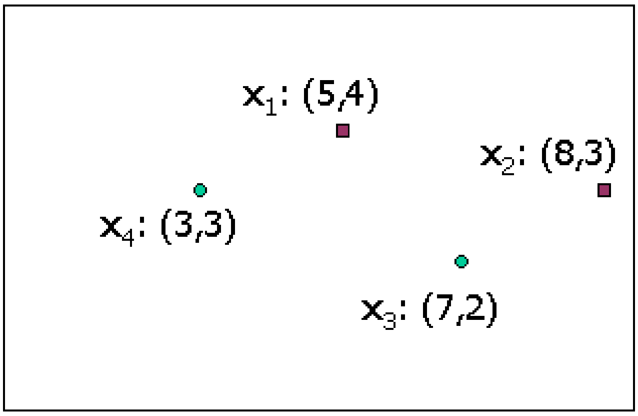
Feedback — Week6B (Basic)

Help Center

You submitted this quiz on **Sat 7 Mar 2015 1:24 PM PST**. You got a score of **3.00** out of **3.00**.

Question 1

The figure below shows two positive points (purple squares) and two negative points (green circles):



That is, the training data set consists of:

($\mathbf{x}_1$,y_1) = ((5,4),+1)
($\mathbf{x}_2$,y_2) = ((8,3),+1)
($\mathbf{x}_3$,y_3) = ((7,2),-1)
($\mathbf{x}_4$,y_4) = ((3,3),-1)

Our goal is to find the maximum-margin linear classifier for this data. In easy cases, the shortest line between a positive and negative point has a perpendicular bisector that separates the points. If so, the perpendicular bisector is surely the maximum-margin separator. Alas, in this case, the closest pair of positive and negative points, $\mathbf{x}_2$ and $\mathbf{x}_3$, have a perpendicular bisector that misclassifies $\mathbf{x}_1$ as negative, so that won't work.

The next-best possibility is that we can find a pair of points on one side (i.e., either two positive or two negative points) such that a line parallel to the line through these points is the maximum-margin separator. In these cases, the limit to how far from the two points the parallel line can get is determined by the closest (to the line between the two points) of the points on the other side. For our simple data set, this situation holds.

Consider all possibilities for boundaries of this type, and express the boundary as **w**.**x**+b=0, such that **w**.**x**+b≥1 for positive points **x** and **w**.**x**+b≤-1 for negative points **x**. Assuming that **w** = (w_1,w_2), identify in the list below the true statement about one of w_1, w_2, and b.

Your Answer		Score	Explanation
b = -19/3			
b = -5			
b = -15/2	✔	1.00	
w_2 = 10/3			
Total		1.00 / 1.00	

Question Explanation

In what follows, assume that v is the vertical dimension and u the horizontal dimension in the diagram. There are only two possibilities: a parallel to either the line through $\mathbf{x}_1$ and $\mathbf{x}_2$, or a parallel to the line through $\mathbf{x}_3$ and $\mathbf{x}_4$.
The line through $\mathbf{x}_1$ and $\mathbf{x}_2$ can be written as v=17/3-u/3. If we move this line down (i.e., lower the constant 17/3) until it meets $\mathbf{x}_3$ or $\mathbf{x}_4$, we find it meets $\mathbf{x}_3$ first, and the parallel line through $\mathbf{x}_3$ is v=13/3-u/3. Thus, the parallel line between these two lines is v=5-u/3.

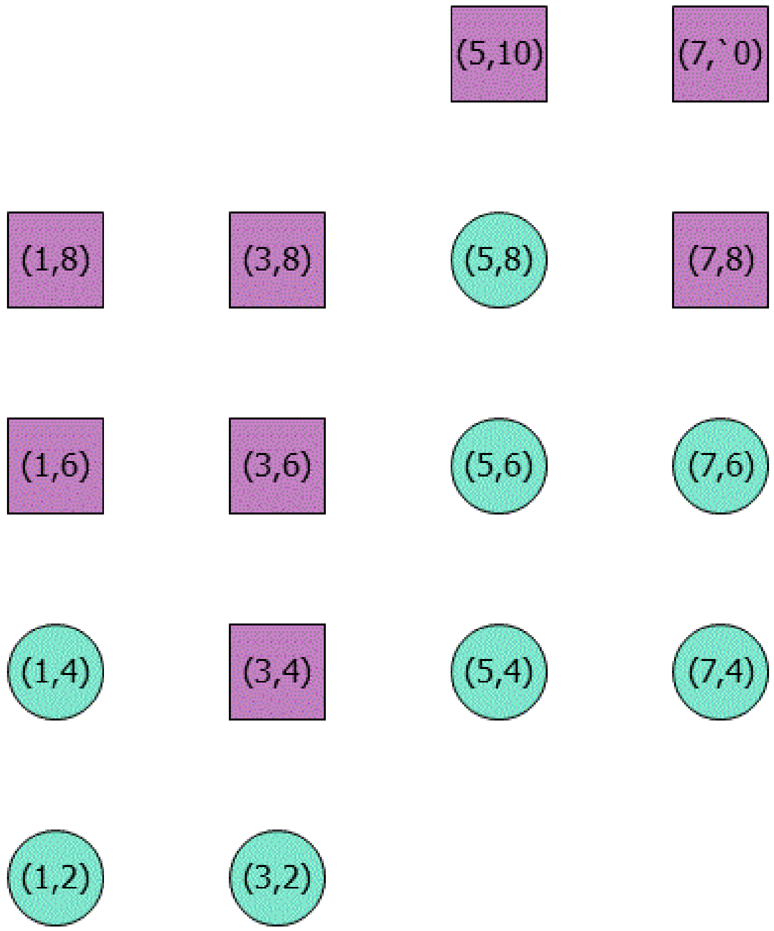
We must put this in the form **w**.**x**+b=0. That is, (1/3,1).(u,v)+(-5)=0. we must scale the vector **w**=(1/3,1) and constant b=(-5) so that if (u,v) is $\mathbf{x}_1$ or $\mathbf{x}_2$, the value of the left side is exactly +1, and if (u,v) is $\mathbf{x}_3$, the value is exactly -1. Since the vertical distance between the boundary and the parallel lines through $\mathbf{x}_1$ and $\mathbf{x}_2$ on one hand, and through $\mathbf{x}_3$ on the other, is 2/3, the scaling factor must be 3/2. That is, **w**=(1/2,3/2) and b=(-15/2). You can check that for these values, **w**.**x**+b is +1 when **x**=$\mathbf{x}_1$ or **x**=$\mathbf{x}_2$, and it is -1 when **x**=$\mathbf{x}_3$. The value is less than -1 when **x**=$\mathbf{x}_4$, as must be the case.

We must do a similar analysis starting with the line through $\mathbf{x}_3$ and $\mathbf{x}_4$, which is v=15/4-u/4. Of the positive points, $\mathbf{x}_2$ is the closer to this line, and the parallel line through $\mathbf{x}_2$ is v=5-u/4. The midpoint of these lines is v=35/8-u/4. When we put this boundary in the **w**.**x**+b form, and scale by 8/5, we are left with the line (2/5,8/5).(u,v)+(-7)=0; i.e., **w**=(2/5,8/5) and b=(-7).

One of these lines is the maximum-margin separator, but which? It is the one with the smaller length of **w**. The length of the vector (1/2,3/2) is sqrt($(1/2)^2$+$(3/2)^2$) = sqrt(5/2). The length of the vector (2/5,8/5) is sqrt($(2/5)^2$+$(8/5)^2$) = sqrt(68/25). Since 5/2<68/25, the former is the smaller. That tells us the maximum-margin separator is v=5-u/3, or in vector form: (1/2,3/2).**x**-(15/2)=0.

Question 2

Consider the following training set of 16 points. The eight purple squares are positive examples, and the eight green circles are negative examples.



We propose to use the diagonal line with slope +1 and intercept +2 as a decision boundary, with positive examples above and negative examples below. However, like any linear boundary for this training set, some examples are misclassified. We can measure the goodness of the boundary by computing all the slack variables that exceed 0, and then using them in one of several objective functions. In this problem, we shall only concern ourselves with computing the slack variables, not an objective function.

To be specific, suppose the boundary is written in the form **w**.**x**+*b*=0, where **w** = (-1,1) and *b* = -2. Note that we can scale the three numbers involved as we wish, and so doing changes the margin around the boundary. However, we want to consider this specific boundary and margin.

Determine the slack for each of the 16 points. Then, identify the correct statement in the list below.

Your Answer		Score	Explanation
○ The slack for (7,8) is 0.			
◉ The slack for (1,4) is 2.	✔	1.00	
○ The slack for (5,4) is 2.			
○ The slack for (3,4) is 0.			

Total	1.00 / 1.00

Question Explanation

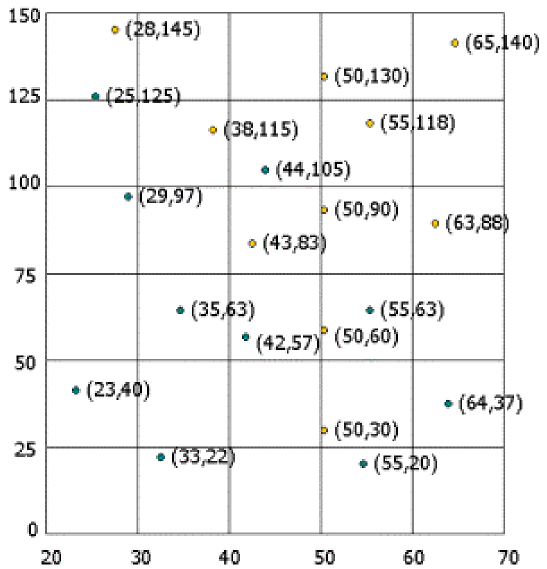
There are four misclassified points: (1,4), (3,4), (5,8), and (7,8). To find the slack for a negative point, we write the requirement that w.x+b≤-1 as w.x+b = -1+ξ. For the point (1,4), we have (-1,1).(1,4)-2 = -1+4-2 = 1 = -1+ξ, from which it follows that ξ=2. A similar calculation gives the same result for the other negative misclassified point, (5,8).

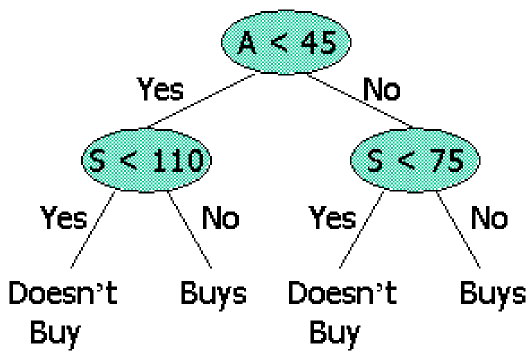
To find the slack for a positive point, we write the requirement that w.x+b≥1 as w.x+b = 1-ξ. For the point (3,4), we have (-1,1).(3,4)-2 = -3+4-2 = -1 = 1-ξ, from which it follows that ξ=2. A similar calculation gives the same result for the other positive misclassified point, (7,8).

It is also possible that a correctly classified point has a slack greater than 0. However, in this case, the correctly classified points that are closest to the boundary, such as (1,2), are right on the margin, and so their slack is 0. For example, (-1,1).(1,2)-2 = -1+2-2 = -1, so no slack is necessary.

Question 3

Below we see a set of 20 points and a decision tree for classifying the points.





To be precise, the 20 points represent (Age,Salary) pairs of people who do or do not buy gold jewelry. Age (appreviated A in the decision tree) is the x-axis, and Salary (S in the tree) is the y-axis. Those that do are represented by gold points, and those that do not by green points. The 10 points of gold-jewelry buyers are:

(28,145), (38,115), (43,83), (50,130), (50,90), (50,60), (50,30), (55,118), (63,88), and (65,140).

The 10 points of those that do not buy gold jewelry are:

(23,40), (25,125), (29,97), (33,22), (35,63), (42,57), (44, 105), (55,63), (55,20), and (64,37).

Some of these points are correctly classified by the decision tree and some are not. Determine the classification of each point, and then indicate in the list below the point that is misclassified.

Your Answer		Score	Explanation
○ (44,105)			
○ (23,40)			
◉ (50,30)	✔	1.00	
○ (55,118)			
Total		1.00 / 1.00	

Question Explanation

The tree classifies as a non-buyer those points with Age
The rest of the space is classified as buyers. There is, however, a non-buyer at (25,125), which is thus misclassified. However, all sixteen other points are classified correctly.