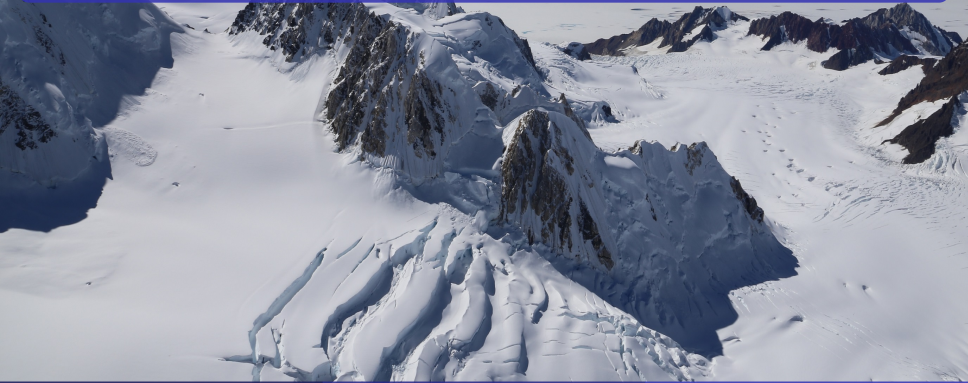
PhD Statistics Seminar - Uni of Leeds

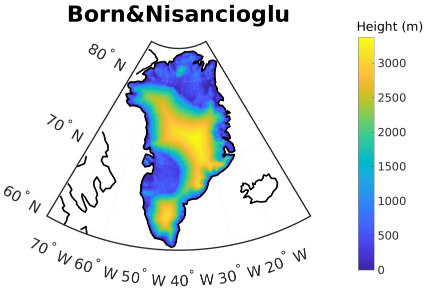
2018/2019

Gaussian Process Emulation to Reconstruct Past Greenland Ice Sheet Morphology



The Problem

Aim : Reconstruct the morphology of Greenland Ice Sheets during the LIG.



Ideal approach : Run complex GCMs (General Circulation Models)

$$\left\{ \begin{array}{c} \text{Ice Sheet} \\ \text{Morphology} \end{array} \right\} \longrightarrow \left\{ \delta^{18}O \text{ values} \right\}$$

and compare predicted outputs to actual $\delta^{18}O$ records.

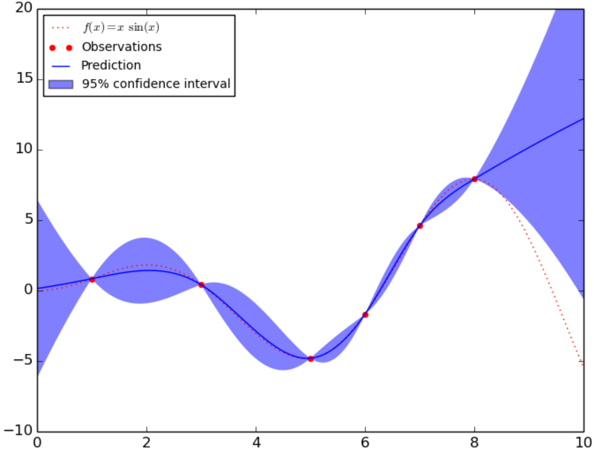
Issue: Model runs are too expensive to perform exhaustive search (around 2 weeks each).

Solution:

- Run simulator on few inputs only
- Accordingly build *cheap* probabilistic model of the input-output relationship: **Emulator**
- Use emulator to match observed data to “best” morphology.

What is an Emulator? Simple example

A Gaussian Process that interpolates the observed outputs (•) of a complex computer model (····).



Our Case

- Inputs: Morphologies (we must paramerise them!)
- Outputs: $\delta^{18}O$ values

To parametrise the input set

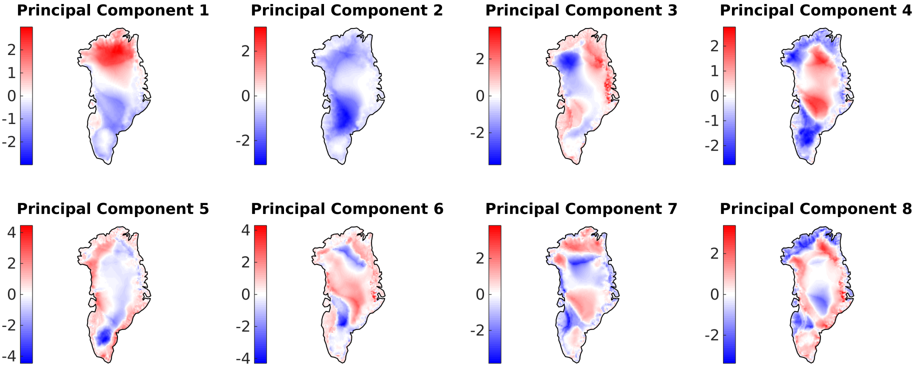
1. Start from $N = 14$ reconstructed morphologies available from previous literature. Each stores height values at p different locations (cells):

$$M_1, \ldots, M_N \in \mathbb{R}^p$$

2. Apply PCA, while accounting for the area associated with the different grid-cells.

3. Obtain

$$P_1, \ldots, P_{N-1} \in \mathbb{R}^p \text{ Principal Components.}$$



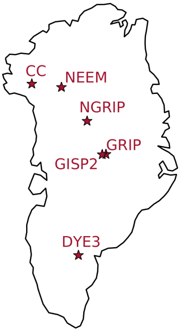
- New morphologies are generated as affine combination of the first 8 PCs, which explain more than 95% of total variance:

$$\left\{\text{Set of Mophologies}\right\} \longleftrightarrow \left\{\text{Elements of } \mathbb{R}^8\right\}$$

- 70 “well-scattered” morphologies are carefully chosen and the simulator is run on them.

Available Data (Ice Core Records)

Ice core records available at 6 different locations, each with uncertainty bands.



Note: For two locations (CC & DYE3), no lower bound can be easily extrapolated from the ice core samples.
We consider three scenarios: a 'tight', a 'middle' and a 'loose' one.

	NEEM	**NGRIP**	**GRIP**	**GISP2**	**CC**	**DYE3**
	$\delta^{18}O$ ‰	$\delta^{18}O$ ‰	$\delta^{18}O$ ‰	$\delta^{18}O$ ‰	$\delta^{18}O$ ‰	$\delta^{18}O$ ‰
Minimum	2.7	2.1	2.2	1.7	?	?
Most likely	3.6	3.1	3.2	2.7	2.5	4.7
Maximum	4.0	3.8	3.5	3.4	4.0	5.2

Record-Compatible Morphologies

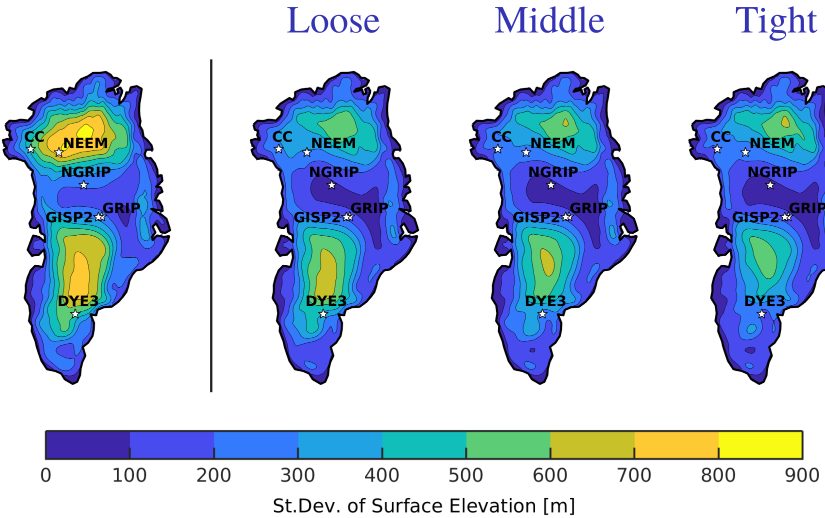
- For each morphology $x \in \mathbb{R}^8$ and one location Loc, compute the discrepancy measure

$$I(x, \mathrm{Loc}) = \frac{m(x) - \mathrm{record}(\mathrm{Loc})}{\sqrt{v(x,x) + \mathrm{var}(\mathrm{Loc})}} \,.$$

- Classify a morphology $x \in \mathbb{R}^8$ as compatible with records iff

$$\max_L \left| I(x, L) \right| < c \qquad (c = 2 \text{ is chosen}).$$

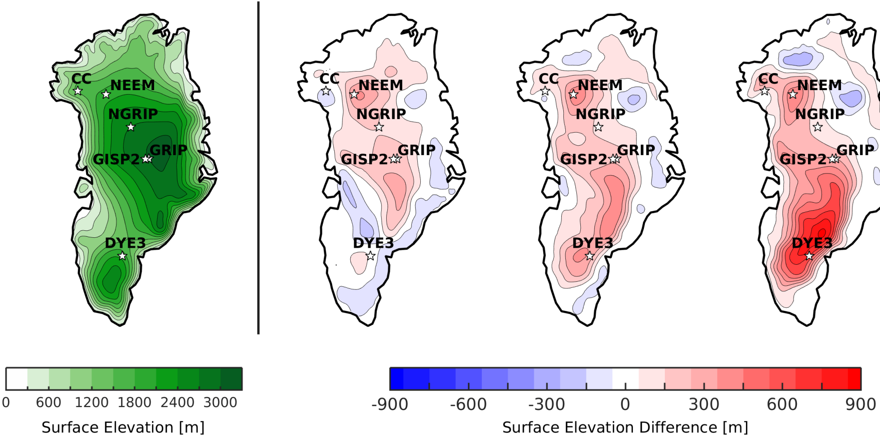
Grid-cell by grid-cell, range of possible heights appears significantly reduced.



Results: Average Surface Height

Average surface height appears massively reduced for tighter scenarios.

Loose Middle Tight



... and happy to answer any curiosity or question!