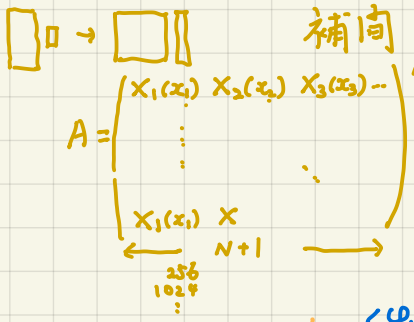


$\cdot \sin x \quad \sin 2x$
$\cdot \cos x \quad \cos 2x$

$$F_I(x) = a_0 + a_1 X_1 + a_2 X_2 + \cdots$$

① 直交関数系

$$\langle \varphi_n, \varphi_m \rangle = \int_a^b \varphi_n(x)\,\varphi_m(x)\,dx = \pi \delta_{mn} = \begin{cases} \pi & n=m \\ 0 & n \neq m \end{cases}$$

Fast Fourier Transform
FFT

② 積分からの係数計算

a_i ?

$$F_I(x) = \sum_{i=1}^{N} a_i \varphi_i(x)$$

$$\int_a^b F_I(x)\,\varphi_m(x)\,dx$$

$$= \int_a^b \sum_{i=1}^{N} a_i \varphi_i(x)\,\varphi_m(x)\,dx$$

$$= \sum_{i=1}^{N} a_i \int_a^b \varphi_i(x)\,\varphi_m(x)\,dx$$

$$= \sum_{i=1}^{N} a_i \delta_{im} = \begin{cases} a_m & m=i \\ 0 & m \neq i \end{cases}$$

積分

$$a_m = \int_a^b F(x)\,\varphi_m(x)\,dx$$

③ 積分→和

$$\sum_{i=1}^{n} F(x_i)\,\varphi_m(x_i)$$

$$= \sum_{i=1}^{n} \sum_{l=1}^{N} a_l \varphi_l(x_i)\,\varphi_m(x_i)$$

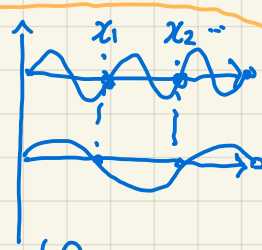
$$= \sum_{l=1}^{N} a_l \sum_{i=1}^{n} \varphi_l(x_i)\,\varphi_m(x_i)$$

$$= \sum_{l=1}^{N} a_l \delta_{ml} \begin{cases} 1 & m=l \\ 0 & m \neq l \end{cases}$$

$$\simeq a_m$$

$$a_m = \sum_{i=1}^{N} F(x_i)\,\varphi_m(x_i)$$

和



選点直交性

$\varphi_{l,m} < n-1$

$\varphi_n(x) = 0$ at $x = x_1$ $x_2 \cdots x_n$

$$\sum_{i=1}^{N} \varphi_l(x_i)\,\varphi_m(x_i) = \delta_{ml}$$

オイラーの関係

$$\cos(a) + I\sin(a) = \exp(aI)$$

$$\cos\left(\frac{2\pi}{N}x\right) + I\sin\left(\frac{2\pi}{N}x\right) = \exp\left(\frac{2\pi}{N}Ix\right)$$

虚数字位