Regressions

MKT 566

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What we will learn

- We continue to talk about **covariation** and learn how to model it using regressions

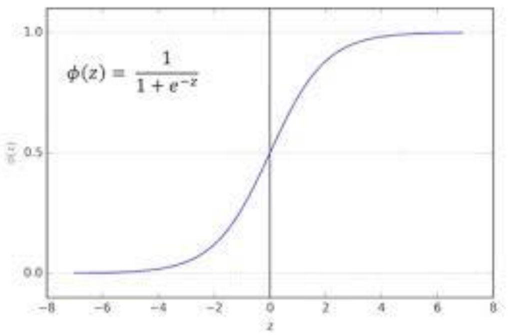
- We are going to cover regressions for binary outcomes, i.e., logistic regressions

- Chapter 3.6 of R for Marketing Students

Binary outcomes: Logistic regression

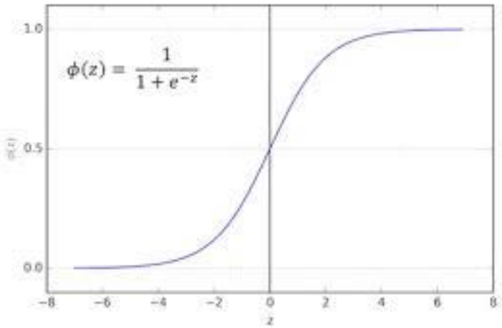
- Let’s assume Y is binary, e.g.: 1 if consumer *i* buys a product, 0 otherwise
  - Linear regression can return predictions outside [0,1]

Binary outcomes: Logistic regression



- We need a different functional form
  - **Logistic** (sigmoid) **function**:
    - Probability of success → $\mathrm{p} = P(Y = 1|X) = \frac{1}{1+e^{z}}$
    - Smoothly “squashes” any real number *z* into a number between 0 and 1

Binary outcomes: Logistic regression



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- So, given our linear predictor: $z = \beta_0 + \beta_1 X$
- Odds of success: $\frac{p}{1-p} = e^{z}$ → $\log\left(\frac{p}{1-p}\right) = z$ (proof)
  - So log odds are modeled as a linear function of X
  - (The concept of "odds" comes originally from gambling, where it's more natural to compare the chance of an event happening vs. not happening)

Proof

1. **Write out the odds**

$$\frac{p}{1-p} = \frac{\dfrac{1}{1+e^{-x}}}{1-\dfrac{1}{1+e^{-x}}} = \frac{1}{1+e^{-x}} \Big/ \frac{(1+e^{-x})-1}{1+e^{-x}} = \frac{1}{1+e^{-x}} \Big/ \frac{e^{-x}}{1+e^{-x}} = \frac{1}{e^{-x}} = e^{x}.$$

2. **Take the natural log**

$$\ln\!\left(\tfrac{p}{1-p}\right) = \ln\!\left(e^{x}\right) = x.$$

So when $p = 1/(1+e^{-x})$, its log-odds $\ln\big(p/(1-p)\big)$ simplifies exactly to x.

Binary outcomes: Logistic regression interpretation

- Logistic regression estimates changes in probability (odds)

- Given $y_i = \beta_0 + \beta_1 X_{1,i} + \epsilon_i$
  - A one unit increase in X_1 **multiplies** the odds of Y by $\boldsymbol{e^{\beta_1}}$, $i.e.$, odds change by $(e^{\beta_1} - 1) * 100\%$
  - (again, for small β_1 we can approximate it with β_1*100%)

Estimation

- Unlike linear regression (which minimizes squared errors), logistic regression **maximizes the likelihood** of observing the actual outcomes, given the model.

Model fit

- R^2 does not work well with binary outcomes
- Binary outcomes are about **yes/no decisions**, not “how much” → the **usual R^2 just doesn’t work well**

- There are alternative measures of fit, e.g.,
  - **Pseudo- R^2**: How much better is this model at predicting 0s and 1s compared to guessing the average?
    - It's useful for checking if your logistic model is doing better than chance.
  - **Log Likelihood**: is a measure of how well the model’s predicted probabilities match the actual 0/1 outcomes.
    - The higher the better

Logistic regression in R

```
# Crate the variable gem which identifies very good listings
airbnb[, gem:=as.integer(star_rating>=4.5 &
reviews_count>20)]
# Predict probability of being a gem using logistic
regression
m_logit = glm(gem ~ price + guests_included + city +
room_type, data = airbnb, family = binomial)
# Print results
stargazer(m_logit, type = "text",
          omit.stat = c("f", "ser", "aic", "bic"))
```

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```
==================================================
                          Dependent variable:
                      ----------------------------
                                  gem
--------------------------------------------------
price                          -0.002***
                               (0.0001)

guests_included                 0.112***
                                (0.008)

cityBoston                      0.167***
                                (0.040)

cityLos Angeles                 0.162***
                                (0.029)

cityMiami                      -0.195***
                                (0.042)

cityNew York City               0.188
                                (0.569)

room_typePrivate room          -0.008
                                (0.026)

room_typeShared room           -0.707***
                                (0.064)

Constant                       -1.393***
                                (0.035)

--------------------------------------------------
Observations                    50,836
Log Likelihood                -25,430.070
==================================================
```