular values. Plot the compressed image.

2.3 Exploring the Effect of Varying (k)

- Exercise 6:
    - o Try different values of (k) to see how the image quality and file size change.
    - o Compare the size of the compressed image to the original by counting the number of non-zero elements in the compressed matrix.