

Exploring, Denoising, and Filtering Time-Series with Singular Spectrum Analysis

Hands-on Experience Using Demographic Data

Damien Delforge
UCLouvain-IRSS-CRED, ADSCIAN

MethodsNET
Methods Excellence Network

MethodsNET 2nd Conference - **September 10, 2025** - Louvain-la-Neuve



Outline



Lecture (45 min)

- SSA examples
- Key steps
- Visualizations
- Significance Testing
- Around SSA



Hands-on experience with SSA (45 min)



SSALib and Rssa



Short Course Materials



https://github.com/dadelforge/MethodsNET2_SSA

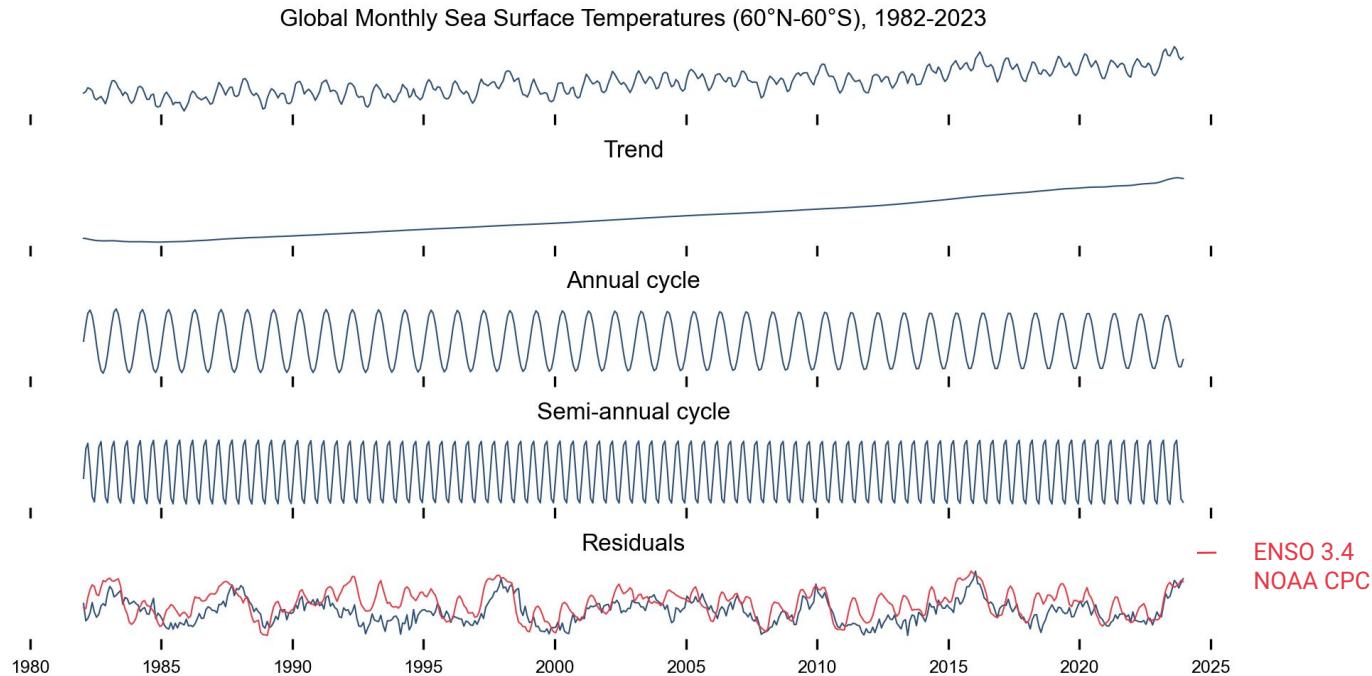




SSA and Time Series Decomposition Examples



Sea Surface Temperature

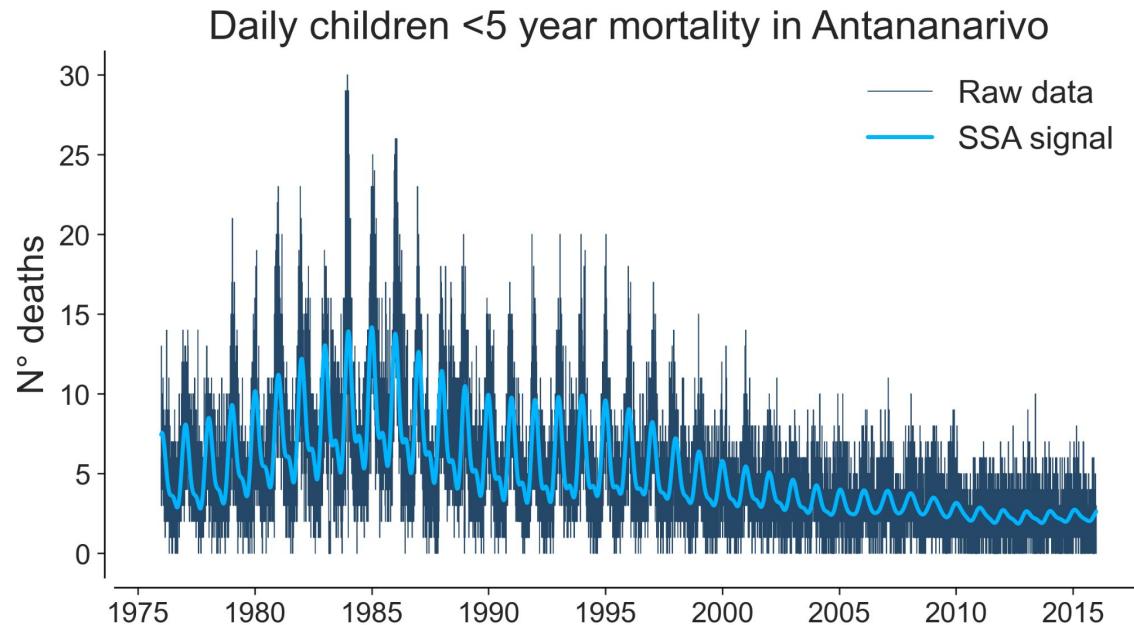


Climate Reanalyzer. Monthly Sea Surface Temperature. Climate Change Institute, University of Maine. Retrieved June 06, 2024 from <https://climatereanalyzer.org/>

Climate Prediction Center (CPC). East Central Tropical Pacific SST (5N-5S) (170-120W). US National Oceanic and Atmospheric Administration (NOAA). January 22, 2024 from <https://www.cpc.ncep.noaa.gov/data/indices/>



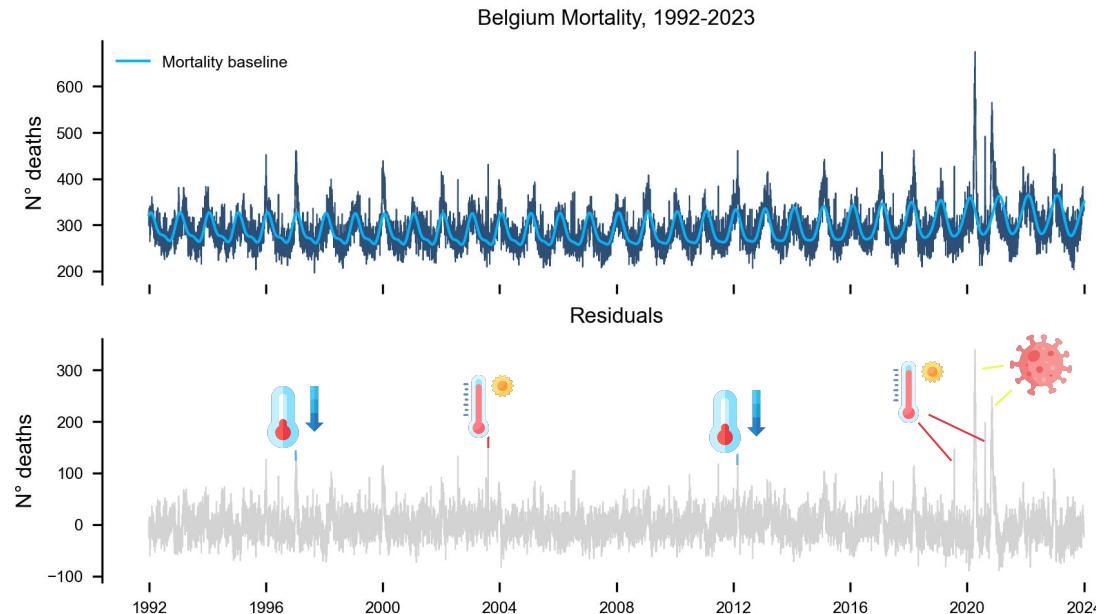
Children Mortality Baseline



Data: courtesy of Masquelier et al. (2019): 10.1186/s12963-019-0190-z



Belgian Mortality



STATBEL. (2024). Number of deaths per day. Available at:
<https://statbel.fgov.be/en/open-data/number-deaths-day>
Accessed: May 2024



SSA Key Steps



Singular Spectrum Analysis (SSA)



- ≈ Principal Component Analysis (**PCA**) for time series
- ≈ Non-parametric decomposition technique
- ≈ Unsupervised learning with supervised steps

Steps

1. Embedding: 2D-matrix representation capturing lagged dependencies
2. Decomposition: Singular (or eigen) Value Decomposition (SVD)
3. Grouping: component selection and grouping (usually supervised)
4. Reconstruction: backward transformation into time domain

PCA



Embedding (1)



The Broomhead & King (1986) trajectory matrix approach

Time series x_1, x_2, \dots, x_N — Time series length

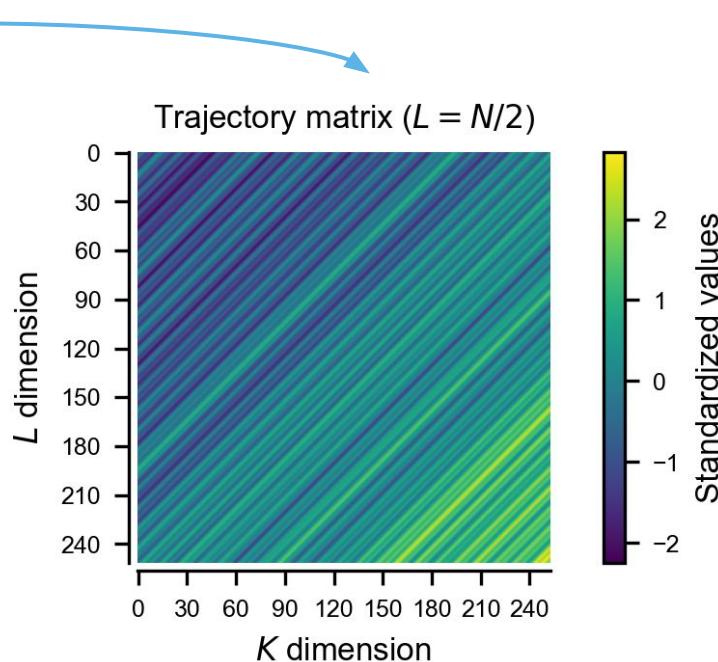
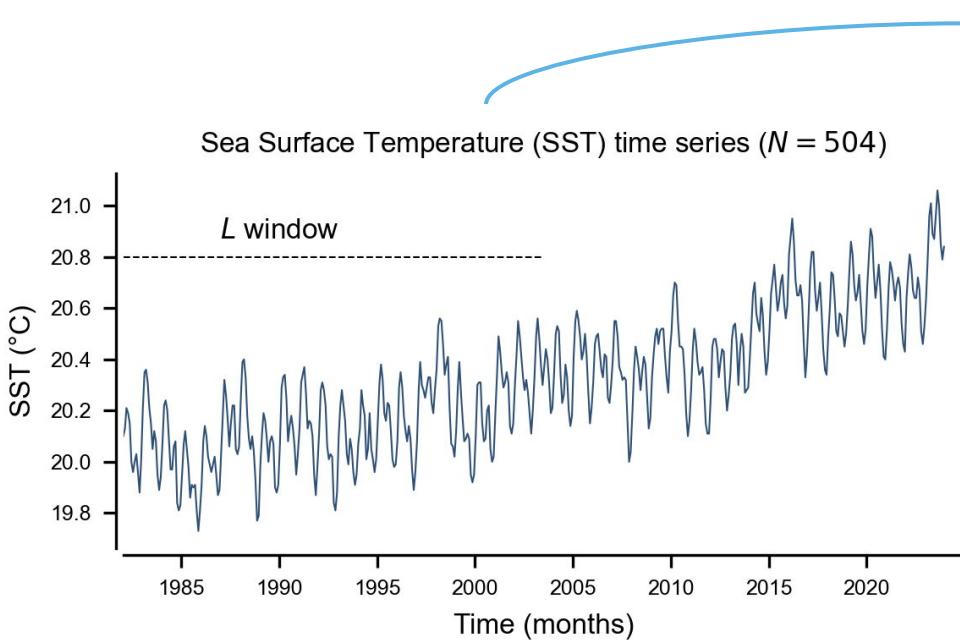
Trajectory Matrix \mathbf{X} =
$$\begin{bmatrix} x_1 & x_2 & \cdots & x_{N-L+1} \\ x_2 & x_3 & \cdots & x_{N-L+2} \\ \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & \cdots & x_N \end{bmatrix}$$
 N° of lags K

Anti-diagonal
Window parameter

Embedding (2)



The Broomhead & King (1986) trajectory matrix approach

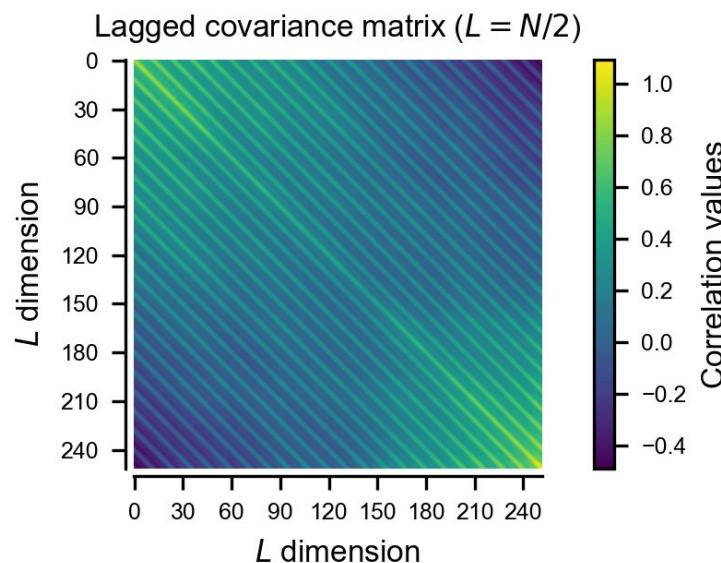
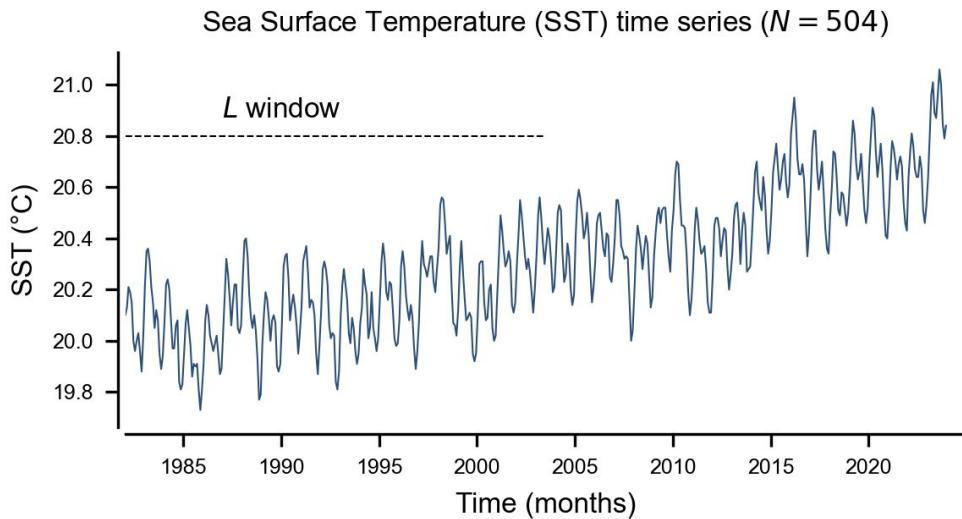




Embedding (3)

$$\frac{1}{K} \mathbf{X} \mathbf{X}^T$$

↗ The Broomhead & King (1986) lagged covariance matrix approach

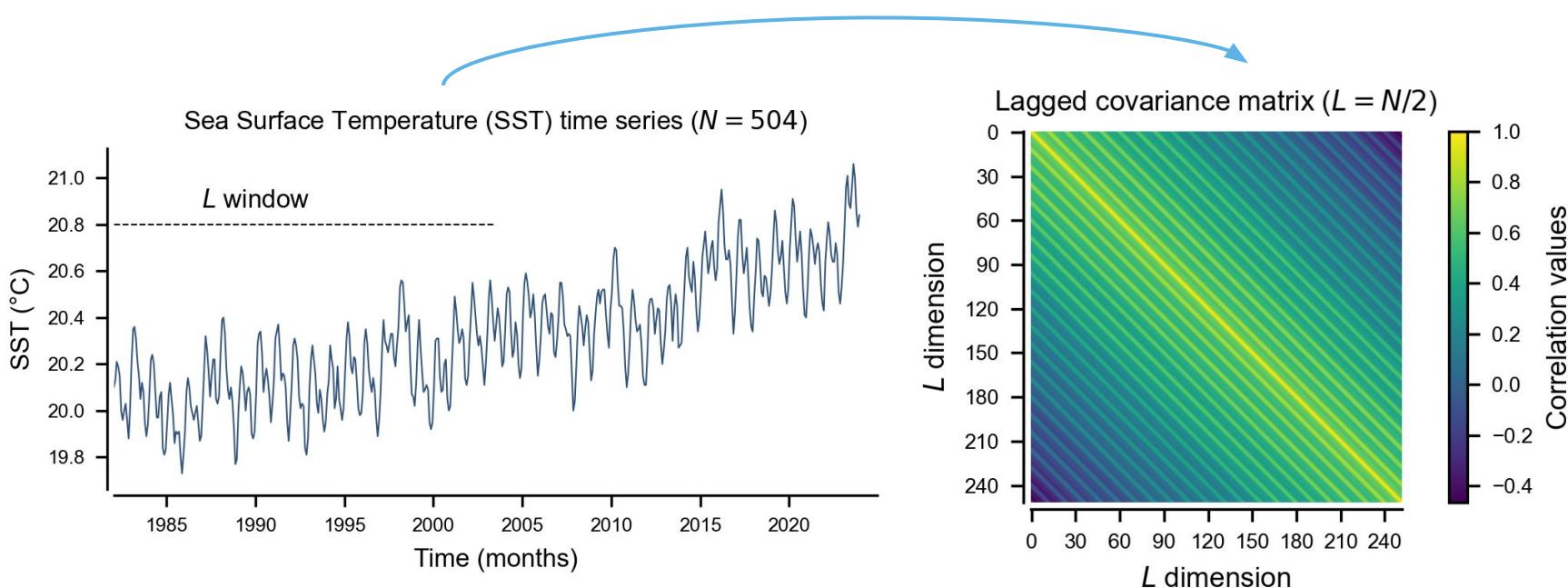




Embedding (4)



The Vautard & Ghil (1989) lagged covariance matrix approach

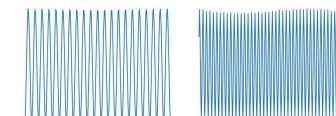
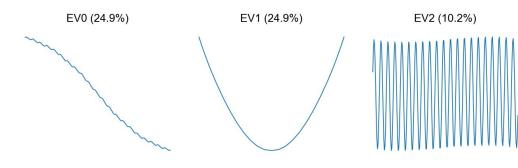
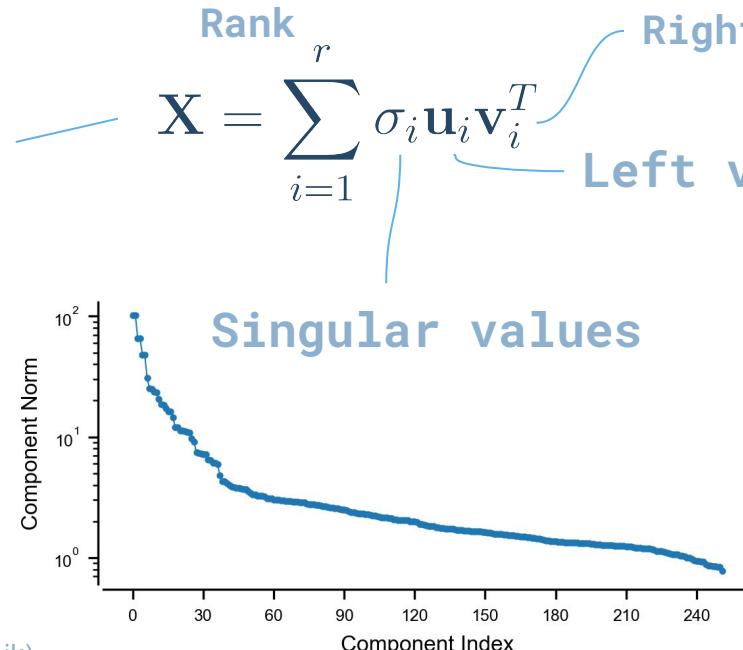
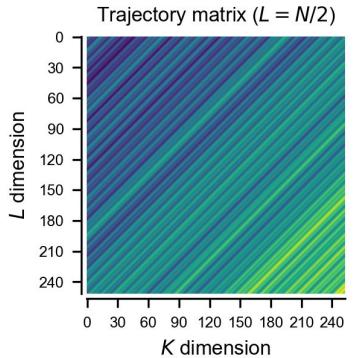




Singular Value Decomposition (SVD)



SVD decomposes a matrix into orthogonal components ranked by importance, revealing the primary patterns in the data.

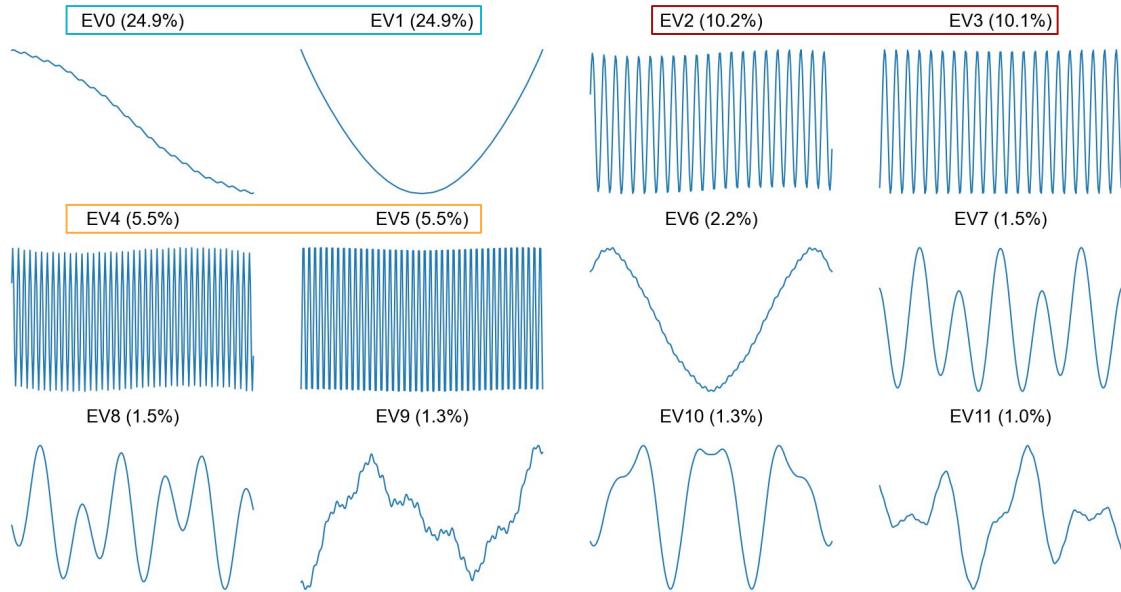
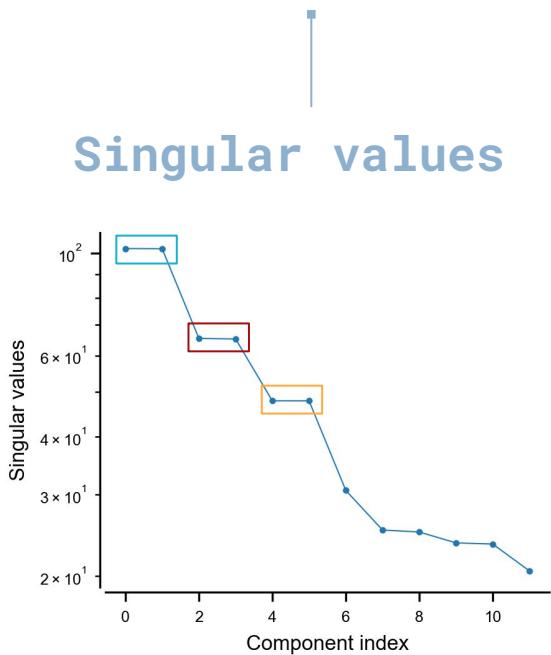




Supervised Grouping

First 12 components

Left eigenvectors

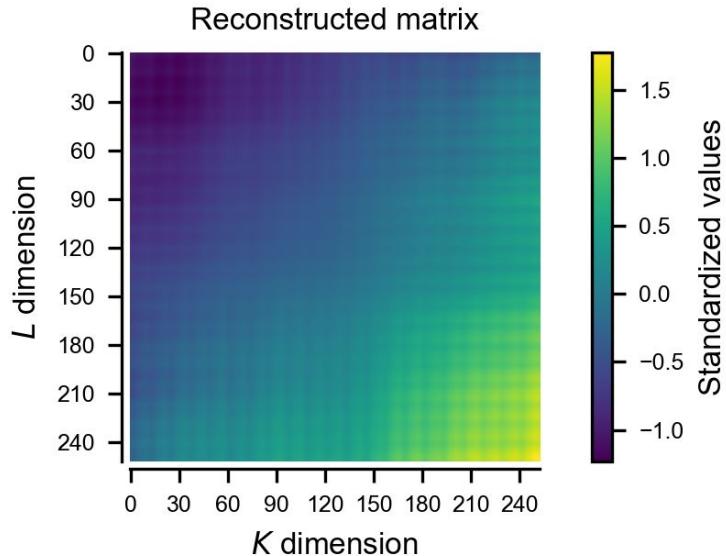




Group Matrix Reconstruction

Trend

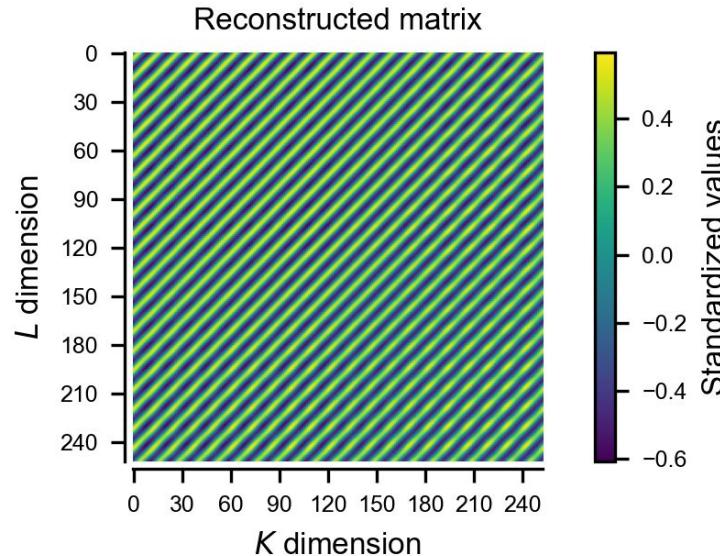
$$G = \{0, 1\}$$



$$\mathbf{X}_G = \sum_{i \in G} \sigma_i \mathbf{v}_i \mathbf{u}_i^T$$

Annual cycle

$$G = \{2, 3\}$$



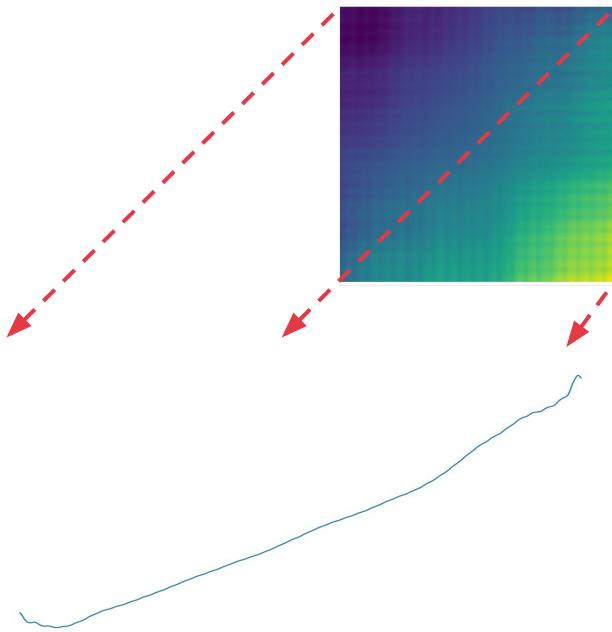


Time Series Reconstruction

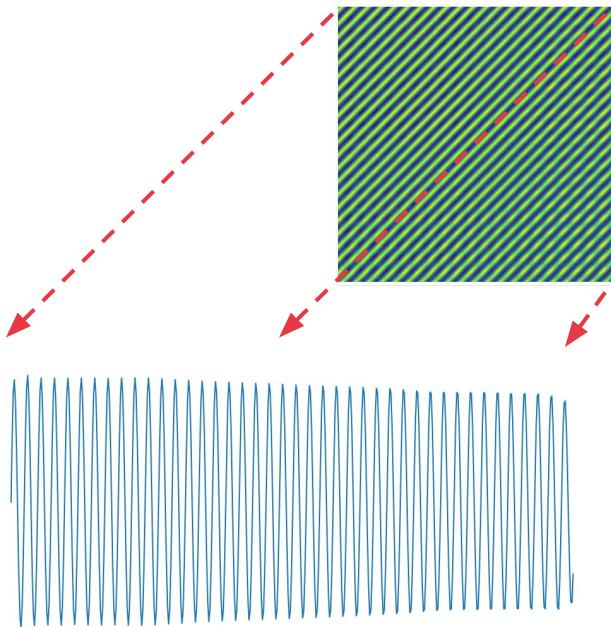


Broomhead & King (1986) anti-diagonal averaging

X_G Trend



X_G Annual cycle





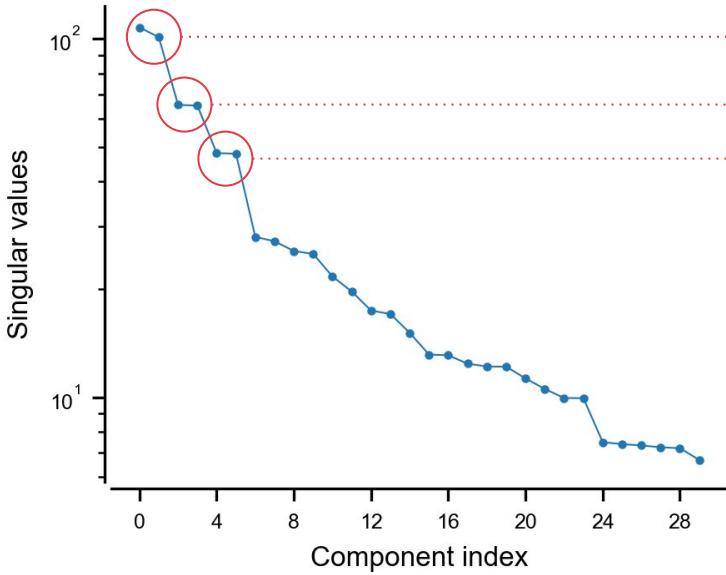
SSA Visualizations for Grouping and Selection



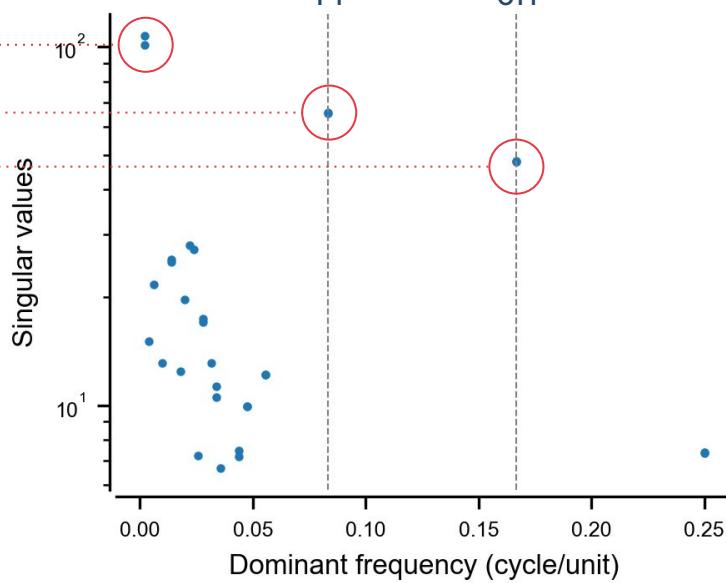
Visualizations: Singular Values



By importance

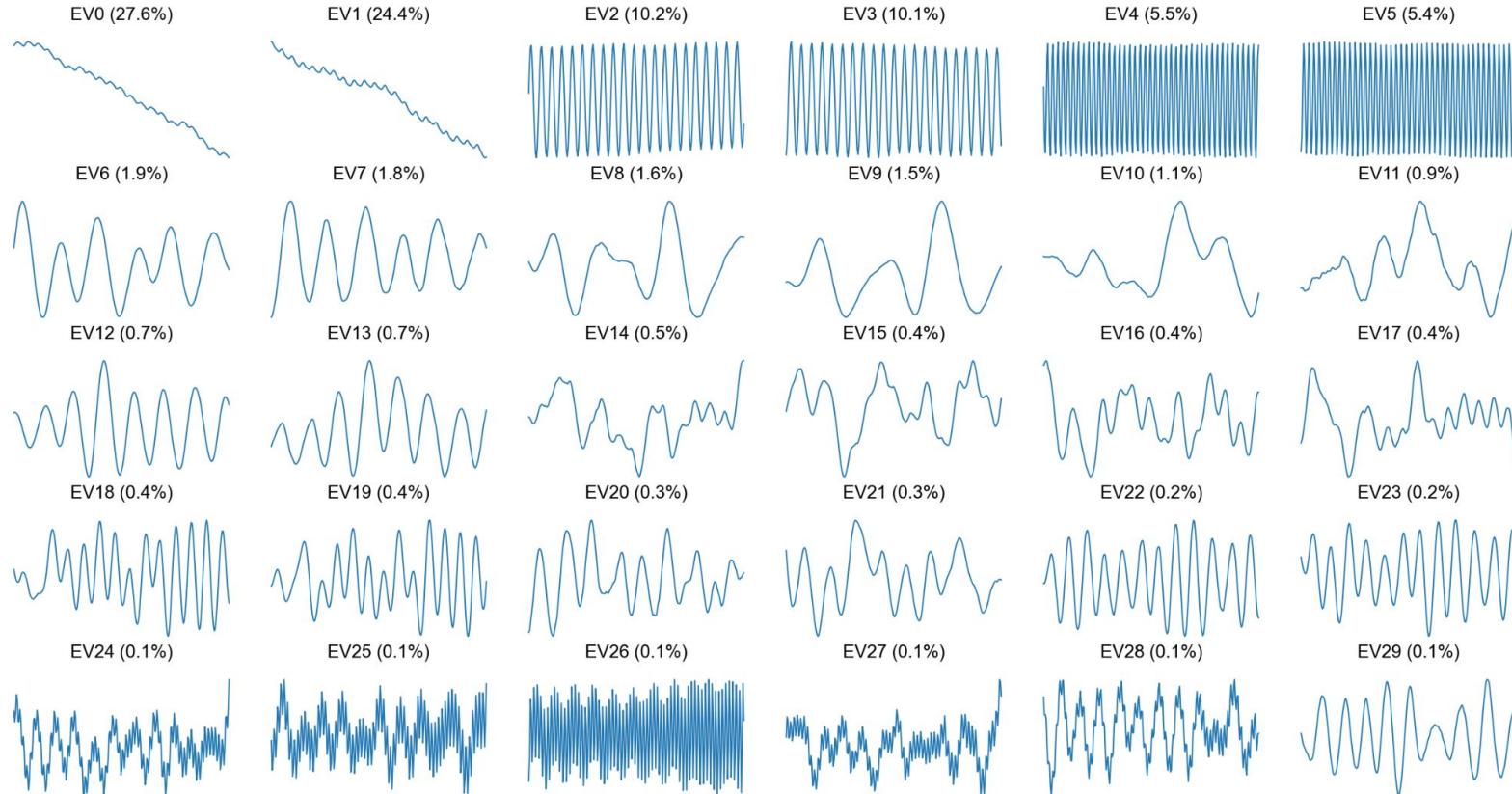


By dominant frequency



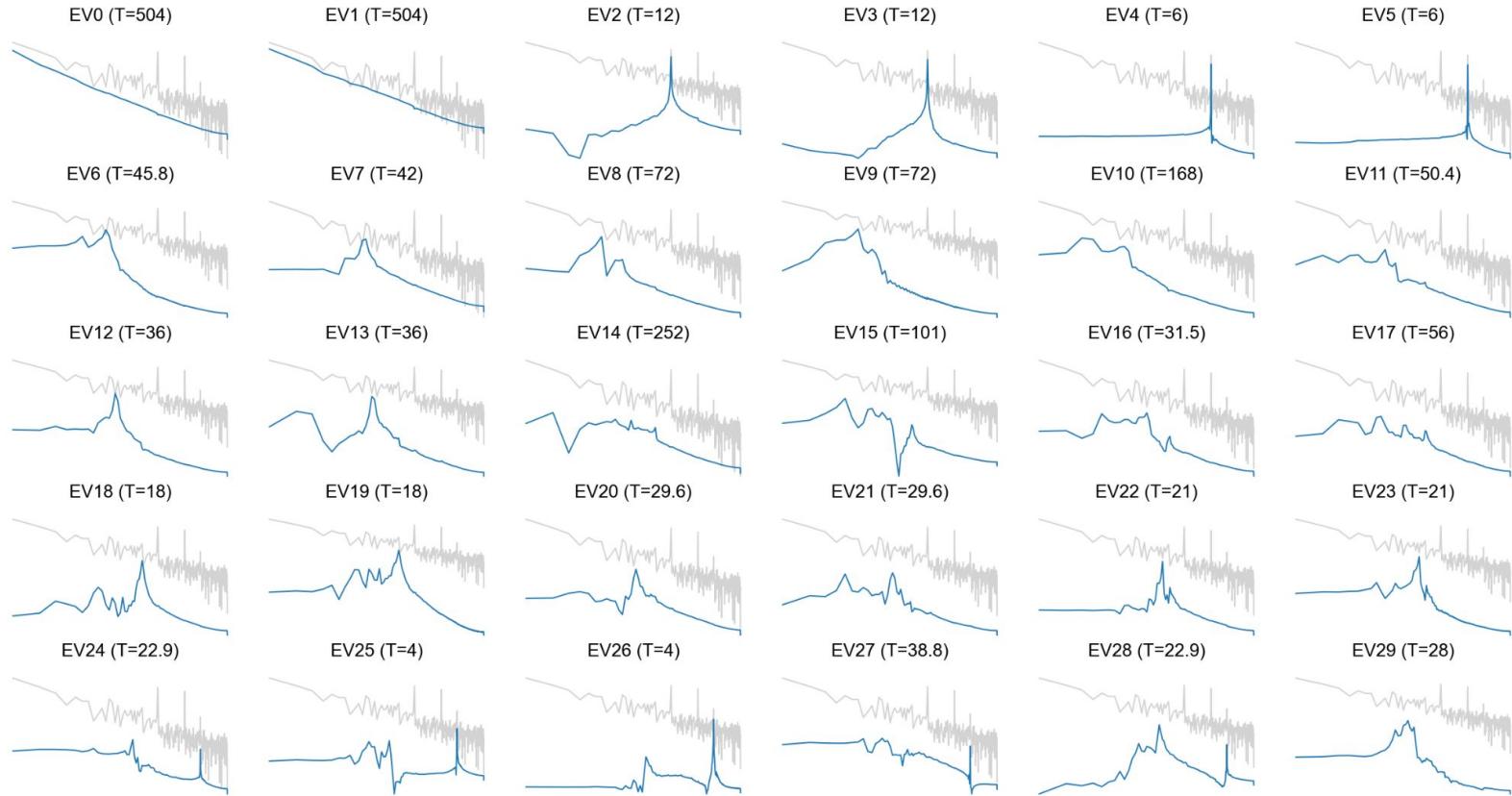


Visualizations: Vectors





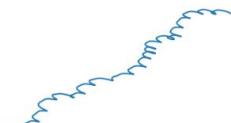
Visualizations: Periodograms





Visualizations: Paired Vectors

EV0 (27.6%) vs. 1 (24.4%)



EV1 (24.4%) vs. 2 (10.2%)



EV2 (10.2%) vs. 3 (10.1%)



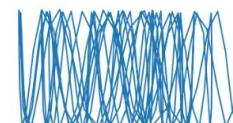
EV3 (10.1%) vs. 4 (5.5%)



EV4 (5.5%) vs. 5 (5.4%)



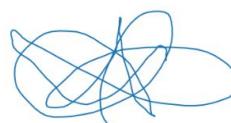
EV5 (5.4%) vs. 6 (1.9%)



EV6 (1.9%) vs. 7 (1.8%)



EV7 (1.8%) vs. 8 (1.6%)



EV8 (1.6%) vs. 9 (1.5%)



EV9 (1.5%) vs. 10 (1.1%)



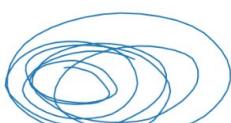
EV10 (1.1%) vs. 11 (0.9%)



EV11 (0.9%) vs. 12 (0.7%)



EV12 (0.7%) vs. 13 (0.7%)



EV13 (0.7%) vs. 14 (0.5%)



EV14 (0.5%) vs. 15 (0.4%)



EV15 (0.4%) vs. 16 (0.4%)



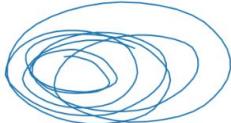
EV16 (0.4%) vs. 17 (0.4%)



EV17 (0.4%) vs. 18 (0.4%)



EV18 (0.4%) vs. 19 (0.4%)



EV19 (0.4%) vs. 20 (0.3%)



EV20 (0.3%) vs. 21 (0.3%)



EV21 (0.3%) vs. 22 (0.2%)



EV22 (0.2%) vs. 23 (0.2%)



EV23 (0.2%) vs. 24 (0.1%)



EV24 (0.1%) vs. 25 (0.1%)



EV25 (0.1%) vs. 26 (0.1%)



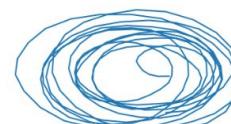
EV26 (0.1%) vs. 27 (0.1%)



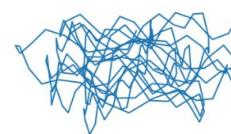
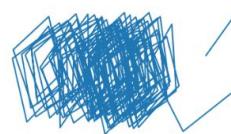
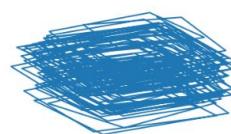
EV27 (0.1%) vs. 28 (0.1%)



EV28 (0.1%) vs. 29 (0.1%)

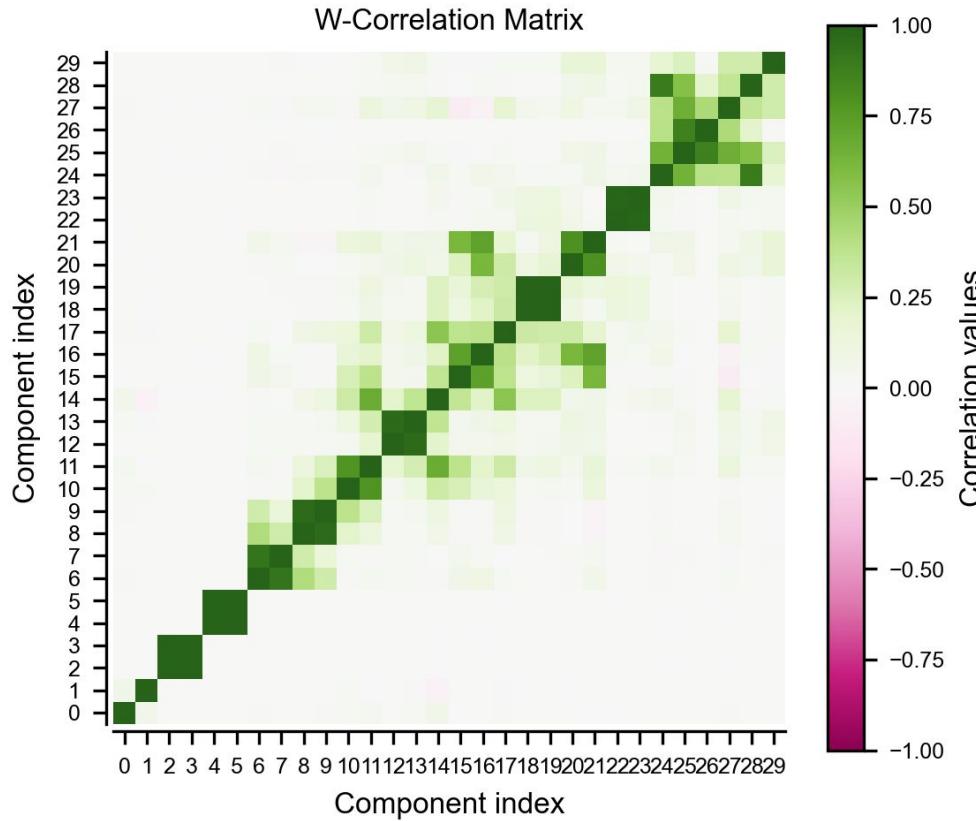


EV29 (0.1%) vs. 30 (0.1%)





Visualizations: Weighted Correlation





Significance Testing with Monte Carlo SSA



Group Selection with Monte Carlo SSA



Monte Carlo SSA generates random **surrogate time series** to distinguish significantly structured components from noise.



1. Perform SSA on time series X_t
2. Fit Autoregressive Model (AR) on time series X_t
3. Create N_{surr} random surrogates with AR model
4. Create N_{surr} matrices using SSA window L
5. Project surrogate matrices onto X_t SSA singular system
6. Compare or test projected norms against SSA singular values



Surrogate Data Generation

Time series

$\xrightarrow{\text{FIT}}$

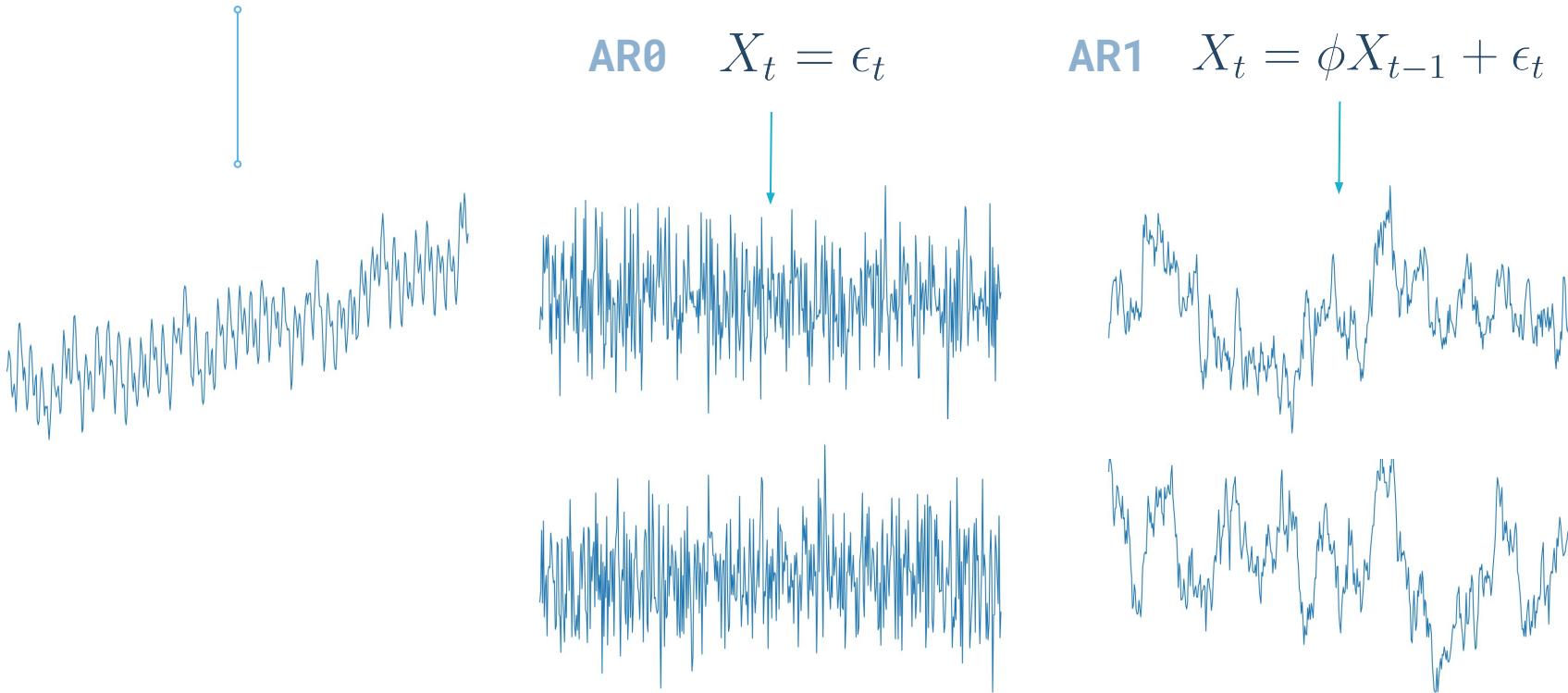
Auto-Regressive (AR) Models

AR0

$$X_t = \epsilon_t$$

AR1

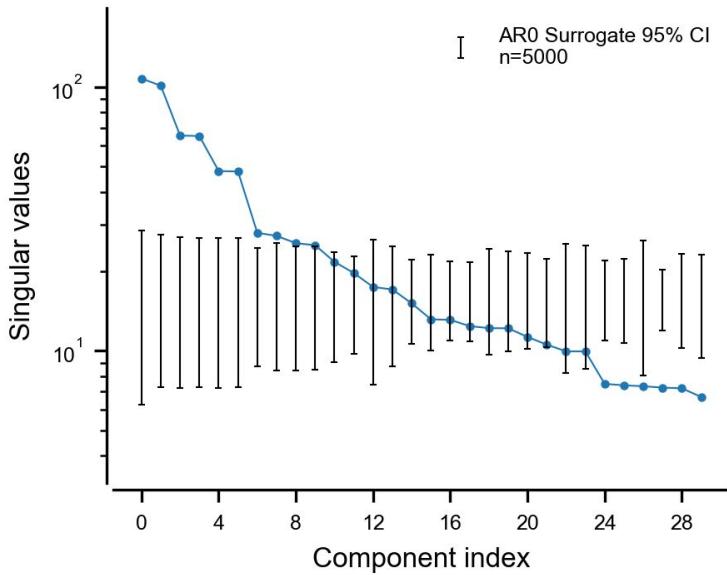
$$X_t = \phi X_{t-1} + \epsilon_t$$



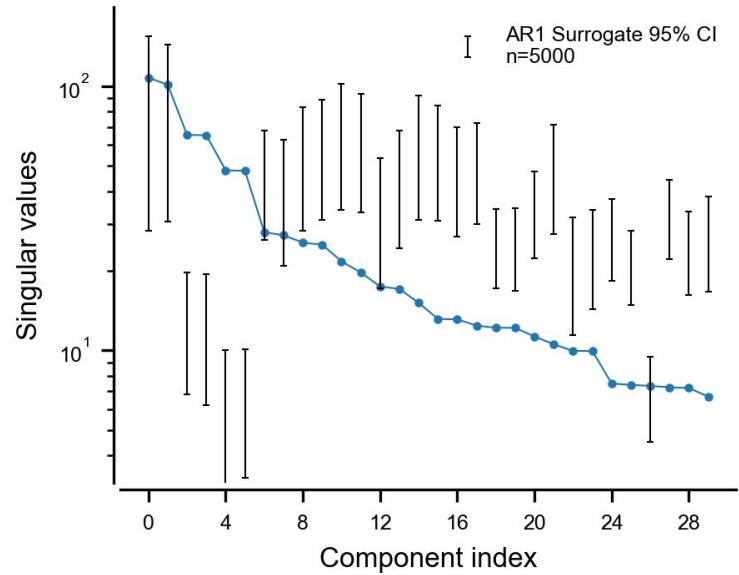


Surrogate Matrices Projection

AR0 Surrogate Model



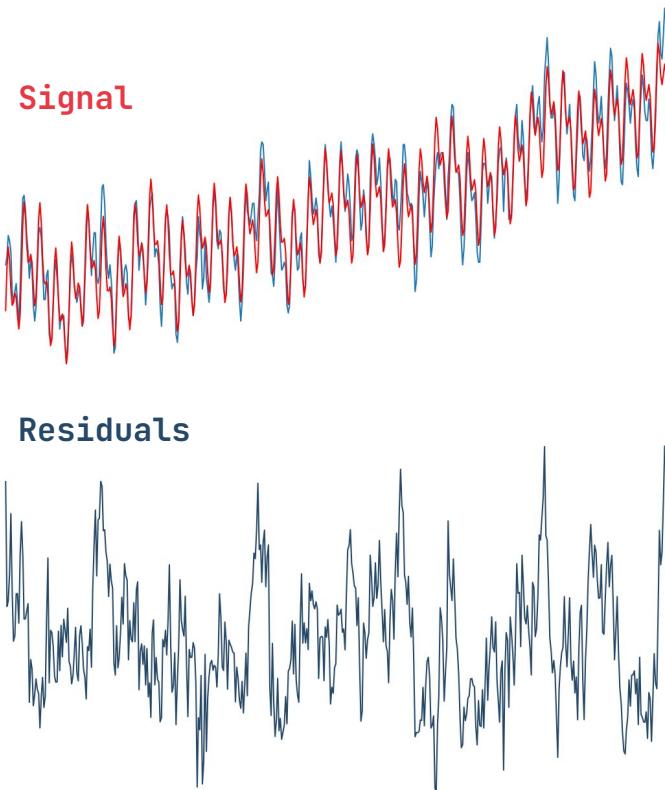
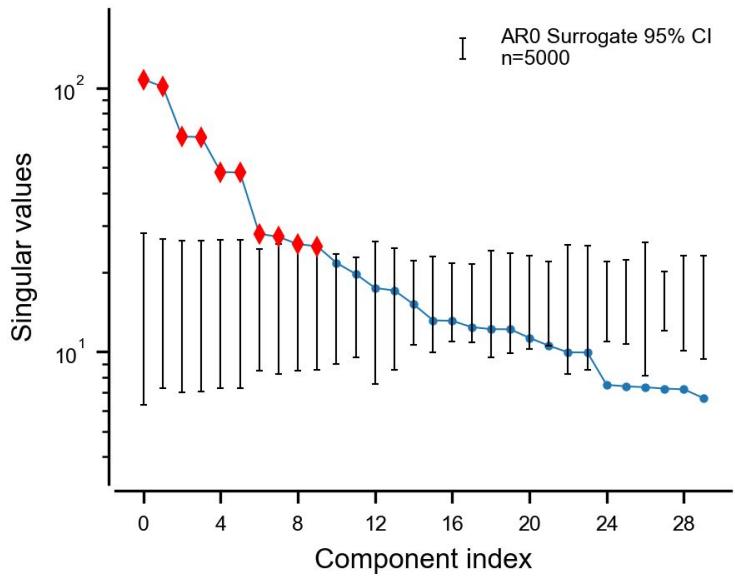
AR1 Surrogate Model



Trend (index 0 and 1) is not statistically significant using an AR1 model

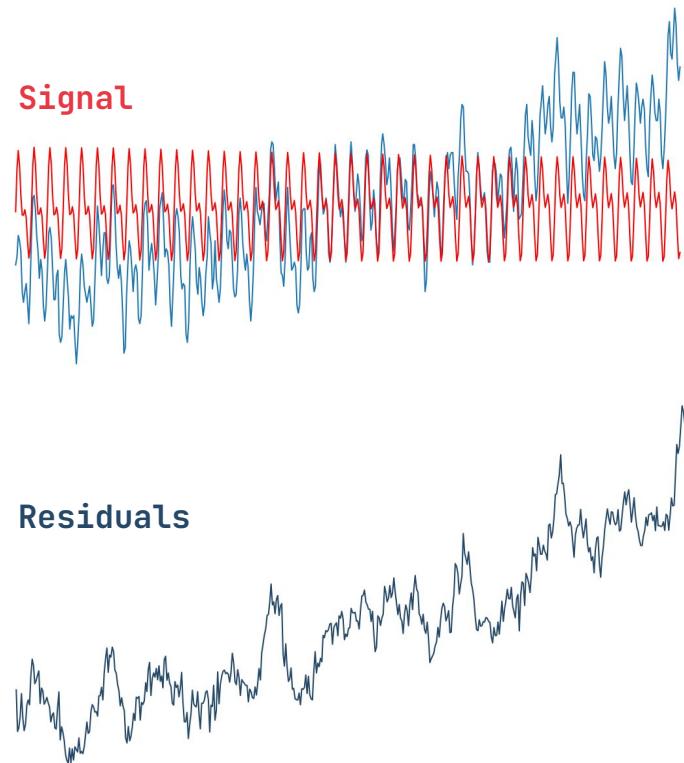
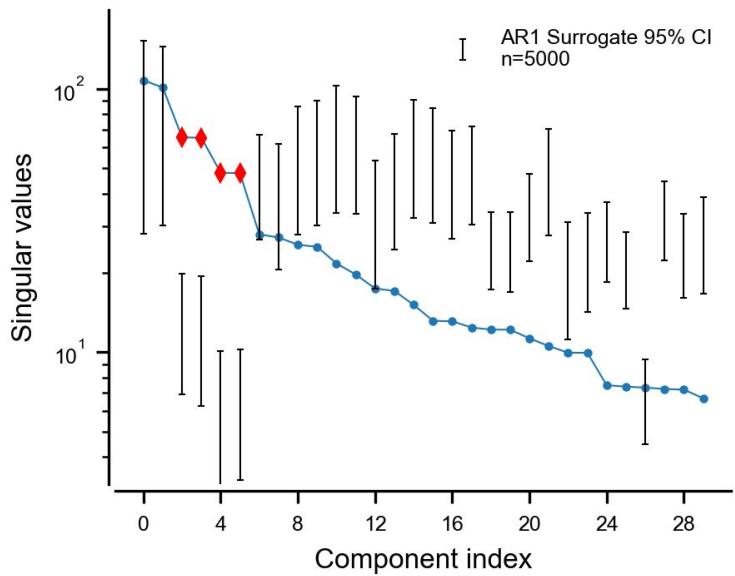


Automated Signal Reconstruction (ARO)





Automated Signal Reconstruction (AR1)

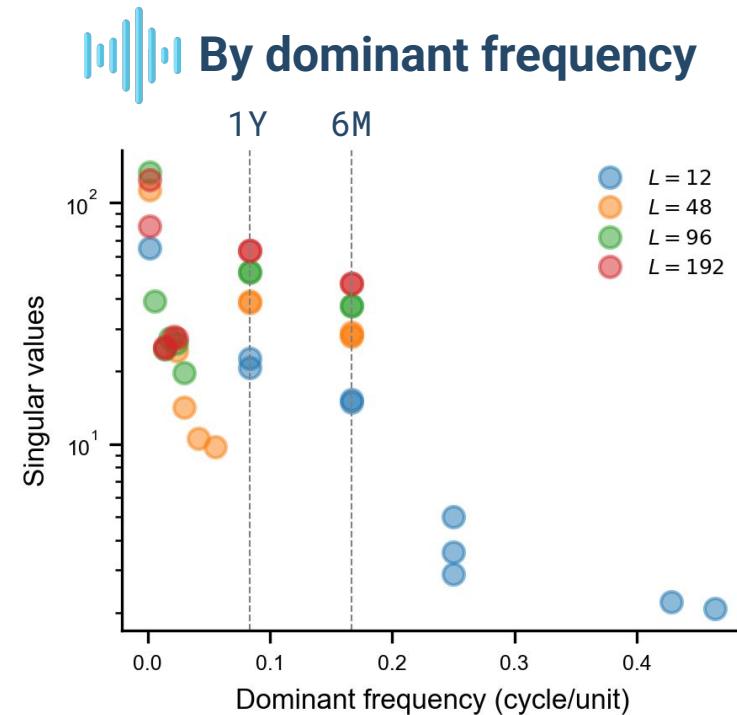
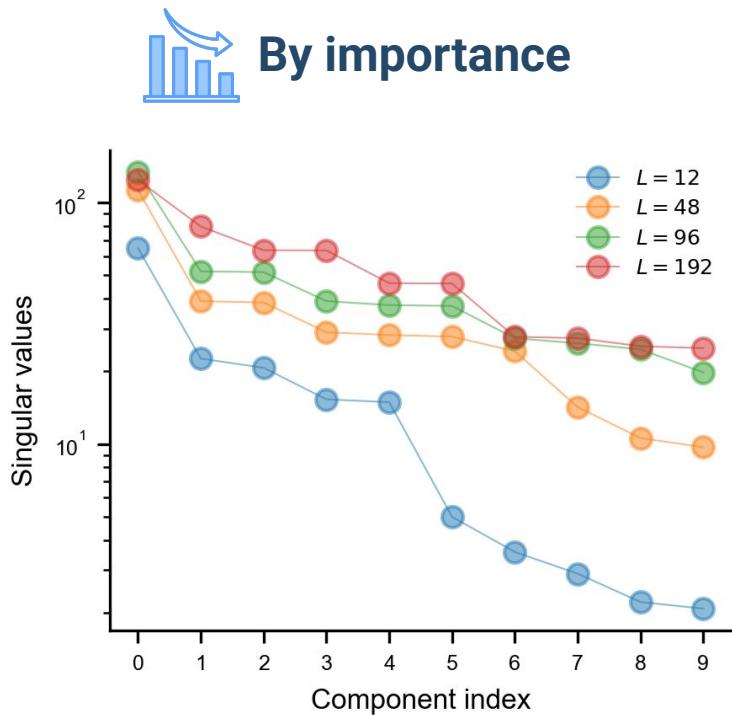




Around SSA: Tips, Limits, Variants, and Perspectives



Embedding Window Sensitivity





Embedding Window Tips

Guiding rules

- Use $L = N/2$ to maximize separability (max value)
- Set L to a multiple of the maximum period of interest
- Lower L is ok for smoothing and denoising purposes
- If $N \gg$, reduce L to lower computational requirements
- Cross-validate L within validation pipeline

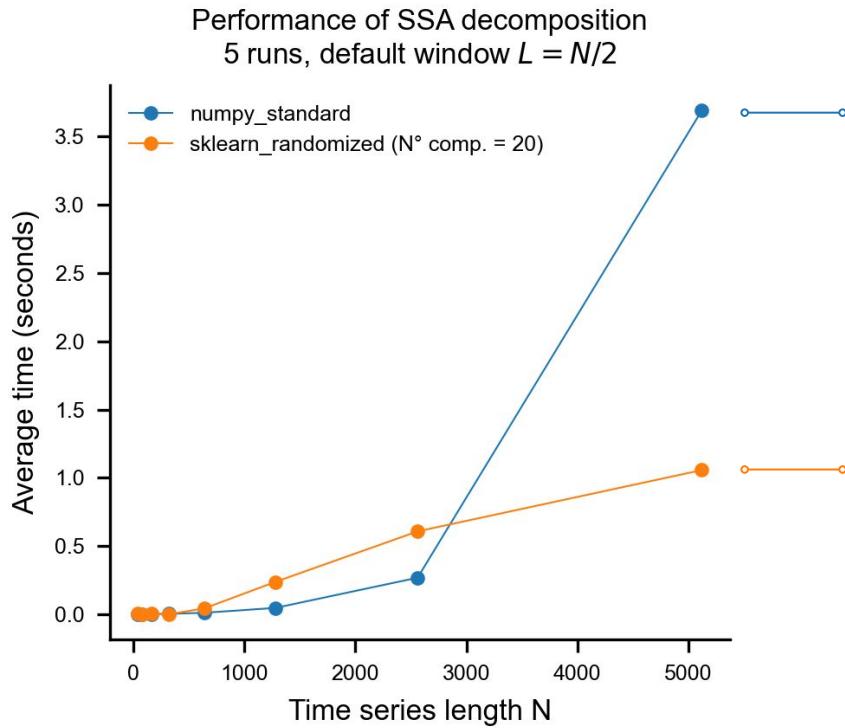


SVD Algorithms

- SVD algorithms differ in performance, stability, accuracy, matrix structural hypothesis, strategies.
- The full SVD algorithm scales as: $O(LK \cdot \min(L, K))$
 - Reduce L and use a covariance matrix: $O(L^3)$
 - Use Randomized or Truncated SVD
- Standard SVD algorithms are not robust to outliers
 - Mask outliers (replace by mean + iterative SVD)
 - Use Robust SVD algorithms



Randomized SVD Performance



[numpy.linalg.svd](#)



[sklearn.utils.extmath.randomized_svd](#)

<https://doi.org/10.1137/090771806>

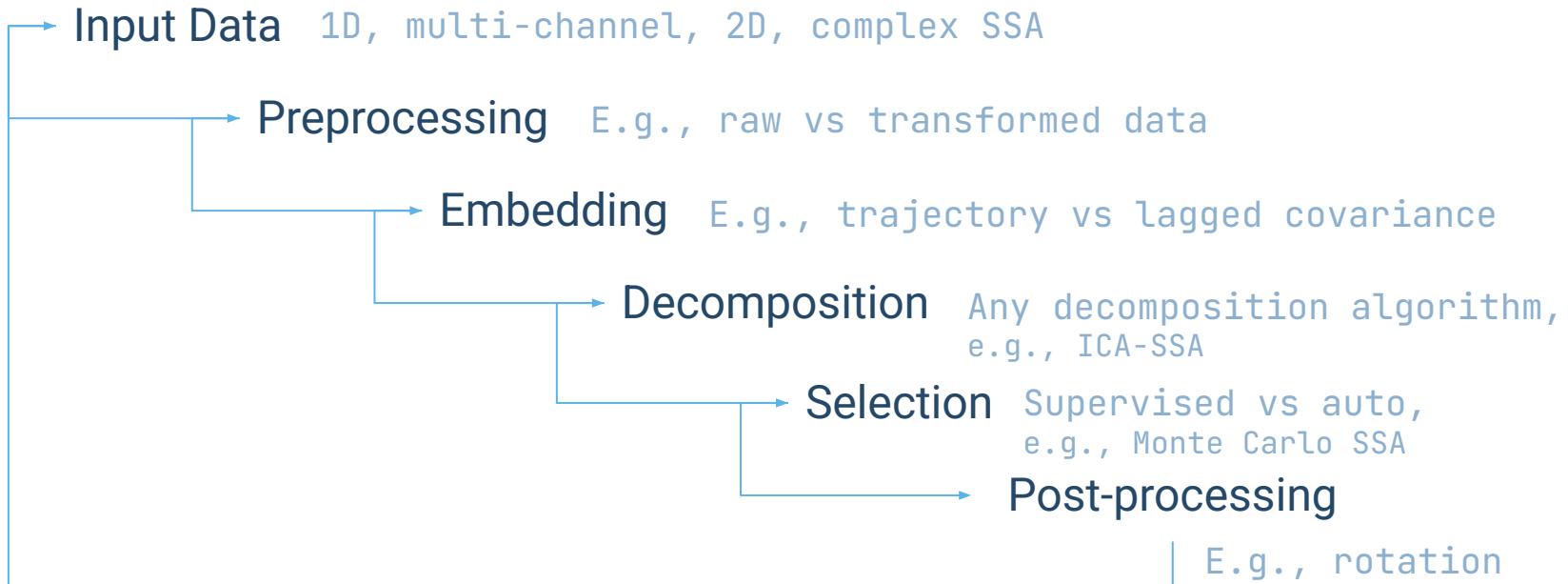


Automated Signal Grouping and Reconstruction

- Monte Carlo SSA
- Common PCA approaches: staircase method, captured variance thresholds, elbow method, ...
- Grouping:
 - Clustering the weighted correlation matrices
 - Clustering or filtering the dominant frequencies
- Cross-validation



SSA Variants



Iterative-SSA



SSA Fitness for Purposes



- Non-parametric, low hypotheses, exploratory framework
- Nonlinear trends and amplitude modulated signal
- Modular, adaptable framework (many SSA variants)
- Subjectivity in grouping >< Automated grouping
- Computationally intensive >< Embedding, SVD algorithm, ...
- Univariate approach >< Multichannel SSA
- Not a standard approach, somehow case-specific
- Relatively difficult to understand



"If the shoe doesn't fit, don't wear it"



Main References

1. Golyandina, N., & Zhigljavsky, A. (2020). Singular Spectrum Analysis for Time Series. Berlin, Heidelberg: Springer. <https://doi.org/10.1007/978-3-662-62436-4>
2. Hassani, H. (2007). Singular Spectrum Analysis: Methodology and Comparison. *Journal of Data Science*, 5(2), 239–257. [https://doi.org/10.6339/JDS.2007.05\(2\).396](https://doi.org/10.6339/JDS.2007.05(2).396)
3. Broomhead, D. S., & King, G. P. (1986). Extracting qualitative dynamics from experimental data. *Physica D: Nonlinear Phenomena*, 20(2), 217–236. [https://doi.org/10.1016/0167-2789\(86\)90031-X](https://doi.org/10.1016/0167-2789(86)90031-X)
4. Vautard, R., & Ghil, M. (1989). Singular spectrum analysis in nonlinear dynamics, with applications to paleoclimatic time series. *Physica D: Nonlinear Phenomena*, 35(3).
[https://doi.org/10.1016/0167-2789\(89\)90077-8](https://doi.org/10.1016/0167-2789(89)90077-8)
5. Allen, M. R., & Smith, L. A. (1996). Monte Carlo SSA: Detecting irregular oscillations in the Presence of Colored Noise. *Journal of Climate*, 9(12), 3373–3404.
[https://doi.org/10.1175/1520-0442\(1996\)009<3373:MCSDIO>2.0.CO;2](https://doi.org/10.1175/1520-0442(1996)009<3373:MCSDIO>2.0.CO;2)



Hands-on Experience with SSA



SSALib Basic Usage (1)



<https://github.com/ADSCIAN/ssalib>

```
>>> pip install ssalib
```



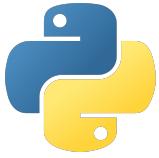
```
from ssalib import SingularSpectrumAnalysis
from ssalib.datasets import load_sst

# Load example data
ts = load_sst()

# Create SSA instance and decompose
ssa = SingularSpectrumAnalysis(ts)
ssa.decompose()
```



SSALib Basic Usage (2)



```
# Visualize results
fig, ax = ssa.plot(kind='values')

# Reconstruct groups
ssa.reconstruct(groups={'trend': [0, 1],
'seasonality': [2, 3]})

# Plot reconstructed series
fig, axes = ssa.plot(kind='timeseries')

# Export
df_ssa = ssa.to_frame()
```



SSALib Tutorial Notebooks



<https://github.com/ADSCIAN/ssalib>

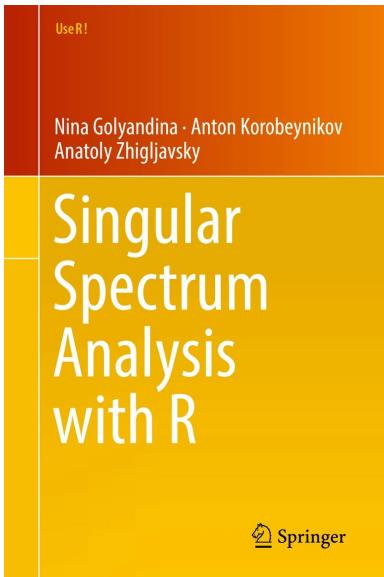
Check the Jupyter notebooks:

- [Tutorial 1: Introduction to SSA](#)
- [Tutorial 2: Plotting Guide](#)
- [Tutorial 3: SVD Matrix Construction and Window Sizes](#)
- [Tutorial 4: Comparison of SVD Solvers and Speed Performances](#)
- [Tutorial 5: Comparison of SSALib and Rssa](#)
- [A1: Testing Significance with MonteCarloSSA](#)





Rssa Basic Usage (1)

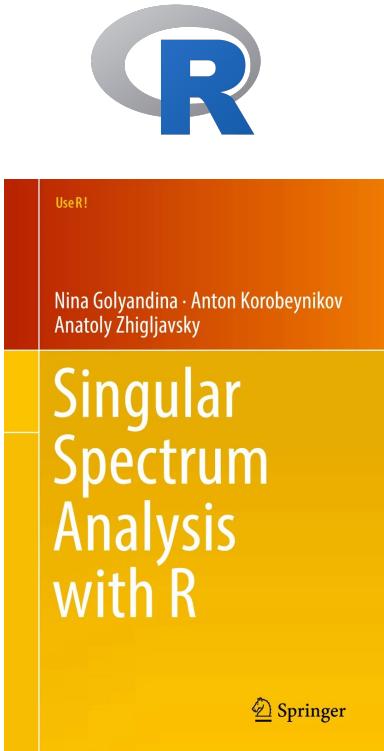


<https://cran.r-project.org/package=Rssa>

```
install.packages("Rssa")  
  
# Load required libraries  
library(Rssa)  
data("co2")  
  
# standardize data  
co2_std = (co2 - mean(co2))/sd(co2)  
s <- ssa(co2_std, L=234)
```



Rssa Basic Usage (2)



```
# View results
plot(s) # plot singular values by default
plot(s, type="vectors") # plot vectors

# Reconstruct groups
r <- reconstruct(
  s, groups=list(c(1, 2), c(3, 4), c(5, 6)))
# Plot the reconstruction
plot(r, add.original = TRUE)
```



Short Course Materials



https://github.com/dadelforge/MethodsNET2_SSA

