

補間

$$A = \begin{pmatrix} x_1(x_1) & x_2(x_2) & x_3(x_3) & \dots \\ \vdots & \vdots & \vdots & \ddots \\ x_1(x_1) & x_2(x_2) & x_3(x_3) & \dots \end{pmatrix}^{N+1}$$

256  
1024  
!

$$F(x) = a_0 + a_1 x_1 + a_2 x_2 + \dots$$

$\sin x$   $\cos x$   $\sin 2x$   $\cos 2x$

### ① 直交関数系

$$\begin{aligned} \langle \varphi_n, \varphi_m \rangle &= \int_a^b \varphi_n(x) \varphi_m(x) dx \\ &= \pi \delta_{mn} = \begin{cases} \pi & n=m \\ 0 & n \neq m \end{cases} \end{aligned}$$

Fast Fourier Transform  
FFT

### ② 積分からの係数計算

$$\begin{aligned} F(x) &= \sum_{i=1}^N a_i \varphi_i(x) \\ \int_a^b F(x) \varphi_m(x) dx &= \int_a^b \sum_{i=1}^N a_i \varphi_i(x) \varphi_m(x) dx \\ &= \sum_{i=1}^N a_i \int_a^b \varphi_i(x) \varphi_m(x) dx \\ &= \sum_{i=1}^N a_i \delta_{im} = \begin{cases} a_m & m=i \\ 0 & m \neq i \end{cases} \end{aligned}$$

$$a_m = \int_a^b F(x) \varphi_m(x) dx$$

### ③ 積分 → 和

$$\begin{aligned} \sum_{i=1}^n F(x_i) \varphi_m(x_i) &= \sum_{i=1}^n \sum_{l=1}^N a_l \varphi_l(x_i) \varphi_m(x_i) \\ &= \sum_{l=1}^N a_l \sum_{i=1}^n \varphi_l(x_i) \varphi_m(x_i) \end{aligned}$$

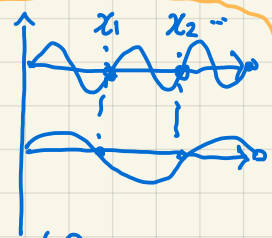
$$\begin{aligned} &= \sum_{l=1}^N a_l \delta_{ml} \\ &= a_m \end{aligned}$$

$$a_m = \sum_{i=1}^N F(x_i) \varphi_m(x_i)$$

積分

和

選点直交性



$$\begin{aligned} \varphi_{l,m} &< n-1 \\ \varphi_n(x) &= 0 \text{ at } x=x_1, x_2, \dots, x_n \\ \sum_{i=1}^N \varphi_l(x_i) \varphi_m(x_i) &= \delta_{ml} \end{aligned}$$

虚数単位

オイラーの関係

$$\begin{aligned} \cos(a) + I \sin(a) &= \exp(a I) \end{aligned}$$

$$\begin{aligned} \cos\left(\frac{2\pi}{N}x\right) + I \sin\left(\frac{2\pi}{N}x\right) &= \exp\left(\frac{2\pi}{N}Ix\right) \end{aligned}$$

