2014/11/28

補間

$\sin x \quad \sin 2x \quad \cdots$
$\cos x \quad \cos 2x \quad \cdots$

$F(x) = a_0 + a_1 x + a_2 x^2 + \cdots$

基底関数

① 直交関数系

$$AI = \begin{pmatrix} \varphi_0(x_0) & \varphi_1(x_0) & \varphi_2(x_0) \cdots \\ \vdots & \ddots & \\ \varphi_N(x_N) & \varphi_1(x_N) & \varphi_2(x_N) \cdots \end{pmatrix} \quad N+1 \text{ 行}, \ N+1 \text{ 列}$$

$$\langle \varphi_n, \varphi_m \rangle = \int_a^b \varphi_n(x)\varphi_m(x)dx = \frac{\pi}{2}\delta_{nm} = \begin{cases} 1 & n=m \\ 0 & n \neq m \end{cases}$$

fast fourier transform

FFT

② 積分から係数計算

$$F(x) = \sum_{i=1}^{N} a_i \varphi_i(x)$$

$$\int_a^b F(x)\varphi_m(x)dx = \int_a^b \sum_{i=1}^{N} a_i \varphi_i(x)\varphi_m(x)dx$$

$$= \sum_{i=1}^{N} a_i \int_a^b \varphi_i(x)\varphi_m(x)dx$$

$$= \sum_{i=1}^{N} a_i \delta_{im} = \begin{cases} a_m & m=i \\ 0 & m \neq i \end{cases}$$

$$a_m = \int_a^b F(x)\varphi_m(x)dx$$

積分 → 和

③ 積分→和

$$\sum_{i=1}^{n} F(x_i)\varphi_m(x_i)$$

$$= \sum_{i=1}^{n} \sum_{l=1}^{N} a_l \varphi_l(x_i)\varphi_m(x_i)$$

$$= \sum_{l=1}^{N} a_l \sum_{i=1}^{n} \varphi_l(x_i)\varphi_m(x_i)$$

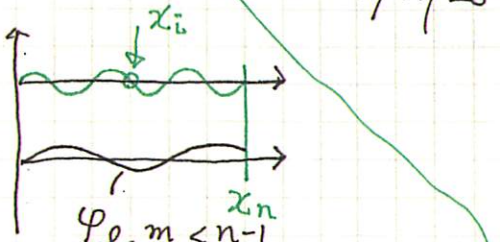
$$= \sum_{l=1}^{N} a_l \delta_{ml} \begin{cases} 1 & m=l \\ 0 & m \neq l \end{cases}$$

$$= a_m$$

$$a_m = \sum_{i=1}^{n} F(x_i)\varphi_m(x_i)$$

和

選点直交性



$\varphi_{l,m} < n-1$

$\varphi_n(x) = 0$ at $x = x_1, x_2, \cdots x_n$

$$\sum_{i=1}^{n} \varphi_l(x_i)\varphi_m(x_i) = \delta_{ml}$$

オイラーの関係

$\exp(aI) = \cos(a) + I\sin(a)$

虚数単位

$\exp\left(\frac{2\pi I}{N}x\right) = \cos\left(\frac{2\pi}{N}x\right) + I\sin\left(\frac{2\pi}{N}x\right)$