Baye's Theorem

$$p(w_i/\vec{x}) = \frac{p(\vec{x}/w_i)\, p(w_i)}{p(\vec{x})}$$

$\vec{x}$ = Feature Vector

w_i = i^{th} class

$$p(A/B) = \frac{p(B/A)\, p(A)}{p(B)}$$

Say our data looks like this,

Features

Sheet1

[redacted]	Sex	Age	No of Siblings	Ticket Fare	Survived
1	male	38	4	25	0
2	female	38	5	25	1
3	female	35	2	25	1
4	female	35	4	25	1
5	male	35	4	25	1
7	male	54	2	15	0
8	male	12	3	25	0
9	female	27	0	25	1
10	female	27	1	25	1
11	female	28	1	10	1
12	female	58	2	15	0
13	male	58	0	15	0
14	male	58	1	15	1
15	female	27	5	30	0
16	female	55	1	30	1
17	male	27	4	25	0
20	male	35	2	30	0
21	male	58	1	15	1
22	female	27	5	30	1
23	male	28	4	25	0
24	female	58	3	15	0
25	female	27	5	25	1

Class / w_i

$\vec{x}$ = Feature vector = [male, 38, 4, 25] { 1st row of above data }

w_i = Class = 1 or 0 = Last column of data

1 → Survived

0 → Not Survived

The above formula says,

tell me the class of ~~doo~~ feature vector if you have the class priors, feature priors and likelihood of that feature.

_____ = _____ × _____ × _____

class prior = $\frac{\text{No of occourance of class "i"}}{\text{Total number of occorance of all class}}$

In above example, the (survived) 0, 1 is the class we are trying to predict.

$\therefore$ class prior of class ("0") = $\frac{\text{No of times "zero" occoured}}{\text{Total numbers of occurance of all ~~class~~ class}}$

$p(w_0)$ ↑ Symbol = $\frac{10}{~~23~~ 22}$

$p(w_1) = \frac{\text{No of occurance of class "1"}}{\text{Total no of occorance of all class}}$

$= \frac{12}{22}$

$\therefore p(w_i) = i^{th}$ class probability

_____ × _____ × _____

$$\text{Feature prior} = \frac{\text{No of occurance of feature } \vec{x}}{\text{Total no of all feature vectors}}$$

Say $\vec{x}$ = [female, 35, 2, 25]

So we need to find how many time $\vec{x}$ occoured in the above data set

∴ Feature prior = $p(\vec{x})$ = $\frac{1}{22}$ ← Such combination occoured only once (Row 3)

(symbol)

Say $\vec{x}$ = [male, 58, 1, 15]

$p(\vec{x})$ = $\frac{2}{22}$ ← such combination occoured only "2" times in entire data set. (Row 14 & 22)

_____ × _____ × _____

Likelihood = class conditional probability

= $p(\vec{x}/w_i)$

= $\frac{\text{No of occurance of } \vec{x} \text{ in the class "i"}}{\text{Total number of feature vectors in that class}}$

Say you are given that, the passenger has already survived ("class 1"), then find the probability of feature vector $\vec{x}$ = [male, 58, 1, 15]

$p(\vec{x}/w_1)$ = $\frac{2}{12}$ → 2 such feature vector in class "1"

↓ No of ~~datap~~ rows with survived == 1

Say $\vec{x}$ = [female, 27, 5, 30]

$p(\vec{x}/w_1) = \frac{2}{12}$ ← such occurance … (row 1st and 23)

↑ Total points with survived == 1

$\therefore p(\vec{x}/w_i)$ = Find probability of occurance of $\vec{x}$ in class i

= Frequency of occurance of $\vec{x}$ in class i / Total number of points in class i

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So, say, for example, given

$\vec{x}$ = [male, 58, 1, 15], then find the class of $\vec{x}$

$\therefore p(w_i/\vec{x}) = p\,\frac{(\vec{x}/w_i)\;\; p(w_i)}{p(\vec{x})}$

Freq of $\vec{x}$ in class i ↙ ; Freq of class i ↖ ; ↑ Freq of $\vec{x}$

So $p(\vec{x})$ = No of occurance of $\vec{x}$ in total dataset / Total no of data points

$= \frac{2}{22}$

$p(w_0)$ = No of occurance of class zero / Total no of data points

$= \frac{10}{22}$

$p(w_1)$ = No of occurance of class 1 / Total no of data points $= 12/22$

$p(\vec{x}/w_0)$ = Prob of occurance of $\vec{x}$ in class zero

$= \frac{0}{10}$ → zero times such feature occured

$= 0$ in class (zero)

10 → No. of datapoints in class (zero)

$p(\vec{x}/w_1)$ = occurance of $\vec{x}$ in class 1

$= \frac{2}{12}$ → 2 times such feature occoured in class (1)

12 → No of data points in class 1

$$\therefore p(w_0/\vec{x}) = \frac{p(\vec{x}/w_0) \times p(w_0)}{p(\vec{x})}$$

$$= \frac{0 \times \frac{10}{22}}{\frac{2}{22}} = 0$$

$$p(w_1/\vec{x}) = p(\vec{x}/w_1) \times \frac{p(w_1)}{p(\vec{x})}$$

$$= \frac{\frac{2}{12} \times \frac{12}{22}}{\frac{2}{22}} = \frac{2}{22} \times \frac{22}{2} = 1$$

$\therefore p(w_0/\vec{x}) = 0.0$ & $p(w_1/\vec{x}) = 1.0$

~~$p(\vec{x}/w_0)$~~

So if you are [male, 58, 1, 15], then you have a higher chance of surviving {In this case completely :)}.

The above values are probability & not absolute numbers. It is by pure chance that we got 0 & 1 as probability. This is a toy example to show you baye's rule.