

GPU Computing Project

Parallel implementation of Dijkstra's Algorithm

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Abstract

The purpose of this paper is to describe the implementation and benchmarking of various parallel implementations of Dijkstra's Algorithm to solve the all-pairs shortest path problem.

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1 Introduction

The problem of shortest path in a graph consists in finding the path, from node A to node B, which minimizes the sum of edges weights of which the path is composed.

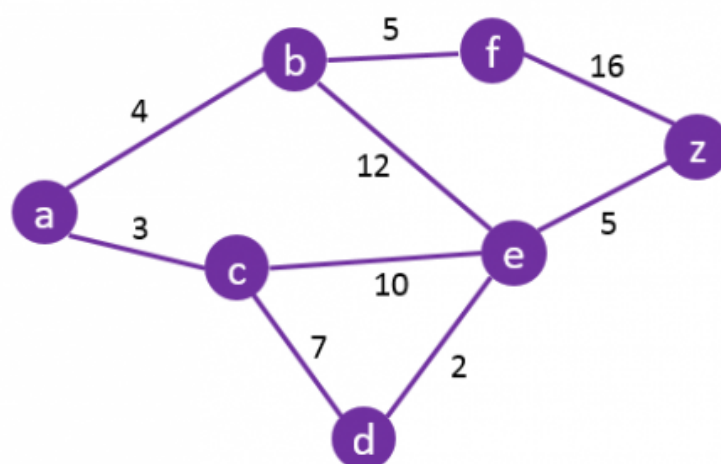


Figure 1: Graph with edge weights

In particular, the algorithm used for this document aim to solve the all-pairs shortest path problem, which consists of finding the lenght of the shortest path for all the pairs of nodes.

The algorithm assumes that there are not negative weights and that the graph is undirected without isolated parts.

This problem is tackled in a lot of practical applications, like road networks path-finding, telecommunication routing and robot navigation.

2 Dijkstra's algorithm overview

To solve the problem introduced before, there are several solutions which have been studied and improved troughout the years. The option chosen for this task has been the Dijkstra's algorithm, which has been implemented using the paper [1] as a guidance.

This is a very well known algorithm, initially designed at the end of the 1950's, and used in a huge number of different applications.

The first version implemented was basically a parallel extension of the sequential algorithm, then a more advanced approach has been taken for the improved version.

2.1 Sequential Version

The basic algorithm solve the problem of shortest path between one Source Node and all the other nodes. The sequential version uses a for loop to launch this version on every node of the graph, so at the end of the loop every node possesses the shortest path to every other node.

The main elements used during the computation are:

- Graph Adjacency Matrix: which represents the graph indicating the weight of the edge which connects a node to another
- Vt Set: used as a stop condition and to check if a node has already a minimal path assigned
- l Array: which contains the minimal paths found for a certain node at a certain moment

We can divide the procedure into two main passages:

- Initialization
- Main loop

During the first phase the Vt set is created containing only the Source Node, then the l Array is initialized, with the direct connections from the Source Node to every other node, putting a symbolic value of "infinity" where a direct connection is not available.

The the main loop begins, and continues until all the nodes are present in the Vt set. The Main cycle, is composed of two internal phases:

- Local minimum search: between the elements of l not included in Vt we look for the minimum weight. Then we add the node selected to Vt.
- Update loop: the value of l, corresponding to the node selected before, gets updated with the minimum between the current value and the sum between the value selected during the minimum search and the weight present in the matrix. This part verifies if the value calculated by the search brings to a path actually shorter than the one stored.

When the Vt set is equal to the set of nodes of the graph, the Main loop terminates and the algorithm returns the l Array, where all the shortest paths are stored.

To solve the all pairs Shortest Paths problem we add an external loop that launch the algorithm for every row of the original Adjacency Matrix, and at the end returns a new matrix, composed of the various l arrays computed for every node.

2.1.1 Complexity

The sequential algorithm perform n times the basic algorithm , which has a complexity of $O(n^2)$:

- Time per Source: $O(n^2)$
- Number of executions: n
- Total Time: $O(n^3)$

2.2 Basic parallel version

The idea behind the initially implemented parallel version is simple: to solve the problem we have to launch the algorithm $|V|$ times, where V is the set of nodes of the graph. Using only one block of the GPU, every thread executes the algorithm for one node of the graph, and at the end insert the results in the correct row of the matrix in Global Memory.

The shared memory is not used because the variables needed for the computation are all locally present inside the thread, which is the main advantage of this first technique, because there are not requirements of any kind on inter-process communications. Each thread works completely isolated from the others.

This implementation requires a thread per node of the graph, which still uses the various for loops present in the sequential version, this fact make this implementation simple to program, with a decent speedup, provided the graph is big enough.

The main problem is that it is not taking full advantage of the Gpu capabilities, which is the weak spot that the second version tries to address.

2.2.1 Complexity

This version improves the performance by performing the basic algorithm in parallel, and has no communication delay:

- Time per Source: $O(n^2)$
- Number of executions: 1

- Total Time: $O(n^2)$

2.3 Improved parallel version

This version aim to exploit more the parallel execution of the GPU, using one Block per node of the graph, and one thread for every node to evaluate in the block. To reach this objective it is necessary to use the shared memory to save the varius data used during the computation of a block.

This time the threads are not isolated and need to cooperate to compute the final array for the block result. The inner for loop now implement a Parallel reduction procedure which compute the local minimum of a node.

There are some critical section of the code, like the stop condition check, that have been assigned exclusively to the first thread of the block, using the thread synchronization directive, because these steps are complex to parallelize, and the istruction executed by multiple threads could cause concurrent acces which would slow down the evaluation.

2.3.1 Complexity

The improved version increases the concurrency by using n processor per instance, so n^2 processors, this means thath every instance is executed by n processors, but we have to take into account the communication delay:

- Time per Source: $O(n^2/p + n\log(p))$
- Number of processors: n
- Total Time: $O(n^3/n) + O(n\log(p)) = O(n^2) + O(n\log(p))$

3 Implementation details

3.1 Sequential version

The sequential version is implemented by the function

```
void shortestPathsSequential(int* matrix, int dimension, int* results)
```

where matrix and results are respectively the Adjacency Matrix of the graph and the result matrix, while dimension is the number of nodes of the graph.

Then the data structures for the computation are allocated and initialized, with Vt having all false values except for the Source node and l having the values of the corresponding matrix row.

```
bool* Vt = (bool*)malloc((dimension) * sizeof(bool));  
int* l = (int*)malloc(dimension * sizeof(int));
```

The main loop start, using a simple function which check the Vt array as a stop condition

```
bool allTrue(bool* vector, int dimension)
```

The search for the local minimum is executed.

```
for (int i = 0; i < dimension; i++) {  
    if (Vt[i] == true) continue;  
    if (l[i] < closestWeight) {  
        closestWeight = l[i];  
        closestIndex = i;  
    }  
}
```

Then the Vt cell of the closest node is set to true and the l array is updated with the minimum between the the current value and the possible better value.

```
Vt[closestIndex] = true;  
for (int i = 0; i < dimension; i++) {  
    if (Vt[i] == true) continue;  
    int uvWeight = matrix[closestIndex * dimension + i];  
    l[i] = min(l[i], l[closestIndex] + uvWeight);  
}
```

When the main loop stops, the result matrix is filled with the values from the l array

```
for (int i = 0; i < dimension; i++) {  
    results[node * dimension + i] = l[i];  
}
```

3.2 Basic parallel version

This simple parallel version is implemented by the function

```
__global__ void shortestPathsParallel(int* matrix, int dimension, int* results)
```

The algorithm is not substantially different from the sequential version, the only change present is that now the reference for the Source node is not the index of a for loop, but the thread Id.

Every thread of the single block used, computes the sequential version without communicating in any way with other threads, not even during the results copying section.

This means that during initialization the Vt index to set is the one corresponding to the tID variable. This id is also used during the update of the l array and the filling of the results matrix.

3.3 Improved parallel version

The last version is implemented by the function

```
__global__ void shortestPathsParallelV2(int* matrix, int* results)
```

This time the code has various significant differences from the sequential one.

Beginning from the initialization phase, the usage of multiple blocks required the utilization of the shared memory, to guarantee to all threads of the block the necessary information with good access speed.

```
extern __shared__ int s[];
int* sharedData = s;

// l vector initialization
int* l = (int*)&sharedData[0];

// minimum vector initialization, first half are values second half are indexes
int* minimum = (int*)&l[bDim];

// Boolean vector simulating the Vt set initialization
bool* Vt = (bool*)&minimum[bDim * 2];

bool* stopCycle = (bool*)&Vt[bDim];
```

The shared memory initialization contains an additional vector, the minimum array, used during the computation of the local minimum, a notable thing about it is the fact that it is conceptually divided in two halves, the first half containing the weight of a certain node, the second half containing the index of that node according to the original matrix.

This is necessary for the correct computation of the parallel reduction, because the standard reduction does not take into account an external index, which is necessary in this application, so there are two assignments instead of one, one for the value, the other for the index.

3.3.1 Parallel Reduction

The Parallel Reduction is a strategy to perform quickly some operations that output a single value from the computing of an array of values. The idea is to maximize the usage of a parallel processor, by reducing the original array in smaller and smaller parts, with each thread evaluating two values per cycle. At the end of the operation the result will lay in the first cell of the array.

This operation does not require additional data structures to work, so it works completely in-place. The general field of application are mathematical operations such as sum or, in this case, a minimum operation.

The standard algorithm of the reduction has been slightly modified to keep track of the node original index between the various passages. This comes with an additional cost, because now the number of comparisons that a thread must perform for every cycle is two instead of one.

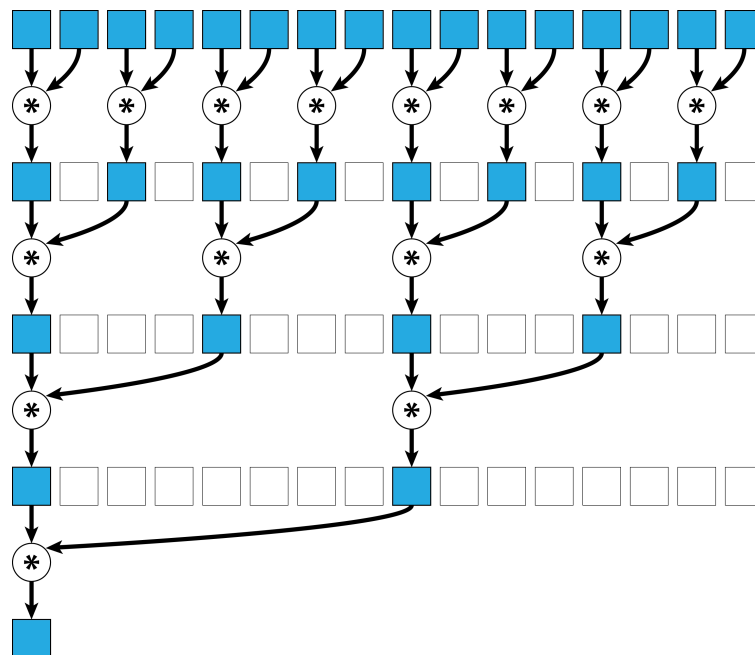


Figure 2: Parallel reduction scheme

```
int localMin = minWithVt(  
    minimum[index],  
    minimum[index + stride],  
    Vt[minimum[index + bDim]],  
    Vt[minimum[index + stride + bDim]]  
);  
int localMinIndex = minIndexWithVt(  
    minimum[index],  
    localMin,  
    minimum[index + bDim],  
    minimum[index + stride + bDim],  
    Vt[minimum[index + bDim]],  
    Vt[minimum[index + stride + bDim]]  
);
```

The two functions above are responsible for finding the minimum and the minimum index, taking the corresponding Vt values into account. This part of the implementation has been particularly tricky, with the algorithm not converging to a solution, due to set errors of the new Vt value.

At the end of the parallel reduction section there will be the value and the index of the minimum edge respectively at `minimum[0]` and `minimum[blockDimension]`.

The update of the l array is executed in the same way as before, with every thread updating its cell of the array.

4 Benchmarking

4.1 Time usage

4.2 Memory usage

4.3 Profiling

References

- [1] Vivek Sarkar. *Parallel Graph Algorithms*. 2008. URL: <https://www.cs.rice.edu/~vs3/comp422/lecture-notes/comp422-lec24-s08-v2.pdf>.