PAPER

Uplink Pilot Allocation for Multi-Cell Massive MIMO Systems*

Wanming HAO[†,††a)], ***Student Member***, **Osamu MUTA**[††], **Haris GACANIN**[†††], ***Senior Members***, ***and*** **Hiroshi FURUKAWA**[††], ***Member***

SUMMARY Pilot contamination due to pilot reuse in adjacent cells is a very serious problem in massive multi-input multiple-output (MIMO) systems. Therefore, proper pilot allocation is essential for improving system performance. In this paper, we formulate the pilot allocation optimization problem so as to maximize uplink sum rate of the system. To reduce the required complexity inherent in finding the optimum pilot allocation, we propose a low-complexity pilot allocation algorithm, where the formulated problem is decoupled into multiple subproblems; in each subproblem, the pilot allocation at a given cell is optimized while the pilot allocation in other cells id held fixed. This process is continued until the achievable sum rate converges. Through multiple iterations, the optimum pilot allocation is found. In addition, to improve users' fairness, we formulate fairness-aware pilot allocation as maximization problem of sum of user's logarithmic rate and solve the formulated problem using a similar algorithm. Simulation results show that the proposed algorithms match the good performance of the exhaustive search algorithm, meanwhile the users' fairness is improved.

key words: *pilot contamination, massive MIMO, pilot allocation*

1. Introduction

Massive multi-input multiple-output (mMIMO) is considered to be a promisiong wireless technology to meet the rapid increase of mobile traffic [2]. The mMIMO systems allow multiple users to access with the same time/frequency resource because the intra-cell interference and uncorrelated noise can be significantly reduced when the number of base station (BS) antennas is very large relative to the number of active users [3]. However, the use of downlink pilots demands that BS send orthogonal pilots, which leads to the huge pilot overhead (i.e., a large number of pilots is needed). In addition, the estimated downlink channel information at user terminals has to be fed back to the BS. On the other hand, if uplink pilot is used, each served user needs to send an orthogonal pilot (i.e., the number of the required pilots in uplink is less than those of downlink). Therefore, the uplink pilot is usually used in mMIMO systems. Meanwhile, time division duplex (TDD) is usually applied to utilize the channel reciprocity for downlink precoding. Due to the short coherence time, the number of available pilots is limited and they need to be reused by users in adjacent cells [4]. The pilot reuse gives rise to the inter-cell interference, i.e., pilot contamination, which is a major performance bottleneck of the mMIMO systems [5].

The pilot contamination problem has been studied widely in the literature [6]–[10]. In [6], a pilot assignment scheme is proposed to mitigate pilot contamination problem, where the allocation of pilot sequences is optimized to maximize the signal-to-interference power ratio (SIR) on the uplink. The work in [7] proposes the users scheduling per cell in order to maximize the spectral efficiency, but for the given number of users in each cell, the approach does not take into consideration the pilot allocation strategy. In [8], a fractional pilot reuse scheme is proposed, where users in different cells are allowed to reuse the same pilot sequence if they are close to their BSs. Otherwise, if users are located far away from BS in different cells, the orthogonal pilot sequences must be used. Thus, the pilot allocation is not considered for users located closely to their BSs. In [9], a graph coloring based pilot allocation is proposed to reduce the pilot contamination. The authors first construct an interference graph according to the strength of potential pilot contamination between any two users in different cells with the same pilot. Then, they allocate pilots among users in order to minimize potential pilot contamination term in the graph. In [10], the authors assume that a subset of pilots is owned by each cell and then, cells may cooperate to utilize pilots from other cells and support more users. However, the pilot-to-user allocation is not considered. Although some pilot allocation schemes [6]–[10] improve system capacity, they are all not global optimum in terms of pilot allocation.

In this paper, we assume that an uplink communication is established in two phases: (i) pilot and (ii) data signaling. Thus, by reducing the interference between utilized pilots from adjacent cells, the (data) uplink user sum rate may be improved. The optimum pilot allocation is decided by a central control unit (CCU) that acts as master BS. Then, we formulate the pilot allocation optimization problem of maximizing the uplink** sum rate of the mMIMO systems.

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[†]The author is with the Institute of Industrial Technology and School of Information Engineering, Zhengzhou University, Zhengzhou, 450001, China.
[††]The authors are with the Kyushu University, Fukuoka-shi, 819-0395 Japan.
[†††]The author is with the Nokia Bell Labs, Belgium.

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a) E-mail: wmhao@hotmail.com

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**A good pilot allocation scheme will reduce the pilot interference and thus improve channel estimation quality and user's rate. In this paper, we evaluated the uplink transmission rate as a manner to clarify the effectiveness of the proposed pilot allocation scheme similar to [6], [9].

To decrease the complexity, we propose an iterative pilot allocation optimization algorithm, where the original problem is transformed into a number of subproblems which can be solved as one-to-one matching problem. The Hungarian algorithm [11] can be applied to find the optimum pilot allocation problem in each subproblem. In addition, to improve the users' fairness, we formulate a users' fairness aware pilot allocation as maximization problem of sum of user's logarithmic rate and use a similar algorithm to obtain the corresponding pilot allocation.

2. System Model

We consider an uplink multi-cell system composed of L hexagonal cells as shown in Fig. 1. The radius of each cell is r_c, and white area in each cell denotes the cell-hole (users are not located within the center disk of radius r_h). One of the BSs works as CCU, while each BS is equipped with M antennas and serves K ($M \gg K$) single-antenna users. We assume that there is time-frequency coherent block of S symbols in each frame. K orthogonal pilot signals $\boldsymbol{\Psi} = [\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \cdots, \boldsymbol{\psi}_K]^T \in C^{K\times K}(\boldsymbol{\psi}_i = [\psi_{i1}, \cdots, \psi_{iK}]^T)$ are reused in adjacent cells due to the limited coherence time, while different users in each cell use orthogonal pilots to avoid severe interference, and we assume that $\boldsymbol{\Psi}\boldsymbol{\Psi}^H = \mathbf{I}_K$. Here, $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively.

In this paper, to avoid the serious pilot interference among users in the same cell, we assume that the number of served users is less than or equal to the number of pilot sequences in one time frame (i.e., different orthogonal pilot can be allocated to each user in the same cell). If the total number of users is larger than that of pilot sequences, the remained users may be allocated the pilot sequences at next time frame. The similar assumption can be found in [6], [9], [10].

2.1 Training Phase

During the training phase, the received signal at the BS of the l-th cell can be expressed as:

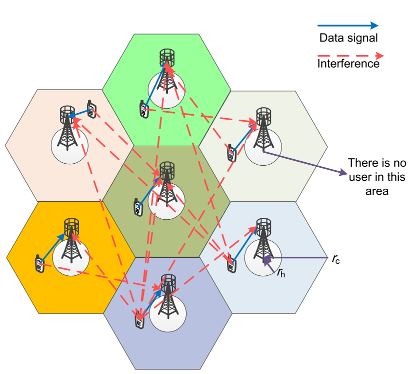

Fig. 1 Uplink interference model for the multi-cell mMIMO system.

$$\mathbf{Y}_l = \sqrt{p_p}\sum_{j=1}^{L}\sum_{k=1}^{K}\mathbf{h}_{ljk}\boldsymbol{\psi}_{jk}^T + \mathbf{Z}_l, \tag{1}$$

where p_p denotes the pilot transmit power, $\mathbf{Z}_l \in C^{M\times K}$ is an independent and identically distributed additive white Gaussian noise (AWGN) defined as $\mathcal{CN}(0,\ \delta_z^2)$, $\mathbf{h}_{ljk} \in C^{M\times 1}$ is the channel coefficient between BS in the l-th cell and the k-th user in the j-th cell. $\mathbf{h}_{ljk} = \sqrt{\beta_{ljk}}\mathbf{g}_{ljk}$, where β_{ljk} and $\mathbf{g}_{ljk} \sim \mathcal{CN}(\mathbf{0},\ \mathbf{I}_M)$ denote the large-scale fading coefficient and small-scale fading vector, respectively. We consider time-domain representation in our system model.

The channel estimate of the k-th user in the l-th cell is obtained by correlating $\mathbf{Y}_l$ with $\boldsymbol{\psi}_{lk}^*$ as follows:

$$\begin{aligned}\tilde{\mathbf{h}}_{llk} &= \mathbf{h}_{llk}\boldsymbol{\psi}_{lk}^T\boldsymbol{\psi}_{lk}^* + \sum_{j\neq l}^{L}\sum_{i=1}^{K}\mathbf{h}_{lji}\boldsymbol{\psi}_{ji}^T\boldsymbol{\psi}_{lk}^* + \frac{1}{\sqrt{p_p}}\mathbf{Z}_l\boldsymbol{\psi}_{lk}^* \\ &= \mathbf{h}_{llk} + \sum_{j\neq l}^{L}\sum_{i=1}^{K} f[\theta(j,i),\theta(l,k)]\mathbf{h}_{lji} + \mathbf{w}_{lk},\end{aligned} \tag{2}$$

where $(\cdot)^*$ denotes the complex conjugate, $\mathbf{w}_{lk}$ denotes the equivalent noise, $\boldsymbol{\psi}_{\theta(j,i)}$ ($\theta(j,i) \in \{1,\cdots,K\}$) denotes that the $\theta(j,i)$-th pilot is used by the i-th user in the j-th cell with $\theta(j,k) \neq \theta(j,k')$ when $k \neq k'$. In the above expression, $f[\cdot] \in \{0,1\}$ represents the pilot reuse index, $f[\theta(j,i),\theta(l,k)] = 1$ when $\theta(j,i) = \theta(l,k)$, else $f[\theta(j,i),\theta(l,k)] = 0$.

2.2 Data Phase

During the data phase, the received signal at the BS of the l-th cell can be expressed as:

$$\mathbf{y}_l = \sqrt{p_t}\sum_{j=1}^{L}\sum_{k=1}^{K}\mathbf{h}_{ljk}x_{jk} + \mathbf{n}_l, \tag{3}$$

where p_t denotes the uplink data transmit power, x_{jk} denotes the data transmitted by the k-th user in the j-th cell with $E[|x_{jk}|^2] = 1$ and $\mathbf{n}_l \sim \mathcal{CN}(\mathbf{0},\ \sigma_l^2\mathbf{I}_M)$ denotes the noise, where $E[\cdot]$ denotes the expectation operator.

Using the channel estimate of the k-th user in (2), the matched-filter (MF) detector is applied to obtain the decision variables of the k-th user as:

$$\begin{aligned}\tilde{x}_{lk} &= \tilde{\mathbf{h}}_{llk}^H\mathbf{y}_l = \underbrace{\sqrt{p_t}\mathbf{h}_{llk}^H\mathbf{h}_{llk}x_{lk}}_{\text{Desired signal}} + \underbrace{\sqrt{p_t}\sum_{n\neq k}^{K}\mathbf{h}_{llk}^H\mathbf{h}_{lln}x_{ln}}_{\text{intra-cell interference}} \\ &+ \underbrace{\sqrt{p_t}\sum_{j\neq l}^{L}\sum_{i=1}^{K}\sum_{m=1}^{L}\sum_{n=1}^{K} f[\theta(j,i),\theta(l,k)]\mathbf{h}_{lji}^H\mathbf{h}_{lmn}x_{mn}}_{\text{pilot contamination}} \\ &+ \underbrace{\sqrt{p_t}\sum_{m\neq l}^{L}\sum_{n=1}^{K}\mathbf{h}_{llk}^H\mathbf{h}_{lmn}x_{mn}}_{\text{inter-cell interference}} + \underbrace{\omega_{lk}}_{\text{uncorrelated noise}}\end{aligned} \tag{4}$$

where $\omega_{lk} = \mathbf{h}_{llk}^H\mathbf{n}_l + \sum_{j\neq l}^{L}\sum_{i=1}^{K} f[\theta(j,i),\theta(l,k)]\mathbf{h}_{lji}^H\mathbf{n}_l + {\mathbf{w}_{lk}}^H\mathbf{n}_l$.

$$r_{lk} = \mathrm{E}\left\{\log_2\left(1+\frac{\left|\mathbf{h}_{llk}^H\mathbf{h}_{llk}\right|^2}{\sum\limits_{n\neq k}^{K}\left|\mathbf{h}_{llk}^H\mathbf{h}_{lln}\right|^2+\sum\limits_{m\neq l}^{L}\sum\limits_{n=1}^{K}\left|\mathbf{h}_{llk}^H\mathbf{h}_{lmn}\right|^2+\sum\limits_{j\neq l}^{L}\sum\limits_{i=1}^{K}\sum\limits_{m=1}^{L}\sum\limits_{n=1}^{K}f[\theta(j,i),\theta(l,k)]\left|\mathbf{h}_{lji}^H\mathbf{h}_{lmn}\right|^2+\frac{|\omega_{lk}|^2}{p_t}}\right)\right\} \tag{5}$$

In (4), the first term denotes the desired signal component, the second term denotes the intra-cell interference, the third term denotes the pilot contamination, the fourth term denotes the inter-cell interference, and the last term denotes the uncorrelated noise after MF filtering. According to (4), the average uplink rate of the user can be expressed as (5) at the top of this page.

3. Problem Formulation and Solution

In this section, we first formulate a pilot allocation optimization problem to maximize uplink sum rate of the system. Then, we propose a low-complexity algorithm to obtain the optimal solution. Next, considering users' fairness, we formulate a fairness aware pilot allocation as maximization problem of sum of user's logarithmic rate and use the similar method to solve the formulated problem.

3.1 Problem Formulation Based on Sum Rate Maximization

We formulate the pilot allocation optimization problem for maximizing uplink sum rate of the system as follows:

$$\begin{aligned} &\max_{\boldsymbol{\theta}}\ R(\boldsymbol{\theta}) = \sum_{l=1}^{L}\sum_{k=1}^{K}(1-\eta)r_{lk} \\ &s.t \quad \theta(l,k)\in\{1,2,\cdots,K\},\forall l,k \\ &\qquad\ \theta(l,k)\neq\theta(l,k'),k\neq k' \end{aligned} \tag{6}$$

where $\boldsymbol{\theta} = [\theta(l,k)]_{LxK}$ denotes the pilot allocation index for each user, $\eta = K/S$. Note that accurate channel state information (CSI) is needed to estimate user's rate r_{lk} for solving the optimization problem (6). However, based on the fact that CSI can not be obtained before determining pilot allocation, it seems that it is not possible to solve the problem (6).

According to [12], when the number of BS antennas M goes to infinity, the uplink rate can be approached using only large-scale fading coefficients as

$$r_{lk}\approx\log_2\left(1+\frac{\beta_{llk}^2}{\sum_{j\neq l}^{L}\sum_{i=1}^{K}f[\theta(j,i),\theta(l,k)]\beta_{lji}^2}\right). \tag{7}$$

It can be observed from (7) that the uplink rate in the optimization problem can be approximated with only the large-scale fading coefficients, which can be easily tracked by the BSs. In this paper, we propose to use approximated rate in (7) for solving the problem (6). The details of the proposed algorithm to solve (6) is mentioned in next subsection.

3.2 Proposed Sum Rate Maximization (SR-M) Algorithm

Problem (6) is known as mixed integer programming (MIP) problem. The challenge of this problem is the discrete nature of the pilot allocation index. Exhaustive search can be used to find the optimum pilot allocation, but it requires high computational complexity given as $O((K!)^L)$. Thus, exhaustive search is not feasible solution for a large number of users in multi-cell mMIMO system.

To decrease the computational complexity, we decouple (6) into L subproblems, where in each subproblem, we aim at optimizing the pilot allocation of K users in one particular cell and fix pilot allocation in other L-1 cells. Based on the above description, we can get one of subproblems as follows:

$$\begin{aligned} &\max_{\boldsymbol{\theta}_m}\quad R_m(\boldsymbol{\theta}_{-m},\boldsymbol{\theta}_m) \\ &s.t.\quad \theta(m,k)=\{1,2,\ldots,K\},\forall k \\ &\qquad\ \ \theta(m,k)\neq\theta(m,k'),k\neq k' \end{aligned} \tag{8}$$

where $R_m(\boldsymbol{\theta}_{-m},\boldsymbol{\theta}_m)=\sum\limits_{l=1}^{L}\sum\limits_{k=1}^{K}(1-\eta)\log_2\left(1+\frac{\beta_{llk}^2}{\sum_{j\neq l}^{L}\sum_{i=1}^{K}f[\theta(j,i),\theta(l,k)]\beta_{lji}^2}\right)$, $\boldsymbol{\theta}_{-m}$ denotes the pilot allocation decision matrix except for the m-th cell, and $\boldsymbol{\theta}_m$ the pilot allocation matrix in the m-th cell. For (8), since pilot allocation in other cells have been decided in advance (at the beginning, we assume that the pilots are randomly allocated in these cells), we just need to allocate pilots to users in the m-th cell for maximizing the sum rate of the system. Exhaustive search is not feasible because the required complexity is given as ($O(K!)$) and significantly increased with a large K.

To reduce the required complexity for finding the optimum solution, we propose a low-complexity pilot allocation scheme. Since we have fixed pilot allocation in other $L-1$ cells, the problem (8) is reduced to a one-to-one matching problem, namely K users select K pilots. Next, we define the one-to-one matching problem as follows:

Definition: We assume that there are K users and K pilots, and we need to allocate the K pilots to K users. The allocation rule is that every user is assigned one pilot and each pilot is only assigned to one user. Each possible allocation between the i-th pilot and the k-th user is associated a utility U_{ik} (the U_{ik} can be regarded as the revenue of the k-th user when it uses the i-th pilot), which is given in Table 1.

Then, the matching problem can be presented by the following optimization problem:

$$\max_{c_{nm}}\quad \sum_{n=1}^{K}\sum_{m=1}^{K}c_{nm}U_{nm}$$

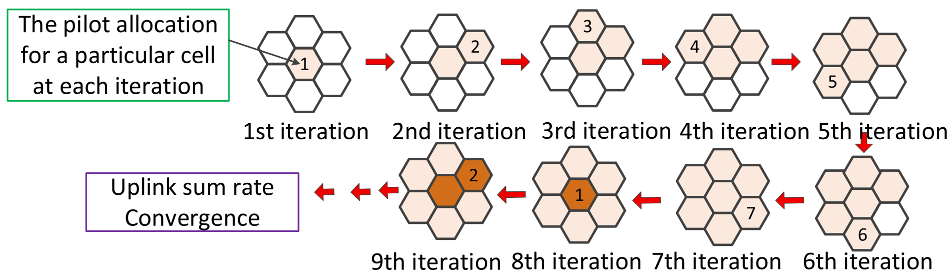

Fig. 2 Iteration diagram for pilot allocation in the proposed algorithm.

Table 1 The utility of pilot allocation.

Pilot \ User	1	2	3	$\cdots$	K
1	U_{11}	U_{12}	U_{13}	$\cdots$	U_{1K}
2	U_{21}	U_{22}	U_{23}	$\cdots$	U_{2K}
3	U_{32}	U_{32}	U_{33}	$\cdots$	U_{3K}
$\vdots$	$\ldots$	$\ldots$	$\ldots$	$\ddots$	$\vdots$
K	U_{K1}	U_{K2}	U_{K3}	$\cdots$	U_{KK}

$$\begin{aligned} s.t. \quad & \sum_{n=1}^{K} c_{nm} = 1, \quad \forall n, \\ & \sum_{m=1}^{K} c_{nm} = 1, \quad \forall m, \\ & c_{nm} \in \{0,1\}, \quad \forall n,m, \end{aligned} \tag{9}$$

where c_{nm} denotes the binary assignment variable, and $c_{nm} = 1$ means that pilot n is allocated to user m, and $c_{nm} = 0$, otherwise. $\sum_{n=1}^{K} c_{nm} = 1$ denotes that each pilot is only allocated to one user, $\sum_{m=1}^{K} c_{nm} = 1$ denotes that each user is only allocated one pilot.

As for the problem (9), the optimal matching problem can be solved by applying the well-known Hungarian algorithm [13], which is a combinatorial optimization algorithm that solves the assignment problem in polynomial time. Therefore, the subproblem (8) can be solved by using the similar method. We rewrite the subproblem (8) as follows:

$$\begin{aligned} \max_{\theta_m} \quad & \sum_{a=1}^{K}\sum_{p=1}^{K} c_{ap} R_m^{ap}(\boldsymbol{\theta}_{-m}, \boldsymbol{\theta}_m) \\ s.t. \quad & R_m^{ap}(\boldsymbol{\theta}_{-m}, \boldsymbol{\theta}_m) = \begin{cases} R_m(\boldsymbol{\theta}_{-m}, \boldsymbol{\theta}_m), \\ \theta(m,a) = p, \end{cases} \\ & \sum_{a=1}^{K} c_{ap} = 1, \quad \forall a, \\ & \sum_{p=1}^{K} c_{ap} = 1, \quad \forall p, \\ & c_{ap} \in \{0,1\}, \quad \forall a,p. \end{aligned} \tag{10}$$

where a and b denote the pilots and users index in the m-th cell, respectively. We can find that the subproblem (10) is also an one-to-one matching problem and the optimum pilot allocation can be obtained by applying the Hungarian algorithm. Next, we move to the next cell and use the same method to optimize pilot allocation for next subproblem. After multiple iterations, the global optimum pilot allocation for problem (6) can be obtained according to the Proposition 1. To describe our proposed algorithm more clearly, we present the iterative diagram in Fig. 2. For example, at the first step, the $m = 1$ in problem (10), namely, we only optimize the pilot allocation at the 1st cell while fixing pilot allocation in other cells. After solving problem (10), we can obtain the uplink sum rate. Then, similar to the first step, we optimize the pilot allocation at the 2nd cell as the second step of Fig. 2. This process is continued until the uplink sum rate is converged. We also summarize the above method in Algorithm 1.

```
Algorithm 1: Proposed SR-M Algorithm
Initialize cell index l, pilot allocation θ_{-l} (assume l = 1),
  tolerance ϵ, iterative index t = 1.
repeat
    Obtain the optimum pilot allocation θ_l at the lth cell
      according to the Hungarian method.
    Get the pilot allocation results θ^(t).
    Compute the uplink rum rate according to R(θ^(t)).
    Update t ← t + 1, l ← l + 1.
    if l > L then
        Update l ← 1.
    end if
until R(θ^(t+1)) − R(θ^(t)) < ϵ;
Obtain optimum pilot allocation θ^(t).
```

Fig. 3 The proposed SR-M algorithm.

Proposition 1: For given L and K, global optimum pilot allocation converges after a finite number of iterations.

Proof: In solving each subproblem (iteration), the pilot allocation is obtained according to the Hungarian method, and the sum rate of the system is maximized in this optimization (iteration). Therefore, the objective of problem (6) increases over each iteration until converges.

3.3 Problem Formulation Based on Users' Fairness

Here, users' fairness means that the users who experience low rate (such as some users located in the cell edge) will be preferentially allowed to use "good" pilot (i.e., less-interfered pilot) in order to improve their rate by pilot allocation scheme. In practice, we not only need to consider the total capacity of the system but also need to concern each user's rate as so to guarantee the good experience for each user. Therefore, we consider the users' fairness as pilot allocation metric. For this purpose, we formulate the pilot allocation optimization problem for maximizing the sum of user's logarithmic rate as follows:

$$\begin{aligned} &\max_{\boldsymbol{\theta}} \; R(\boldsymbol{\theta}) = \sum_{l=1}^{L}\sum_{k=1}^{K} \log\left((1-\eta)r_{lk}\right) \\ s.t \quad &\theta(l,k) \in \{1, 2, \cdots, K\}, \forall l, k, \\ &\theta(l,k) \neq \theta(l,k'), k \neq k'. \end{aligned} \tag{11}$$

As for problem (11), we can use the similar algorithm to problem (6) to obtain the corresponding pilot allocation. The algorithm consists of the following four steps:

1. Divide problem (11) into L subproblems.
2. Optimize pilot allocation for users in one cell while fixing pilot allocation in others cell.
3. Move to the next cell and do the same optimization as step 2.
4. Repeat steps 2 and 3 until sum logarithmic rate $\log\left((1-\eta)r_{lk}\right)$ converges.

We call the above algorithm as user's fairness aware (UF-A) algorithm. Since the similar algorithm in Sect. 3.2 (i.e., algorithm 1) is applied, we omit the detailed explanations of the algorithm. The related results will be presented in simulation section (Sect. 4) directly.

3.4 Complexity Analysis

It can be easily verified that the computational complexity of solving problems (6) and (11) is the same. We note here that, in each iteration, the complexity of the Hungarian method is $O(K^3)$ [13]. Therefore, if the sum rate converges to maximization after ξ iterations, the computational complexity of the proposed algorithm is $O(\xi K^3)$. Thus, we can find that the computational complexity of the proposed algorithm $O(\xi K^3)$ is much lower than the exhaustive search method which requires higher complexity ($O((K!)^L)$) especially for a larger number of users K.

Here, we assume that the pilot allocation algorithm is executed during pilot allocation phase after uplink pilot transmission. Therefore, the convergence of the algorithm must be guaranteed unless the computation speed of algorithm is enough to finish all calculation during a given time period. On the other hand, the user's mobility speed may affect the coherence time (or one frame duration), namely the higher mobility speed leads to shorter coherence time. Thus, if users' mobility is high, the number of available orthogonal pilots is decreased but it does not affect its convergence if channel state is time-invariant during one frame. In general, the user's channel state information is invariable within coherence time (or one frame duration) [4], [6], [9], [12].

4. Numerical Results

In this section, we evaluate the average uplink rate per user† of the proposed pilot allocation schemes. We consider an $L = 7$ typical hexagonal cellular network where each BS is equipped with M antennas, and there are K users in each cell. Therefore, the proposed algorithm works to maximize total sum rate of 7 cells as defined in problem (6). We assume that cell radius is $r_c = 500$ meters, and cell-hole radius $r_h = 100$ meters (as shown in Fig. 1). The large-scale fading coefficient captures the path-loss effect as follows $\beta_{ljk} = 1/d_{ljk}^{\alpha}$ [14], [15], where d_{ljk} denotes the distance between the l-th BS and the k-th user in the j-th cell, and $\alpha = 3.8$ is the path-loss exponent. Users are distributed randomly within each cell, and Monter-Carlo method is applied with 10^4 simulation for single user having random location in each trail. Note that (5) is used to compute the uplink rate of each user, while the approximated user-rate in Eq. (7) is used to solve the problems (6) and (11). The system parameters are summarized in Table 2.

Table 2 Simulation Parameters.

Parameters	Value
Radius of cell r_c	500 m
Radius of cell hole r_h	100 m
Number of users K	$2 \leq$ K ≤ 8
Number of BS antennas M	$10 \leq$ M ≤ 500
Number of cells L	7
Transmit power of users	0 dBm
Time-frequency coherent block size S	100 symbols
Bandwidth	20 MHz
Noise Power	−174 dBm/Hz

In fact, similarly to [16], the inter-cluster interference should be also considered (as shown in Fig. 4). Since there is no any cooperation among clusters, the cluster cannot know necessary information of adjacent clusters. Next, we propose the following approximate scheme. We only consider the interference from adjacent outer-cluster cells due to the very slight interference for non-adjacent outer-cluster cells. During the pilot allocation phase, the BS'location is assumed as the user's location in adjacent clusters. We will show the rationality of such an assumption (i.e., approximation) in our simulation.

Figure 5 shows the average uplink rate versus number of BS antennas. "Unlimited number of BS antennas" denotes the achievable average uplink rate per user when the number of BS antennas goes to infinity. "Ideal case (the inter-cluster interference does not occur)" denotes that the inter-cluster interference is not considered. "With perfect knowledge of inter-cluster interference" denotes that the users' locations are assumed known in other clusters. Here, we only illustrate the performance of the proposed SR-M algorithm. Compared with Ideal case, it can be verified that the average rate is lower when the inter-cluster interference is considered. On the other word, the inter-cluster inter-

†Here, the average uplink rate per user is caculated as "average uplink rate per user $= \frac{\text{average uplink sum rate}}{\text{total number of users}}$", where average uplink rate per user is given as the average uplink sum rate normalized by the total number of users. In this paper, although we employ the average user rate to focus on each user rate, it is clear that we can know the average sum rate by using the above equation easily.

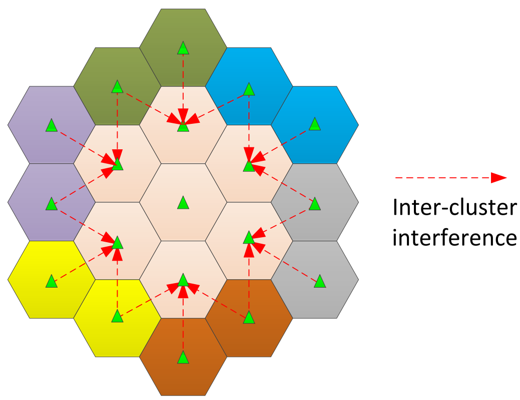

Fig. 4 The model of the inter-cluster interference, where each cluster consists of 7 cells.

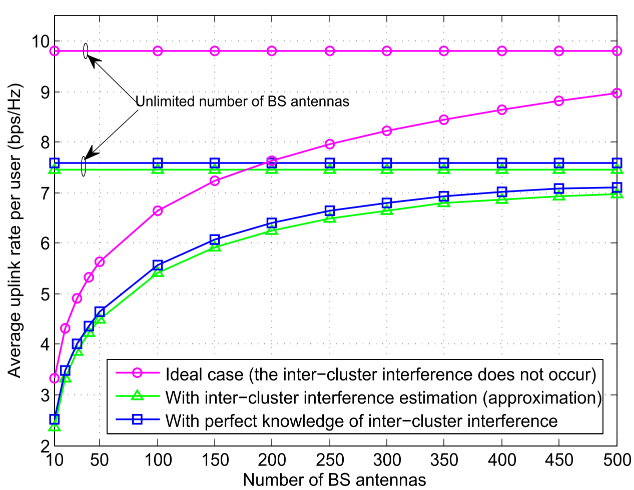

Fig. 5 The average uplink rate versus the number of BS antennas (K = 4).

ference largely affects the rates of users located in 6 outside cells. Therefore, the inter-cluster interference should be considered during the evaluation of the transmission performance. In addition, the gap between our proposed approximate scheme and perfect scenario is small.

Figure 6 plots the average uplink rate versus number of BS antennas with different algorithms when the number of users in each cell is 4. It can be clearly found that the average uplink rate increases with M under all algorithms, and the average uplink rate under the proposed SR-M algorithm is almost the same with that under the exhaustive search algorithm. In exhaustive search scheme, the best pilot allocation to maximize the average uplink sum rate is selected among all possible candidates. In random allocation scheme, pilot allocation is randomly determined regardless of the achievable uplink sum rate. We can find that the average uplink rate of the proposed UF-A algorithm is lower than that of the proposed SR-M algorithm and is higher than that of the random allocation algorithm. The reason is that the achievable sum rate has to be sacrificed for improving the users' fairness with the proposed UF-A algorithm. Meanwhile, the average uplink rate with the random pilot

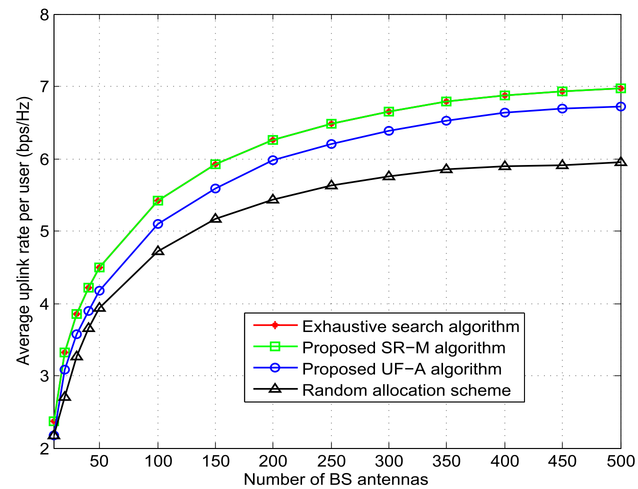

Fig. 6 Comparison of the average uplink rates as a function of the number of BS antennas (K = 4).

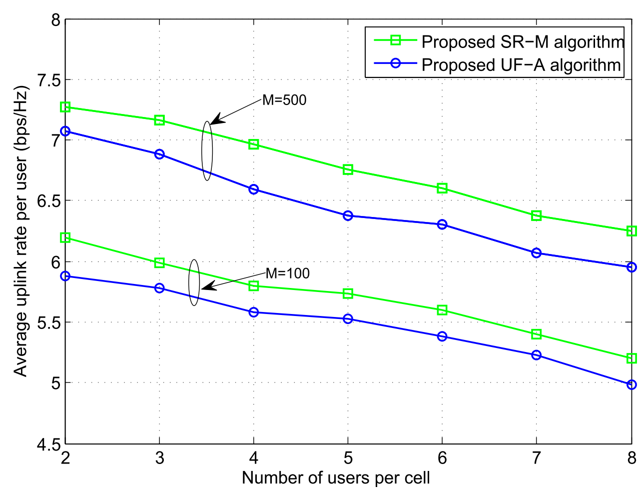

Fig. 7 The average uplink rate versus the number of users per cell.

allocation scheme is the lowest compared with that of others' algorithms.

Figure 7 shows that the average uplink rate versus the number of users in each cell with different algorithms. We can find that the average uplink rate decreases with K increases. In fact, there are two reasons for this result. The first is that (1-η) decreases as K increases, which reduces the uplink rate per user. The second is that the degree of freedom (DoF) of the BS antennas decreases with the number of serviced users increases, which leads to the decline of the average rate. It is also easy to understand that more number of BS antennas leads to higher rate. Although the average uplink rate decreases with the number of users, the uplink sum rate increases, and we can get it by the proposed low complexity algorithms according to Fig. 6. On the other hand, we can get that the uplink sum rate of the system will increase when it services more users, but the average uplink rate per each user will decrease, which lowers each user's experience. Therefore, in practice, the tradeoff between number of serviced users and each user's experience

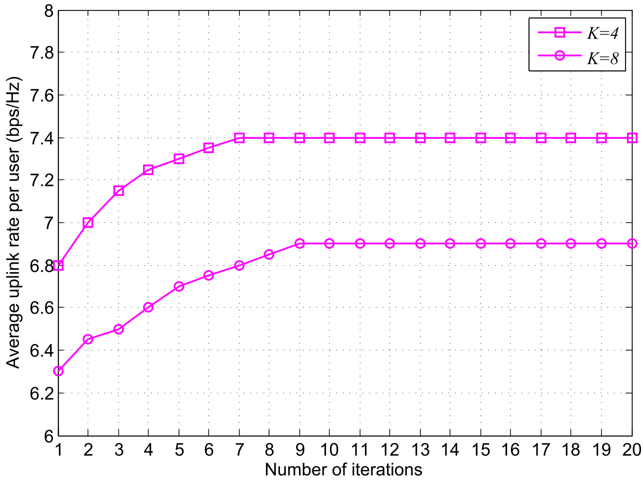

Fig. 8 The average uplink rate versus the number of iterations.

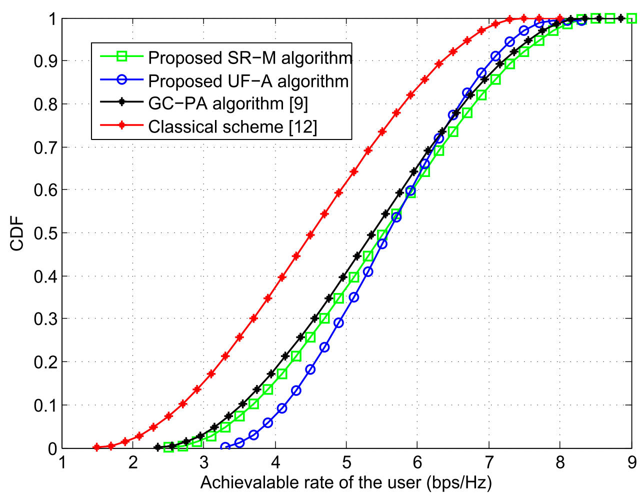

Fig. 9 CDF versus users' uplink achievable rate (bps/Hz) (K=4,M=100).

needs to be considered.

Figure 8 plots the average uplink rate versus the number of iterations under different K. When the number of users per cell is $K = 4$, we need 7 iterations to converge for all cells. The required number of iterations slightly increases as the number of users per cell, i.e., 9 iterations are needed when $K = 8$. The results indicate that the required convergence time will increase as the number of users increases.

Figure 9 shows the cumulative distribution function (CDF) curve of users' uplink achievable rate with $K = 4$ and $M = 100$. The graph coloring based pilot allocation (GC-PA) [9] and classical random pilot allocation scheme [12] are compared with our proposed schemes. We can find that that the uplink rate with our proposed SR-M algorithm is higher than that with GC-PA algorithm. Meanwhile, it can be verified that the user's rate is more concentrated with UF-A algorithm than that with SR-M algorithm, which means that the UF-A improves the users' fairness. In addition, it is clear that the classical scheme has the worst performance compared with other algorithms.

5. Conclusions

In this paper, we proposed an optimum pilot allocation scheme to improve uplink sum rate in mMIMO systems. First, we formulated the pilot allocation optimization problem for maximizing uplink sum rate of the system. Due to the high complexity involved in solving the original problem, we transformed the formulated problem into several subproblems. In each subproblem, we obtained the optimum pilot allocation by applying the Hungarian method. Through multiple iterations, the global optimum pilot allocation was found. For improving users' fairness, we formulated the maximization problem of sum of user' logarithmic rate and applied the similar algorithm to obtain the corresponding pilot allocation. Simulation showed that the proposed algorithms obtain the optimal performance, meanwhile the users' fairness is also improved.

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Wanming Hao received the M.S. degree from the School of Information Engineering, Zhengzhou University, China, in 2015, the Ph.D. degree in Graduate School of Information Science and Electrical Engineering, Kyushu University, Japan, in 2018. Now, he joined in the Institute of Industrial Technology and School of Information Engineering, Zhengzhou University. His research interests include broadband wireless communication, cognitive radio, cooperative communication, massive MIMO, HetNet and cloud radio access networks.

Osamu Muta received B.E. degree from Ehime University, in 1996, M.E. degree from Kyushu Institute of Technology, Japan, in 1998, and Ph.D. degree from Kyushu University in 2001. In 2001, he joined the Graduate School of Information Science and Electrical Engineering, Kyushu University as an assistant professor. Since 2010, he has been an associate professor in Center for Japan-Egypt Cooperation in Science and Technology, Kyushu University. His current research interests include signal processing techniques for wireless communications and powerline communications, MIMO, and nonlinear distortion compensation techniques for high-power amplifiers. He received the 2005 Active Research Award from IEICE technical committee of radio communication systems, the 2014, the 2015, and the 2017 Chairman's Awards for excellent research from IEICE technical committee of communication systems, respectively. Dr. Muta is a member of IEEE.

Haris Gacanin received his M.E.E. and Ph.D.E.E. from Graduate School of Electrical Engineering, Tohoku University, Japan, in 2005 and 2008, respectively. Since April 2008 until May 2010 he has been working first as Japan Society for Promotion of Science (JSPS) postdoctoral research fellow and then as an Assistant Professor at Graduate School of Engineering, Tohoku University. Currently, he is with Nokia, Belgium. His research interest is in the fields of wireline and wireless communications with focus on wireless network coding, channel estimation and equalization, cognitive radio, MIMO, wireless sensor networks, dynamic resource allocation, iterative receivers. He is member of IEEE and senior member of IEICE and was Chair of the IEICE Europe Section. He is a recipient of the 2010 KDDI Foundation Research Grant Award, the 2008 Japan Society for Promotion of Science (JSPS) Postdoctoral Fellowships for Foreign Researchers.

Hiroshi Furukawa received the B.E. degree in information engineering from KyushuInstitute of Technology in 1992 and the Ph.D. degree in electronics engineering from Kyushu University in 1998. From 1992 to 1996, he had been with Kyushu-Institute of Technology. From 1996 to 2003, he had been with the Networking Research Laboratories, NEC. Since 2010, he is a professor with Kyushu University. He received the Young Engineer Award of IEICE in 1995. Dr. Furukawa is a member of IEEE.