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1. List of changes

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Sylwia Mól	20-Nov-2017	Structure changed
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Malwina Molendowska	23-Jan-2018	Final Module 1 documentation added
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Klaudia Gugulska	23-Jan-2018	Module 2 implementation description added
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2. Assumptions

The aim of this project is to create the system for MRI data pre-processing and post-processing using Python, SciPy, Cython and Qt5 (detailed description of tools available in chapter *Implementation*, section *Tools*). The project is divided into 11 modules (more: chapter *Structure*).

The application is called SieMRI (the abbreviation for Siemens MRI). It allows to reconstruct MRI images, correct intensity inhomogenities, estimate non-stationary noise and remove it using one of two methods, show diffusion tensors, strip the skull, show specific parts of human brain, upsample the image or visualize brain in 3D or using specific plane. MRI image reconstruction starts automatically when user choose the file with data. Other functionalities are non-obligatory - the user decide if specific functionality should be used.

3. Structure

As mentioned in chapter *Assumptions* the project is divided into 11 modules. Each module includes one system's functionality, as follows:

- Module 1.: MRI images reconstruction using Cartesian SENSE algorithm.
- Module 2.: Intensity inhomogeneity correction on MRI reconstructed images.
- Module 3.: Non-stationary noise estimation (a homomorphic approach)
- Module 4.: Non-stationary noise filtering #1 with LMMSE filter.
- Module 5.: Non-stationary noise filtering #2 using unbiased non-local means filter.
- Module 6.: Diffusion tensor imaging (WLS, NLS algorithms)
- Module 8.: Skull stripping
- Module 9.: MRI segmentation of the human brain
- Module 10.: Upsampling based on non-local means approach
- Module 11.: Brain 3D visualization using marching cubes method
- Module 12.: Oblique imaging

Module 7. is excluded due to limited human resources and time.

The input and output signals for each module are listed in the table.

Module	Input	Output	Before that module
1	k-space signals	x-space fully reconstructed data	—
2	reconstructed image	image with intensity correction	image reconstruction
3	reconstructed image	estimated noise map	reconstruction
4	reconstructed, normalized image	Rician noise-free image	intensity correction
5	image, noise map	Rician noise-free image	reconstruction, noise estimation
6	reconstructed, normalized image; gradient-sequence vectors	estimated diffusion tensor 3D 6-channel image	modules 1-5, 8*
8	reconstructed, normalized, filtered image	image without non-brain tissues	modules 1- 4 or 5
9	image without non-brain tissues	segmentated image	skull stripping
10	320x240 image	640x480 image	denoised data
11	segmentated image	new window of application	segmentation
12	many images from one examination	one image from non-standard perspective	denoised data

Dependencies between modules are shown on the picture. 11th module is divided into two parts, because every part of module requires other pre-procesing activities.

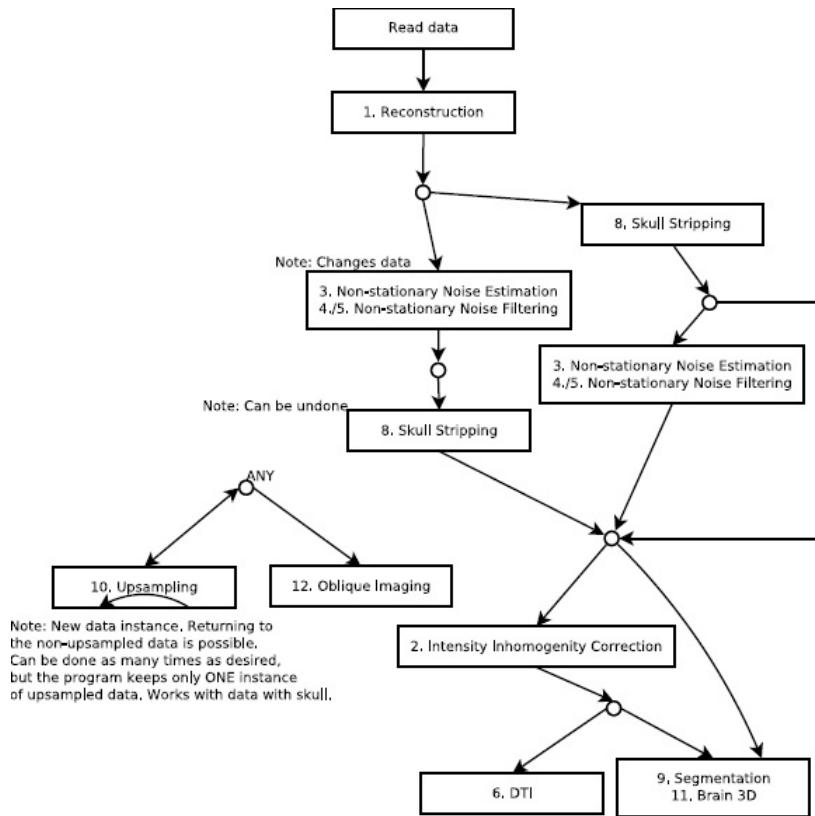


Figure 3.1. Dependencies between modules

4. User guide

4.1. Requirements

what user need to use this app - e.g. windows version etc
Currently conda is sufficient.

4.2. Instruction

instructions for user - GUI screens etc

5. Detailed description

5.1. Module 1. MRI reconstruction

The aim of this module is to formulate mathematical algorithm, which enables proper data reconstruction for images obtained with parallel MRI scans. The reconstruction is performed with use of Sensitivity Encoding (SENSE) algorithm in least squares (LS) solution context and Tikhonov regularization method.

Generally, parallel MRI acquisitions are targeted to diminish time needed for data sampling. The usage of multiple coils enabled simultaneous acquisition of signals. A further step, which is acquiring partial data from **k**-space, leads to craved time savings, meanwhile maintaining full spatial resolution as well as contrast at the same time. However, the approach of omitting lines in acquisition step results in data aliasing, i.e. folded images that need further data processing.

To clearly mark out how data is processed in this module, we list following reconstruction steps: i) the application of 2D Fourier Transform transform (2D FFT) to **k**-space data (acquired raw signals) from multiple coils. The result is a set of **x**-space images with folded pixels, ii) the sensitivity maps estimation of coils profiles (the information is needed to properly unfold subsampled data) and iii) the proper unfolding data process with usage of SENSE reconstruction algorithm and its alterations.

The most crucial step in processing is estimation of sensitivity coil profiles as a successful image reconstruction with use of pMRI algorithms highly depends on accurate sensitivity coil assessment. As sensitivity information varies from scan to scan it is impossible to obtain absolute maps. To obtain reliable knowledge, reference scans have to be conducted each time an examination is performed. These low-resolution information helps to estimate coil profiles with use of the many methods i.e. dividing each component coil image by a 'sum of squares' image.

In basic formulation, SENSE algorithm is applied to Cartesian MRI data undersampled uniformly by a factor r (i.e. $r = 2$ means that every other line in **k**-space is skipped). After Fourier transformation, each pixel in **x**-space image received in l -th coil can be seen as weighted sum of r pixels from full FOV, each multiplied by corresponding localized values of maps. The distance between those 'aliased' points in the full FOV is always equal to the desired FOVy value divided by subsampling rate. Obviously, depending on subsampling rate the number of folded pixels changes. Basically, the signal in one pixel at a certain location (x, y) received from l -th component coil image D_l^S with chosen subsampling rate r can be written as:

$$D_l^S(x, y) = S_l(x, y_1)D^R(x, y_1) + S_l(x, y_2)D^R(x, y_2) + \dots + S_l(x, y_r)D^R(x, y_r), \quad (5.1)$$

where index l counts from 1 to L (number of coils) and index i counts from 1 to r . Eq.(5.1) can be rewritten as:

$$D_l^S(x, y) = \sum_{i=1}^r S_l(x, y_i)D^R(x, y_i) \quad \text{for } l = 1, \dots, L. \quad (5.2)$$

Including all L coils the above equation can be rewritten in a matrix form:

$$\mathbf{D}^S(\mathbf{x}) = \mathbf{S}(\mathbf{x})\mathbf{D}^R(\mathbf{x}), \quad (5.3)$$

The vector $\mathbf{D}^S(\mathbf{x})$ denotes the aliased coil image values at a specific location $\mathbf{x} = (x, y_i)$ and has a length of L , $\mathbf{S}(\mathbf{x})$ is a $L \times R$ matrix and represents the sensitivities values for each coil at the r superimposed positions and $\mathbf{D}^R(\mathbf{x})$ lists the r pixels from full FOV image to be reconstructed. The closed-form solution of the problem is as follows:

$$\widehat{\mathbf{D}^R(\mathbf{x})} = (\mathbf{S}^H(\mathbf{x})\mathbf{S}(\mathbf{x}))^{-1}\mathbf{S}^H(\mathbf{x})\mathbf{D}^S(\mathbf{x}), \quad (5.4)$$

where $\widehat{\mathbf{D}^R(\mathbf{x})} = [\widehat{D^R(x, y_1)}, \dots, \widehat{D^R(x, y_r)}]^T$ and $\mathbf{S}^H(\mathbf{x})$ is the conjugate transpose of the $\mathbf{S}(\mathbf{x})$ matrix. The final reconstruction image is defined as:

$$M(\mathbf{x}) = \left| \widehat{\mathbf{D}^R(\mathbf{x})} \right|. \quad (5.5)$$

The ‘unfolding’ process can be performed as long as inversion of $\mathbf{S}(\mathbf{x})$ matrix is possible. Therefore, we cannot set the value of subsampling rate exceeding the number of coils L . To restore full FOV data, SENSE algorithm has to be recalled for each pixel in aliased \mathbf{x} -space image.

A regularization approach is defined as an inversion method that introduces additional information in order to stabilize the solution. This method is beneficial as it roughly matches the desired solution and is less sensitive to perturbations of the data. Tikhonov regularization is a common approach to obtain an inexact solution to a linear system of equations. In particular, the Tikhonov regularized estimate reads as follows:

$$\widehat{\mathbf{D}_{reg}^R} = \underset{\mathbf{D}^R}{\operatorname{argmin}} \left\{ \left\| \mathbf{D}^S - \mathbf{SD}^R \right\|^2 + \lambda^2 \left\| \mathbf{A}(\mathbf{D}^R - \mathbf{D}) \right\|^2 \right\}, \quad (5.6)$$

where λ is a regularization parameter ($\lambda > 0$) and \mathbf{D} is a prior image known as a regularization image. Selection of the parameter λ and \mathbf{D} can be performed using different procedures. In this module \mathbf{A} is assumed to be an identity matrix. The first term provides fidelity to the data and the second introduces prior knowledge (e.g. median filtered initial guess of LS SENSE) about the expected behaviour of \mathbf{D}^R . The Tikhonov regularization problem is given by:

$$\widehat{\mathbf{D}_{reg}^R} = \mathbf{D} + (\mathbf{S}^H\mathbf{S} + \lambda\mathbf{A}^H\mathbf{A})^{-1}\mathbf{S}^H(\mathbf{D}^S - \mathbf{SD}). \quad (5.7)$$

A reasonable value for λ can be picked using many technique, i.e. the L-curve criterion or generalized cross-validation.

Module input: Synthetic MR images are brain MRI slices coming from BrainWeb are normalized to [0-255] (all with intensity non-uniformity INU=0). Only T1- and T2-weighted data is used. The dataset is free of noise and the background areas are set to zero. The slice thickness equals 1 mm. These images are used then to simulate synthetic noisy accelerated parallel Cartesian SENSE MRI data according to following steps (the data simulation is performed with use of eight receiver coils ($L = 8$)): i) simulated sensitivity maps (divided into the ratio 3:1 for real and imaginary parts, respectively) are added to fully-sampled \mathbf{x} -space data, ii) correlated complex Gaussian noise with different values of standard deviations is added to each coil image, iii) 2D FFT and data subsampling with chosen reduction factor r is performed and iv) 2D iFFT is applied to recover data in \mathbf{x} -space. Then, data reconstruction process is conducted.

Module output: The output is full resolution reconstructed data performed with two different algorithms: SENSE (LSE) and Tikhonov regularization.

5.2. Module 2. Intensity inhomogeneity correction

The Intensity inhomogeneity of the same tissue varies with the location of the tissue within the image. In other words it refers to the slow, nonanatomic intensity variations of the same tissue over the image domain. It can be due to imaging instrumentation (such as radio-frequency nonuniformity, static field inhomogeneity, etc.) or the patient movement. This artifact is particularly severe in MR images captured by surface coils. Although intensity inhomogeneity is usually hardly noticeable to a human observer, many medical image analysis methods, such as segmentation and registration, are highly sensitive to the spurious variations of image intensities.

The aim of this module is estimation of intensity inhomogeneity field in MR image using surface fitting method. The method fits a parametric surface to a set of image features that contain information on intensity inhomogeneity. The resulting surface, which is usually polynomial or spline based, represents the multiplicative inhomogeneity field that is used to correct the input image.

The steps of this approach are:

1. Extract a background image from the corrupted MRI image, for example, by smoothing the image with a Gaussian filter of a large bandwidth (about 2/3 the size of the MRI image) to filter out all the image details that correspond to highfrequency components.
2. Select few data points from the background image and save their coordinates and graylevel values into a matrix $D = (xi, yi, gi), i = 1, 2, \dots, n$. It is recommended not to select points from the regions where there is no MRI signal since this regions has no bias field signal.
3. Select a parametric equation for the fitted surface . It is better to fit simple surfaces such as low order polynomial surfaces since they are very smooth and their parameters are very easy to estimate.
4. Estimate the parameters of the surface that best fits the data in matrix D by the method of nonlinear least-squares.
5. Use the fitted equation to generate an image of the bias field signal.
6. Divide the corrupted MRI image by the estimated bias field image in step 5.

Even though different surfaces can reasonably fit the data very well and it is not possible to tell which surface is most likely represents the actual bias field signal, however, in practice the bias signal estimated by fitting a smooth 2-dimensional polynomial surface to a background image can be used effectively to restore the corrupted MRI image.

Fitting of the surface can be done by means of the Levenberg-Marquardt algorithm for nonlinear least squares fitting of a function $f(x, y; a_1, \dots, a_m)$ of known form to n data points $(x_1, y_1, g_1), \dots, (x_n, y_n, g_n)$. For example, a polynomial surface of degree three can be fitted which has the following equation: $f(x, y; a) = a_1x^3 + a_2y^3 + a_3$, where $a = a_1, a_2, a_3$ is the parameter vector that define the surface. If we substitute the data points in the nonlinear function we get an overdetermined set of equations, i.e.,

$$\left\{ \begin{array}{l} g_1 = f(x_1, y_1; a_1, a_2, \dots, a_m) \\ \vdots \\ g_n = f(x_n, y_n; a_1, a_2, \dots, a_m) \end{array} \right\} \quad (5.8)$$

These equations can be solved to obtain the unknown parameter vector (a_1, a_2, \dots, a_m) by minimizing the sum of the squares of the differences between the data and the fitted function

$$Q(\mathbf{a}) = \frac{1}{2} \sum_{i=1}^n (g_i - f(x_i, y_i; a_1, a_2, \dots, a_m))^2 \quad (5.9)$$

Let $r_i(\mathbf{a}) = (g_i - f(x_i, y_i; a_1, a_2, \dots, a_m))$, which is the residual vector of point i , then equation(??) can be written as:

$$Q(\mathbf{a}) = \frac{1}{2} \sum_{i=1}^n (r_i(\mathbf{a}))^2 \quad (5.10)$$

According to the Levenberg-Marquardt algorithm, Eq(??) can be solved iteratively to find the values of the parameters vector (\mathbf{a}) starting from an initial estimate of the parameter vector (\mathbf{a}_0) using:

$$\mathbf{a}_{i+1} = \mathbf{a}_i - (H + \lambda \text{diag}[H])^{-1} \bigtriangledown Q(\mathbf{a}_i) \quad (5.11)$$

where H and $\bigtriangledown Q(\mathbf{a}_1)$ are Hessian matrix and the gradient of Eq. ?? both evaluated at a_i , diag is the diagonal elements of the Hessian matrix. At each iteration, the algorithm tests the value of the residual error and adjusts λ accordingly.

List of References

[1], [2]

5.3. Module 3. Non-stationary noise estimation

Magnetic Resonance Imaging (MRI) is known to be affected by several sources of quality deterioration, due to limitations in the hardware, scanning times, movement of patients, or even the motion of molecules in the scanning subject. Among them, noise is one source of degradation that affects acquisitions. The presence of noise over the acquired MR signal is a problem that affects not only the visual quality of the images, but also may interfere with further processing techniques such as registration or tensor estimation in Diffusion Tensor MRI.

The aim of this module is estimation of non-stationary noise maps based on MRI image. The module contains a homomorphic method for the non-stationary noise of Gauss and Rice as well. For the high SNR (signal to noise ratio) algorithm for Gaussian case should be used and for the lower value algorithm for Rician case. In the first case image is corrupted with Gaussian noise which has mean equal to zero spatially-dependent variance $\sigma^2(x)$:

$$I(x) = A(x) + N(x; 0, \sigma^2(x)) = A(x) + \sigma(x) \cdot N(x; 0, 1) \quad (5.12)$$

The goal is to estimate $\sigma(x)$ from the final image $I(x)$.

The variance of the noise $\sigma^2(x)$ slowly differs across the image. Therefore the first step is to remove mean of the image in order to avoid contribution of $A(x)$:

$$I_n(x) = I(x) - E\{I(x)\} = \sigma(x) \cdot N(x; 0, 1) \quad (5.13)$$

where $E\{I(x)\}$ is expected value in each point in the image. It is estimated as local mean using 3×3 window.

The next step is to separate signals $\sigma(x)$ and $N(x)$ by applying the logarithm, but to do so the absolute value of $I_n(x)$ is needed:

$$\log |I_n(x)| = \log \sigma(x) + \log |N(x)| \quad (5.14)$$

where $\log |N(x)|$ has its energy distributed in all frequencies and $\log\sigma(x)$ is a low frequency signal.

$\log\sigma(x)$ is the interesting part, so to get rid of $\log |N(x)|$ low pass filter has to be used:

$$LPF \{\log |I_n(x)|\} \approx \log\sigma(x) + \delta_N \quad (5.15)$$

where δ_N is a low pass residue of $\log |N(x)|$ that must be removed from the estimation. Throughout various calculations the final outcome of the LPF was delineated as:

$$LPF \{\log |I_n(x)|\} \approx \log\sigma(x) - \log\sqrt{2} - \frac{\gamma}{2} \quad (5.16)$$

with γ being Euler-Mascheroni constant.

Taking the exponential of every part of foregoing formula estimator of $\sigma(x)$ can be defined as:

$$\widehat{\sigma(x)} = \sqrt{2}e^{LPF\{\log|I(x)-E\{I(x)\}|\}+\gamma/2} \quad (5.17)$$

The Rician case is similar to the described above Gaussian case, but a bit more complicated. The signal $A(x)$ is corrupted with complex Gaussian noise with zero mean and spatially-dependent variance $\sigma^2(x)$, which has module following a nonstationary Rician distribution:

$$I(x; A(x), \sigma(x)) = |A(x) + N_r(x; 0, \sigma^2(x)) + j \cdot N_i(x; 0, \sigma^2(x))| \quad (5.18)$$

To simplify, the dependence of $I(x)$ with $A(x)$ and $\sigma(x)$ are removed.

First couple of steps in Rician case are congenial to the steps taken in Gaussian case. The local mean calculated using 3×3 window is subtracted from the image so it can be centered. After that logarithm of the absolute value of centered image is filtered by low passing filter, giving as a result:

$$\log |I_n(x)| = \log\sigma(x) + \log |G(s_0(x))| LPF \{\log |I_n(x)|\} \approx \log\sigma(x) + \delta_R \quad (5.19)$$

where σ_R is a low pass residue of $\log |G(s_0(x))|$. Again after various operation previous formula presents as:

$$LPF \{\log |I_n(x)|\} \approx \log\sigma(x) - \log\sqrt{2} - \frac{\gamma}{2} + \varphi(s_0(x)) \quad (5.20)$$

with $\varphi(s_0(x))$ being a Rician/Gaussian correction function. Finally the estimator for $\sigma(x)$ is defined as:

$$\widehat{\sigma(x)} = \sqrt{2}e^{LPF\{\log|I(x)-E\{I(x)\}|\}} e^{\gamma/2-\varphi(s_0(x))} \quad (5.21)$$

5.4. Module 4. Non-stationary noise filtering 1

The objective of fourth module is to remove Rician noise from MR images. If both real and imaginary parts of signal are corrupted with zero-mean uncorrelated Gaussian noise with equal variance, the envelope of magnitude signal will follow a Rician distribution. Many processes allow to remove noise, here the method is used to denoise MR images is the linear minimum square error estimator (LMMSE). The main purpose of LMMSE is to find a closed-form estimator for a signal that follows a Rician distributions. It is more efficient than optimization-based solutions. The estimator uses information of the sample distribution of local statistics of the image such as the local mean, the local variance and the local mean square value. In this method, the true value for each noisy pixel is estimated by a set of pixels selected from a local neighborhood.

The LMMSE estimator

Scalar parameter θ is to be estimated based on the data set $\{x[0], x[1], \dots, x[N - 1]\}$. The unknown parameter is modeled as the realization of a random variable. We assume only a knowledge of the first two moments. θ may be estimated from x is due to the assumed statistical dependence of 0 on x as summarized by the joint PDF $p(x, B)$, and in particular, for a linear estimator we rely on the correlation between θ and x . We now consider the class of all linear estimators of the form:

$$\hat{\theta} = \sum_{n=0}^{N-1} a_n x[n] + a_N \quad (5.22)$$

and choose the weighting coefficients a_n 's to minimize the Bayesin MSE (mean square error):

$$Bmse(\hat{\theta}) = E[(\theta - \hat{\theta})^2] \quad (5.23)$$

where the expectation is with respect to the PDF $p(x, O)$. The resultant estimator is termed the linear minimum mean square error (LMMSE) estimator.

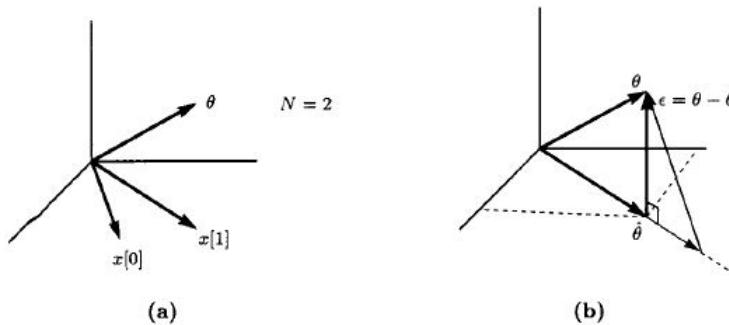


Figure 5.1. Geometrical interpretation of LMMSE estimator

The LMMSE estimator admits a geometrical interpretation, the random variables $\theta, x[0], x[1], \dots, x[N - 1]$ as elements in a vector space as shown symbolically in Fig. 5.1a. This approach is useful for conceptualization of the LMMSE estimation process. As before, we assume that

$$\hat{\theta} = \sum_{n=0}^{N-1} a_n x[n] \quad (5.24)$$

where $a_N = 0$ due to the zero mean assumption. The weighting coefficients a_n should be chosen to minimize the MSE:

$$E[(\theta - \hat{\theta})^2] = E[(\theta - \sum_{n=0}^{N-1} a_n x[n])^2] \quad (5.25)$$

But this means that minimization of the MSE is equivalent to a minimization of the squared length of the error vector $\epsilon = \theta - \hat{\theta}$. The error vector is shown in Fig. 5.1b for several candidate estimates. Clearly, the length of the error vector is minimized when ϵ is orthogonal to the subspace spanned by $\{x[0], x[1], \dots, x[N - 1]\}$.

The LMMSE estimator for MR image

The LMMSE estimator for a 2-D signal with Rician distribution is defined:

$$\widehat{A_{ij}^2} = E\{A_{ij}^2\} + C_{A_{ij}^2 M_{ij}^2}^{-1} (M_{ij}^2 - E\{M_{ij}^2\}) \quad (5.26)$$

where A_{ij} is the unknown intensity value in pixel ij , M_{ij} the observation vector, $C_{A_{ij}^2 M_{ij}^2}^{-1}$ the cross-covariance vector and $C_{M_{ij}^2 M_{ij}^2}$ the covariance matrix. If the estimator is simplified to be pointwise, vectors and matrices become scalar values. To calculate the second order moment we assume local ergodicity, the expectation ($E\{\cdot\}$) may be replaced by sample estimator, that can be defined:

$$\langle I_{ij} \rangle = \frac{1}{|\eta_{ij}|} \sum_{p \in \eta_{ij}} I_p \quad (5.27)$$

with η_{ij} a square neighbourhood around the pixel ij . The finally equation for LMMSE is defined:

$$\widehat{A_{ij}^2} = \langle M_{ij}^2 \rangle - 2\sigma_n^2 + K_{ij}(M_{ij}^2 - \langle M_{ij}^2 \rangle) \quad (5.28)$$

with K_{ij}

$$K_{ij}^2 = 1 - \frac{4\sigma_n^2(\langle M_{ij}^2 \rangle - 2\sigma_n^2)}{\langle M_{ij}^4 \rangle - \langle M_{ij}^2 \rangle^2} \quad (5.29)$$

The estimated noise is needed to use the LMMSE filter. The map of noise is prepared one module before. The use of the LMMSE method should make the filtering process computationally far more efficient and easier to implement.

Module I/O

Module input: Reconstructed, normalized and corrected data, noise maps. **Module output:** Images without Rician noise.

5.5. Module 5. Non-stationary noise filtering 2

Magnetic Resonance images are endangered of being corrupted by noise and artifacts. Since they are used as a basis for medical diagnosis their quality has to be at highest possible level. Noise can be dealt with by changing the parameters of images acquisition, however it increases the scanning time, which is undesirable in medical imaging. To overcome this obstacle, post-processing methods like filtering are employed for denoising. In domain of MRI denoising many filters may be used, though here emphasis is put on Unbiased Non-Local Means (UNLM) filter, which is an extension of NLM filter. In order to understand Unbiased version of this algorithm, the basic one has to be presented.

Having image Y , the NLM algorithm calculates the new value of point p accordingly to the equation:

$$NLM(Y(p)) = \sum_{\forall q \in Y} w(p, q)Y(q) \quad (5.30)$$

$$0 \leq w(p, q) \leq 1, \sum_{\forall q \in Y} w(p, q) = 1$$

It can be seen that value of p is calculated as weighted average of pixels in the image (q), having fulfilled restrictions from 5.30. To determine before mentioned average the similarity between square neighbourhoods windows centered around pixels p and q are calculated. The size of the window can be determined by the user, defined by parameter R_{sim} . Equation 5.31 shows how to determine this similarity.

$$w(p, q) = \frac{1}{Z(p)} e^{-\frac{d(p, q)}{h^2}} \quad (5.31)$$

$Z(p)$ is the normalizing constant which also uses exponential decay parameter h and the weighted Euclidean distance measure for pixels in each neighbourhood, called d .

$$Z(p) = \sum_{\forall q} e^{-\frac{d(p, q)}{h^2}} \quad (5.32)$$

$$d(p, q) = G_p \|Y(N_p) - Y(N_q)\|_{R_{sim}}^2 \quad (5.33)$$

In above equation G_p stands for a Gaussian weighting function that has a 0 mean and standard deviation usually equal to 1.

Once NLM filter is fully explained, unbiased extension of it can be examined. It builds on the properties of MRI signal. According to [3] the magnitude signal of MRI follows a Rician distribution. Furthermore, for low intensity regions the Rician distribution approaches to a Rayleigh one, whilst for high intensity it shifts towards Gaussian. It was investigated that this bias can be handled by filtering the squared MRI image, since it is not longer signal-dependent [3]. As a consequence the bias, which equals $2\sigma^2$ [4] can be deleted with ease. The blueprint for UNLM can be summarized in:

- noise estimation - which can be done by calculating standard deviation of background in the image. To distinguish background and the body on the MRI scan the Otsu thresholding method [5] can be successfully used. In this version noise maps are used, to achieve non-stationary noise filtration,
- calculating NLM values for each point of image as in 5.30,
- assessing the unbiased value of each point accordingly to the equation 5.34.

$$UNLM(Y) = \sqrt{NLM(Y)^2 - 2\sigma^2} \quad (5.34)$$

In above equation σ refers to value of non-stationary noise in the signal, more precisely it is a value for each pixel from the original image, stored in a form of noise map.

UNML implementation for diffusion weighted data becomes a bit less trivial task, due to the new dimension of data associated with different gradients for each slice. Based on assumption that gradients in similar directions present related behaviours, UNLM for DWI can be formulated as:

$$Y_i(p) = \sqrt{\sum_{j \in \Theta_i^N} \sum_{q \in N_p} w_i^j(p, q) M_j^2(q) - 2\sigma^2} \quad (5.35)$$

Where weights are calculated as for structural data and M is a vector containing gray values.

$$d_i^j(p, q) = (M_i(N_p) - M_j(N_q))^T G_p (M_i(N_p) - M_j(N_q)) \quad (5.36)$$

However it was reported in [6] that denoising diffusion weighted data using UNLM method gives no significant results, so gradients related data can be ignored, or filtered as structural data.

It is worth mentioning that UNLM filter's performance is highly dependent on parameter values. The optimal values of them were examined in [3] and same values are adapted in presented implementation. Properly-tuned filter can significantly increase SNR of the scans while preserving body structures.

Module input: Previously reconstructed, normalized and corrected data, noise maps.

Module output: Image with deleted Rician noise by unbiased non-local means filter.

5.6. Module 6. Diffusion tensor imaging

The aim of this module is to estimate brain tissue diffusion tensor from Diffusion Weighted Images (DWI). DWI are obtained using a different pulse sequence than anatomical MRI images and as such differ in their information content. Concretely, Diffusion Tensor Imaging (DTI) gives an insight into tissue microstructure based on DWI, which are obtained by probing tissues using gradient-pulse excited water molecules [7]. This enables indirect measurements of structural orientation and the degree of anisotropy as water molecules diffuse differs between tissues.

In DTI-MRI the measured signal is defined as [8]:

$$S(b, \mathbf{g}) = S_0 \exp(-b\mathbf{g}^T \mathbf{D}\mathbf{g}) \quad (5.37)$$

where $S(b, \mathbf{g})$ is the measured signal, S_0 is the reference signal without diffusion gradient attenuation, b is the diffusion weight scalar, \mathbf{g} is the diffusion encoding gradient vector of unit length and \mathbf{D} is the diffusion tensor.

The diffusion tensor \mathbf{D} is symmetric and describes molecular mobility along each direction:

$$\mathbf{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix} \quad (5.38)$$

Furthermore, the Eq. (5.37) can be rewritten as:

$$\ln S(b, \mathbf{g}) = \ln S_0 - b\mathbf{g}^T \mathbf{D}\mathbf{g} \quad (5.39)$$

which established a linear relationship between model parameters and measured quantity.

Estimating \mathbf{D} from the above equation can be done using Weighted Least Squares (WLS) or Nonlinear Least Squares (NLS) algorithms. In order to correctly estimate the diffusion tensor it is necessary to acquire at least seven DWI - one for each direction of diffusion and one in the absence of diffusion gradient.

Let $\boldsymbol{\gamma}$ be the parameters vector of DTI model:

$$\boldsymbol{\gamma} = [\ln S_0, D_{xx}, D_{yy}, D_{zz}, D_{xy}, D_{yx}, D_{xz}]^T \quad (5.40)$$

Furthermore let \mathbf{W}_i be the design matrix of our model. In this case it represents the diffusion vector in a given direction for i -th measurement:

$$\mathbf{W}_i = \left[1, -b_i g_{ix}^2, -b_i g_{iy}^2, -b_i g_{iz}^2, -2b_i g_{ix}g_{iy}, -2b_i g_{iy}g_{iz}, -2b_i g_{ix}g_{iz} \right] \quad (5.41)$$

Then equations 5.37 oraz 5.39 can be rewritten as:

$$S(\boldsymbol{\gamma}) = \exp(\mathbf{W}_i \boldsymbol{\gamma}) \quad (5.42)$$

$$\ln(S(\boldsymbol{\gamma})) = \mathbf{W}_i \boldsymbol{\gamma} \quad (5.43)$$

The aforementioned expressions can be used to estimate the vector $\boldsymbol{\gamma}$ using Least Squares Regression models. It is important to note that tensor built using the elements of $\boldsymbol{\gamma}$ will be symmetric.

5.6.1. Weighted Least Squares estimation

For Weighted Least Squares (WLS) model the cost function is defined as [9]:

$$f_{WLS}(\gamma) = \frac{1}{2} \sum_{i=1}^m \omega_i^2 \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right)^2; \quad y_i = \ln(S(\gamma)_i) \quad (5.44)$$

where m is the total number of measurements while ω is the measurement weight associated with WLS method.

The aim of Least Squares Regression methods is to minimize the cost function f_{WLS} with respect to γ , which coincides with the function gradient $|\nabla f_{WLS}|$ equal to zero.

In general, vector-valued function gradient is defined as a vector whose elements can be calculated using the following formula:

$$|\nabla f_{WLS}|_k = \frac{\partial f_{WLS}}{\partial \gamma_k} \quad (5.45)$$

For WLS cost function:

$$\begin{aligned} \frac{\partial f_{WLS}}{\partial \gamma_k} &= \sum_{i=1}^m \frac{\partial}{\partial \gamma_k} \omega_i^2 \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right) \\ &= \sum_{i=1}^m \omega_i^2 \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right) \frac{\partial}{\partial \gamma_k} \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right) \\ &= \sum_{i=1}^m \omega_i^2 \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right) \sum_{j=1}^7 (-W_{ij}) \frac{\partial}{\partial \gamma_k} \gamma_j \\ &= \sum_{i=1}^m \omega_i^2 \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right) (-W_{ik}) \end{aligned} \quad (5.46)$$

or in matrix form:

$$\nabla f_{WLS} = -\mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} (\mathbf{y} - \mathbf{W} \boldsymbol{\gamma}) \quad (5.47)$$

where $\boldsymbol{\omega}$ is a diagonal matrix with non-zero elements equal to WLS model weights.

The Hessian of vector-valued function is defined as a matrix whose elements computed as:

$$|\nabla^2 f_{WLS}|_{lk} = \frac{\partial^2 f_{WLS}}{\partial \gamma_l \partial \gamma_k} \quad (5.48)$$

For WLS cost function:

$$\begin{aligned} \frac{\partial^2 f_{WLS}}{\partial \gamma_l \partial \gamma_k} &= \frac{\partial}{\partial \gamma_l} \frac{\partial f_{WLS}}{\partial \gamma_k} \\ &= \frac{\partial}{\partial \gamma_l} \sum_{i=1}^m \omega_i^2 \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right) (-W_{ik}) \\ &= \sum_{i=1}^m \omega_i^2 \frac{\partial}{\partial \gamma_l} \left(y_i - \sum_{j=1}^7 W_{ij} \gamma_j \right) (-W_{ik}) \\ &= \sum_{i=1}^m \omega_i^2 (-W_{il}) (-W_{ik}) \\ &= \sum_{i=1}^m \omega_i^2 W_{il} W_{ik} \end{aligned} \quad (5.49)$$

or in matrix form:

$$\nabla^2 f_{WLS} = \mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} \mathbf{W} \quad (5.50)$$

One can notice that Eq. (5.47) does not depend on model parameters, meaning that it can be solved using its normal equation:

$$\begin{aligned} \nabla f_{WLS} = 0 &\iff -\mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} (\mathbf{y} - \mathbf{W}\boldsymbol{\gamma}) = 0 \\ \mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} \mathbf{W} \boldsymbol{\gamma} &= \mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} \mathbf{y} \\ \boldsymbol{\gamma} &= (\mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} \mathbf{W})^{-1} \mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} \mathbf{y} \end{aligned} \quad (5.51)$$

Before continuing, it is important to note that there exists no simple choice of $\boldsymbol{\omega}$ weights. The authors of [9] suggest using the measured signal \mathbf{S} , in [10] one can read that weights should be the Linear Least Squares estimate of signal in each voxel, while the authors [11] argue for the use of signal variance estimation using Rice's noise estimate. As one can see, WLS model complexity varies depending on the choice of weights vector computation method.

5.6.2. Nonlinear Least Squares estimation

Nonlinear Least Squares cost function is defined as:

$$f_{NLS}(\boldsymbol{\gamma}) = \frac{1}{2} \sum_{i=1}^m \left(S_i - \exp \left(\sum_{j=1}^7 W_{ij} \gamma_j \right) \right)^2 = \frac{1}{2} \sum_{i=1}^m r_i^2 \quad (5.52)$$

Same as before, we will derive NLS cost function gradient and Hessian matrix.

$$\begin{aligned} \frac{\partial f_{NLS}}{\partial \gamma_k} &= 2 \frac{1}{2} \sum_{i=1}^m r_i \frac{\partial}{\partial \gamma_k} (r_i(\boldsymbol{\gamma})) \\ &= \sum_{i=1}^m (-r_i) \frac{\partial}{\partial \gamma_k} \left(\exp \left[\sum_{j=1}^7 W_{ij} \gamma_j \right] \right) \\ &= \sum_{i=1}^m (-r_i) \exp \left(\sum_{j=1}^7 W_{ij} \gamma_j \right) \frac{\partial}{\partial \gamma_k} \left(\sum_{j=1}^7 W_{ij} \gamma_j \right) \\ &= \sum_{i=1}^m (-r_i) \exp \left(\sum_{j=1}^7 W_{ij} \gamma_j \right) \left(\sum_{j=1}^7 W_{ij} \frac{\partial}{\partial \gamma_k} \gamma_j \right) \\ &= \sum_{i=1}^m (-r_i) \exp \left(\sum_{j=1}^7 W_{ij} \gamma_j \right) W_{ik} \\ &= \sum_{i=1}^m (-r_i) \hat{S}_i W_{ik} \end{aligned} \quad (5.53)$$

where \hat{S}_i is the voxel signal estimate of NLS model. Let $\hat{\mathbf{S}}$ be a diagonal matrix with non-zero elements set to \hat{S}_i . Then f_{NLS} computation can be rewritten in matrix form:

$$\nabla f_{NLS} = -\mathbf{W}^T \hat{\mathbf{S}} \mathbf{r} \quad (5.54)$$

as one can see from the Eq. (5.54), the value of gradient depends on NLS signal estimate, meaning that contrary to WLS method, it is not possible to compute NLS estimate without resorting to iterative methods.

NLS Hessian matrix elements can be computed as:

$$\begin{aligned}
 \frac{\partial^2 f_{NLS}}{\partial \gamma_l \partial \gamma_k} &= \frac{\partial}{\partial \gamma_l} \frac{\partial f_{NLS}}{\partial \gamma_k} \\
 &= \frac{\partial}{\partial \gamma_l} \sum_{i=1}^m (-r_i) \hat{S}_i W_{ik} \\
 &= - \sum_{i=1}^m \left(\hat{S}_i W_{ik} \frac{\partial}{\partial \gamma_l} r_i + r_i W_{ik} \frac{\partial}{\partial \gamma_l} \hat{S}_i \right) \\
 &= - \sum_{i=1}^m \left(\hat{S}_i W_{ik} \frac{\partial}{\partial \gamma_l} [S_i - \hat{S}_i] + r_i W_{ik} \frac{\partial}{\partial \gamma_l} \hat{S}_i \right) \\
 &= - \sum_{i=1}^m \left(\hat{S}_i W_{ik} \frac{\partial}{\partial \gamma_l} [-\hat{S}_i] + r_i W_{ik} \frac{\partial}{\partial \gamma_l} \hat{S}_i \right) \\
 &= - \sum_{i=1}^m \left(\hat{S}_i W_{ik} [-\hat{S}_i W_{il}] + r_i W_{ik} \hat{S}_i W_{il} \right) \\
 &= \sum_{i=1}^m \left(\hat{S}_i W_{ik} \hat{S}_i W_{il} - r_i W_{ik} \hat{S}_i W_{il} \right) \\
 &= \sum_{i=1}^m W_{ki} \left(\hat{S}_i^2 - r_i \hat{S}_i \right) W_{il}
 \end{aligned} \tag{5.55}$$

or in matrix form:

$$\nabla^2 f_{NLS} = \mathbf{W}^T (\hat{\mathbf{S}}^T \hat{\mathbf{S}} - \mathbf{R} \hat{\mathbf{S}}) \mathbf{W} \tag{5.56}$$

where \mathbf{R} is the diagonal matrix whose non-zero elements are equal to NLS model residuals (r_i).

Usually, iterative approaches to function minimization compute the Taylor series of a given function and try to find a vector δ such that $f_{NLS}(\gamma + \delta) < f_{NLS}(\gamma)$. If additionally:

$$\delta = -(\nabla^2 f_{NLS})^{-1} \nabla f_{NLS} \tag{5.57}$$

then this approach is called Newton's method in optimization.

5.6.3. Diffusion Biomarkers

Regardless of the chosen estimation method, the estimated vector $\hat{\gamma}$ is reshaped to be a symmetric 3x3 matrix \mathbf{D} .

There exists a couple of strategies for visualizing this high-dimensional array containing estimated tensor in each voxel of each slice. One of these approaches is to compute the eigenvalue decomposition of \mathbf{D} and compute a series of images called Diffusion Biomarkers which can be used to extract additional information about tissue microstructure.

Let us define four biomarker images:

1. MD (Mean Diffusivity) is the mean of tensor eigenvalues. MD is an inverse measure of membrane density and is very similar for both gray (GM) and white matter (WM) and is higher for cerebrospinal fluid (CSF). MD is sensitive to cellularity, edema and necrosis [12].

$$MD = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3} \tag{5.58}$$

2. RA (Relative Anisotropy) exhibits high degree of contrast between anisotropic (WM) and isotropic tissues.

$$RA = \sqrt{\frac{(\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2}{3MD}} \quad (5.59)$$

3. FA (Fractional Anisotropy) is a summary measure of microstructural integrity. It is highly sensitive to microstructural changes without considering the type of change [7].

$$FA = \sqrt{\frac{3}{2} \sqrt{\frac{(\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}} \quad (5.60)$$

4. VR (Volume Ratio)

$$VR = \frac{\lambda_1 \lambda_2 \lambda_3}{MD^3} \quad (5.61)$$

where $\lambda_1, \lambda_2, \lambda_3$ are the tensor eigenvalues. One can notice that for each biomarker image to be real-valued, eigenvalues must be non-negative. There are a couple of strategies aimed at solving this problem [8]:

- Substituting negative eigenvalues with zero.
- Substituting eigenvalues with their absolute value.
- Computing Cholesky-parametrization of input vectors to WLS and NLS estimation methods.

One can also compute a 3D map using the eigenvectors corresponding to tensor eigenvalue of highest magnitude. The resulting map is a measure of the direction of principal diffusion, which along with it's magnitude (eigenvalue), is the input signal to Tractography.

Additionally, the aforementioned 3D map can be displayed as a RGB image, because eigenvectors obtained from eigenvalue decomposition of a symmetric real matrix are orthogonal. The intensity of resulting color image is then weighted using one of computed biomarker images, usually Fractional Anisotropy.

5.7. Module 8. Skull stripping

Preliminary processing to isolate the brain form extra-cranial or non-brain tissues such as e.g. the eye sockets, skin from MRI head scans is commonly referred as skull stripping. Skull stripping methods which are available in the literature are broadly classified into five categories: mathematical morphology-based methods, intensity-based methods, deformable surface-based methods, atlas-based methods, and hybrid methods. Each skull stripping method has their own merits and limitations. The aim of this module is to remove pieces of skull from MRI Image using at pleasure chosen algorithm from the literature. Skull stripping is performed with use of two methods:

- Brain Surface Extractor BSE proposed by Stattuck et al. [1]
- Marker-controlled watershed algorithm by Abdallah and Hassan and by Segonne et al. [2], [3]

Before this methods, there is applying preprocessing function to estimate global parameters: CSF - an upper bound for the intensity of the cerebrospinal fluid, COG - the coordinates of the centroid of the brain, BR - the average brain radius and ratio to brain diameter in axis x to brain radius in axis y. These estimated parameters are useful in this to methods to optimization and working. BSE procedure consist of three steps:

1. The MRI is processed with anisotropic diffusion filter to smooth nonessential gradients.

2. The filtered image has applied a Marr-Hildreth edge detector to identify important anatomical boundaries.
3. Using a sequence of morphological and connect component operation to define object by previous boundaries.

Anisotropic Diffusion filtering is applied to smooth noisy regions, which can obscure boundaries or their edges can be indistinguishable from the other in the image. To implement this filter is used an image processing method by Perona and Malik. They demonstrated that using the gradient of image intensity as an estimate of edge strength produces good results. The filtered image is modeled as the solution to the anisotropic diffusion equation

$$\frac{\partial I}{\partial t} = \nabla \cdot (c(\mathbf{p}, t) \nabla I) = c(\mathbf{p}, t) \nabla^2 I + \nabla c \cdot \nabla I \quad (5.62)$$

where \mathbf{p} is a point in R^2 , ∇ and ∇^2 represent the gradient and Laplacian operators and $\nabla \cdot$ indicates the divergence operator. The function is defined as:

$$c(\mathbf{p}, t) = g(\| \nabla I(\mathbf{p}, t) \|) = e^{-\| \nabla I(\mathbf{p}, t) \|^2 \kappa_d^2} \quad (5.63)$$

where κ_d is the diffusion constant. (5.63) gives preferences to high-contrast edges. Number of iteration and the diffusion parameter κ_d is selected empirically.

To locate the anatomical boundaries in MRI brain volumes, it is used the Marr-Hildreth edge detectors. It is based on a low-pass filtering step with a symmetric Gaussian kernel, followed by the localization of zero-crossing in the Laplacian of the filtered image. The Marr-Hildreth operator is defined as:

$$C(k) = \nabla^2(I(k) * g_\sigma(\mathbf{p})) \quad (5.64)$$

where C is the output contour image, I is an input image, $*$ is the convolution operator, g_σ is a Gaussian kernel with variance σ^2

$$g_\sigma = \frac{1}{\sqrt{2\pi}\sigma} e^{-\|\mathbf{p}\|^2/2\sigma^2} \quad (5.65)$$

where \mathbf{p} is a point in the image, and ∇^2 is the Laplacian operator. To find pixels in the contour image, C where zero-crossing occur, a binary image E is produced that separates the image into edge-differentiated segments. Small values for sigma produce narrow filters, resulting in more edges in the image, on the other hand increasing this value makes the blurring kernel wider and only strong edges remain. This detector output is an image, which edge pixels are black and nonedge ones are white.

Morphological processing's task is to select the pixels corresponding to the brain tissue from the original image. The output of edge detector often does not distinguish meninges or blood vessels from the brain tissue due to noise, low contrast between brain and meninges or true anatomical continuity. The first step is morphological erosion, which delete narrow connections without globally damaging image. After that the largest connected region centered in the volume consist entirely of brain tissue. Second step is selection of this region. Third step, is binary dilatation to restore the brain due to previous erosion, which decrease brain surface. Because of imperfections in the edge boundaries detection, image after dilatation may consist of pits in its surface or small holes within the surface. The last step of morphological processing is closing, which fill small pits and close of some holes that occur.

Marker-controlled watershed segmentation follows this procedure:

1. Computation a segmentation function.
2. Computation the foreground markers.

3. Computation the background markers.

4. Computation of the watershed transform using of the foreground markers and the background markers.

First step is applied to find the edges in the image, it is processed with Sobel edge filter. The gradient is high at the borders of the object and low inside. Morphological techniques are used to compute the foreground markers, opening by reconstruction and closing by reconstruction to clean up the image. These operation will create flat regional maxima inside each object that can be used to find regional maxima and next modified them by a closing followed by an erosion to obtain good foreground markers. The background pixels after binarization are black, but their markers should be to close to the edges of the object. this can be done by computing the watershed transform of the distance transform of the binary image and the looking for the watershed ridges lines of the result. With the foreground markers and the background markers there is applied watershed based segmentation.

Brain Surface Extraction and Marker-controlled Watershed Segmentation is working separately. The base method is BSE. Marker-controlled watershed segmentation is compute, when BSE brain mask is larger than a mask based on preprocessing estimated BR.

List of References

[8_dti_1], [8_dti_2], [8_dti_3], [8_dti_4],

5.8. Module 9. Segmentation

Image segemntation is very important part of medical processing. Segmentation is the process clustering an image pixels into region of interest with similar properties. Many image segmentation algoritm is used in medical applications to segment body organs and tissues. Some of the application consist of surgical planning, tumor detection and segmentation, brain development study, heart segme tation, etc. For example, MRI segmentation is commonly used for measuring and visualizing different brain structures, for delineating lesions, for analysing brain development, and for image-guided interventions and surgical planning.

This process can prove useful in clustering the specific brain regions in MRI image. Partition the image into different region colours can vizualize grey matter, white metter and cerebrospinal fluid in brain image.

In this documenation, we focuse only on brain segmentation and describe an algoritm, which is used in application.

Image segmentation The real purpose of image segmentation is to subdivided an image into nonoverlapping, semantically meaningful and homogeneous regions with similar features, such as: colour, depth, intensity or texture. The basic part of structural brain MRI analysis is MRI data classification into appropriate tissue types, recognition and description of specific anatomical structures. Classification means assignment to each element in the image a tissue class. The problems of classification and segmentation are interlinked, because classification is the result of segmentation and a classifier implicitly segments an image. In brain MRI segmentation, image pixels are typically classified into three main tissue types: cerebrospinal fluid (CSF), gray matter (GM) and white matter (WM)(Fig. 5.2).

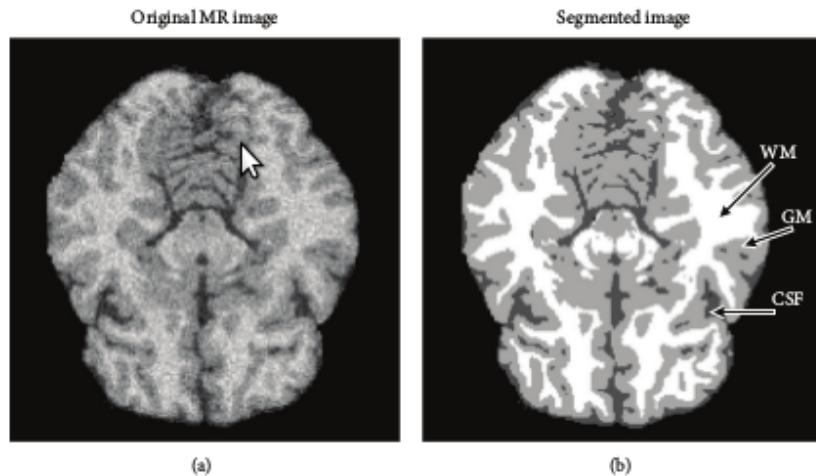


Figure 5.2. The brain MRI segmentation with a) original MRI image and b) image after segmentation with three labels: CFS, GM and WM

Most of the image segmentation research has focused on 2D images. For the data, which is defined in 3D space - a series of MRI images, then typically each image “slice” is segmented individually in a “slice-by-slice” manner. In practice, 2D image segmentation methods can be extended to 3D space, but often with the cost of an increased complexity of the method and slower computational time.

MRI preprocessing To prepare brain MRI images for segmentation, it is necessary to perform several preprocessing steps (Fig. 5.3). One of the most important features for brain MRI segmentation is the intensity of brain tissue. However, intensity-based segmentation algorithms will lead to wrong results when intensity value are corrupted by MRI artifacts. The most important preprocessing steps are:

- the MRI bias field correction,
- image registration,
- skull stripping and nonbrain tissue removing – brain extraction

Trained medical experts can make visual MRI analysis to certain levels of intensity inhomogeneity, in contrast, the performance of intensity-based segmentation methods and automatic MRI analysis decreases greatly in the presence of the bias field (Fig. 5.4).

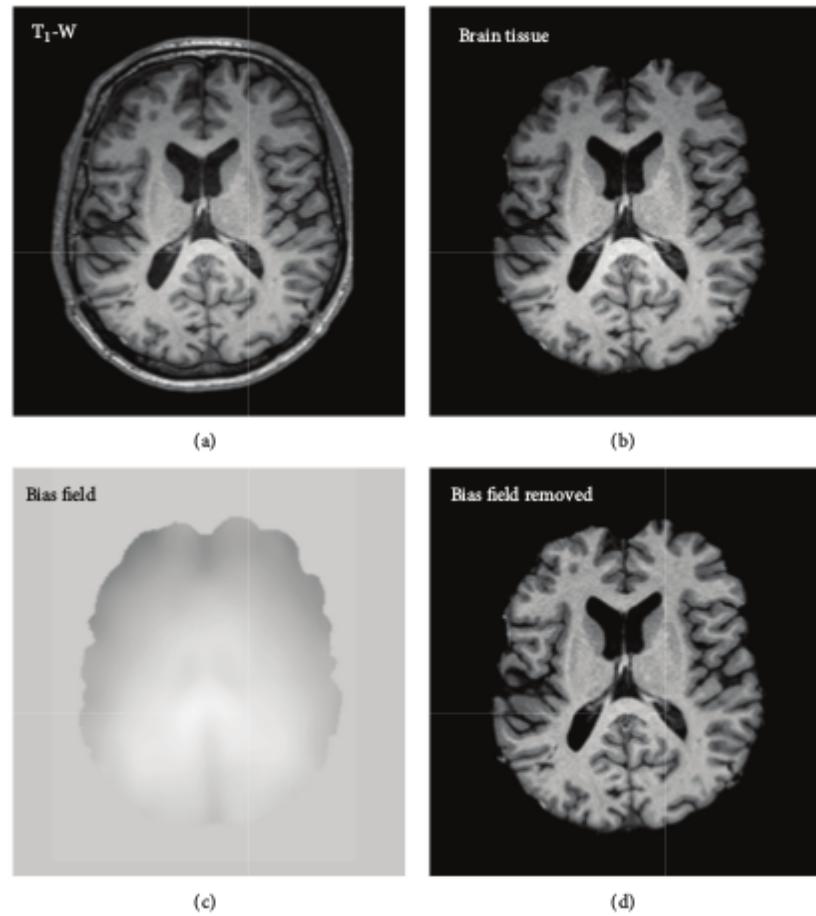


Figure 5.3. Preprocessing steps: a) the original T1 brain MRI image, b) the brain tissue image after skull stripping, c) the bias field, d) the brain tissue image after the bias field correction

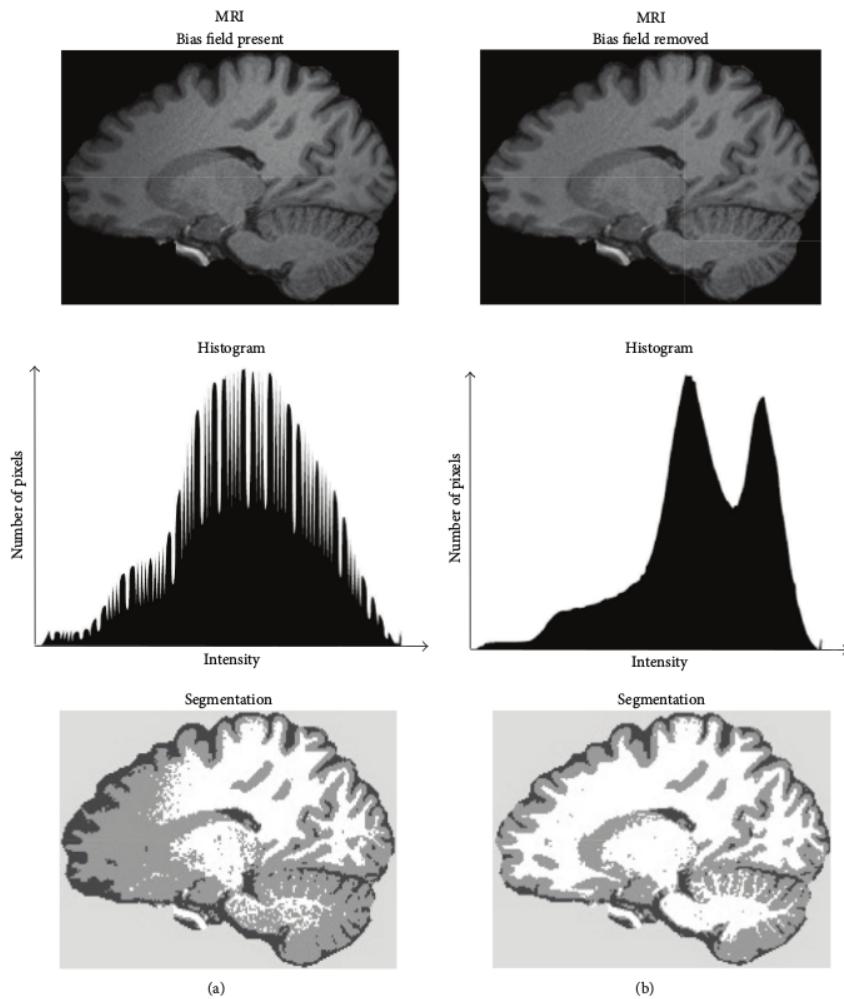


Figure 5.4. Influence of the bias field on brain MRI segmentation

The second, very important preprocessing step is to remove nonbrain tissue. Tissues such as skull, fat or neck have intensities overlapping with intensities of brain tissues. Which means, that only skull stripping is not enough to segmentation whole brain image. The result of removing nonbrain elements can be either a binary mask , which has a value of 1 for brain voxels and 0 for the rest of tissues or a new image with just brain voxels. The brain voxels comprise GM, WM, and CSF of the cerebral cortex and subcortical structures, including the brain stem and cerebellum.

MRI segmentation methods Automated analysis of MR images is challenging due to intensity inhomogeneity, variability of the intensity ranges and contrast, noise or other image-artifacts. There is no single algoritm that can be suitable for all images. Most of the segmentation methods developed for one class of images can be easily applied/extended to another class of images. For example, the theory of graph cuts, although firstly developed for binary images, can be modified and used for MRI segmentation of the brain tissue. The segmentation methods, with application to brain MRI, may be grouped as follows:

- manual segmentation,
- intensity-based methods (including thresholding, region growing, classification, and clustering),
- atlas-based methods,

- surface-based methods (including active contours and surfaces, and multiphase active contours),
- hybrid segmentation methods.

Segmentation method used in application In application, the Gaussian Mixture Model (GMM) based on Expectation-Maximization (EM) algorithm is used to extract specific brain tissues. This is one of clustering methods, which are unsupervised segmentation methods that partition an image into clusters of pixels/voxels with similar intensities without using training images. In fact, clustering methods use the available image data to train themselves. Both, the segmentation and training, are done in parallel by iterating steps.

Gaussian mixture model Often the data we are trying to model is much more complex, than simple distributions, like Bernoulli or Gaussian. For instance, it might be multimodal. This means that there are several different modes, or regions of high probability mass, and regions of smaller probability mass in between. An example can be image. In this case, it is possible to model the data in terms of a mixture of several components, where each component has a simple parametric form (such as a Gaussian). The Gaussian mixture model is simple linear superposition of Gaussian components, aimed at providing a richer class of density models than the single Gaussian. Mixture model is probabilistic model, which can represent the presence of subpopulations within an overall population. The Gaussian distribution in mixture model (Fig. 5.5) can be written in the form ??.

The probability given in mixture of K Gaussians is

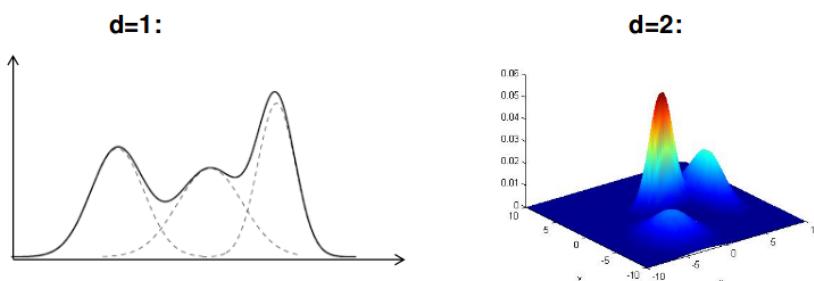


Figure 5.5. Gaussian mixture model examples for 1 and 2 dimension

Gaussian mixture model can be attached to set of data with an unknown distribution, by estimate the parameters (theta) of GMM model that fits the data. The solution is to maximize the likelihood $p(X|\theta)$ of the data with regard to the model parameters.

List of References

[13]

5.9. Module 10. Upsampling

Image spatial resolution is limited by several factors. There are for example time of acquisition, organ motions, signal to noise ratio or medical equipment which do not meet the specialized requirements. Specialists usually make relatively small number of 2-D slices. It takes less time but it also results in large slice thickness and spacing between slices. Another example are voxel-wise analysis. The method requires all image volumes of one subject to be analysed in one space. If they are not acquired from the same location (because of the organ motions) the interpolation must be involved.

Interpolation is a method of constructing new points based on the existing ones. In other words finding a value of a new point in High Resolution (HR) image based on points in Low Resolution (LR) pictures.

In classical interpolation techniques pixels in LR data y can be related to the corresponding x values of HR data:

$$y_p = \frac{1}{N} \sum_{i=1}^N x_i + n, \text{ where}$$

y_p is the pixel of LR image at location p ,

x_i is each one of the N High Resolution pixels contained within this LR pixel,

n is some additive noise.

In classical interpolation techniques the High Resolution pixels x are calculated as weighted average of existing LR pixels y .

$$x_i = \frac{1}{M} \sum_{j=1}^M w_{ij} * y_j, \text{ where}$$

w are weighted calculated as Euclidean distance between M existing surrounding LR pixels and the coordinates of new ones.

The biggest problem is that to find High Resolution data from the Low Resolution values. Unfortunately, there is an infinite number of values that meet that condition. Interpolation methods can be divided into three basic techniques.

The first group are the most common ones, like linear or spline-based interpolation. These techniques assume that it is possible to count the value of a new point by determination some kind of generic function. The main disadvantage is that they are correct only for images of homogeneous regions. As is well known, brain consists of grey substance, white substance and cerebral spinal fluid, so above-mentioned methods are not appropriate for MRI images interpolation. The second one is Super Resolution technique. It is commonly used to increase image resolution on functional MRI (fMRI) and Diffusion Tensor Imaging (DTI). The method is based on acquisition of multiple Low Resolution (LR) images of the same object. It is time consuming and not adequate for clinical applications.

The last but not least method is non-local patch-based technique which is based on self-similarity of a single image. It is possible to improve resolution by extracting information from a single image instead of acquiring several pictures.

First of all, there is a constraint which says that the original LR image has to be equal to the downsampled version X of the reconstructed image. It is named subsampling consistency constraint which is written as:

$$y_p - \frac{1}{N} \sum i = 1^N X_i = 0$$

To make the above equation real, the noise has to be removed. The presence of noise has to be minimized on the LR image by applying a filter, for example MNLM3D filter for 3D MR images based on Non-Local Means filter. The filter removes the noise effectively without affecting the image structure.

The aim of this module is to increase MRI image resolution by upsampling. The input data is a single 320x240 image. After the processing it is going to be twice as big. To be more specific 640x480 pixels. The process can be named as double upsampling.

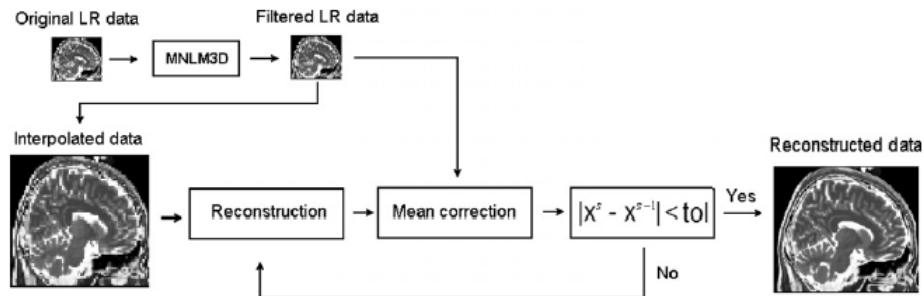


Figure 5.6. Diagram of the patch-based non-local algorithm [14]

In a nutshell, the first step of the algorithm is to divide every pixel of LR image into more pixels. Then the patch is designed. It is a rectangle which will be the area of interest. The pixels that are inside the area are taken into account during calculation the values of new points. After that an appropriate estimator is used to correctly calculate the values of new pixels. Every pixel has to be classified into one of three groups (grey substance, white substance or cerebral spinal fluid).

In order to show how the algorithm is designed the following pictured were generated using exemplary data. At the beginning there is an image 256x256 pixels.



Figure 5.7. DICOM Low Resolution image 256x256

The first step is to add new pixels to increase the resolution M times horizontally and N times vertically. The pixels are equal to zero. After this process an appropriate estimator has to be chosen to make a decision about the values of new points.

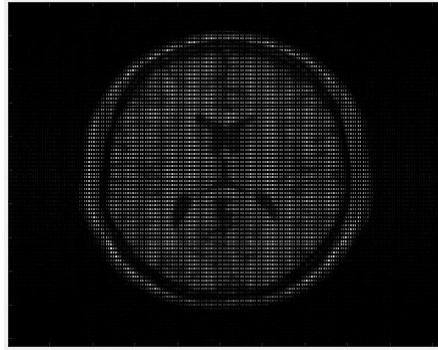


Figure 5.8. DICOM image with new points with N=2 and M=2

Patch-based non-local regularization is used to reconstruct new voxels. The voxel under study is reconstructed using all similar voxels which are placed inside the patch. Contrary to classical techniques which use local neighbourhood of the reconstructed voxel, the proposed method uses the only relevant information, the context of the voxel.

Another step in the algorithm diagram (Fig. 5.6) is mean correction. It has to be applied to ensure that the downsampled reconstructed HR X image is equal to the original y LR one. Thus the local mean value of X fits with the value of the y by adding the corresponding offset.

$$X = X + NN * (D(X) - y), \text{ where}$$

D is an operator that downsamples HR into original LR,

NN is a Nearest Neighbour operator that interpolates LR image to HR.

The Regularization and Correction steps are repeated until no important difference between two consecutive reconstructions. Tolerance is determined by Mean Absolute Difference .

List of References

[14] [9art2]

5.10. Module 11. Brain 3D

To prepare tree dimension visualization of the cerebral cortex algorithm of marching cubes is used.

The input data is multiple 2D slices of MR image. The marching cubes algorithm create a polygonal representation of constant density surfaces from a 3D array of data. To select the cerebral cortex is used output data from segmentation made in module 9. The structure of cortex is represented by value 3 in segmentation mask.

The space of the image is divided into a regular grid of cubes. In each iteration one cube is considered. At each vertex of cube is determined how the surface intersects this cube. The density value are compared with the limit value - surface constant. If the data value is bigger than suface constant, one is assinged to a cube's vertex. There are 256 combinations of cube orientation relative to the surface, but we can distinguish 15 basic patterns, that repeat as symmetrical reflections, produces all possibilities (Fig. 5.9). If all values are less than the constant value, then the cube does not form any polygon. Otherwise, the edges of the polygon are defined (by linear interpolation) at the edges that intersect the surface. Using central differences, a unit normal at each cube vertex is calculated and then normal to each trangle vertex is interpolated. The output of the algorithm is the triangle vertices and vertex normals. [15]

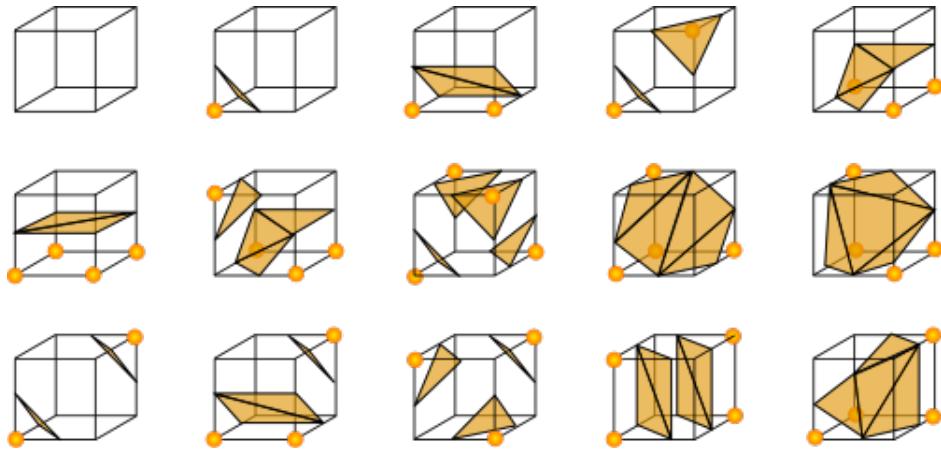


Figure 5.9. Triangulation for the 15 patterns.

To visualization the model, obtained by marching cubes, the VTK library is used, which enables building the three-dimension model. The VTK is object-oriented library. The classes of VTK is dedicated to processing and visualization data.[16]

The second part of this module includes visualization of the brain's cross-section on arbitrarily defined plane. To enable selecting of intersection plane there was used object of VTK class, which allows set plane in elected direction by using computer mouse. When the plane is moved by user, in the real time the three-dimensional model is clipped in the place of selected plane. To improve quality of visualization the cross-section image, there is also possibility to see the image imposed on the three-dimensional model.

5.11. Module 12. Oblique imaging

Oblique Imaging is a technique to create non-perspective projections from 3D or multiple 2D images. There is not much information about this technique in theoretical sources.

Algorithm

Input data is a 3D array, where index is X, Y or Z value and and value of the array in those indices is pixel value of the image. Proposed algorithm involves creating a grid, rotating and translating it, checking which points to interpolate and which can be taken from MRI slices directly and interpolating needed points.

Algorithm step by step:

- create a grid
- select 3 angles and rotate this grid relative to the X, Y and Z axes under those angles
- translate this grid relative to X, Y and Z axes - this grid gives information about x, y and z indices needed for the oblique image
- based on created grid check points, that can be taken from MRI slices directly, and which to interpolate
- create an image base on interpolated points and points from MRI slices

1. Creating a grid

Grid is created based on normal vector $\vec{v1} = [0, 0, 1]$. Two vectors perpendicular to it are: $\vec{v2} = [1, 0, 0]$ and $\vec{v3} = [0, 1, 0]$ are used in grid generation. For given $i = [1, 2, \dots, m]$ and $j = [1, 2, \dots, m]$ every point can be designated from equotation:

$$[x, y, z] = \vec{v2} * i + \vec{v3} * j \quad (5.66)$$

2. Rotating grid

To rotate a grid rotation matrices are used.

- OX rotation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi & \sin \Phi \\ 0 & -\sin \Phi & \cos \Phi \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \quad (5.67)$$

- OY rotation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos \Phi & 0 & -\sin \Phi \\ 0 & 1 & 0 \\ \sin \Phi & 0 & \cos \Phi \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \quad (5.68)$$

- OZ rotation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos \Phi & \sin \Phi & 0 \\ -\sin \Phi & \cos \Phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \quad (5.69)$$

In order to save computational time, it is not the created grid itself, that is being rotated, but two vectors $\vec{v2}$ and $\vec{v3}$

3. Translating grid

To translate every point of grid i units in X direction, j units in Y direction and k units in Z direction given equotation is used:

$$[x, y, z] = x + [1, 0, 0] * i + y + [0, 1, 0] * j + [0, 0, 1] * k \quad (5.70)$$

4. Points interpolation

If after grid rotation and translation any points can not be taken directly from MRI slices it is essential to interpolate them. Maximum number of points from slices taken to interpolate is 8. Their distance to the point to be interpolated is computed using:

$$d = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \quad (5.71)$$

Where:

x_0, y_0, z_0 are coordinates of point to be interpolated

x_i, y_i, z_i are coordinates of the point from surroundings

After that interpolated point value is computed using equotation:

$$p_v = s_p / n_p \quad (5.72)$$

Where p_v is point value and s_p is sum of values of points in surroundings and n_p is number of points

5. Module I/O

- **Input** - the input of this module is structural data in form of 3D array ($x, y, slicenumber$)
- **Output** - the output of this module is one oblique image in form of 2D array

6. Implementation

6.1. Tools

Python - The main programming language of this project is Python. The utilized language version is **Python 3.5.4** (last "bugfix" release of Python 3.5).

VTK - The Visualization Toolkit (VTK) is an open-source software system for 3D computer graphics, modeling, image processing, volume rendering, scientific visualization, and information visualization. The project uses version **7.0**, as it is compatible with python 3.5 (compatibility assured by menpo project¹).

Cython - Cython is an optimising static compiler for both the Python programming language and the extended Cython programming language. It allows to represent the most computationally complex operations with more efficient code. This project uses version **0.26.1**.

PyQT - PyQt brings together the Qt C++ cross-platform application framework and the cross-platform interpreted language Python. This project uses its version **5.6** to create Graphical User Interface (GUI) for the application.

SciPy - SciPy is a collection of open source packages, which serves to import .mat MATLAB data into NumPy arrays and perform computations on them.

Conda - Conda is an open source package management system and environment management system that runs on Windows, macOS and Linux. Conda quickly installs, runs and updates packages and their dependencies. This project uses **Anaconda3** distribution to provide standarized enviroment with all of the required packages for our developers.

Pyinstaller - PyInstaller is a program that freezes (packages) Python programs into stand-alone executables. Despite of numerous package-specific problems, it appeared feasible to deploy the project using this tool.

6.2. Module 1. MRI reconstruction

Classical least squares estimation provides only the minimization of data error, which is an invalid solution. Conventionally, the LSE solution has a huge norm and thus it is a valueless outcome of a reconstruction procedure. To this end, we introduced basic Tikhonov regularization method that seem to partially overcome the problem. However, we firstly implemented LSE approach for later usage of the results obtained with this procedure in restoration of the images from a ‘nearby’ well-posed problem (regularization). For Tikhonov regularization, additional quadratic penalty term allows controlling the norm of the solution, which benefits in introduction of smoothness prior to estimated solution.

¹<http://www.menpo.org/>

Firstly, we checked the data dimension that we load to the programme, as we could receive data starting from 3D to 5D. The implementation works for single slice structural and diffusion data (from many gradients) as well as for a whole set of brain slices. First two dimensions, provides information about data resolution i.e. 128×256 , which means that the images were subsampled with factor $r = 2$. In case of structural data, the third dimension would be the number of slices (then, the forth is number of coils images) or simply number of coils images. For diffusion data, in 5D case the third dimension stands for number of slices, the forth - number of diffusion gradients and the fifth - number of coils images. The third dimension can be equal to one, so we get 4D data i.e. diffusion data for one slice. We also tested whether input dataset contains matrices with sensitivity maps profiles of a proper size, corresponding to maximal dimension of an image i.e. $256 \times 256 \times 8$, for case of images acquired with 8 coils. Furthermore, the input files should also contain subsampling factor r and number of coils L , if not, they can be calculated having the dataset's dimensional information.

Secondly, we performed 2D inverse Fourier Transform (2D IFT) on each of the dataset image, to transform them from **k**-space to **x**-space. This stage of an algorithm is presented in Fig. 6.1.

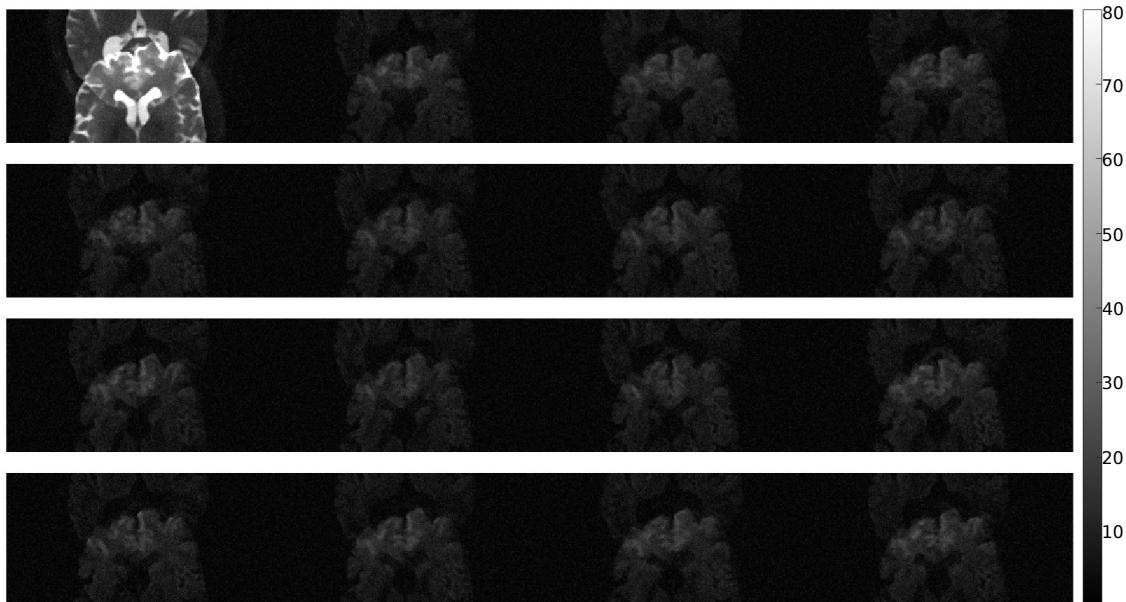


Figure 6.1. Subsampled diffusion dataset in **x**-space domain (data for one particular slice, acquired for 15 diffusion weighting gradients).

Thirdly, we used absolute value operator right before performing the SENSE reconstruction main step. As we are provided with sensitivity maps profiles (Fig. 6.2), we can easily reconstruct the subsampled data as it is shown in Fig. 6.3. The key idea is to evaluate the algorithm pixel by pixel. According to equation 5.4, the construction of defined \mathbf{D}^S vector is simple. Basically, the \mathbf{D}^S is a column vector of $L \times 1$ size, containing pixels withdrawn from each L coil image at specified spatial location. The matrix \mathbf{S} we construct in a subsequent way: the l – th row of \mathbf{D}^S contains values of the sensitivity maps profiles, corresponding to data position in a subsampled image and moved by a subsampled FOV value (i.e. for $r = 4$, $FOV_y = 256/4 = 64$) $r - 1$ times. As a result, depending on the number of coils and subsampling rate value, we obtained a matrix of $L \times r$ size. In LSE sense, the result of computation of for defined \mathbf{D}^S and \mathbf{S} is column vector \mathbf{D}^R ($r \times 1$ size).

However, the LSE approach is not an optimal solution, so we implemented the regularization approach which uses the *a priori* information of searched solution obtained with LSE algorithm. The images obtained after first

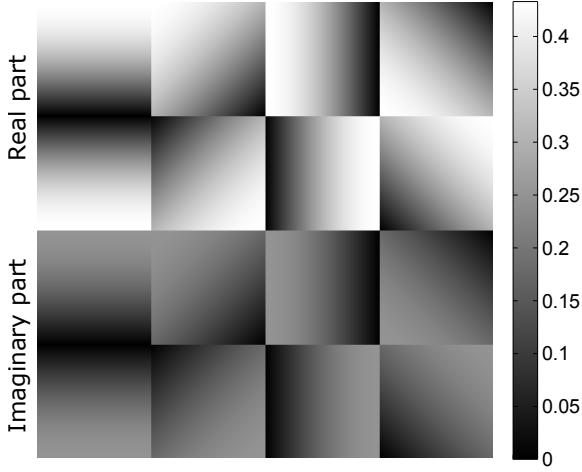


Figure 6.2. The real and imaginary parts of sensitivity maps used to reconstruct data ($L = 8$).

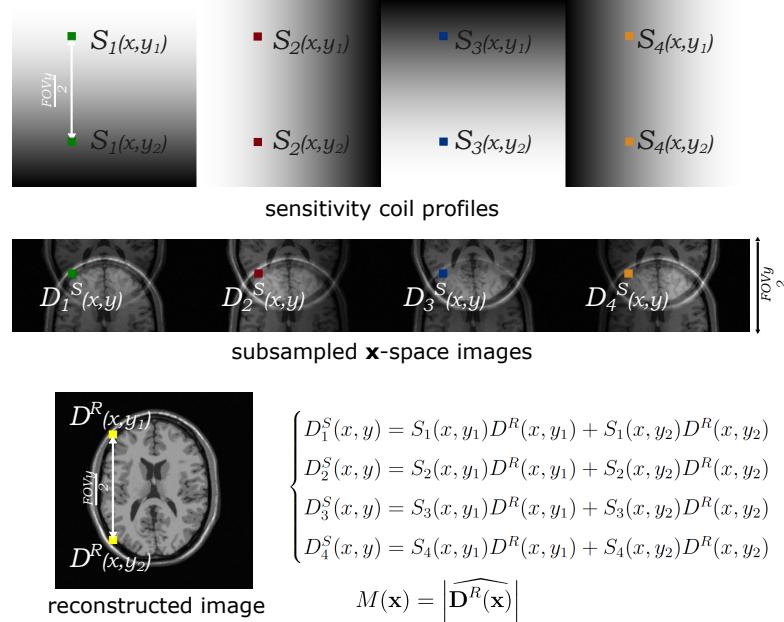


Figure 6.3. SENSE algorithm graphical explanation of Cartesian sampling using four receiver coils ($L = 4$) and the subsampling rate $r = 2$. Two yellow pixels of the reconstructed image are unfolded using coil sensitivity profiles and the corresponding folded pixels (points marked with green, red, blue and orange).

reconstruction are median filtered with window size 3x3. We introduced this additional information to the solution of derived objective function according to Eq. 5.7. In this case, \mathbf{D}^S and \mathbf{S} are constructed in the same manner as for LSE case, however we regularized the solution with λ parameter. The choice of appropriate value of regularization parameter allows controlling the balance between both components (bias-variance tradeoff). For this scenario, we empirically picked λ parameter as a constant value. As the reconstruction is performed pointwisely, we introduced extra information about expected result as a vector \mathbf{D} of $rx1$ size, containing pixels from reconstructed median filtered LSE images corresponding spatially to those to be reestimated with Tikhonov approach. The result is

similar, i.e., we derived values of r reconstructed points, which differ from those obtained with basic LSE approach. Fig. 6.4 presents exemplary result of Tikhonov reconstruction algorithm implemented in the software.

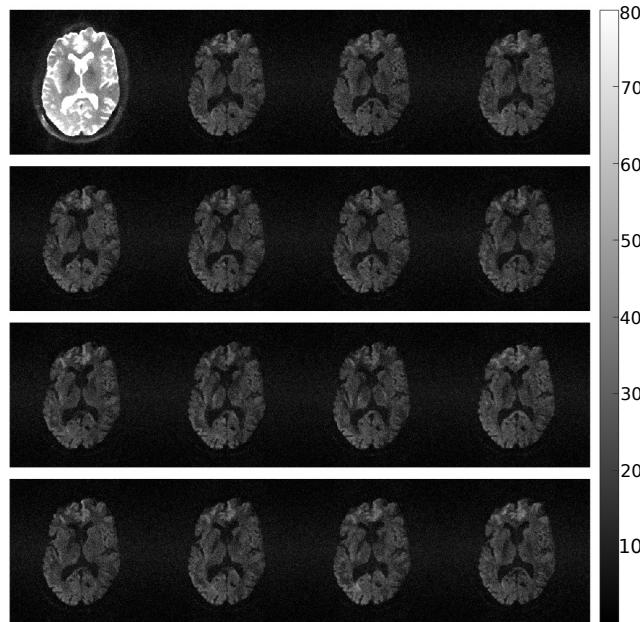


Figure 6.4. Reconstructed diffusion dataset in x-space domain (data for one particular slice, acquired for 15 diffusion weighting gradients).

6.3. Module 2. Intensity inhomogeneity correction

The aim of this module is to remove intensity inhomogeneity from MR image. Usually after MR imaging there are brighter and darker spots in the image of brain. The bias is very undesirable especially for the next modules such as segmentation and 3D imaging. The expected result of this module is that every kind tissue would be presented by approximately one shade of gray. As we never know how the bias map signal will look like, a method of removing it, which is able to adapt to any shape of the inhomogeneity map is required. To achieve the goal, the following steps are taken. Firstly, the reconstructed image with intensity inhomogeneity is read (Fig. 6.5).

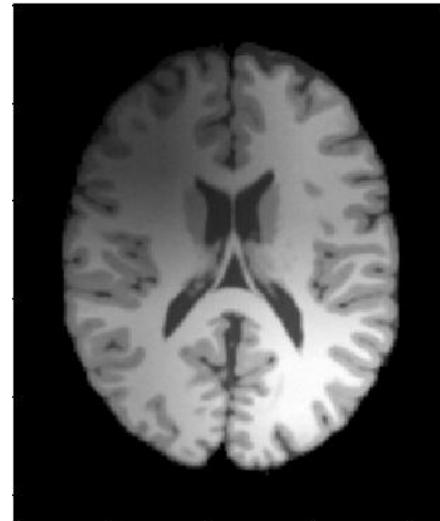


Figure 6.5. The original image with visible intensity inhomogeneity

The next step is extracting the background of the image with Gaussian Filter of a size equal 2/3 the size of MRI image. In this step all the details corresponding to high frequency components are filtered out. The gaussian filter used in this part takes two variables. The first one is the size of the bandwidth and the second is the standard deviation sigma. For a picture of size 256x256 the size of the filter is set as 170x170 and the sigma is set as 20. Many tests have shown that too large sigma would reduce too much details from the background while too little sigma would leave too much details (Fig. 6.6). The appropriate background extraction is shown in the Fig. 6.7.

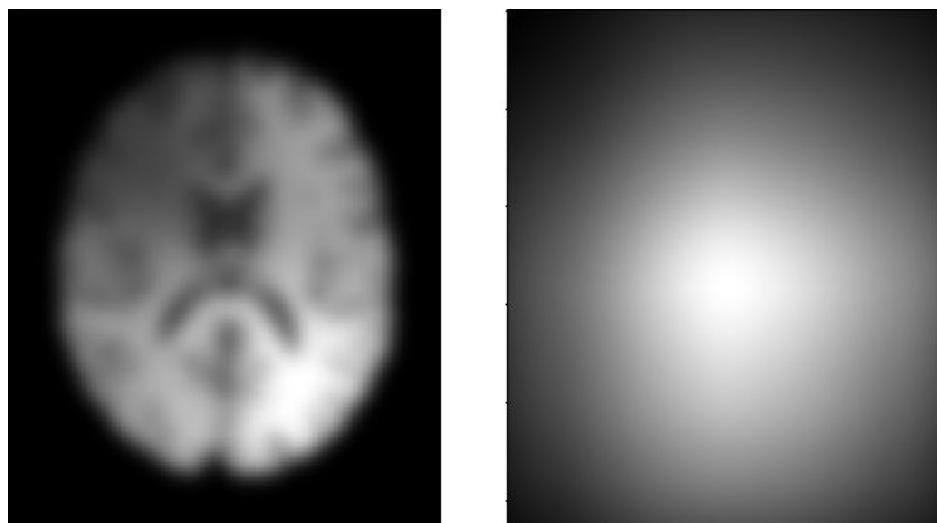


Figure 6.6. The filtered image with sigma = 5 (left) and sigma = 150 (right)

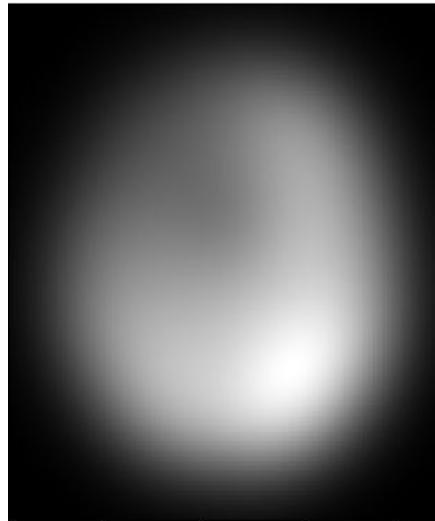


Figure 6.7. The image after Gaussian filtering with sigma = 20

Next there are 150 datapoints randomly selected from the background image. Every datapoint has 3 dimensions: x and y coordinates and z as a level of gray. They are used in next steps to fit a surface to the background. Following 3rd degree polynomial was set as the parametric equation of the surface:

$$f(x, y) = a + (b * x) + (c * y) + (d * x^2) + (e * y^2) + (f * x * y) + (g * x^3) + (h * y^3) + (i * x^2 * y) + (j * x * y^2) \quad (6.1)$$

The fitting of the surface was performed with an `curvefit` function from the `scipy` module. **describe in a few words how the function works** The equation is used to generate an image of bias field signal. The result is shown in the fig. 6.8.

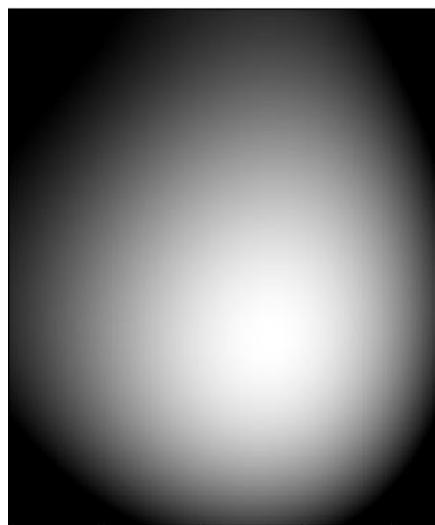


Figure 6.8. The surface fitted to the filtered image

The last step is dividing each element of the original image by the corresponding element of the image of bias field performed with numpy function `divide()`. To avoid dividing by 0, all the pixels in the bias field image that were equal 0 are changed into value 1. The final result of all the following operations is shown in the fig. 6.9.

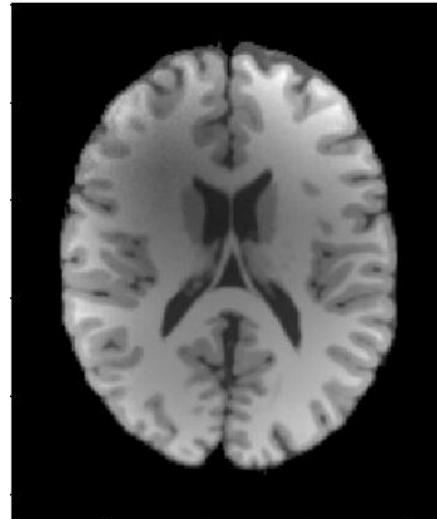


Figure 6.9. The result with intensity bias reduced

6.4. Module 3. Non-stationary noise estimation

Algorithm responsible for estimating spatially variant noise map in this application was implemented based on [17]. In the process implementation some changes were made in the original code in order to improve its effectiveness. The changes were then approved after tests that proved usefulness of those changes. The whole algorithm can be divided into couple of main stages:

- estimation of expected value in each point in the image,
- subtraction expected value from corrupted image and doing so getting the noise,
- calculation of SNR (*signal to noise ratio*),
- introductory processing of noise,
- filtering,
- correction.

Below is the exemplary MRI image corrupted with complex noise.

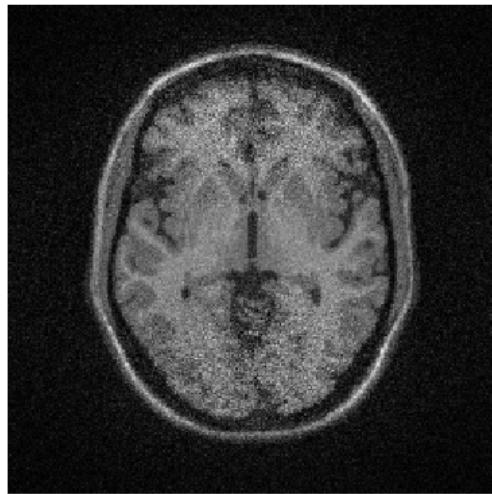


Figure 6.10. Corrupted MRI image.

Estimation of expected value in each point in the image.

Expected value in each point of the image is estimated as a local mean using 3×3 window. The local mean is calculated with the filter designed as a function called *filter2b* which receives two arguments as input, first one is window, second is image.

```
def filter2b(h,I0):
    Mx = np.size(h,1)
    My = np.size(h,0)
    Nx = (Mx-1)/2
    Ny = (My-1)/2
    Nx = int(float(Nx))
    Ny = int(float(Ny))
    It = np.pad(I0, [Nx, Ny], 'edge')
    filt = signal.convolve2d(It, np.rot90(h), mode='valid')
    return filt
```

Listing 6.1. Filtering function.

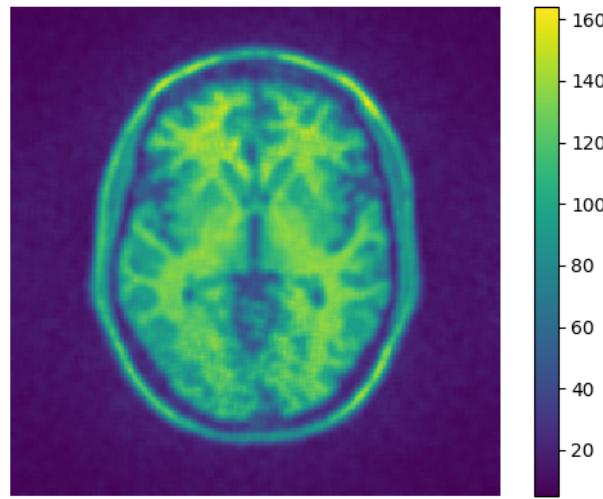


Figure 6.11. Local mean of image presented above.

Getting the noise.

Getting a noise from the corrupted image is accomplished by subtracting local mean from the corrupted image.

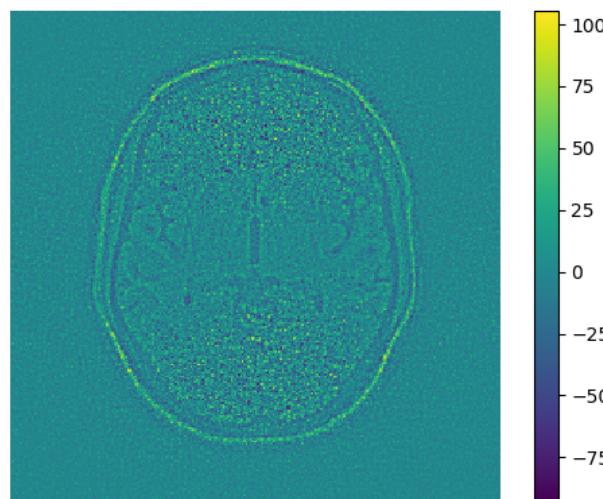


Figure 6.12. Noise.

Calculation of SNR.

This stage is more complicated than the others. Another function called *appro* was designed responding accordingly to the necessities of this stage. The purpose of this function is approximation of Besseli which helped during calculations of SNR.

```
def appro(z):
    cont = (z<1.5)
    z8 = np.multiply(8, z)
    z8[z==0] = 0.0001
    Mn = 1 - (3/z8) - (15/2*np.power(z8, 2)) - ((3*5*21)/6*np.power(z8, 3))
    Md = 1 + (1/z8) + (9/2*np.power(z8, 2)) + ((25*9)/6*np.power(z8, 3))
    M = Mn/Md
    M = M.flatten()
    i = 0
    if (sum(sum(cont))>1):
        for x in np.nditer(z, op_flags=['readwrite']):
            if x<1.5 and x!=0:
                x[...] = (iv(1, x))/(iv(0, x))
            elif x==0:
                x[...] = 0
            else:
                x[...] = M[i]
            i = i +1
    return z
```

Listing 6.2. Appro.

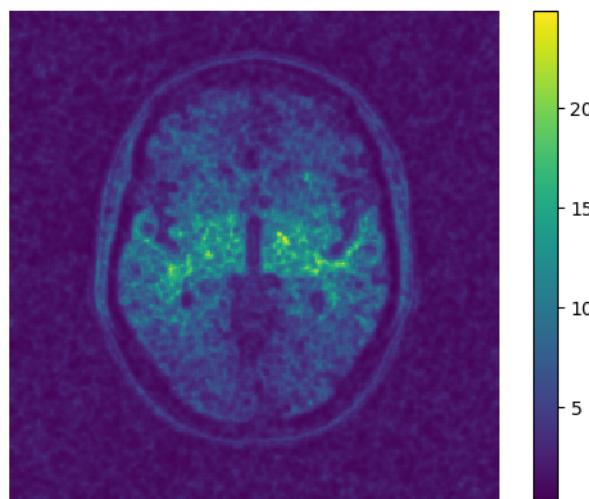


Figure 6.13. Signal to noise ratio.

Indtrodutory processing of noise.

This stage cosnists of getting absolute value of each point of the noise and after that calculating logarithm of these values.

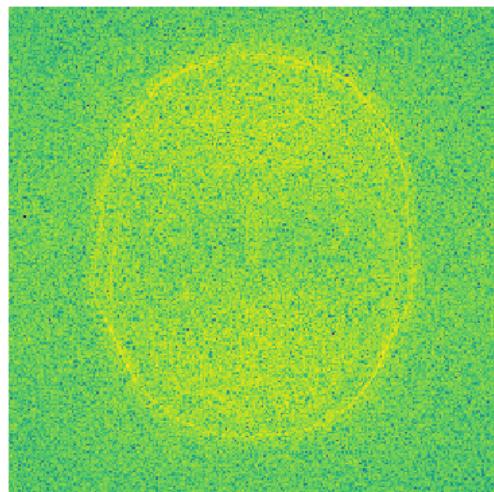


Figure 6.14. Preprocessed noise.

Filtering.

Filtering occurs two times in the alogrithm. First time after introductory processing of the noise and second time after Gaussian correction and in both cases it's low pass filtering. Filtering process is based on discrete cosine transform (*dct*) and inverse discrete cosine transform (*idct*), both performed in the horizontal and vertical direction. The result of *dct* is matrix with the same shape as input image (in this case noise), this matrix contains frequency components of the input whith the lowest arround top-left corner. The result of *dct* is muliplied by specially prepared Gaussian mask to remove high frequencies. Then the *idct* of the result of this multiplication is performed to restore image from the new frequency components.

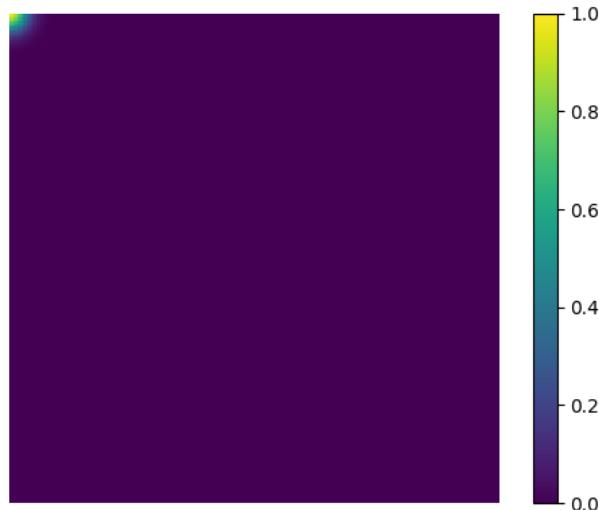


Figure 6.15. Prepared Gaussian mask.

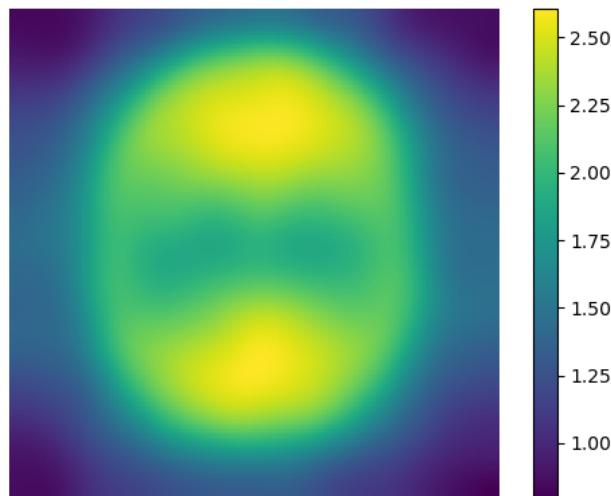


Figure 6.16. Result of first low pass filtering.

Correction.

Correction consists of two parts. First one is Rician/Gaussian correction and second one removes residues from low pass filtering. For needs of Rician/Gaussian correction SNR was calculated earlier.

```
coefs = [-0.2895, -0.0389, 0.4099, -0.3552, 0.1493, -0.0358, 0.0050, -0.00037476, 0.000011802]
correct = np.zeros((x,y)) #x and y are dimensions of the image
```

```
for i in range(0,8):
    correct = correct + np.multiply(coefs[i], np.power(SNR, i))
temp = (SNR<=2.5)
correct = np.multiply(correct, temp)
noise = noise - correct
```

Listing 6.3. Rician/Gaussian correction.

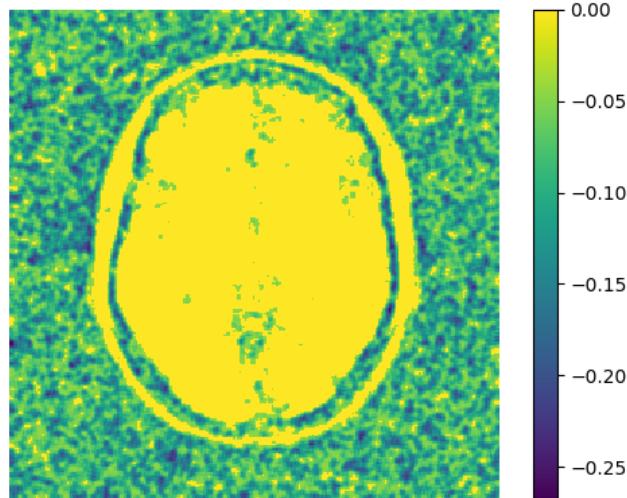


Figure 6.17. Correction matrix.

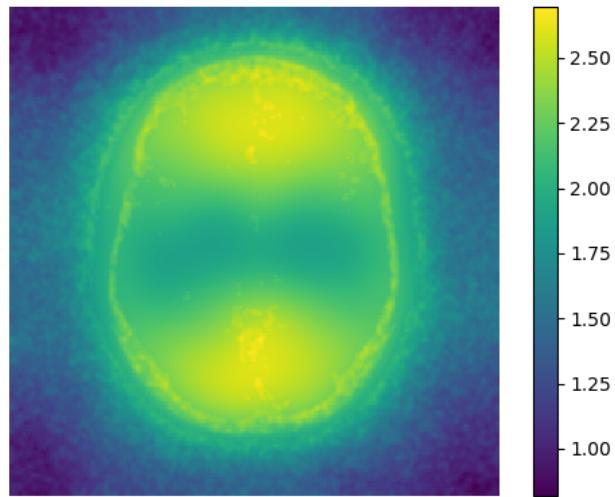


Figure 6.18. Result of Rician/Gaussian correction.

After this part noise is filtered again with low pass filter.

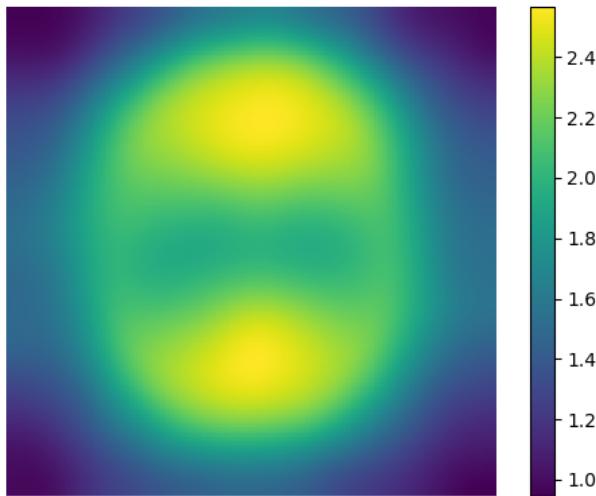


Figure 6.19. Result of second low pass filtering.

In the second part of correction residues from filtering are being removed.

```
eg = 0.5772156649015328606/2
noise = np.multiply(2, noise)
noise /= np.sqrt(2)
temp = np.exp(eg)
noise = np.multiply(noise, temp)
```

Listing 6.4. Removing of residues from filtering.

The last part of the algorithm is getting exponential value of each point in the calculated matrix and it results in getting estimated spatially variant noise map.

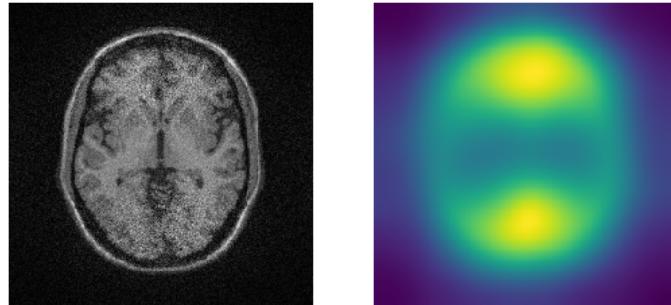


Figure 6.20. Corrupted MRI image (left) with estimated map of its noise (right).

6.5. Module 4. Non-stationary noise filtering 1

Linear minimum mean square error method consist of three step:

- Estimate the second or fourth order moment
- Calculate the K parameter
- Estimate new signal without noise

The code was prepared in simens class template. The main programme is divided to part of code for structural data and diffusion data. The data, which is received, includes whole set of brain slices. The structural data has three dimensions - first two dimensions are the number of pixels in MR image. The third includes the number of slices. The LMMSE filter is called for every slice in whole data set. Diffusion data has one more dimension, which described the number of diffusion gradients. The LMMSE filter is called also for every slice and every diffusion gradients. In this data is used one more for loop to iterate data by every number of gradient.

Module is divided into two functions and one main function. One of them is prepared to estimate the second and the fourth order moment. The second function includes estimating new image and calculating K parameter. K-th order moment is calculated using the estimator of k-th raw sample moment for two-dimensional data:

$$\langle I_{ij}^k \rangle = \frac{1}{|\eta_{ij}|} \sum_{p \in \eta_{ij}} I_p^k \quad (6.2)$$

This case is solved by convolution data with square neighbourhood divided by size of the window. The size of neighbourhood window is chosen experimentally. We used the python function *convolve2d*, which convolve two 2-dimensional arrays with output size determined by mode. The mode is checked to 'same', because we need the same size of arrays. The inputs of first function are image and square neighbourhood window, the function return second and fourth order moment (two-dimensional array).

Next step is to calculate the K parameter, which is defined:

$$K_{ij}^2 = 1 - \frac{4\sigma_n^2(\langle M_{ij}^2 \rangle - 2\sigma_n^2)}{\langle M_{ij}^4 \rangle - \langle M_{ij}^2 \rangle^2} \quad (6.3)$$

With numpy array we could easily perform transformations on arrays without using loops.

The last step of programme is estimated new image without noise. The estimator is defined:

$$\widehat{A_{ij}^2} = \langle M_{ij}^2 \rangle - 2\sigma_n^2 + K_{ij}(M_{ij}^2 - \langle M_{ij}^2 \rangle) \quad (6.4)$$

After estimation $\widehat{A_{ij}^2}$, we calculate the absolute value pixels in the new image, because we need positive value to square root. Then, we calculate the positive square-root of an array. The inputs of second function are image, noise map and square neighbourhood window, the function return new, estimated image without noise. The result of main function is returned to next module as class.

The programme is really fast, the duration time is approximately 0.046 second for one image. To improve working of method, we checked the different size of square neighbourhood window to adjust it to receive the best results. To compare result we used the relative error:

Window	Relative error
3	0,07
4	0,02
5	0,005
6	0,002
7	0,011
10	0,014

The best result for filtering image is when window size equals 6. When the size is higher than 6 the good filtering results occurs in background of image, but noise appears in the contour of brain.

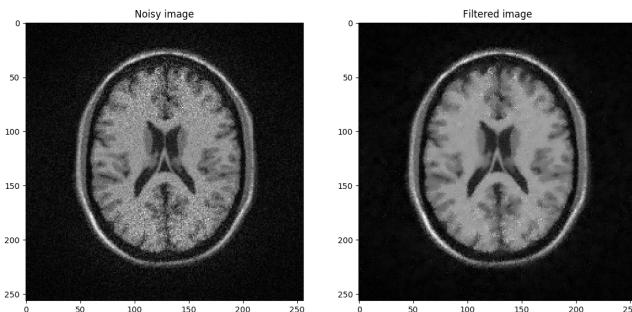


Figure 6.21. Results of LMMSE estimation with size window equal 6

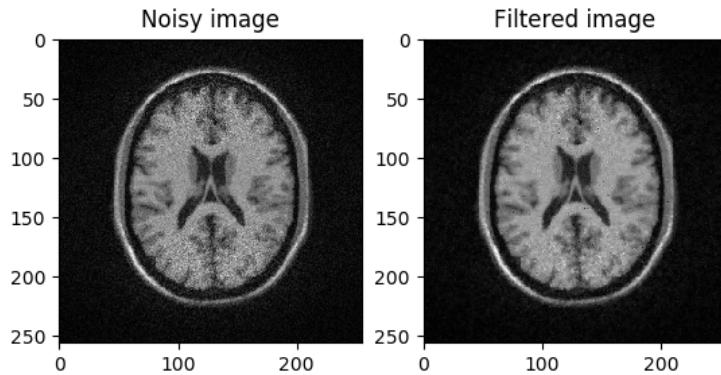


Figure 6.22. Results of LMMSE estimation with size window equal 3

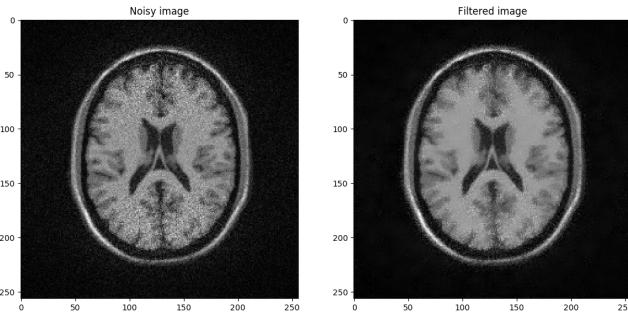


Figure 6.23. Results of LMMSE estimation with size window equal 10

6.6. Module 5. Non-stationary noise filtering 2

Unbiased Non-Local Means algorithm was implemented based on [3]. Some, it is believed, improvements to original idea were introduced and described below. Implementation of the UNLM algorithm was divided into several steps:

- rewriting NLM prototype from MATLAB to Python,
- refactoring to UNLM,
- testing different Gaussian kernels,
- optimization of algorithm in order to reduce execution time,
- adjusting the algorithm to fit both types of data (structural and diffusion weighted).

Because some works and further improvements were done in parallel it is hard to describe implementation in chronological order.

Comparison of different Gaussian kernels

Once the main body of algorithm was ready, it was decided to run some tests to examine the influence of various Gaussian kernels on the output of the UNLM function. The main requirement for the Gaussian kernel was the fact that the overall sum of all elements within kernel should be equal to 1. Another, for sake of sanity, requirement built on the shape of the kernel surface - the slope should be gentle in order to avoid 'overfitting' to the central pixel.

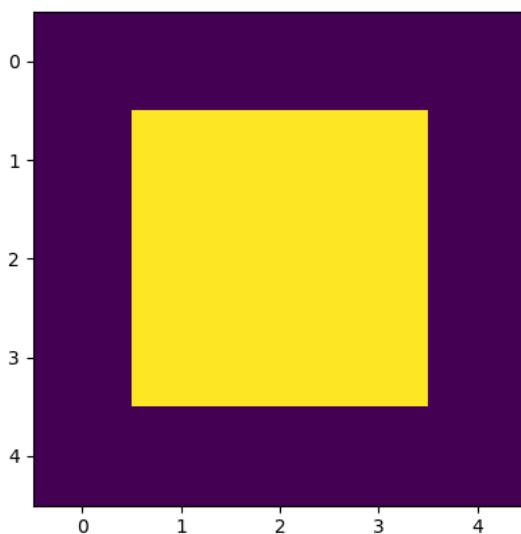


Figure 6.24. First tested Gaussian kernel.

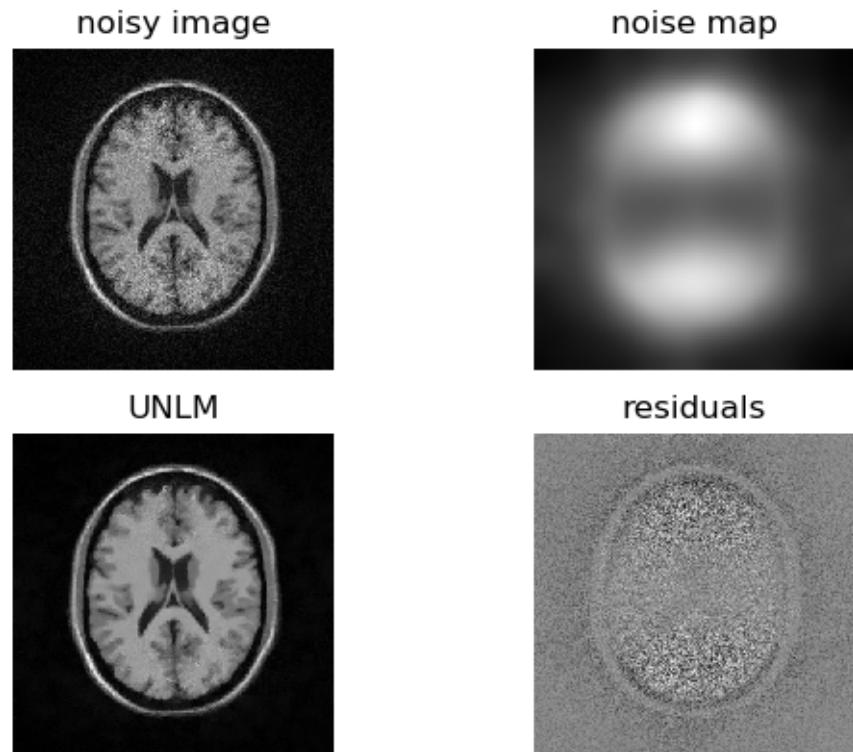


Figure 6.25. Results obtained for first kernel.

First kernel was presented in the article describing the algorithm [3]. However, kernel was considered as not optimal due to its two-level shape. To overcome the problem of wrong shape another attempt took place. Second generated kernel had desired shape, yet values didn't meet the requirement of summing to 1.

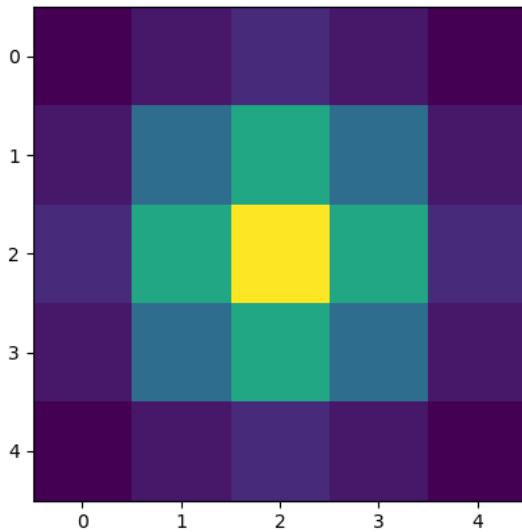


Figure 6.26. Second tested Gaussian kernel.

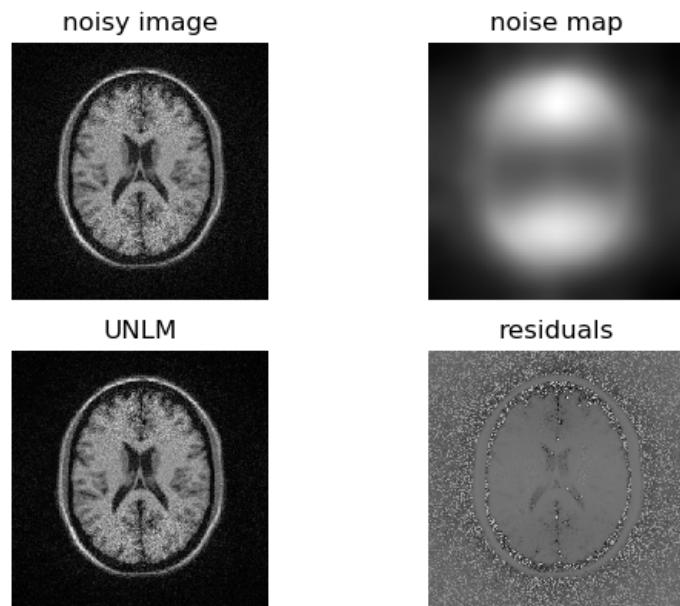


Figure 6.27. Results obtained for second kernel.

Third kernel, again was disappointing. It had a shape of first kernel, but represented different values, since it was generated in other manner.

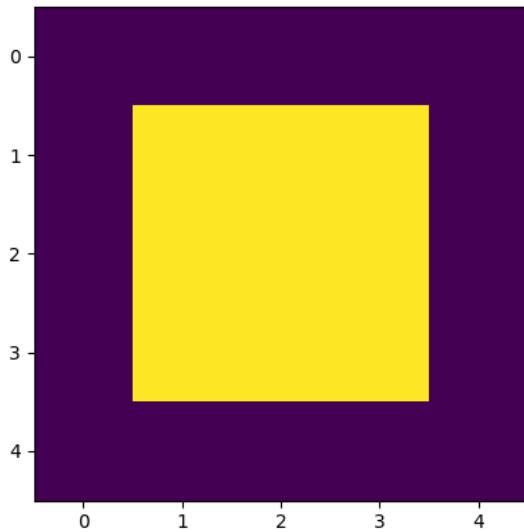


Figure 6.28. Third tested Gaussian kernel.

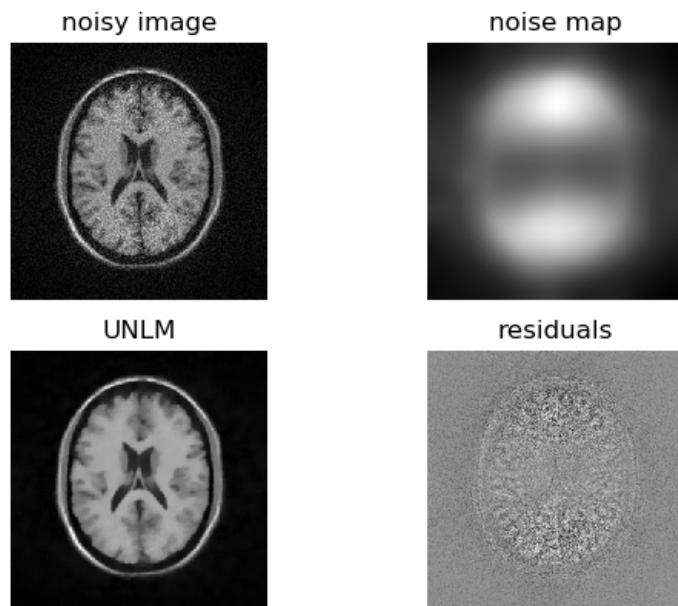


Figure 6.29. Results obtained for third kernel.

Fourth kernel, had all properties that were desired. It was decided to use this particular one in implementation of the UNLM algorithm.

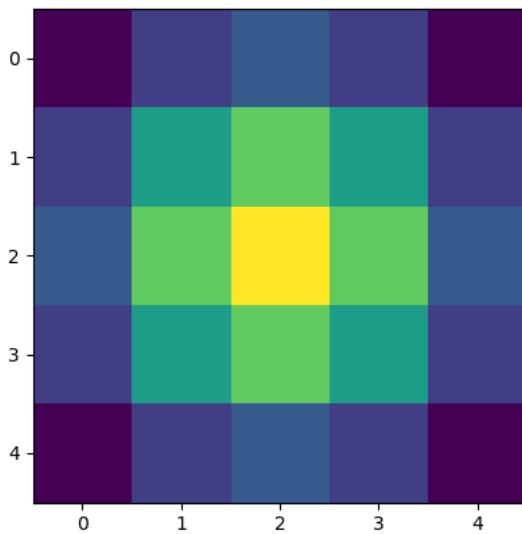


Figure 6.30. Fourth, and final, tested Gaussian kernel.

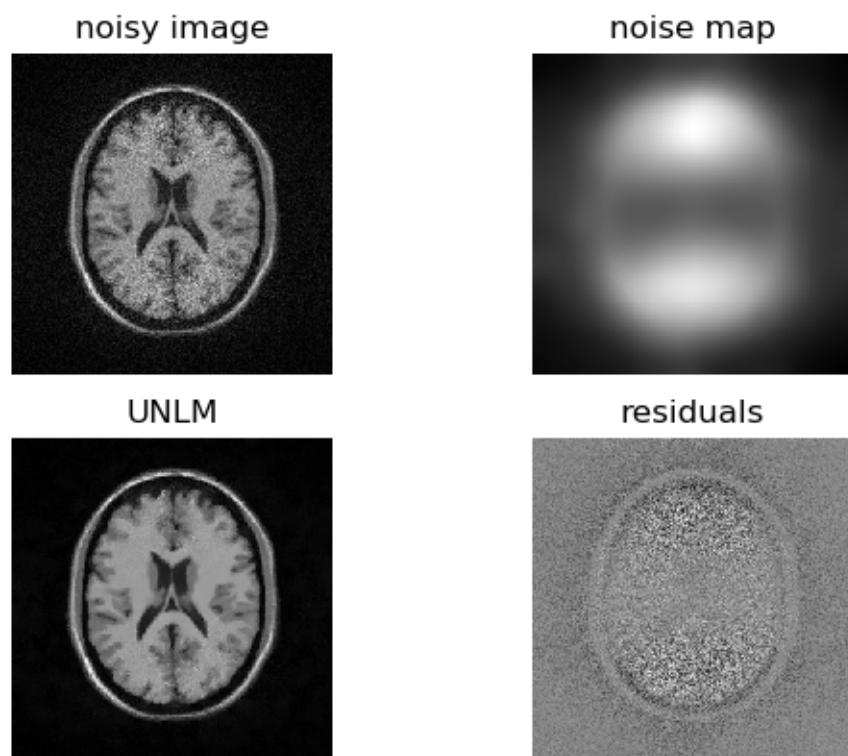


Figure 6.31. Results obtained for fourth kernel.

Someone with an eye for detail can spot the differences between results achieved whilst using all kernels. Most eye-catching distinction lies in boundaries of the brain and skull.

Following code snippet presents the implementation of Gaussian window, that was finally chosen after running some tests. This particular window has a special characteristic of being shallow - the central pixel's value doesn't stand out from values in close vicinity.

```
def gk4(rsim):

    # function that prepares Gaussian window used to penalize
    # pixels based on distance within window
    kerlen = 2 * rsim + 1
    nsig = 2
    interval = (2*nsig+1.) / kerlen
    x = np.linspace(-nsig-interval/2., nsig+interval/2., kerlen+1)
    kern1d = np.diff(st.norm.cdf(x))
    kernel_raw = np.sqrt(np.outer(kern1d, kern1d))
    kernel = kernel_raw / kernel_raw.sum()
    return kernel
```

Listing 6.5. Code used for Gaussian kernel generation.

Reducing execution time

After line-by-line translation of code from MATLAB to Python filtration of single slice lasted for 180 seconds. That was unacceptable, so few flaws in the code had to be identified. It's worth mentioning that single slice filtering in MATLAB took 19 seconds on average.

The first faced incorrectness was the fact of not fully using the numpy library utilities. Changing 'pure python' functions like *sum* to numpy equivalents allowed reduction of time to 80 seconds per slice. That run time was still insufficient. After code profiling with *cProfile* library it was identified, that nested for loops used to move around image were most time consuming. As a result it was decided to get rid off loops and change tedious loop-based architecture to multi-dimensional matrices operations. That operation resulted in achieving 3.5 seconds on average. Involving matrices operations allows to fully exploit numpy implementation (which itself is based on C, so it is faster than Python).

Furthermore, an attempt to use threads in order to speed up calculations on multi-dimensional data took place. For that test, a synthetic 3D data was generated. Processing each slice in a for loop takes $3.5 \times \text{number of slices}$ seconds. Involving threads should reduce the time, but unfortunately it was doubled. So a tough decision of processing slices in classic loop has been made. Other possibilities of parallel computing were not taken into consideration due to the compatibility issues with GUI.

Unbiased Non-Local Means algorithm

Having reduced the computation time and having chosen suitable penalty function (Gaussian kernel) algorithm was considered finished. Some of the most important code features are presented in snippets below.

```
# precompute possible window2
window2_recomp = np.zeros((m, n, rsearch, rsearch))
for i in range(2, m):
    for j in range(2, n):
```

```
window2_precomp[i, j, :, :] = img_ext[i - rsim:i + rsim + 1, j - rsim:j + rsim + 1]
```

Listing 6.6. Overcoming computation of same matrices in every iteration.

During thorough analysis it was discovered that q pixel windows can be calculated only once. Fact of precomputing windows allowed to save resources, which are really important in computationally complex problems.

Moving widows, which are visualized later in this section are changing positions in each iteration. Code snippet below shows how p pixel window and search space window are determined.

```
# p pixel loop
for i in range(0, m):
    for j in range(0, n):
        ii = i + rsim
        jj = j + rsim
        window1 = np.asarray(img_ext[ii - rsearch-1:ii + rsearch + 1, jj - rsearch:jj + rsearch + 1])

# limit the search space
pmin = max(ii - rsearch-1, rsim)
pmax = min(ii + rsearch - 1, m)
qmin = max(jj - rsearch-1, rsim)
qmax = min(jj + rsearch - 1, n)
```

Listing 6.7. Calculation of p pixel window and search space area.

Using precomputed set of all possible q pixel widows and using only those that fit the search space was a smart trick presented below.

```
window2_slice = window2_precomp[pmin:pmax, qmin:qmax, :]
```

Listing 6.8. Calculation of q pixel windows within search space area.

To explain the code and present it in more picturesque way, one can look at the image filtering as moving set of 2D windows on image and calculating some metrics, according to their positions and values. There are two types of windows: neighbourhood and search space. Size of neighbourhood window is indirectly defined by rsim parameter, which was assigned by default to 2. That results in neighbourhood window of size 5x5, because $5 = 2 * rsim + 1$. Search space window is dependent of rsearch parameter, which was assigned to 5 giving 11x11 window. The values of these parameters were set based on analysis conducted in [3]. Authors discovered that these sizes give best results in filtration.

Neighbourhood windows are centered around pixels p and q, which are pixel being filtered and filtering pixel correspondingly. Figure below presents the idea of moving windows.

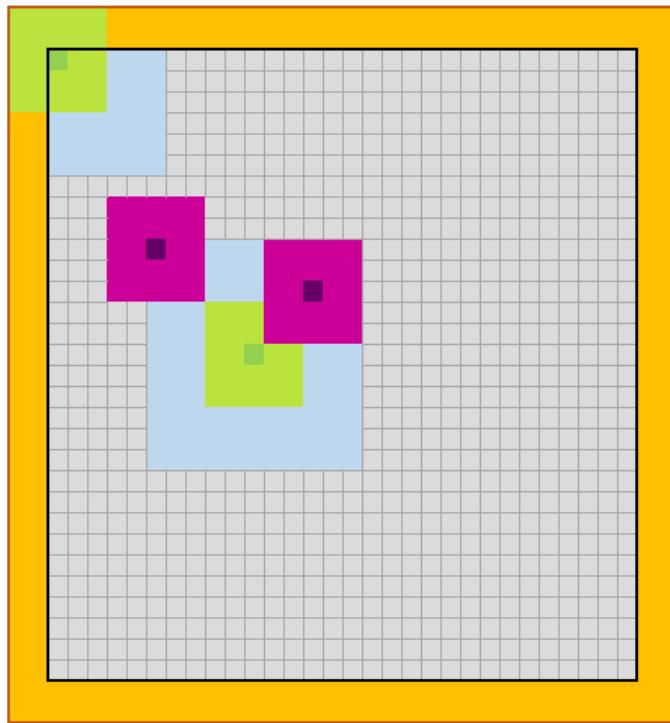


Figure 6.32. Graphical representation of moving windows.

Given figure has to be explained. Gray area represents 2D picture, the grid on in resembles pixels. Dark green square (pixel) is the p pixel and the light green area around it is the neighbourhood window defined by $rsim$. Similarly, purple area and dark purple pixel resembles window of pixel q . The size of the image has to be extended by $rsim$ in each direction to make calculations on edges of original image possible. This scenario is presented in the top left corner of picture. Pixel p is within area of image but its window doesn't fit into original image. As a consequence of extending the image, the orange area is temporarily added to the image. The values in the added space are assigned by symmetrical padding near edges. Last thing that has to be deciphered is blue square. It is the search space window. It narrows the possibilities of q pixel positions taken into consideration whilst calculating distances between p and q windows.

It is worth mentioning that there exist special case, when q pixel is in the same position as p pixel. If that situation would have been treated equally to other, what would introduce some bias towards 'self-distance'. According to the equation used to calculate distance (d) between windows, one has to calculate element-wise difference of windows. Having both windows in the same position would result in distance equal to 0, which further can have negative influence on the output. To overcome this problem, maximum weight found within the window is assigned to a weight calculated based on 'zero' distance.

Adjusting the algorithm for both types of data

Denoising using UNLM method is used by both data types, structural and diffusion weighted. There are some differences in implementation of the algorithm for both types, however the same code is used for them in this module. It is caused by analysis of diagrams presented in [6]. They showed that taking gradient information gives no significant information in filtering result. On this occasion, the only difference in handling both types of data lays in main function of the module. More precisely, after making decision which data is being processed loops work

on different dimensions. Following code snippet presents the *run_module* function, where data is being accurately handled.

```
def run_module(mri_input, other_arguments=None):

    if isinstance(mri_input, smns.mri_diff):
        [m, n, slices, grad] = mri_input.diffusion_data.shape
        data_out = np.zeros([m, n, slices, grad])

        for i in range(slices):
            for j in range(grad):
                data_out[:, :, i, j] = unlm(mri_input.diffusion_data[:, :, i, j], mri_input.noise_map[:, :, i, j])

        mri_input.diffusion_data = data_out

    elif isinstance(mri_input, smns.mri_struct):
        [m, n, slices] = mri_input.structural_data.shape
        data_out = np.zeros([m, n, slices])

        for i in range(slices):
            data_out[:, :, i] = unlm(mri_input.structural_data[:, :, i], mri_input.noise_map[:, :, i])

        mri_input.structural_data = data_out

    else:
        return "Unexpected_data_format_in_module_number_5!"

    return mri_input
```

Listing 6.9. *run_module* function.

One decision must be finally justified. The whole bigger product is targeted (theoretically) at physicians that usually have no knowledge of image filtering. On this occasion it was decided to reduce number of parameters responsible for algorithm outcome to 0. That solution should reduce possibilities of getting badly-processed data, as optimal parameters are set by default.

6.7. Module 6. Diffusion tensor imaging

Preprocessing and Module I/O

In order to improve diffusion tensor estimation it is imperative to remove artifacts. In addition to standard MRI pre-processing, one needs to correct for artifacts arising the use of diffusion-gradient pulse sequences and longer acquisition time. While hardware manufacturers try to proactively diminish some of these effects, software processing is still mandatory.

Module Input:

- 3D structural data array of shape X x Y x Z, where XY - pixel image intensities, Z - chosen slice, which is the T1- or T2-weighted image corresponding the the given DWI acquisition
- 4D diffusion data array of shape X x Y x Z x M, where XY - pixel image intensities, Z - chosen slice, M - applied diffusion gradient direction

- b_value, a scalar value corresponding to applied diffusion gradient sequence magnitude
- 2D gradients matrix of shape M x 3, where each row corresponds to a normalized (x, y, z) components of diffusion gradient sequence vectors
- optionally - 3D binary mask of shape X x Y x Z, corresponding to the brain area detected by Module 8 (Skull Stripping); if not supplied, DTI is computed on each input data voxel

Module Output:

- list of size Z, corresponding to each slice; every list element is a dictionary of biomarker images: MD, RA, FA, VR of shape X x Y, and biomarker FA_rgb of shape X x Y x 3

6.7.1. Initialization

In order to abstract DTI implementation from end-user, all classes and methods other than the main function `run_module` are private to module source code script. It is important to note that prior to running the module one has to provide the module with input data object, as well as `SOLVER` and `FIX_METHOD` parameters. `SOLVER` passed as an argument decides whether to use `WLS` or `NLS` estimation, while `FIX_METHOD` decides how to "fix" negative eigenvalues.

As mentioned in the detailed description chapter, '`ABS`' takes absolute value of each eigenvalue, while '`CHOLESKY`' ensures that the estimated tensor is positive definite. Eigenvalues of positive definite matrices are always non-negative. '`ABS`' is a post-estimation fix, meaning that it does not modify the default estimation algorithm (i.e. it is applied after `WLS` or `NLS` computation), while '`CHOLESKY`' directly modifies the expressions for `WLS` and `NLS` cost function gradients and Hessian matrices.

After passing all required arguments to the `run_module` function, they are reshaped internally in order to be compatible with module. Concretely, `DTISolver` class instance, computing the DTI proper, assumes that input data argument is a concatenated 3D array of both structural and diffusion images, which are stored separately in the original data structure. Moreover, `b_value` and gradient fields are reshaped to be lists corresponding to each slice of the new data array (that is: `b_value` is repeated in length while both have zeros appended that correspond to structural images). Finally, all of the above is done separately for each slice and DTI module performs its computation slice-by-slice due to memory constraints.

6.7.2. WLS estimation

`WLS` with the `ABS` fix method is a fast yet simple method of module pipeline computation based on diffusion tensor estimation. As such these parameters were set as default for DTI.

Diffusion tensor estimate was computed by implementing the equation:

$$\gamma = (\mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} \mathbf{W})^{-1} \mathbf{W}^T \boldsymbol{\omega}^T \boldsymbol{\omega} \mathbf{y} \quad (6.5)$$

using NumPy matrix broadcasting operations, effectively abstracting away array reshaping. Weights vector $\boldsymbol{\omega}$ is calculated using a separate function in order to avoid changing every piece of code referring to `WLS` weights in case they change. The following implementation assumes the simplest of models presented in the Detail Description chapter, that is weights being equal to the measured signal.

6.7.3. NLS estimation

In case of NLS estimation, in addition to implementing gradient and Hessian matrix computation methods:

$$\nabla f_{NLS} = -\mathbf{W}^T \hat{\mathbf{S}}_r$$

$$\nabla^2 f_{NLS} = \mathbf{W}^T (\hat{\mathbf{S}}^T \hat{\mathbf{S}} - \mathbf{R} \hat{\mathbf{S}}) \mathbf{W} \quad (6.6)$$

It is important to devise an iterative scheme because gradient result depends on NLS diffusion tensor estimate. For that reason an algorithm based on [9] has been implemented. The method itself is called a Modified Newton's Algorithm and can be summarised as in Fig.6.33.

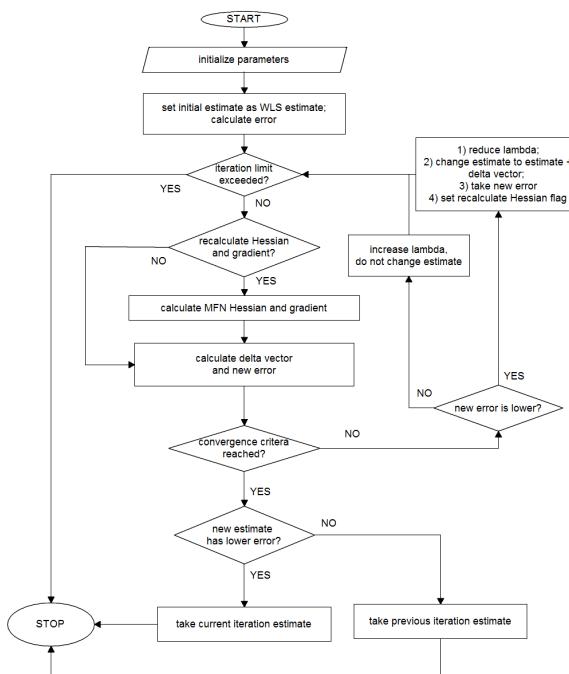


Figure 6.33. Modified Newton's method for iterative computation of NLS estimate (based on [9])

The following parameters (collectively known in code as MFN parameters) were set:

- MFN_MAX_ITER = 3 - iteration limit
- MFN_ERROR_EPSILON = 1e-5 - first convergence criterion (error change is small)
- MFN_GRADIENT_EPSILON = 1e-5 - second convergence criterion (vanishing gradient)
- MFN_LAMBDA_MATRIX_FUN = 'identity' - regularization matrix added to Hessian matrix
- MFN_LAMBDA_PARAM_INIT = 1e-4 - initial regularization matrix multiplier

Delta estimate is calculated using the following formula:

$$\delta = -(\nabla^2 f_{NLS} + \lambda I)^{-1} \nabla f_{NLS} \quad (6.7)$$

with λ parameter increasing and decreasing by a factor of 10, depending on whether newly calculated estimate yields lower error (decreasing for lower error, increasing otherwise).

Convergence is established using the following formulas:

$$\begin{aligned} |f_{NLS_{new}} - f_{NLS_{old}}| &< \text{MFN_ERROR_EPSILON} \\ \delta^T \nabla f_{NLS} &< \text{MFN_GRADIENT_EPSILON} \end{aligned} \quad (6.8)$$

6.7.4. Biomarkers computation

As mentioned previously, the estimate obtained from NLS or WLS methods can be reshaped to a 3x3 matrix:

$$\mathbf{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix} \quad (6.9)$$

assuming our estimate is equivalent to:

$$\gamma = [lnS_0, D_{xx}, D_{yy}, D_{zz}, D_{xy}, D_{yx}, D_{xz}]^T \quad (6.10)$$

Tensor estimation results for a sample 126x126x55 slice are presented on Fig.6.34.



Figure 6.34. Diffusion tensor estimate using the WLS-ABS method for a 126x126x55 test image.

We can then compute the eigenvalue decomposition of \mathbf{D} and obtain eigenvalues and eigenvectors for each pixel of input image. Eigenvalues are sorted in descending order and saved for later computation (Fig.6.35). Moreover, eigenvectors corresponding to the highest eigenvalue are saved to separate variable, since they represent the direction of principal diffusion.

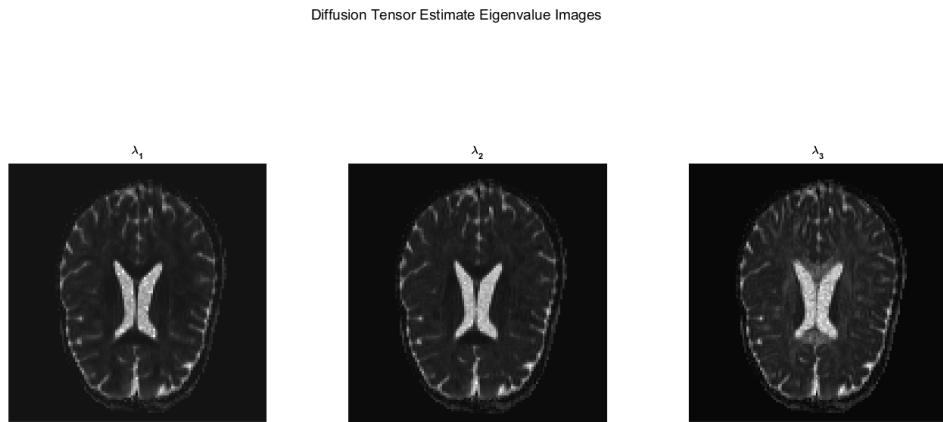


Figure 6.35. Diffusion tensor estimate eigenvalues of sample imgae; eigenvalues sorted in descending order from left to right.

Biomarkers are then computed using the following formulas:

$$MD = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3} \quad (6.11)$$

$$RA = \sqrt{\frac{(\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2}{3 MD}} \quad (6.12)$$

$$FA = \sqrt{\frac{3}{2} \sqrt{\frac{(\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}} \quad (6.13)$$

$$VR = \frac{\lambda_1 \lambda_2 \lambda_3}{MD^3} \quad (6.14)$$

Computed biomarkers of the sample image are presented on Fig.6.36. It is important to note that skull is visible because the brain area was selected by hand instead of relying on Module 08 output.

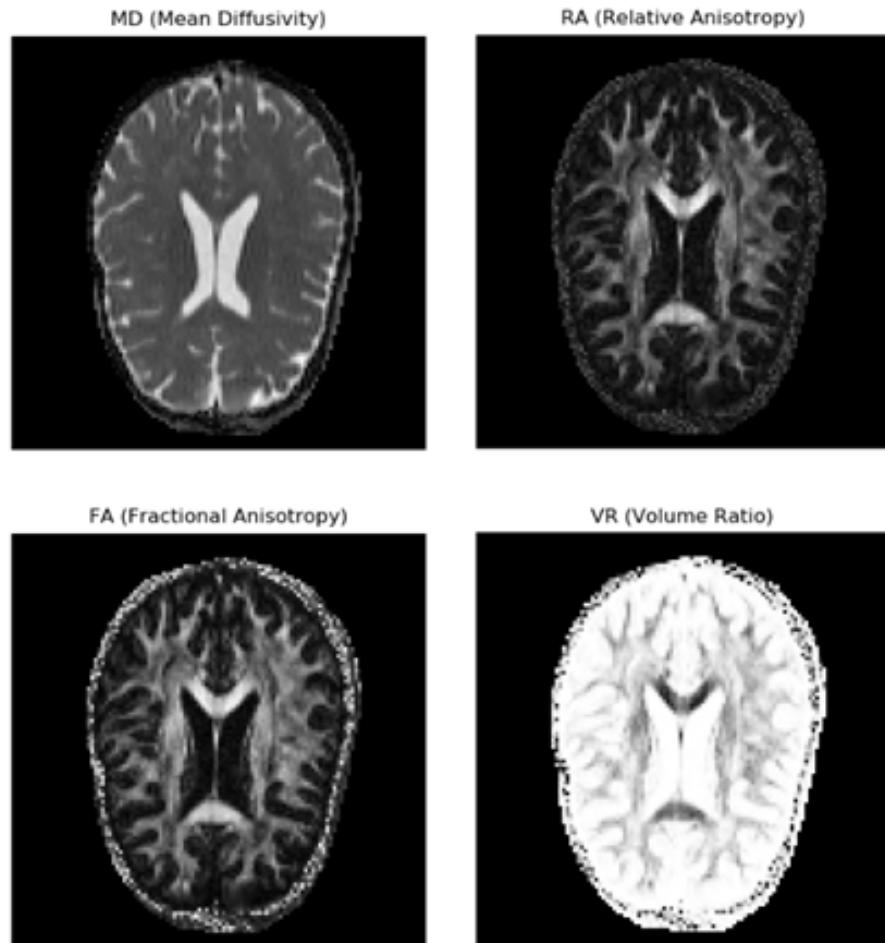


Figure 6.36. Diffusion biomarkers of sample image. Top left: MD, top right: RA, bottom left: FA, bottom right: VR.

Figure 6.37 presents the fifth biomarker image present in output dictionary - FA-weighted principal direction of diffusion color map. Color-encoded direction for this slice is presented as follows:

- red - transversal (left-right)
- green - anterior-posterior (front-back)
- blue - crano-caudal (head-feet)

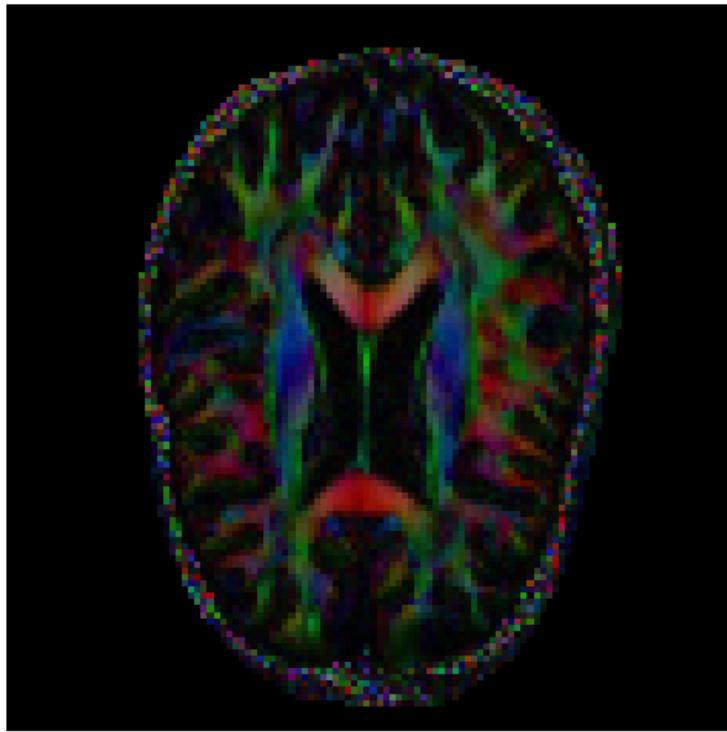


Figure 6.37. FA-weighted principal direction of diffusion of sample image.

6.7.5. Implementation caveats

Module source code was implemented and tested on a single sample image outlined above. It constituted a perfect basis for algorithm evaluation because it was already preprocessed, without the need to wait for other modules implementation. Even though it lacked a skull stripping mask, hand-drawn shape was good enough for testing, even though skull outline is visibly present, particularly on Fig.6.37.

It was difficult to determine which estimation method - WLS or NLS - is objectively better. WLS was significantly faster and it was not much different from NLS results. This might have been due to the fact, that the diffusion-encoding gradient matrix was unusually large (7 structural images and 48 DWIs). Testing on preprocessed raw data, supplied for application evaluation has shown differences between methods, although both estimates are valid (to the Author's best knowledge) as it is difficult to compare any estimation methods without reference.

Initially developed using nested for-loops, DTI module has been vectorized to speed up computations. For a supplied 256x256x16 DWI stack a single slice is being processed in less than 3 seconds for WLS-ABS, while other methods scale linearly with chosen number of iterations (starting at 6 seconds, then 3 seconds per MFN iteration). It must be noted that the module was not tested with skull stripping mask, which could have drastically reduced computation time. Moreover computation time heavily depends on hardware constraints.

6.8. Module 8. Skull stripping

Module 8 has implemented own class **SkullStripping** with consist of methods required to skull strip the image. Implemented methods:

- **def recon (self,marker,mask)** - which is used to grey image reconstruction based on morphological grey dilation, **input:** marker - image to reconstruction, mask - mask image **output:** recon - reconstructed image
- **def strel (self, type, size)** - which creates an logical array to morphological operation, **input:** type - string with type structuring element, two option implemented 'disk' and 'array', size - array size **output:** array - a logical array
- **def csf counting(self)** - which estimates CSF based on histogram **input:** self - class SkullStripping object consisting of MRI image **output:** csf - estimated CSF parameter
- **def binarization(self, image, threshold, upper=1, lower=0)** - which changes grey image to binary image **input:** image - image to binarization, threshold - threshold for binarization, upper - value of pixels over the threshold, lower - value of pixels under the threshold, output: bin image - binary input image
- **def radius counting(self, bin image)** - which estimates BR and ratio diameter in axis x to diameters in axis y **input:** bin image - binary image **output:** r - BR estimated parameter, st - ratio
- **def preprocessing(self)** - which consist methods to estimate CSF, BR, ratio, and COG and crop image based on BR parameter **input:** self - class SkullStripping object consisting of MRI image **output:** cropped image - image cropped based on BR parameters, csf - estimated CSF parameter, COG - coordinates of the centroid of the brain, r - estimated BR parameter, st - ratio diameters in axis x to diameters in axis y
- **def grad mag sobel(self, image)** - which computes gradient magnitude using the Sobel edge mask **input:** image - input image **output:** gradmag - gradient magnitude input image
- **def anisodiff(self, image, niter=1,kappa=50,gamma=0.1, step(1.,1.),option=1)** - which filter using of anisotropic diffusion filter **input:** image - input image, niter- amount of iteration, kappa, gamma, step - parameters of filter, option - method of filtering **output:** image out - image after filtration
- **def edges marr hildreth(self, image, sigma)** - which is the Marr-Hildreth edge detector **input:** image - input image, sigma - edge detector parameter **output:** zero crossing - binary image with black boundaries of object in image and white the others object
- **def background marker(self, preproc image, csf)** - which marks the background objects **input:** preproc image - image especially after preprocessing, csf - CSF parameter **output:** bgm - markers of background
- **def foreground marker(self, preproc image, csf)** - which marks the foreground objects **input:** preproc image - image especially after preprocessing, csf - CSF parameter **output:** fgm4 - markers of foreground
- **def watershed(self, preproc image, csf, cog)** - which is used to marker-controlled watershed segmentation **input:** preproc image - image especially after preprocessing, csf - CSF parameter, cog - COG parameter **output:** brain - skull stripping mask
- **def bse(self, cog)** - which is used to brain surface extraction **input:** self - class SkullStripping object consisting of MRI image, cog - COG parameter **output:** brain - skull stripping mask
- **def run(self, verbose=False)** - which is used to run module08 **input:** self - class SkullStripping object consisting of MRI image, verbose - True or False plotting result of method **output:** skull stripping mask - output of the module

To connect with project it is implemented function:

- **def main8(mri input,verbose=False)**

Method **run()** scheme:

1. Preprocessing compute CSF, COG, BR, ratio, and cropped image using of **preprocessing()**.
2. Checking BR and ratio to eliminate slice without the brain tissue.
3. Brain Surface Extraction using of **bse()**.
4. Checking skull stripping mask to eliminate too large mask based on BR parameters. If it is not too large return skull stripping mask.
5. **(OPTIONAL if mask from BSE is too large.)** Marker-controlled watershed segmentation using of **watershed()**.

Evaluation of Module 8's results are presented on figures bellow:

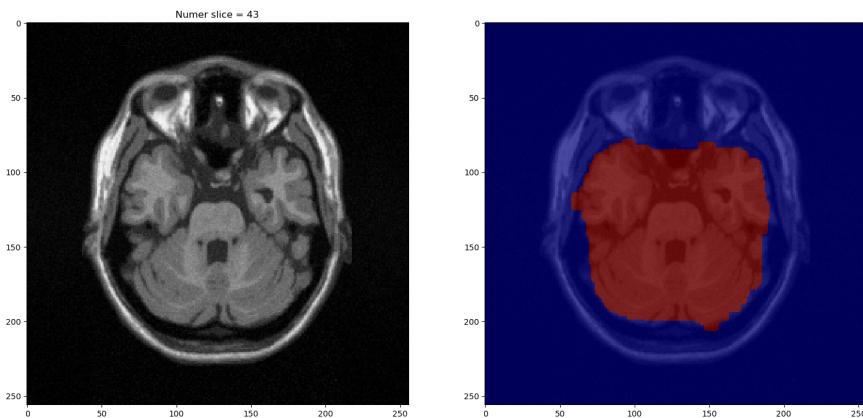


Figure 6.38. MRI image with eyes and input image with skull stripping mask

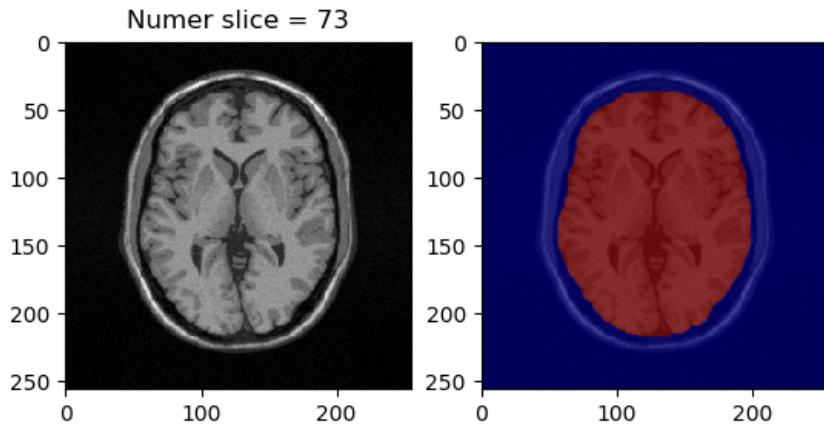


Figure 6.39. MRI image and input image with skull stripping mask

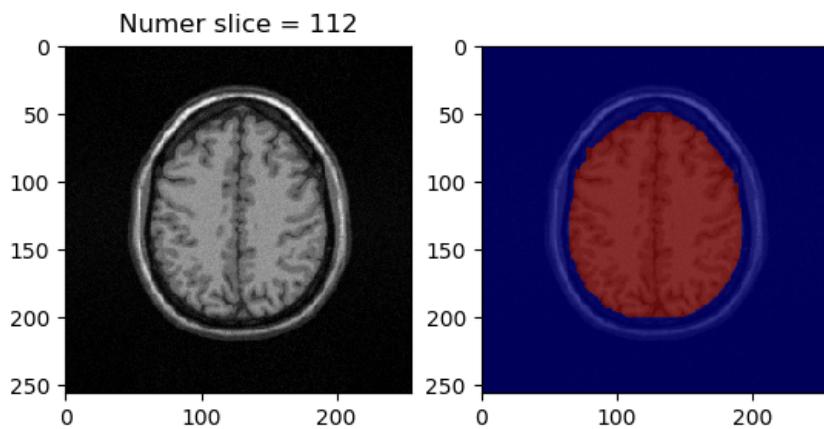


Figure 6.40. MRI image and input image with skull stripping mask

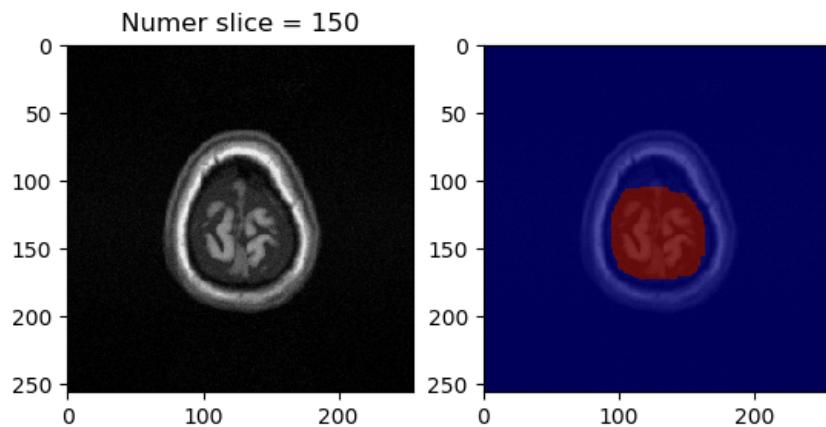


Figure 6.41. Image of upside part of brain and input image with skull stripping mask

6.9. Module 9. Segmentation

Input and output data

An input data is NxM brain MRI image after skull stripping and with whole slices (Fig. 6.42).

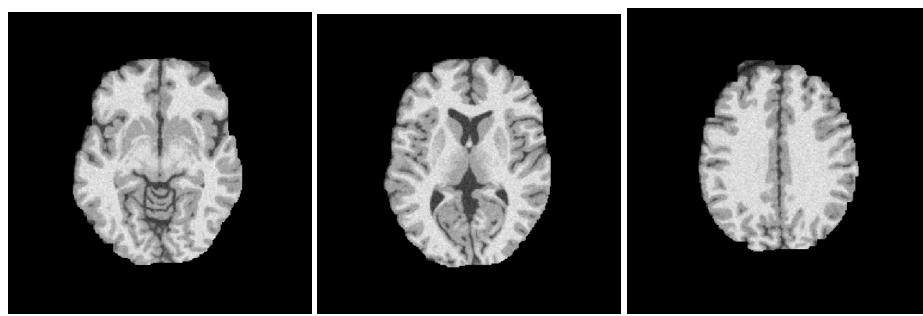


Figure 6.42. Example of input data

An output data is NxM segmentation mask of image without nonbrain tissues (6.43).

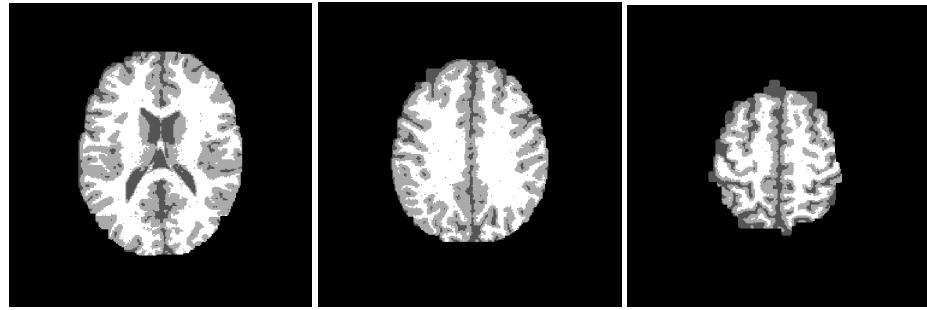


Figure 6.43. Example of output data - segmentation masks

Libraries In module, the basic python libraries are used:

- math
- scipy
- numpy

Implementation This module is used to brain MRI segmentation based on skull stripping images from Module 8. It consists of four functions:

- imHist (image),
- imPart (skullFreeImage, firstPitch, lastPitch, rows, columns, pitches)
- gmm (x,mu,v,p),
- segmentation (skullFreeImage).

imHist function It is a function used for create image histogram. An image histogram is a chart that shows the distribution of intensities in an indexed or grayscale image. This is the first step of segmentation in every single slice of brain MRI image. In the beginning, to calculate the total number of image pixels, an array of image value is changing to vector. Next, by iterating by vector length, histogram is creating step by step (Fig. 6.44). 30/5000 The function argument is the **NxM single image**

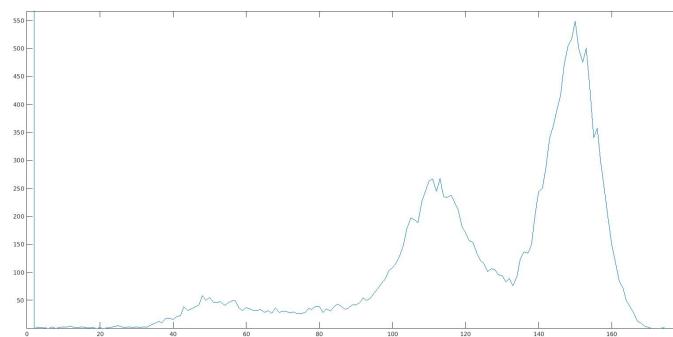


Figure 6.44. Image histogram

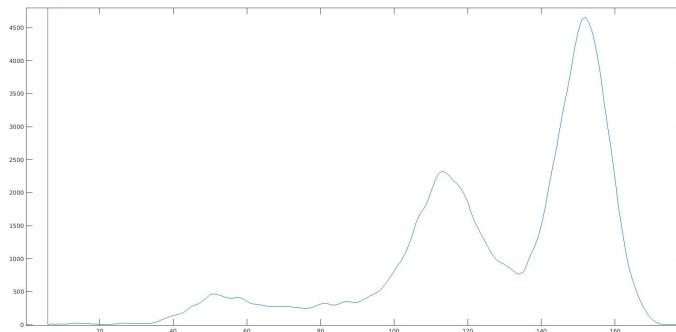
```

for i in range(lengthImage):
    # create histogram of non-zero image values
    f = int(floor(image[i]))
    # round floor
    if f >0:
        if f < maxValue:
            odds=image[i]-f
            # difference between image and round floor image value
            a1=1-odds
            imageHistogram[f-1]=imageHistogram[f-1]+a1
            imageHistogram[f]=imageHistogram[f]+odds
            ...
            imageHistogram=np.convolve(imageHistogram,[1,2,3,2,1])# smoothing the histogram

```

Listing 6.10. Create image histogram

At the end, histogram is smoothing by convolution with window [1,2,3,2,1] and noramlize by the sum of histogram values (6.45).

**Figure 6.45.** Image histogram after smoothing by convolution

imPart function It is a small function to get only image slices, which doesn't contain nonbrain tissues. In this case, function gets 70 slices, from 70 to 140. Getting greater amount can make issues. The function argument are: **image after skull stripping, number of first getting slice, number of last getting slice, image rows, image columns, number of slices**

```

for pitch in range (pitches):
    if pitch < firstPitch:
        continue
    if pitch >= firstPitch:
        if pitch < lastPitch:
            mageToSeg [:,:,rPit] = skullFreeImage [:,:,pitch]
            rPit = rPit+1
        else:
            break

```

Listing 6.11. Getting part of image to segmentation

gmm function The gmm function calculate mixture model gaussian distribution and probability for each cluster of image data, based on mixture model initialize parameters The function argument are: **x - histogram values vector, mu - expectation values vector, v - variance vector, p - probability**

```

for i in range(mu.size):
    differ=x-mu[i]
    amplitude = p[i]/(math.sqrt(2*math.pi*v[i]))
    app = amplitude*(np.exp((-0.5*(differ*differ))/v[i]))
    probab[:,i] = app

```

Listing 6.12. gmm function

segmentation function The segmentation function is the most important function in this module. It's utomatic algorithm, matching the Gaussian mixture model to the image histogram, in order to group pixels into proper cluster. To achieve the match, the expectation-maximization algorithm is used.

The first step is inicialize Gaussian mixture model parameters vectors. They need to have the same numbers of element as numer of clusters. The initial parameters are choosen arbitrarily. It is important, to set different expected values to provide various algorithm starting points.

```
# alghoritm parameters inicialization
```

```

clustersNum = 4                                # number of clusters
mu = arange(1,clustersNum+1)*imMax/(clustersNum+1)      # expected value of each clusters
v = np.ones(clustersNum)*imMax                  # variation of each clusters
p = np.ones(clustersNum)/clustersNum            # probability of each clusters

```

Listing 6.13. Segmentation - parameters inicialization

After inicialization, the algorithm starts its operation. It consists of two steps. The first one - expectation (step E) - use gmm function to deremine the gaussian distribution and check relative density using inicialized (in the first time) or updated distribution parameters. It calculate also the log-likelihood based on histogram values.

```
#Expectation Step
```

```

probab = gmm(x,mu,v,p)                         # use gmm function -
                                                 # get probability
distrDens = probab.sum(axis=1)                   # relative density
llh = (hx*np.log(distrDens)).sum()              # the log-likelihood base on
                                                 # histogram data

```

Listing 6.14. Segmentation - step E

The second step - maximization (step M) - in which the model parameters are restimated using the distribution from step E

```
#Maximization Step
```

```

for j in range(clustersNum):
    hxsh = hx.shape
    probabxh = probab.shape
    distsh=distrDens.shape
    resp = hx*probab[:,j]/distrDens                # compute the responsibilities
    p[j]=resp.sum()
    mu[j]=(x*resp/p[j]).sum()                      # compute the weighted of
                                                 # expected values
    differ=x-mu[j]
    v[j]=(differ*differ*resp/p[j]).sum()            # compute the weighted varainces

```

Listing 6.15. Segmentation - step M

The algorithm's operation is shown below

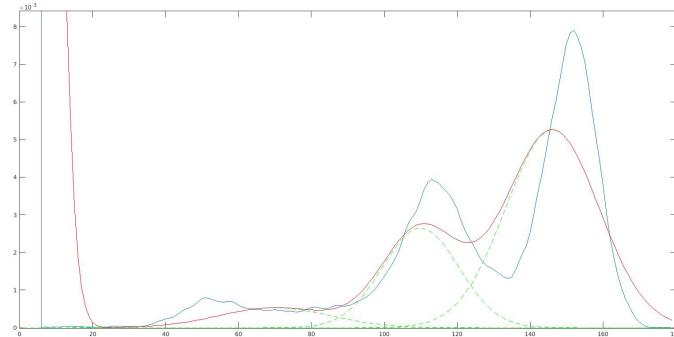


Figure 6.46. The first step - Gaussian mixture model after parameters initialzation

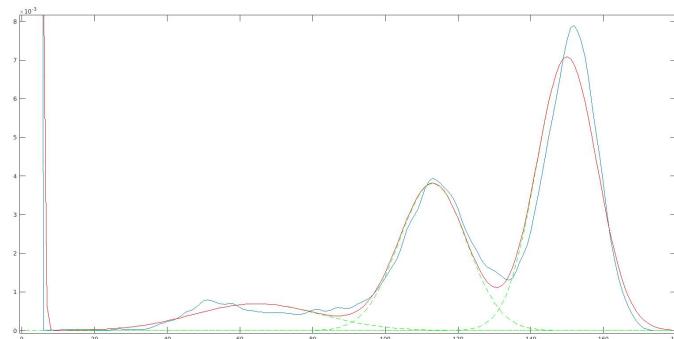


Figure 6.47. After few iteration - fitting Gaussian mixture model

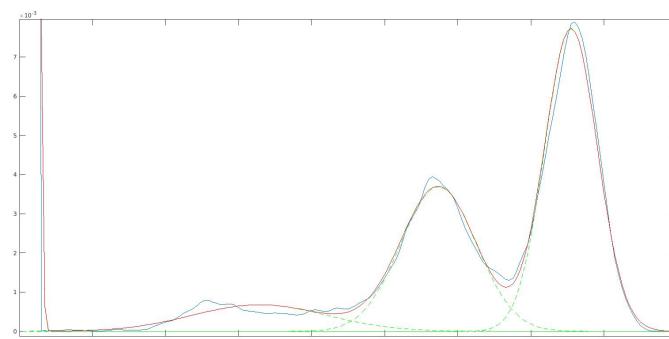


Figure 6.48. Result of matching

The last part of segmentation function is part, which creates image maske after segmenation (). Mask can be divided into 3 separated binary mask, represent white matter, gray matter and cerebrospinal fluid (Fig. 6.49).

```
# Creat image mask
for i in range(rows):
    for j in range(columns):
```

```

for k in range(clustersNum):
    c[k] = gmm(image[i,j], mu[k], v[k], p[k])
a = (c==c.max()).nonzero()
imageMask[i,j]=a[0]

```

Listing 6.16. Create image mask

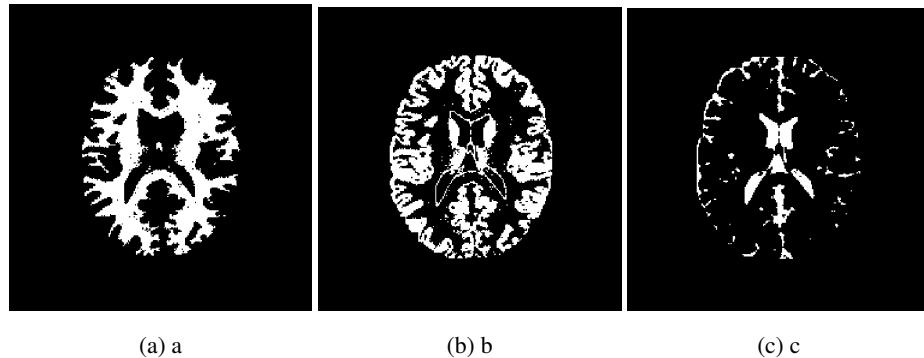


Figure 6.49. Isolated brain structure: a) white matter, b) gray matter, c) cerebrospinal fluid

6.10. Module 10. Upsampling

The implementation began with making a decision about the input data. The input image has to be 2 dimensional without a noise. The uploaded image will be considered as a 2 dimensional array of double values. Other parameters that will be needed are vertical and horizontal extensions. It has to be a total number. The size of the input array will be multiplied by the number. For example, the image 256x256 pixels with extensions equal to 3 will be 768x768 pixels as an output. The function needs also the window size. In one iteration, the function will take into consideration only pixels which are inside the window. The loop will stop only when the whole image will be interpolated. To see the results the *plotting* parameter has to be a boolean *True* value.

The below picture shows the block diagram of the algorithm (Fig. 6.50) with comments added.

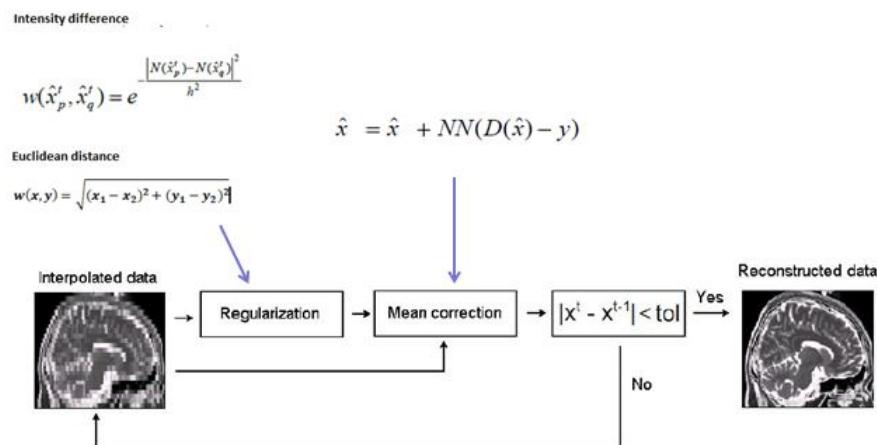


Figure 6.50. Algorithm block diagram with comments

The following steps of the implementation will be discussed:

1. Initial interpolation

To form the input image into the output with chosen size the initial interpolation is needed. As showed in [14] the spline interpolation is used. In short, spline interpolation is an interpolation where interpolant is a special kind of piecewise polynomial called spline. It is recommended, because of the fact that the method makes small error even with low degree polynomials for the spline. Taking into consideration the edges the extrapolation was applied. To be more specific, the last row n is equal to the previous one $n-1$. The same with the first row, the first column and the last column of the input. To show the results, the extensions are set as 2. The input data is 256x256 pixels. The undermentioned picture (Fig. 6.51) shows the result of the initial interpolation. The scale is included so it is easy to see that the size is twice as big. Now, the zeros (Fig. 5.8) have nonzero values, so the next steps can be made.

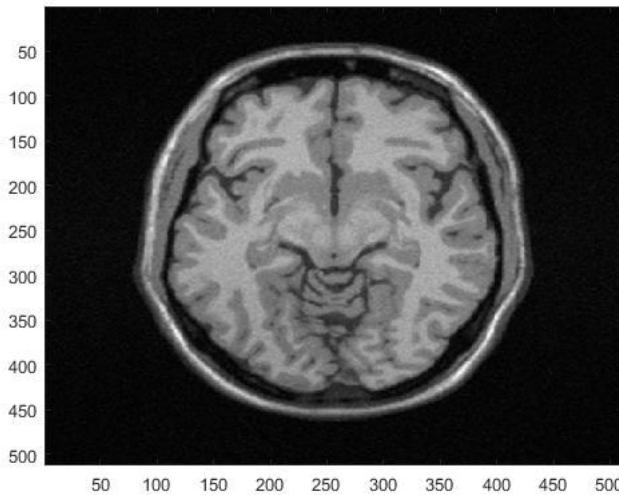


Figure 6.51. The image after initial interpolation 512x512 pixels

2. Regularization

The main functionality is the regularization step. The function makes the square window which contains some pixels. In one iteration of a loop, only the pixels which are inside the window are considered. Next, the window moves and takes another pixels. It is repeated till the end of the image. The main goal is to determine of the weight which will be used at the end.

The first weight is calculated as the difference between the pixels intensities.

$$w_1(x, y) = e^{\frac{|N(x) - N(y)|^2}{h^2}}, \text{ where}$$

$N(x), N(y)$ are window and image intensity values,

h is a level, which is equal to the half of the standard deviation of the input image.

The next part of the whole weight is an Euclidean distance weight. It is simply calculated as the Euclidean distance between every pixel in the window and every pixel in the image. It is based on the below equation.

$$w_2(x, y) = \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{h^2}, \text{ where}$$

h is a level, which is equal to the half of the standard deviation of the input image.

The total weight is calculated as $w_{total} = w_1 * w_2$. The output image is determined as the result of the sum of the multiplication of the weight and the window, divided by the sum of the weight.

3. Mean correction

The mean correction step is a place in the function where the correctness of the equation $X = X + NN * (D(X) - y)$. It was abovementioned, that the downsampled HR has to be equal to the input data.

4. **Tolerance checking** At the beginning it was established that only the 2D images will be taken into consideration and the upsampling function will be applied only once, so the step is excluded. There were experiments to upsample the data several times, but the result were not satisfied.

6.11. Module 11. Brain 3D

The implementation of 11th module included visualization of cortex structure in three-dimensional model and visualization of elected cross-section of model.

As input parameter, module get ouput of 9th module. This is the mask of segmentated data, which includes cortex structure as value 3. The data about cortex structure is seperated from 3D array. 11 th module is displayed in new window of application, so implementation includes not only functionality requirements but also designing of graphical user interface.

To prepare design of user interface used Qt Designer program.

Module is selecting by user in main window of application. If input data is correct, new window is opened and reconstrucion of three-dimensional model is initialized automatically.

First input data – 3D array – is converted to vtkFloatArray, which makes vtkImageData object. Then basing on vtkImageData there is used vtkMarcingCubes class, which makes reconstruction. As a thresholds there was used value equal to 1, because of fact that all values not equal to zero are included to cortex structure.

After reconstrucion model is displayed in BRAIN 3D window. To enable displaying VTK objects in Qt Application there is set frame in which is inserted the object of VTK library by using QVTKRenderWindowInteractor. It is dedicated vtkWidget to display VTK in QT ibrary.

To visualization data there are used following classes:

- vtkRenderer – which enables rendering process: transforming geometry, light and camera view into an image
- vtkMapper – which maps data to graphics primitives
- vtkActor – adjusting data to 3D scenery
- vtkInteractorStyleTrackballCamera – setting possibility of interaction with model

BRAIN 3D window enables three options:

- preview 3D model
- clip model
- clip model and show plane.

Automatically after initialization there is loaded firts mode: preview 3D model. Mode: clip model, enables to setting intersection plane and cut model in place of it. Intersection plane is vtkImagePlaneWidget object. It allows to

interactive set the plane by computer mouse. Interaction of plane is activated whenever user press „clip model” button. When interaction event is detected the model is automatically clip by vtkClipPolyData class.

Mode: clip model and show plane call the same function as previous mode, but with input parameter planemode - True (default – False). It expands functionality of displaying the cross-section corresponding to the elected plane. It improves readability of intersection plane. It is based on the same objects of VTK library, but using additional methods of it.

User has possibility fluently switch modeLs by pressing appropriate buttons. Application has also help window, with short user guide.

6.12. Module 12. Oblique imaging

The implementation of 12th module is divided in 2 parts: one part is core functions for data manipulation and the other one is visualisation. Visualisation part is also responsible for creating a new window with sliders and boxes with which it is possible to change both rotation and translation of the grid .

As module input, module gets structural data as 3D array ($x, y, slicenumber$). Based on this input a grid is created. For user convenience this grid is 1.2x bigger than maximum size of oblique image that can be obtained from the data. Both a shape of the data and this grid are visualized.

After visualisation it is possible to change rotation and translation of grid. Every change of rotation or translation automatically updates visualisation.

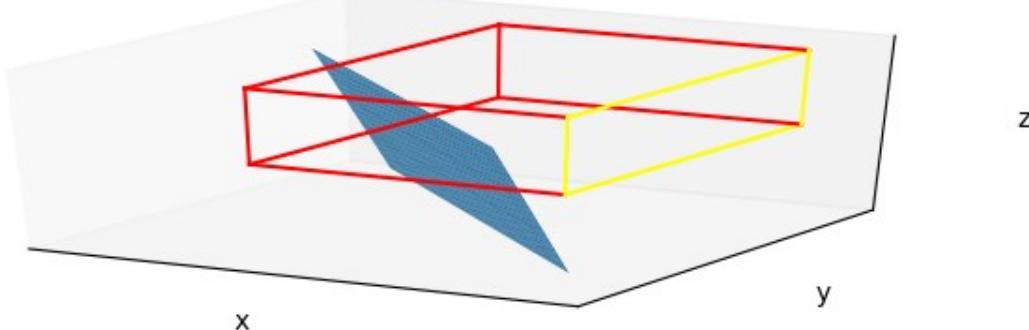


Figure 6.52. Visualization of data shape and grid

After clicking button 'Get oblique image' function that interpolates points needed for oblique image executes returning oblique image.

Interpolation algorithm looks as follows:

- check if point for x,y,z from grid exists, if it exists copy values to output array
- if not, get 8 points by rounding x,y,z grid values up and down. By that coordinates for every possible point from surroundings are gotten. Create variable for storing number of points in surroundings.
- sum these points values
- compute distance from interpolated point to each of those

- if distance is longer than established (1) subtract value(s) from sum
- divide sum of values by number of points in surroundings.

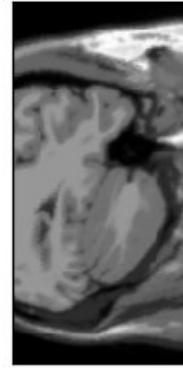


Figure 6.53. Oblique image created based on grid shown on previous figure

The only library used in core part is numpy. To visualise the data matplotlib is used and PyQt is responsible for creating window.

7. Tests

7.1. Module 1. MRI reconstruction

The code reliability was checked by running following unit tests:

- **test_none_input_data** - if the module receives empty data field, the function should rise an error,
- **test_none_sensitivity_maps** - this test examines scenario when input data structure lacks sensitivity maps profiles, which cannot be estimated with this code implementation - in this case, the function should rise an error,
- **test_invalid_input_subsampling_factor** - as information about subsampling factor is needed to properly perform reconstruction, this unit test checks what happens if this field is empty in the structure; as this information can be obtained from input data dimensions (first two dimensions) the module tackles the problem or throws an error if calculated subsampling rate is different than value 2 or 4,
- **test_invalid_input_num_coils** - as information about number of coils is needed to properly perform reconstruction, this unit test checks what happens if this field is empty in the structure; as this information can be obtained from input data dimensions (last dimension) the module handles the problem.

The results of above mentioned tests performed for provided datasets were as it was expected.

7.2. Module 2. Intensity inhomogeneity correction

7.3. Module 3. Non-stationary noise estimation

In order to test effectiveness of implemented algorithm the results of it were compared to results of algorithm prepared to perform calculations contained in [17]. The algorithm for the article was implemented in Matlab.

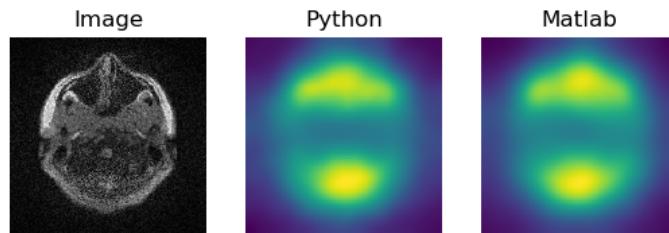


Figure 7.1. Image (left), noise map form Python algorithm(middle), noise map form Matlab algorithm(right).

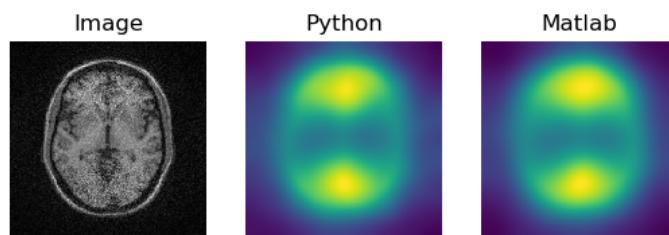


Figure 7.2. Image (left), noise map form Python algorithm(middle), noise map form Matlab algorithm(right).

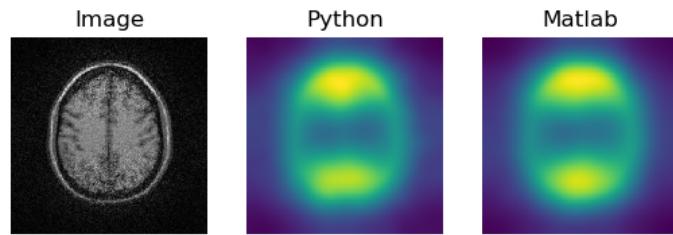


Figure 7.3. Image (left), noise map form Python algorithm(middle), noise map form Matlab algorithm(right).

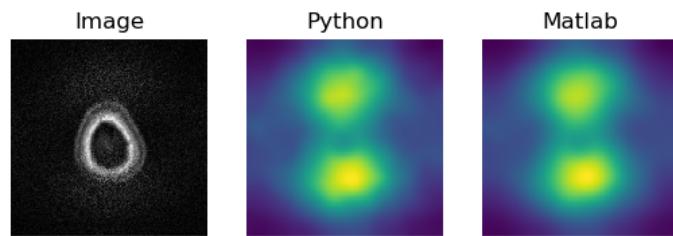


Figure 7.4. Image (left), noise map form Python algorithm(middle), noise map form Matlab algorithm(right).

Results from Python algorithm are not perfect equivalent of the results from Matlab algorithm however they are very similar and seem to be suitable for further processing of the image.

7.4. Module 4. Non-stationary noise filtering 1

The unit tests was implemented to check how the programme behaves in different conditions. The few tests

- **Test_empty_input_data** - test checks how the code behave in case receiving empty dataset,
- **Test_incorrect_input_data** - test checks how the code behave, when incorrect data is passed to the function,
- **Test_size_input_data** - test checks if the size of data is changed on the output of code,
- **Test_none_noise_map** - if code received empty data of noise maps, the function should rise an error,
- **Test_LMMSE_function_changes** - test checks if the input data was changed in the LMMSE-filter function.

The results of tests were as it was expected and code behaves in correct way.

7.5. Module 5. Non-stationary noise filtering 2

Checking the code reliability was done by designing and implementing unit tests with special library suited to handle them. That way of testing allows to reuse once written tests in situations where the code being tested has changed.

To cover that basics, 5 different test were implemented to check how code behaves on extreme conditions. Tests included:

- **test_empty_input_data** - this test checks whether the function will rise an error or it would work, when an object with empty data field is passed as an argument,
- **test_invalid_input_data** - this test examines scenario when data of invalid size is passed to the function,
- **test_size_check** - due to the fact that inside function's code size of an image is temporarily changed aim of this test is to check if size of the function's output is equal to size of the input,
- **test_value_change** - as it was decided in class architecture, filtering algorithm overwrites the data in the class field, instead of working on separate field, this test checks if the output has actually different values than input,
- **test_map_absence** - one of two things needed to run the algorithm (except scan data itself) are noise maps, in the scope of this test lies checking what happens if object with no maps is passed to the body of the function.

As during tests, data in the object is overwritten special method *setUp* to 'refresh' object to its original state was added. Results of all test were positive and the code behaves as it was expected.

7.6. Module 6. Diffusion tensor imaging

In order to verify whether any changes to the module source code result in module no longer working properly, a series of unit tests have been designed and implemented. This automates the procedure of testing module functionality after every change. This addition resulted not only in validating functionality, but also allowed to detect unexpected behaviour and edge cases, which lead to changes in source code.

7.6.1. High-level tests

Methods of the `Module06Tests` class are designed to verify high-level functionality of DTI module. Specifically, it allows to check whether the resulting biomarkers are calculated as expected. All of the following methods are tested for each solver-fix method combination (that is: WLS-ABS, WLS-CHOLESKY, NLS-ABS, NLS-CHOLESKY).

Interface Tests

This group of methods test module behaviour when receiving incorrect data from GUI:

- `test_invalid_input_data` - checks whether module raises error when input data structure strutural data field is empty
- `test_invalid_solver_type` - checks whether module raises error when input solver is neither 'WLS' nor 'NLS'
- `test_invalid_fix_method` - checks whether module raises error when input fix method is neither 'ABS' nor 'CHOLESKY'

Data Object Tests

This group of methods test whether input data instance is modified during module execution:

- `test_object_modified_in_place` - tests whether module output is a view of input data memory instead of a new object (it is in order to be less memory-intensive)
- `test_input_biomarker_field_changed` - verifies if input object biomarkers field is modified during module execution
- `test_input_other_fields_not_changed` - compliments the aforementioned methods by checking whether all fields but biomarkers where changed during module execution or not

Output Tests

This group of methods check the properties of resulting biomarkers field:

- `test_biomarker_fields_check` - verifies whether all expected biomarkers (MD, RA, FA, VR, FA_rgb) have been calculated during module execution and written to input data class instance
- `test_biomarkers_for_each_slice` - tests whether biomarkers dictionary is calculated for each slice present in input data and returned as Python list of dictionaries
- `test_biomarker_shape` - validates the shape of biomarker images (whether XY dimensions are the same as input data as well as whether they are gray or in color)

7.6.2. Low-level tests

Methods of the `DTISolverTests` class are designed to verify low-level functionality of DTI computation by testing the behaviour of class constructor of `DTISolver` and verifying methods called during pipeline module execution abstracted from the end user.

Constructor Tests

This group verifies class constructor for valid and invalid input information:

- `test_solver_object_created` - verifies the output of class constructor for valid input data
- `test_input_invalid_bvalue` - checks whether an error is raised when constructor is presented with invalid (empty) `b_value`
- `test_input_invalid_gradients` - checks whether an error is raised when constructor is presented with invalid (empty) gradient matrix
- `test_input_invalid_data` - checks whether an error is raised when constructor is presented with invalid (empty) data array
- `test_input_negaive_data` - checks whether an error is raised when constructor is presented with invalid (negative values) data array

Skull Stripping Mask Tests

This group of methods verifies solver class object behaviour when presented with invalid skull stripping masks. Since this input information is optional, instead of raising an error in response to invalid mask, a default mask (all pixels in image are valid for computation) is used during execution:

- `test_input_skull_mask_empty` - verifies that the default mask is used if input mask is empty
- `test_input_invalid_mask` - verifies that the default mask is used if input mask is not binary

Eigenvalue Computation Tests

This group of methods the sign of eigenvalues with and without 'ABS' or 'CHOLESKY' fix. For proper biomarker computation it is important to have non-negative diffusion tensor eigenvalues.

- `test_eigenvalues_abs_fix` - compares eigenvalue computation with 'ABS' fix method (expected output: non-negative eigenvalues when using the fix method, no guarantee otherwise)
- `test_eigenvalues_cholesky_fix` - compares eigenvalue computation with 'CHOLESKY' fix method (expected output: non-negative eigenvalues when using the fix method, no guarantee otherwise)

Biomarker Computation Tests

This last group contains only one test method, designed to check module behaviour when computing biomarkers:

- `test_negative_eigenvalue_biomarker_warning` - checks whether a warning is issued during biomarker computation if input eigenvalues are negative (no error is raised)

7.7. Module 8. Skull stripping

```

class Module08Tests(unittest.TestCase):

    def setUp(self):
        dataset_name = DATASETS[0]
        self.dwi = smns.load_object(file_path=DATASETS_ROOT + dataset_name)

    def test_strel(self):
        ss = SkullStripping(None)
        se = ss.strel('disk', 1)
        disk = numpy.array([[1., 0., 1.], [0., 0., 0.], [1., 0., 1.]])
        self.assertEqual(type(se), type(disk))
        numpy.testing.assert_array_equal(se, disk)

    def test_radius_counting(self):
        ss = SkullStripping(None)
        se = ss.strel('disk', 6)
        r, st = ss.radius_counting(se)
        r_check = (6 * 2 + 1) / 2
        st_check = 1.0
        self.assertEqual(r, r_check)
        self.assertEqual(st, st_check)

    def test_binarization(self):
        ss = SkullStripping(None)
        image = numpy.array([[0.6, 0.7, 0.9], [0.2, 0.4, 0.3], [0.1, 0, 0.8]])
        threshold = 0.5
        bw_check = numpy.array([[1, 1, 1], [0, 0, 0], [0, 0, 1]])
        bw = ss.binarization(image, threshold)
        numpy.testing.assert_array_equal(bw, bw_check)

    def test_main8_incorrect_input(self):
        mri_input = None
        result = main8(mri_input)
        self.assertEqual(result, "Unexpected data format in module number 8!")

    @mock.patch("core.inc.module08.SkullStripping.preprocessing")
    def test_run_preprocessing_frist_condition(self, mock_obj):
        mock_obj.return_value = (None, None, None, 10, None)
        ss = SkullStripping(None)
        self.assertEqual(ss.run(), 0)
        mock_obj.return_value = (None, None, None, 100, 2.0)
        self.assertEqual(ss.run(), 0)

    def test_main8_adding_skull_striping_mask(self):
        self.dwi.skull_striping_mask = []
        input_skull_striping_mask = self.dwi.skull_striping_mask
        output_object = module08.main8(self.dwi)
        self.assertNotEqual(output_object.skull_striping_mask, input_skull_striping_mask)

```

```

def test_main08_not_changing_other_inputs(self):
    self.dwi.skull_stripping_mask = []
    inputs = copy.deepcopy(self.dwi.__dict__)
    output_object = module08.main8(self.dwi)
    inputs['skull_stripping_mask'] = output_object.skull_stripping_mask
    self.assertTrue(
        all([
            numpy.array_equal(inputs[key], getattr(output_object, key))
            for key in inputs.keys()
        ])
    )

def test_main08_mask_size(self):
    self.dwi.skull_stripping_mask = numpy.zeros_like(self.dwi.mri_input.diffusion_data[:, :, 0, 0])
    input_skull_stripping_mask = self.dwi.skull_stripping_mask
    output_object = module08.main8(self.dwi)
    numpy.testing.assert_equal(output_object.skull_stripping_mask.shape(),
        input_skull_stripping_mask.shape())

```

Listing 7.1. Implemented tests.

7.8. Module 9. Segmentation

Unity tests are designed to checked if smaller unit of module is running correctly. In this module, the unity tests are using to test each function in module for different conditions. implemented unity test provide information, what happens in module after changing the source code.

functionality of imHist function Unity test is checking functionality for different data type, data with only 0 values, empty dataset. Module returns correctly data only for double data type. Unexpected result should be shown to the user.

functionality of gmm function Unity test is checking functionality for different data type, data with only 0 values, empty dataset and missing arguments. Checking the function behavior for different data sizes is also very important. Valid, because of using matrix operation. Module returns correctly data only for double, nonzero array data, with the same size of expected values vector (mu), variation vector (v), probability vector (p). Unexpected result should be shown to the user.

functionality of imPart function

- Checking value of lastPitch, if the value is greater than pitches or smaller than firstPitch, function returns an error.
- Checking value of firstPitch, if the value is negative, greater than lastPitch or pitches, function returns an error.
- Checking the correct size of image data for the selected part. If the separated part of the data does not have a size equal to [rows x column x difference between the lastPitch and the firstPitch], the function should not return the image and returns an error

functionality of segmentation function

- Loading an image automatically starts the algorithm and calls all functions.

- An unexpected action of any of the functions immediately interrupts the program and the function returns an error.
- Checking the size of initialized parameters vectors, if are not the same, function returns an error.
- Testing exit condition of while loop in function. If the loop operation repeats over 100 times, the function will be immediately interrupted and return an error.
- Checking the correct size of segmentation image mask. It should be the same as the size of the data after calling the function partIm, in other way function returns an error.
- Checking mask contents after segmentation. The mask should contain four data clusters for: background, cerebrospinal fluid, gray matter, white matter, with values, respectively, 1, 2, 3 and 4, which is corresponding to image histogram.

7.9. Module 10. Upsampling

Results

To visualize the upsampling functionality several output with different parameters are showed (Fig. 7.5, 7.6).

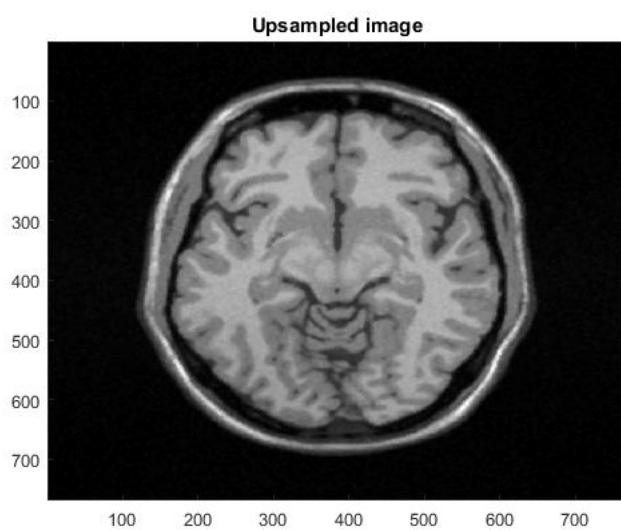


Figure 7.5. The upsampled image with extensions equal to 3 and window equal to 2

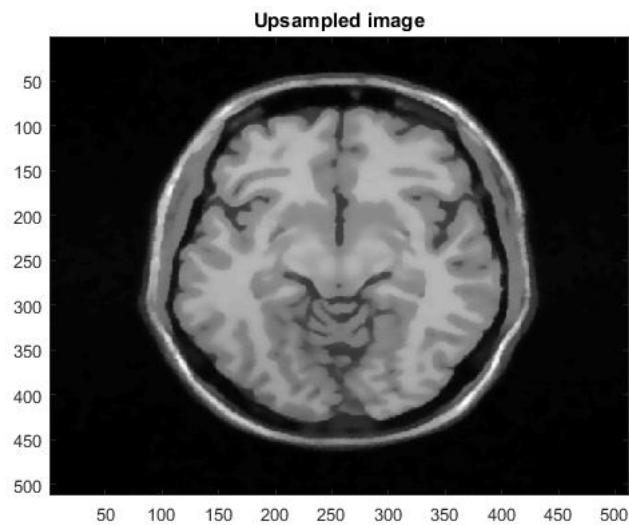


Figure 7.6. The upsampled image with extensions equal to 2 and window equal to 5

To better see the comparison (Fig.7.7, 7.8)

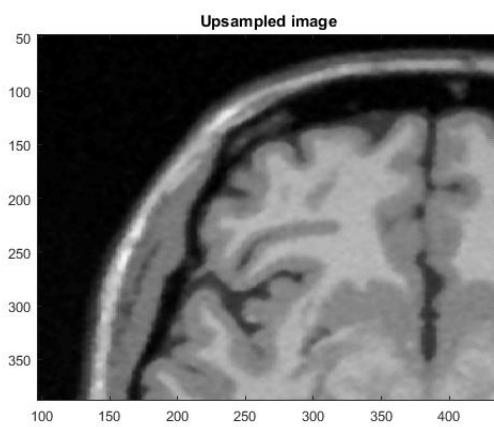


Figure 7.7. The upsampled image with extensions equal to 3 and window equal to 2

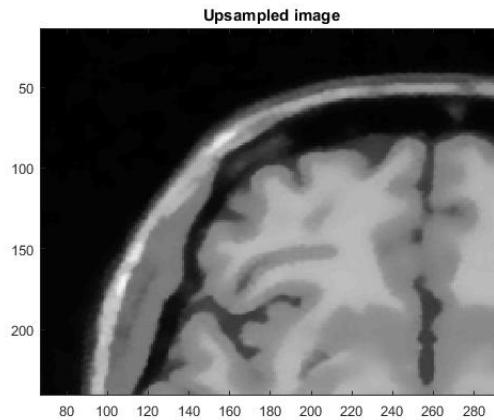


Figure 7.8. The upsampled image with extensions equal to 2 and window equal to 5

Comparison with the other methods

To confirm the better functionality of the proposed method the results will be compared. The other are Linear, Nearest Neighbour, Cubic and Spline interpolation methods (Fig 7.9, 7.10, 7.11, 7.12). To better see the comparison (Fig. 7.13).

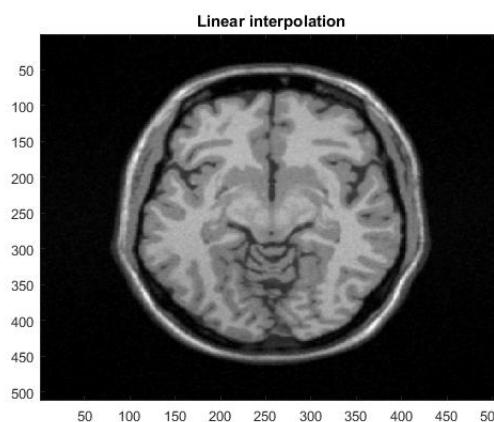


Figure 7.9. Linear interpolation

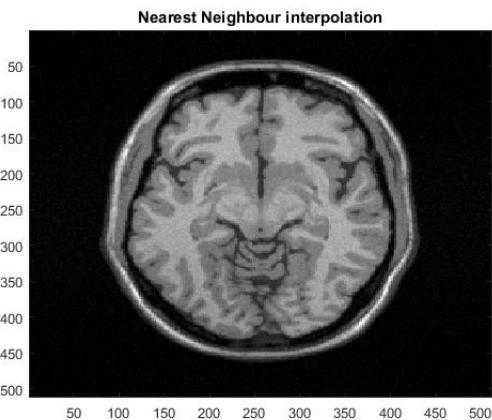


Figure 7.10. Nearest neighbour interpolation

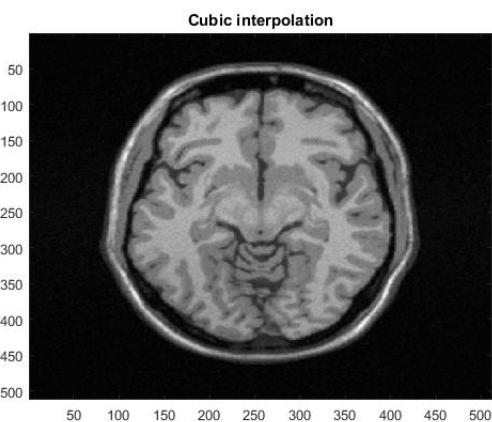


Figure 7.11. Cubic interpolation

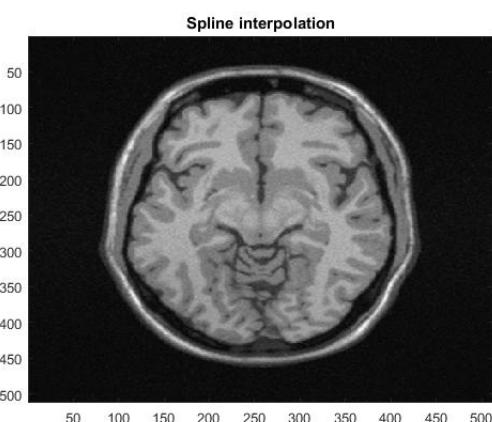


Figure 7.12. Spline interpolation

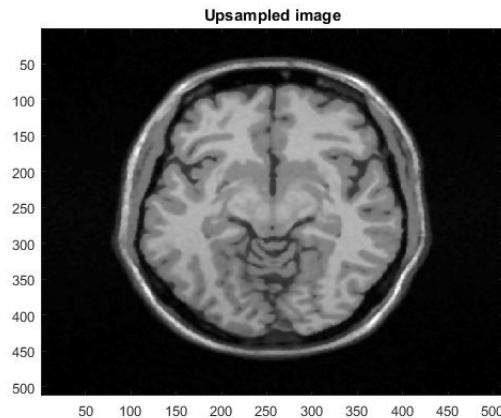


Figure 7.13. Proposed method

Conclusions

In conclusion, implementation of a new interpolation method met with success. Unquestionably, the proposed method gives better results than other popular interpolation methods.

7.10. Module 11. Brain 3D

11th module is interactive module, where subsequents steps are conditioned by user action. This is the reason why, the verification of functionality of module included mainly qualitative tests. Module 11 is displayed in new window. After initialization model of cortex basing on segmentation data is shown. Render window is interactive, user can set camera view by moving computer mouse. It gives possibility to see model from various perspectives (Fig. 7.14 - 7.15).



Figure 7.14. Preview model obtained by marching cubes algoritm.

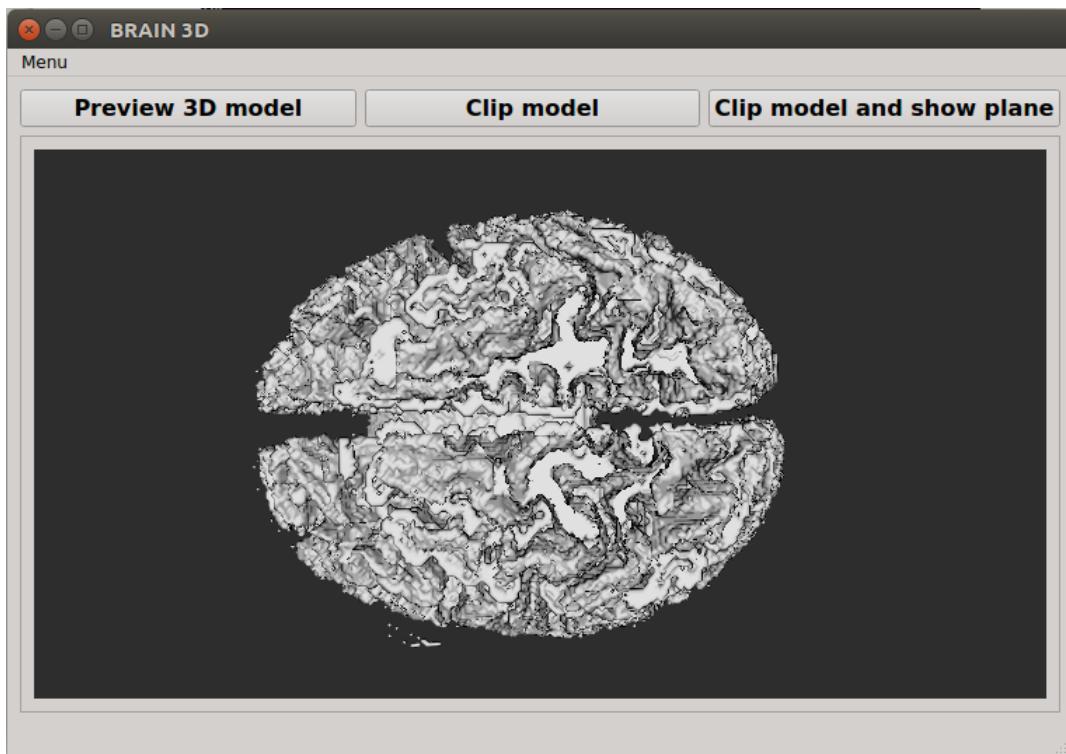


Figure 7.15. Preview model obtained by marching cubes algoritm.

There are three buttons, corresponded to three mode of 11th module. After pressing „Clip model” user can clip model in elected cross-section. To set the intersection plane user has to press middle mouse button. During

mouse movement model is automatically clipped.

To improve readability of intersection plane user can display cross-section, by choosing „Clip model and show plane”. In this mode user can also clip model and select plane in the same way as in previous mode.

Examples of clipping model in elected cross-section is shown on the figures 7.16 - 7.21.

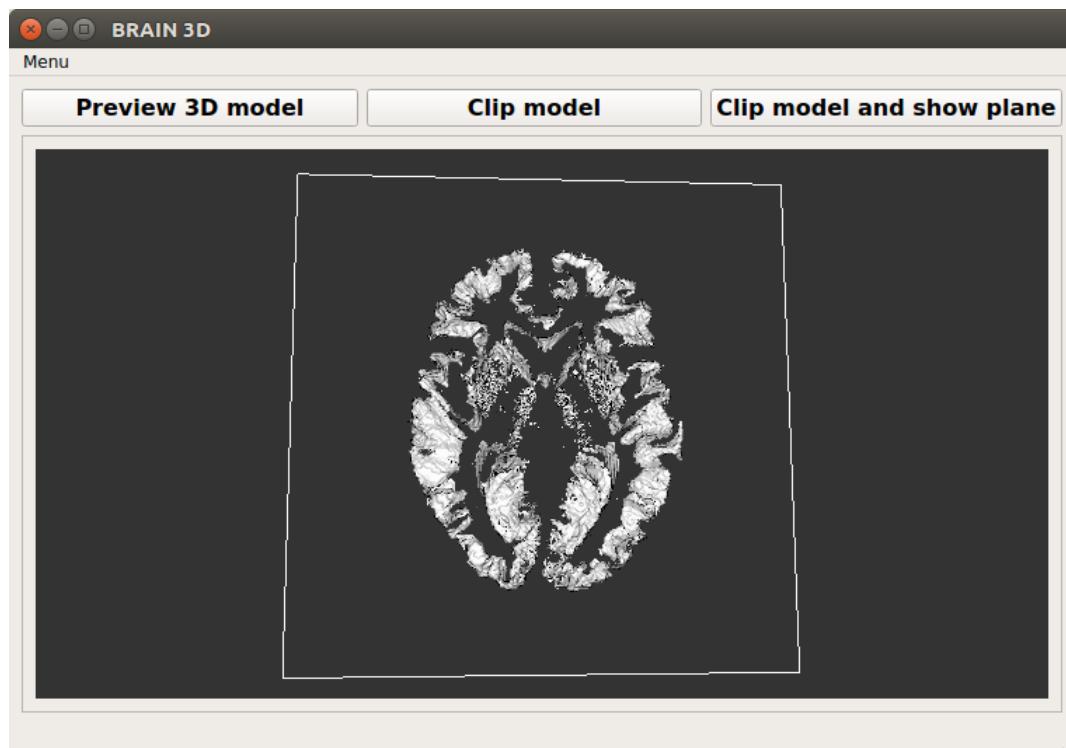


Figure 7.16. Clipped model in transverse plane.

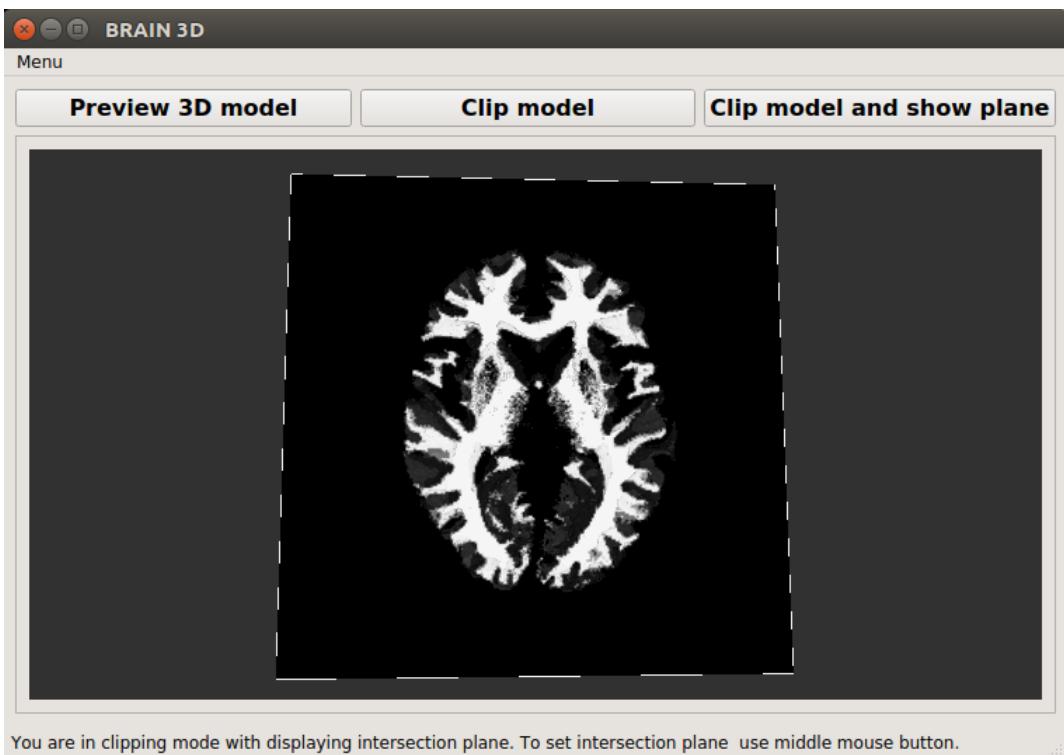


Figure 7.17. Clipped model in the same transverse plane with cross-intersection image.

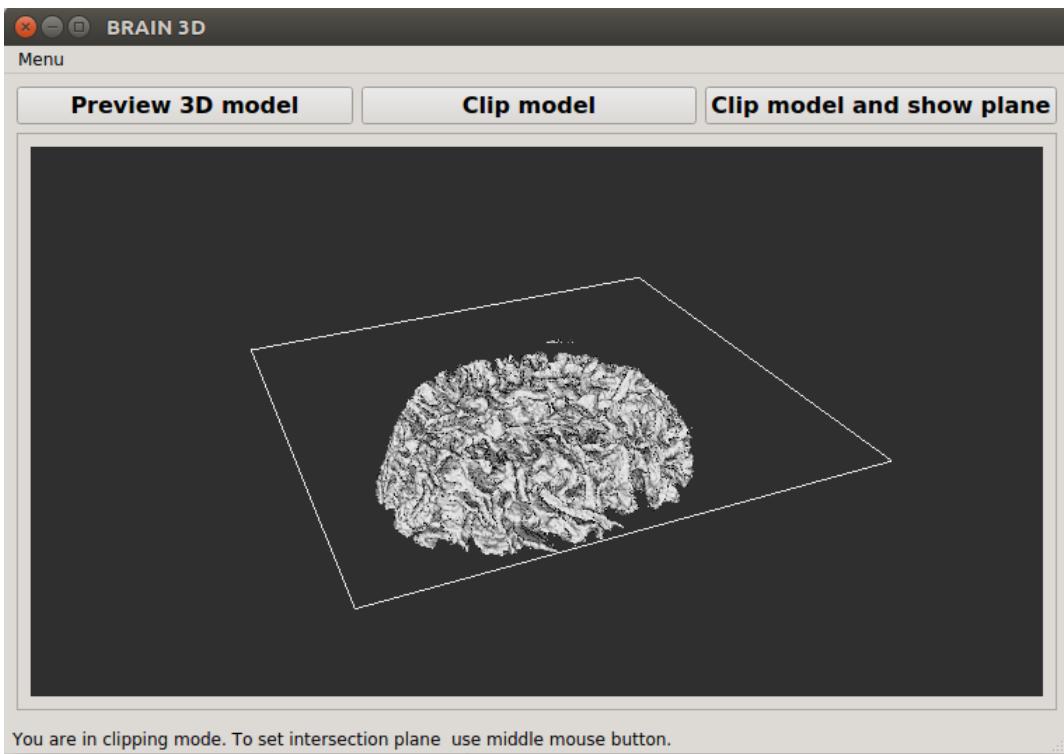


Figure 7.18. Clipped model in transverse plane.

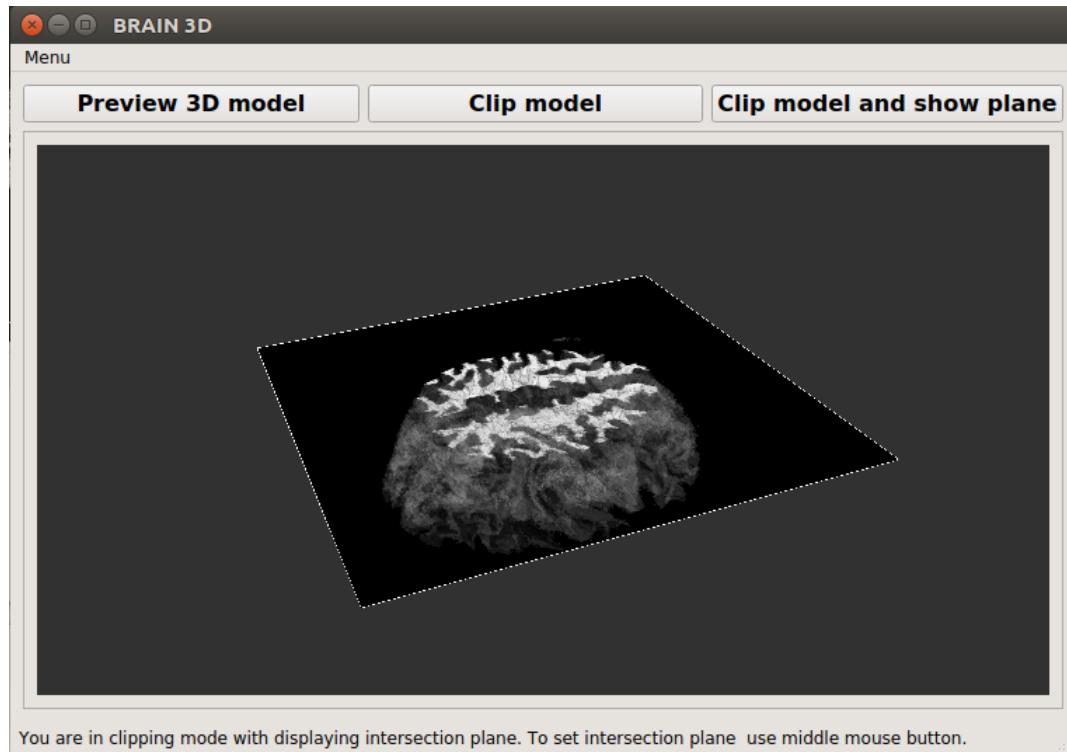


Figure 7.19. Clipped model in the same transverse plane with cross-intersection image.

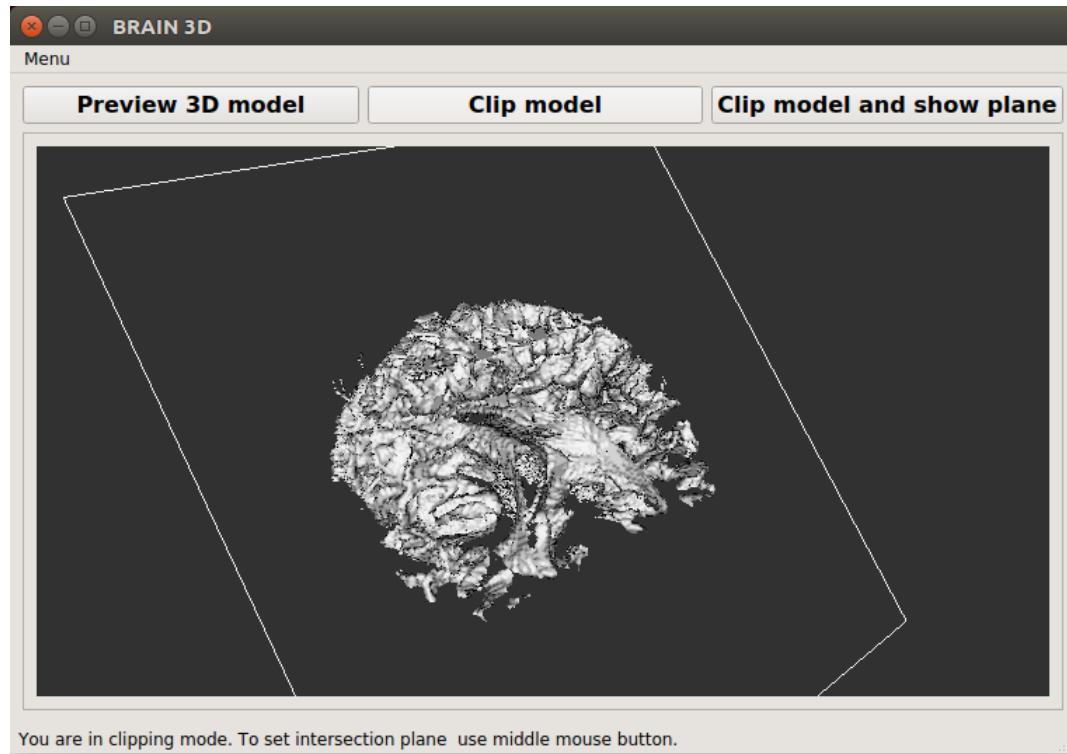


Figure 7.20. Clipped model in elected plane.

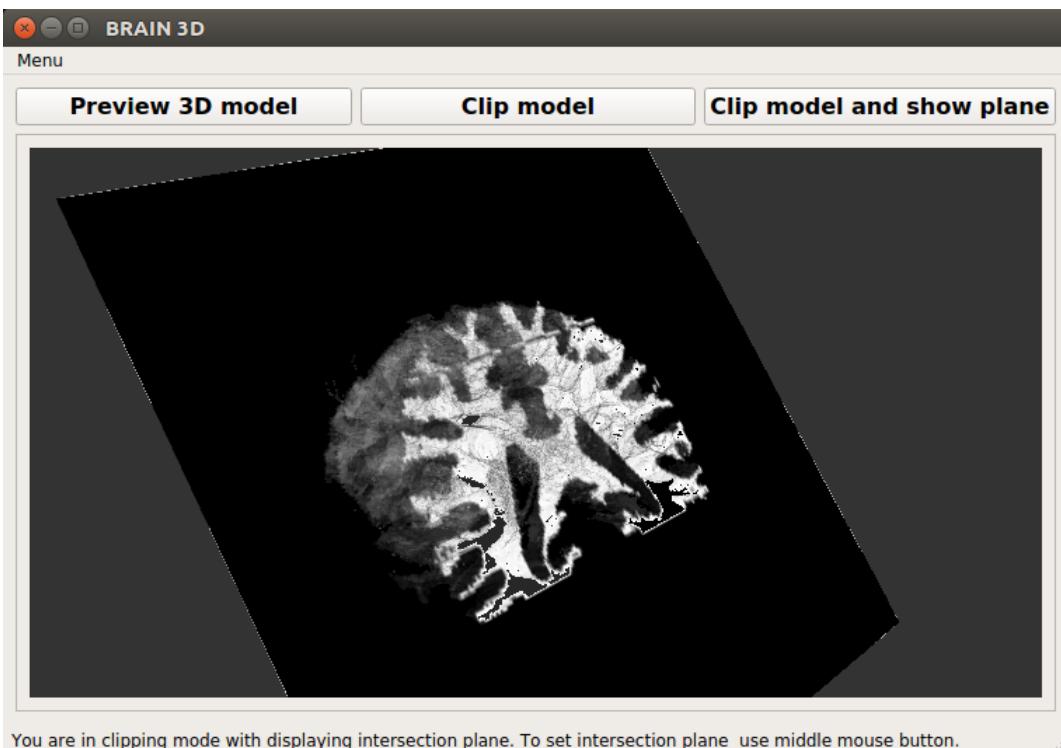


Figure 7.21. Clipped model in the same elected plane with cross-intersection image.

In menu user has option to display help window with short user guide. 7.22

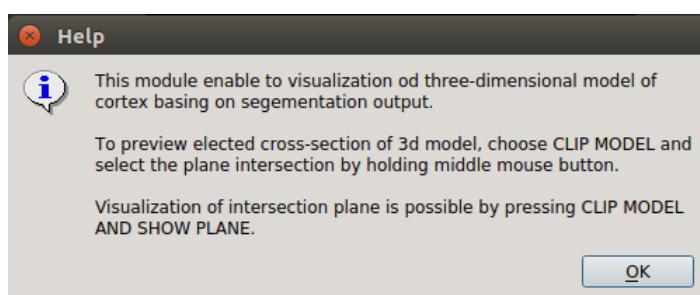


Figure 7.22. Help window with short user guide.

Automatically tests are also implemented. Unit tests cover the verification of input data and output of reconstruction function. There are three test cases:

1. **test invalid input data format** - checking if program raises error when the input data contains empty segmentation parameter of main class.
2. **test invalid input data type** - checking if program raises error when the input data contains array in inappropriate dimension as segmentation parameter of main class.
3. **test generate model** - checking if function returns object of vtkImageData and vtkPolyData type.

```
test_generate_model (__main__.Module11Tests) ... ok
test_invalid_input_data_format (__main__.Module11Tests) ... ok
test_invalid_input_data_type (__main__.Module11Tests) ... ok
-----
Ran 3 tests in 0.023s
OK
```

Figure 7.23. Result of unit tests.

7.11. Module 12. Oblique imaging

After clicking to run module 12 a new window is opened.

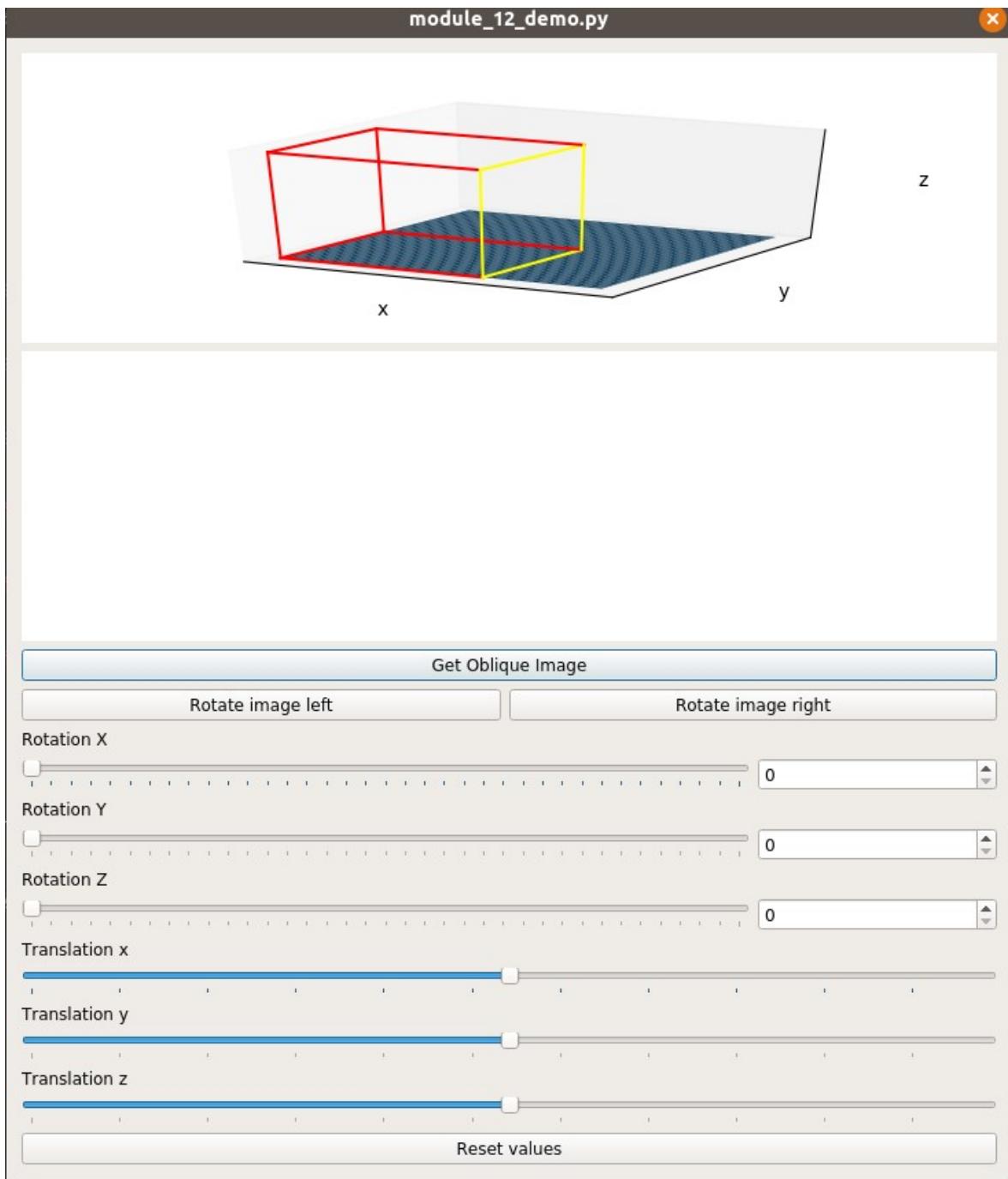


Figure 7.24. Module 12 window

In this window there are 2 plotting space 4 buttons : 'Get Oblique Image', 'Rotate image left', 'Rotate image right' and 'Reset values'. There are also 6 sliders: 'Rotation X', 'Rotation Y', 'Rotation Z', 'Translation X', 'Translation Y', 'Translation Z' and 3 spinboxes responsible for 'Rotation X', 'Rotation Y', and 'Rotation Z'.

As can be seen on previous figure only grid and shape of the data in form of red and yellow lines (yellow lines mean the front of the data) is displayed and not the actual data itself. It is because displaying the data in matplotlib and updating the figure that displays it with the grid every time someone uses a slider or write a different value in a box might result in a really slow performance.

The view of this plot is default from the start of the module and can be changed manually at any moment. In order to change it user needs to press left button on the figure and slide in any direction.

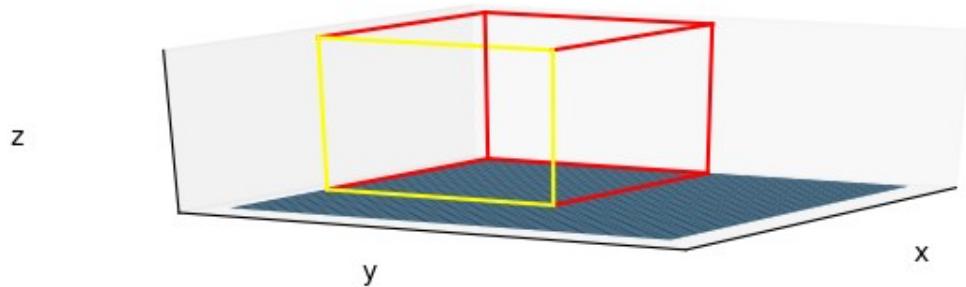


Figure 7.25. Manually changed view

As previously mentioned at the start of the module the grid is created based on two vectors $\vec{v_2} = [0, 1, 0]$ and $\vec{v_3} = [0, 0, 1]$. It can be changed at any moment of this module running.

Grid rotation relative to the X, Y or Z axis can be performed by changing slider value or changing spinbox value, that is next to the corresponding slider. Changing slider value changes spinbox value and vice versa.

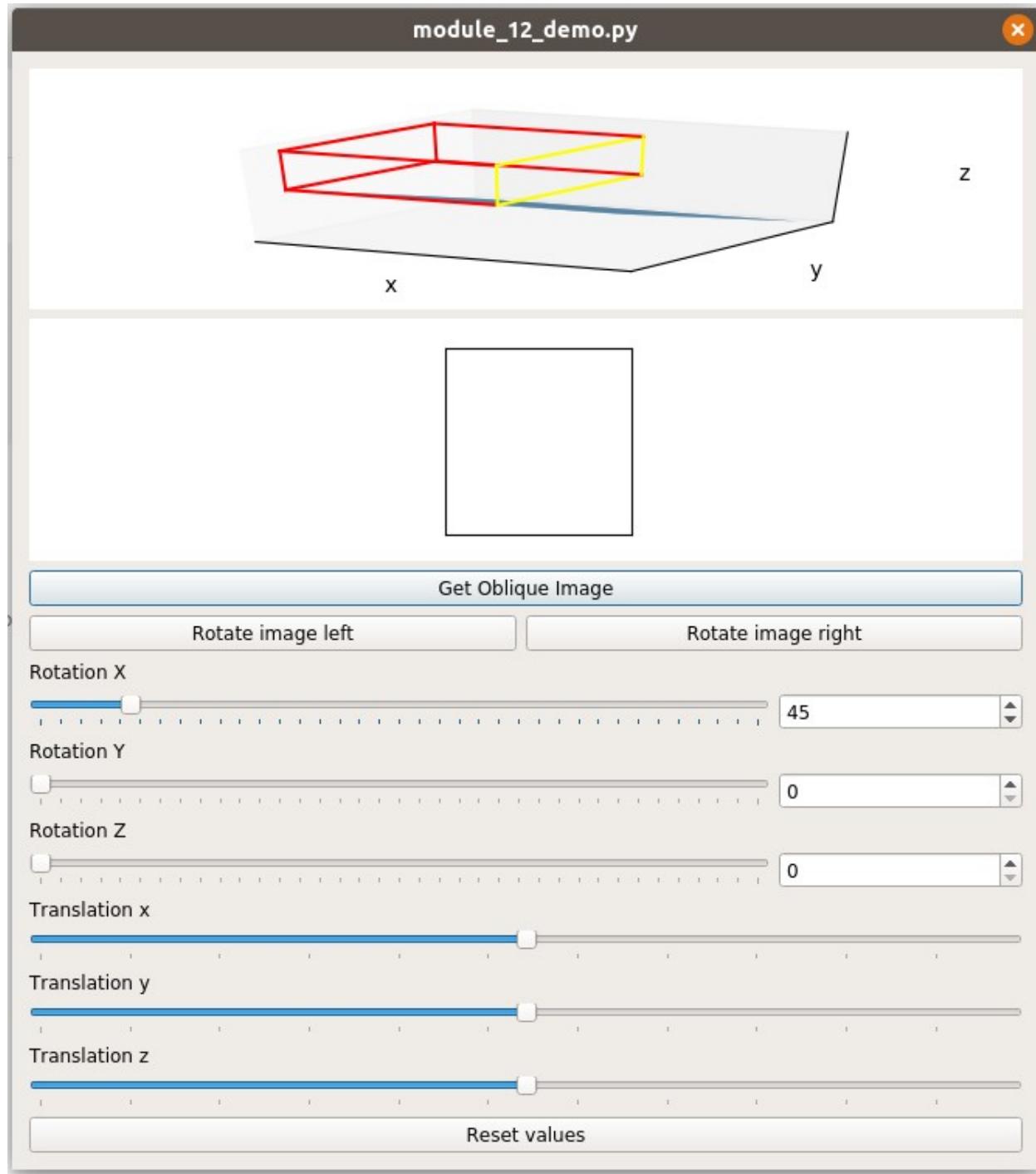


Figure 7.26. Grid rotated 45 degrees relative to X axis

As can be seen on figure above after changing value of 'Rotation X' value in spinbox is the same as the one of the slider. Since the grid after only this rotation is under the slices and has 0 common points getting oblique image is impossible right now.

It is essential to translate the grid relative to Z axis. This and any other translation can be performed only by changing slider value. There are no spinboxes for that operation, because contrary to rotation, translation by any number of units doesn't mean anything if the scale is not seen

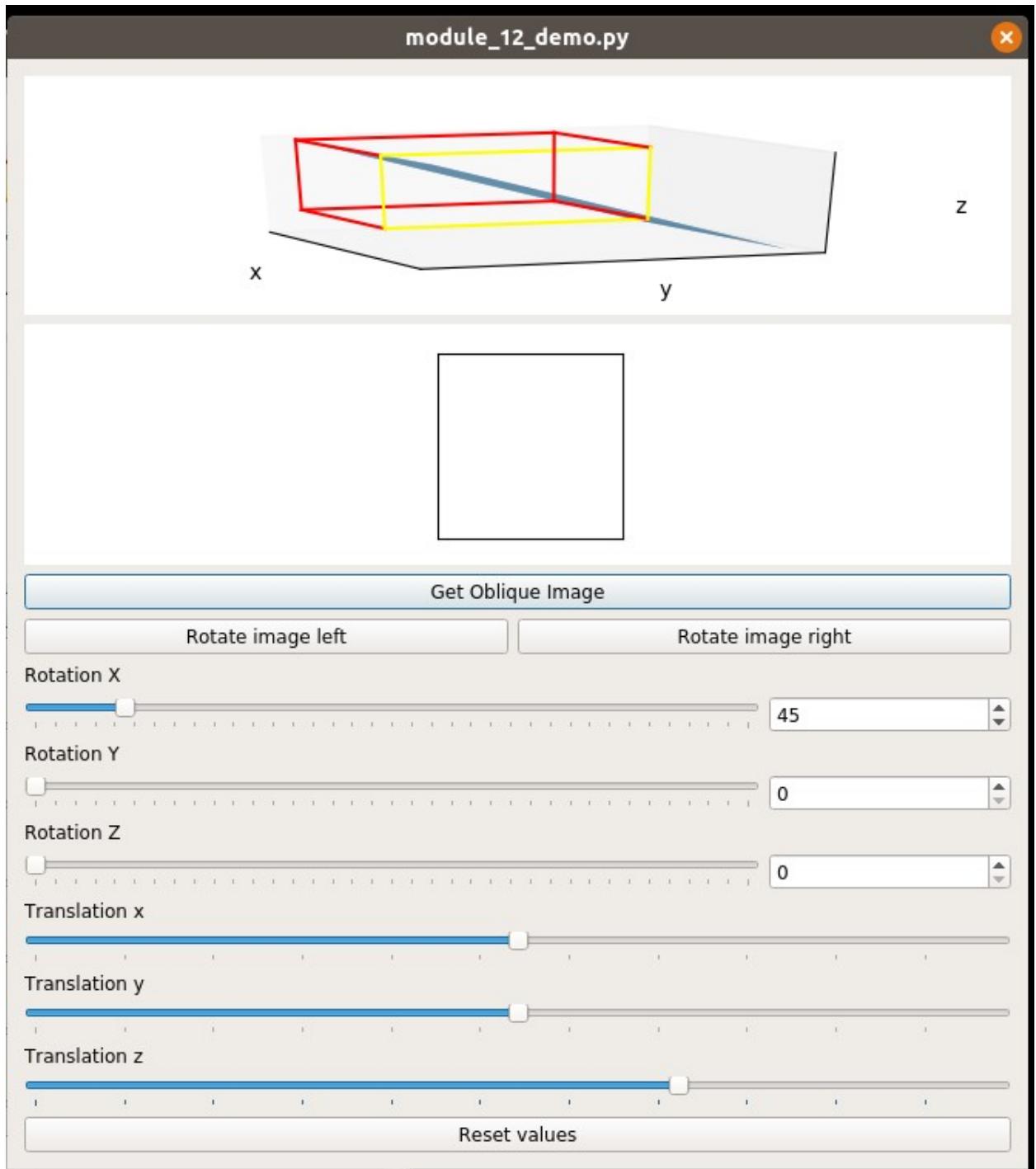


Figure 7.27. Grid rotated 45 degrees relative to X axis and translated relative to Z axis

After changing the translation the grid is placed so that getting oblique image is possible. After clicking button 'Get Oblique Image' following image appears.

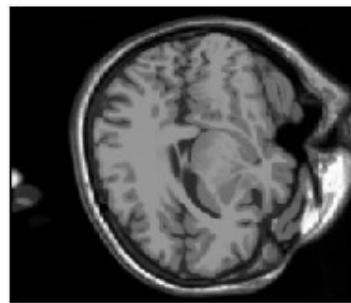


Figure 7.28. Oblique Image obtained with parameters from previous figures

For convenience of the user it is possible to rotate this image right or left by clicking 'Rotate image right' and 'Rotate image left' respectively.



Figure 7.29. Oblique Image rotated right



Figure 7.30. Oblique Image rotated left

Of course it is possible to rotate image infinitely number of times and every 4 times the image will be in starting orientation.

Clicking button 'Reset values' resets values of all sliders and spinboxes, resets upper plot to default but doesn't clear oblique image.

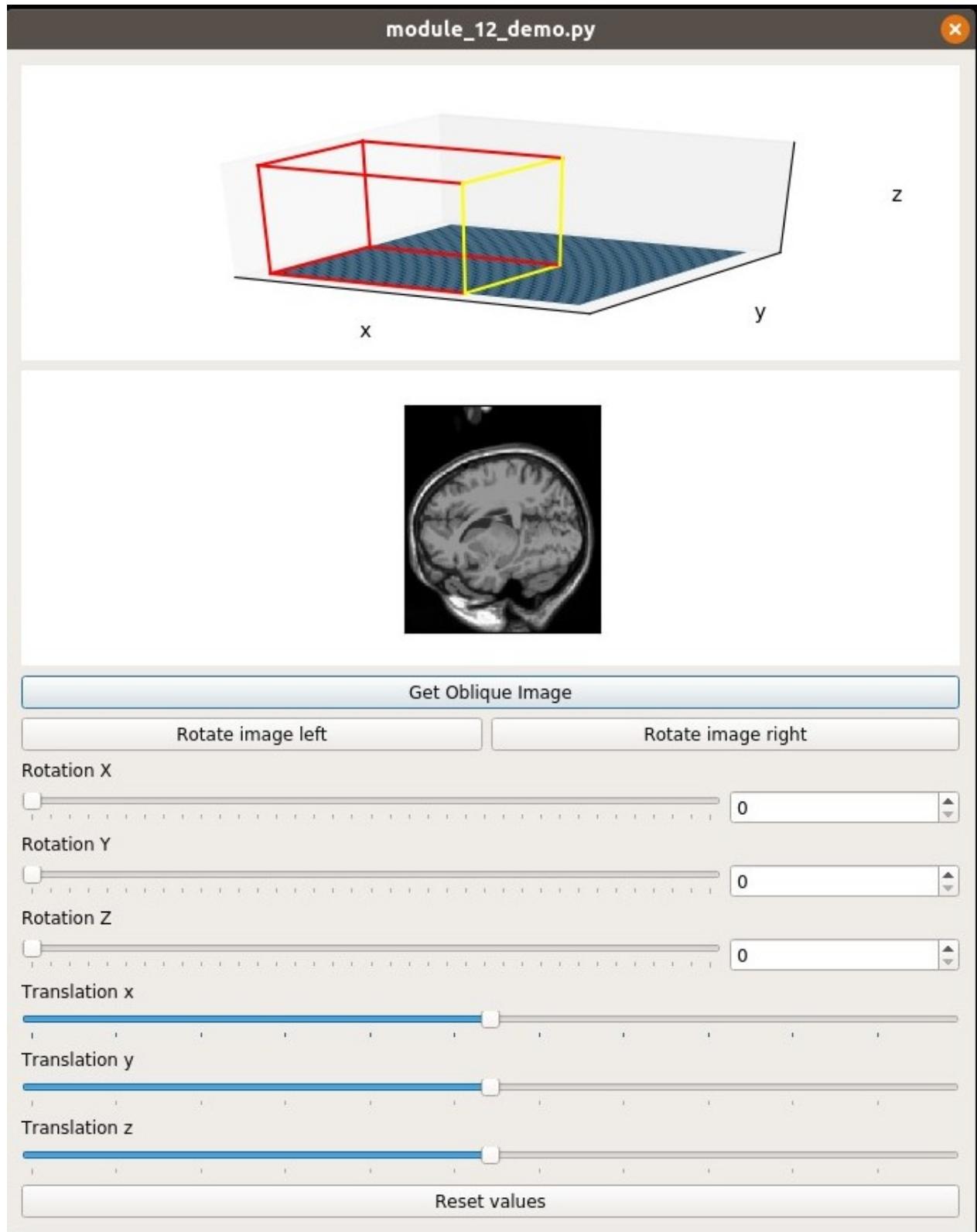


Figure 7.31. Window after clicking 'Reset values' button

Other results

It is necessary to check if the module works in expected way, therefore plots and images for 3 settings are presented:

- Default settings

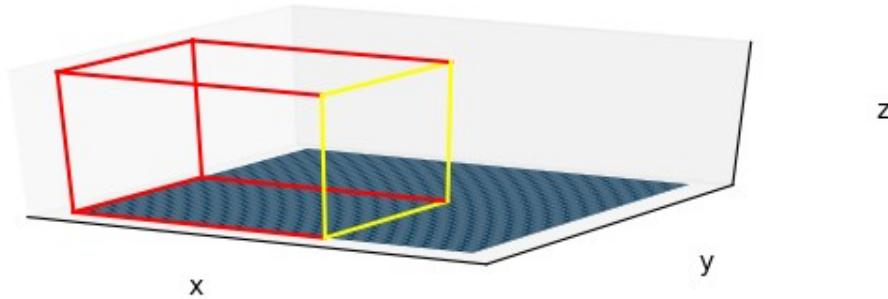


Figure 7.32. Plot with default settings

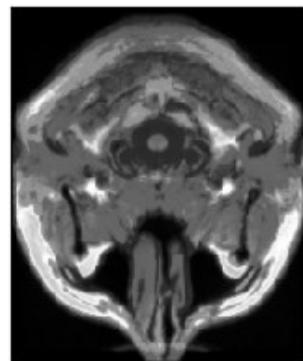


Figure 7.33. Image from default settings

- 290 degrees rotation relative to X axis, 30 degrees rotation relative to Y axis, translated manually

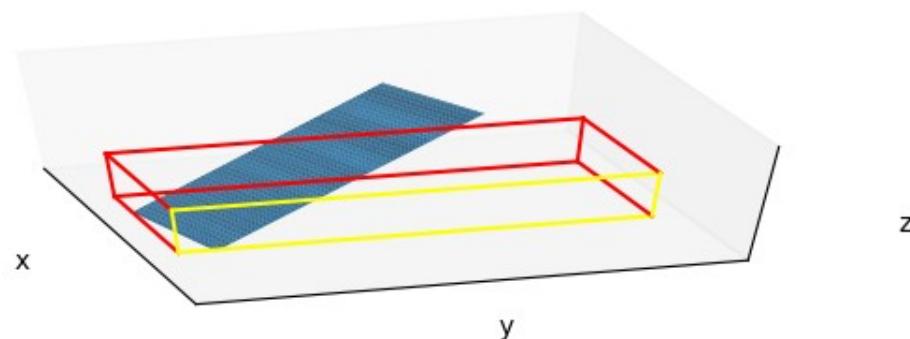


Figure 7.34. Plot with 290 degrees X, 30 degrees Y rotation and manual translation

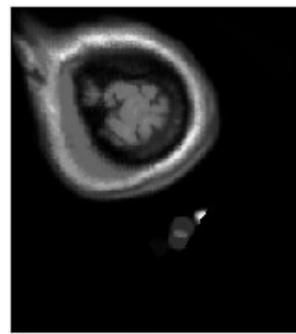


Figure 7.35. Image from 290 degrees X, 30 degrees Y rotation and manual translation

- 290 degrees rotation relative to X axis, 30 degrees rotation relative to Y axis, 45 degrees relative to Z axis, translated accordingly

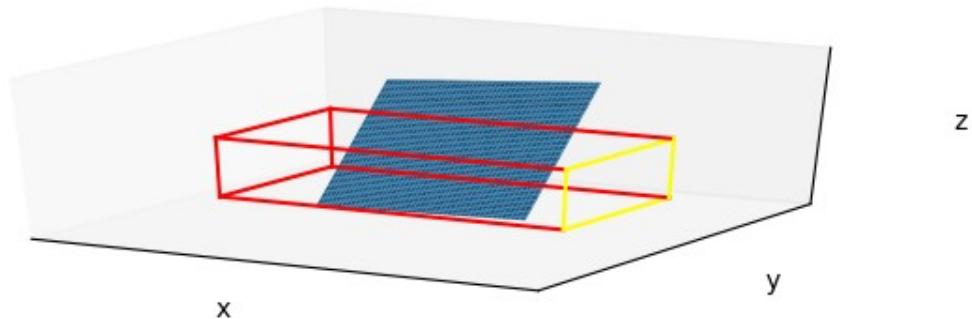


Figure 7.36. Plot with 290 degrees X, 30 degrees Y rotation, 45 degrees Z rotation and manual translation

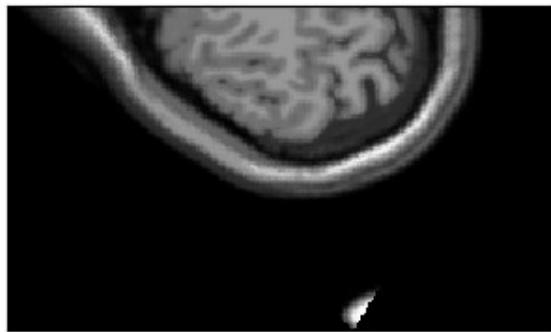


Figure 7.37. Image from 290 degrees X, 30 degrees Y rotation, 45 degrees Z rotation and manual translation

Unfortunately it is impossible to tell if the images are interpolated correctly, because the developer does not have access to other software that performs such action. But knowing what is in places where interpolation occurs

if proper structures are visualized can tell if the module works properly. In order to do that the following grid was set.

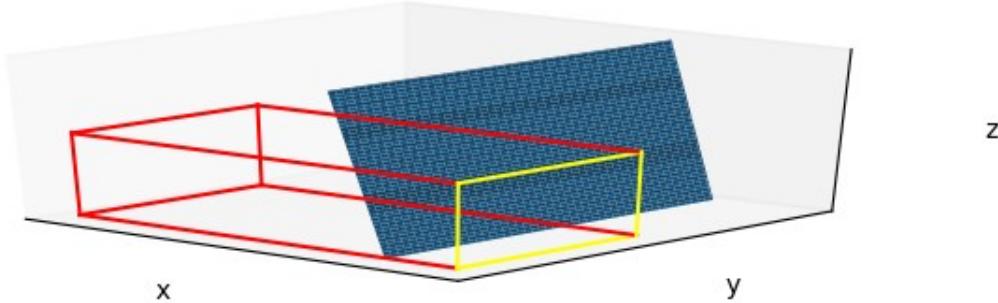


Figure 7.38. Plot with 95 degrees X rotation and proper translation.

Based on this grid an oblique image was created.

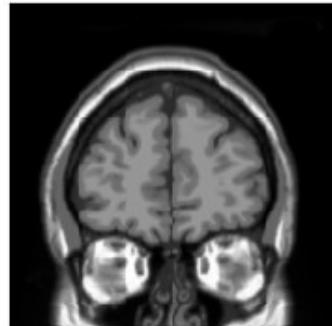


Figure 7.39. Oblique image from previous settings

In the picture we can see image of face, with 95 degrees X rotation at least some points were needed to be interpolated and the image looks as we could've expected. Therefore it is believed the module works just as it should.

7.12. Application

-whole app tests

8. Authors

Authors of this project are students of Biomedical Engineering, AGH UST, Krakow, Poland.

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Maciej Gryczan	GUI engineer
Adrian Stopiak	Vizualization engineer
Malwina Molendowska	1st module developer
Klaudia Gugulska	2nd module developer
Kacper Turek	3rd module developer
Magdalena Rychlik	4th module developer
Alicja Martinek	5th module developer
Mateusz Pabian	6th module developer
Anna Grzywa	8th module developer
Magdalena Kucharska	9th module developer
Eliza Kowalczyk	10th module developer
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