

GLMNET

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What is GLMNET?

- **GLMNET** is an R package (part of the `glmnet` library) for fitting *regularized generalized linear models*.
- It efficiently handles **high-dimensional data** (many features, sparse matrices).
- Supports various models:
 - Linear Regression
 - Logistic Regression (binary or multinomial)
 - Poisson Regression, Cox, etc.
- Adds a **penalty term** to control model complexity → prevents overfitting.

Penalty

GLMNET minimizes:

$$\text{Loss}(\boldsymbol{\beta}) = \underbrace{\text{DataFit}(\boldsymbol{\beta})}_{\text{depends on model}} + \lambda \underbrace{[\alpha \|\boldsymbol{\beta}\|_1 + (1 - \alpha) \|\boldsymbol{\beta}\|_2^2]}_{\text{Elastic Net penalty}}$$

Where:

- **λ (lambda)** = regularization strength
- **Penalty** = penalty term (Lasso, Ridge, or both)
- **Smaller λ** → more flexible, coefficients can take large values, can overfit
- **Larger λ** → more shrinkage (some coefficients are shrunk towards zero, simpler model)

Penalty: Choosing α

- The penalty itself depends on a parameter called α :
 - 1: Lasso Regression → sets some coefficients **exactly to 0** (feature selection)
 - 0: Ridge regression → **shrink** coefficients **towards 0**
 - $0 < \alpha < 1$: Elastic Net → mix of both
- Rule of thumb:
 - Start with **Elastic Net** (e.g., alpha = 0.5).
 - Prefer **Ridge** for collinearity + no need for selection.
 - Prefer **Lasso** when you value **sparse, interpretable** models and suspect many features are noise.

Penalty: Choosing Lambda (λ)

- GLMNET automatically fits models for many λ values along a **regularization path**.
- Typically, use **cross-validation** to pick the best one.
- `cv.glmnet()` returns:
 - **lambda.min** $\rightarrow \lambda$ giving the best performance (lowest error / highest AUC)
 - **lambda.1se** \rightarrow largest λ within 1 standard error of the best (simpler, more stable)