

補間

$$\begin{matrix} \sin x & \sin 2x \\ \cos x & \cos 2x \end{matrix}$$

$$F(x) = a_0 + a_1 x_1 + a_2 x_2 \dots$$

$$A = \begin{pmatrix} x_1(x_0) & x_2(x_2) & x_3(x_3) & \dots \\ \vdots & \vdots & \vdots & \vdots \\ x_1(x_N) & x_2(x_N) & \dots & \dots \end{pmatrix}$$

← $N+1$ 166 1024 i →

① 直交関数系

fast Fourier transform
FFT

$$\begin{aligned} \langle \varphi_n, \varphi_m \rangle &= \int_a^b \varphi_n(x) \varphi_m(x) dx \\ &= \pi \delta_{mn} = \begin{cases} \pi & n=m \\ 0 & n \neq m \end{cases} \end{aligned}$$

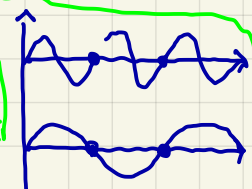
② 積分から係数計算

$$\begin{aligned} F(x) &= \sum_{i=1}^N a_i \varphi_i(x) \\ \int_a^b F(x) \varphi_m(x) dx &= \int_a^b \sum_{i=1}^N a_i \varphi_i(x) \varphi_m(x) dx \\ &= \sum_{i=1}^N a_i \int_a^b \varphi_i(x) \varphi_m(x) dx \\ &= \sum_{i=1}^N a_i \delta_{im} = \begin{cases} a_m & m=i \\ 0 & m \neq i \end{cases} \\ a_m &= \int_a^b F(x) \varphi_m(x) dx \end{aligned}$$

③ 積分 → 和

$$\begin{aligned} \sum_{i=1}^N F(x_i) \varphi_m(x_i) &= \sum_{i=1}^N \sum_{l=1}^N a_l \varphi_l(x_i) \varphi_m(x_i) \\ &= \sum_{l=1}^N a_l \sum_{i=1}^N \varphi_l(x_i) \varphi_m(x_i) \\ &= \sum_{l=1}^N a_l \delta_{ml} = a_m \end{aligned}$$

選点直交性



$$\begin{aligned} \varphi_{l,m} &< n-1 \\ \varphi_n(x) &= 0 \text{ at } x=x_1, x_2, \dots, x_n \\ \sum_{i=1}^N \varphi_l(x_i) \varphi_m(x_i) &= \delta_{ml} \end{aligned}$$

$$a_m = \sum_{i=1}^N F(x_i) \varphi_m(x_i)$$

和

対偶の関係

$$\begin{aligned} \cos(a) + I \sin(a) &= \exp(aI) \text{ 虚数} \end{aligned}$$

積分

$$\begin{aligned} \cos\left(\frac{2\pi}{N}x\right) + I \sin\left(\frac{2\pi}{N}x\right) &= \exp\left(\frac{2\pi}{N}Ix\right) \text{ 単位} \end{aligned}$$