PRIORITY QUEUE REPORT

Davide Di Mauro

Politecnico di Torino

Student ID: 306089

s306089@studenti.polito.it

Abstract

The main objective of this homework is to simulate a Galton-Watson process where the number of children of a node is described by a Poisson random variable with parameter lambda.

Assumptions

1. The number of children of a node is described by a Poisson random variable with parameter λ

Simulation Parameters

The process has 3 input parameters: the λ of the Poisson random variable.

1. The λ of the Poisson random variable. λ = [0.6, 0.8, 0.9 0.95, 0.99, 1.01, 1.05, 1.1, 1.3]
2. The number of runs of each simulation to estimate the probability of survival q_i
3. The maximum number of generations of the tree

Algorithm

- **for each** λ **in** [0.6, 0.8, 0.9 0.95, 0.99, 1.01, 1.05, 1.1, 1.3]:
  - **for each** run **in** range(n_runs):
    - n_childs = 1
    - **for each** generation:
      - n = extract n_childes values from a Poisson(λ)
      - n_childs = sum(n)
      - compute stop_condition
      - **if** λ > 1 and stop_condition == 0:
        - **break**
      - **if** n_childs == 0
        - **break**
    - save result
  - compute probability_of_survival
- **return** probabilities_of_survival

Computing the Moment Generating Function

$$P(X = i) \sim Poisson(i, \lambda) = \frac{\lambda^{\,i}}{i!} e^{-\lambda}$$

$$\phi_X(z) = \sum_{k=0}^{\infty} P(X = k) z^k = \cdots = e^{\lambda(z-1)}$$

$$\phi_{X_i} = \phi_Y\left(\phi_{X_{i-1}}(z)\right)$$

$$\begin{cases} q_i = \phi_{X_i}(0) \rightarrow PROBABILITY\ OF\ EXTINTION\ GENERATION\ i \\ \phi_{X_i}(0) = \phi_Y\left(\phi_{X_{i-1}}(0)\right) \end{cases}$$

Thus: $q_i = \phi_Y(q_{i-1})$

Where: $q_1 = \phi_1(0) = \phi_Y(0) = e^{-\lambda}$

$q_2 = \phi_2(0) = \phi_Y\left(\phi_Y(0)\right) = e^{\lambda(e^{-\lambda}-1)}$

...

Stop Condition

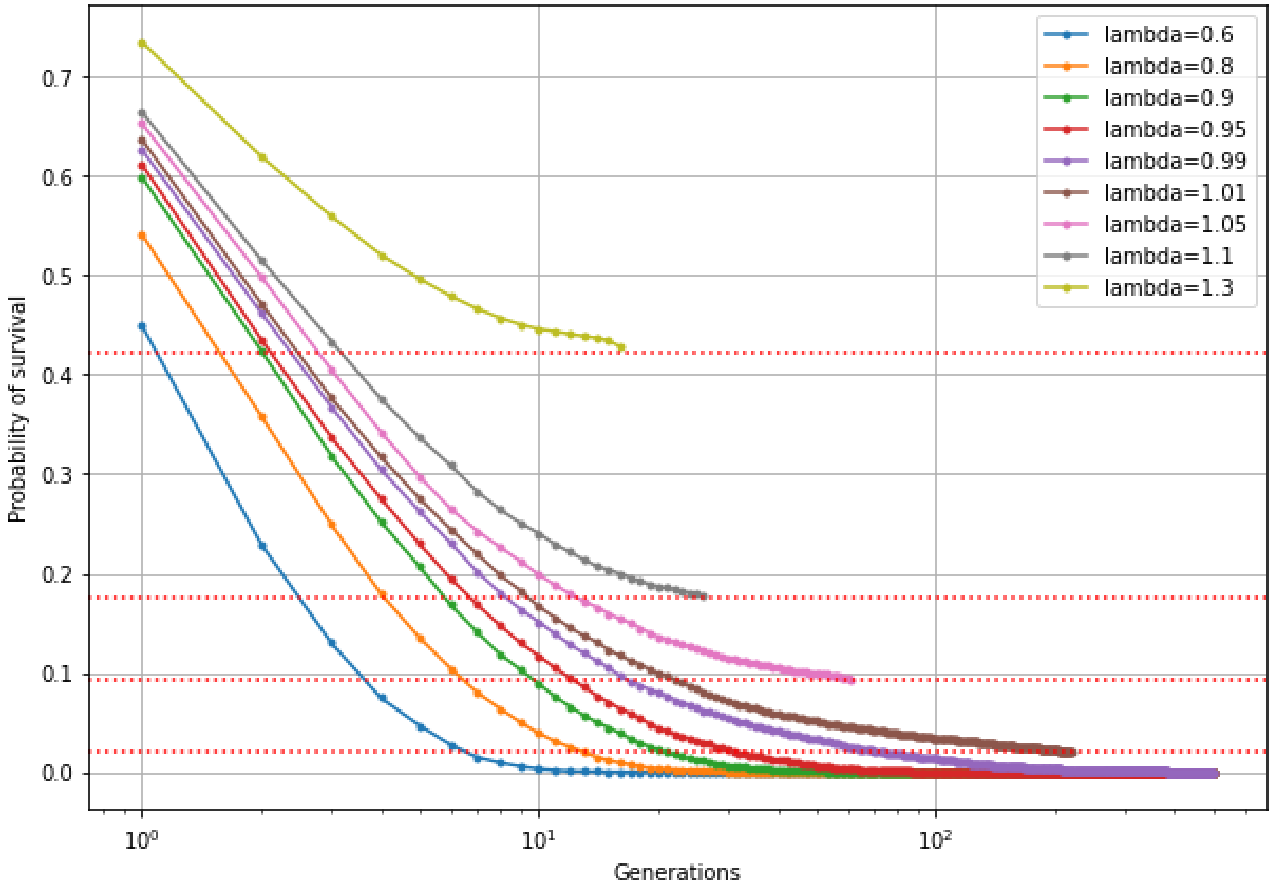
In order to efficiently simulate this process, we need to implement a stopping condition in the case where $\lambda > 1$ since the tree never goes extinct and grows really fast. To do so, we compute the probability of having 0 children at a generic generation n, which is the probability that all nodes have no children.

We stop our simulation when this probability is so close to 0 that python automatically rounds it to 0 $(< 1e - 200)$

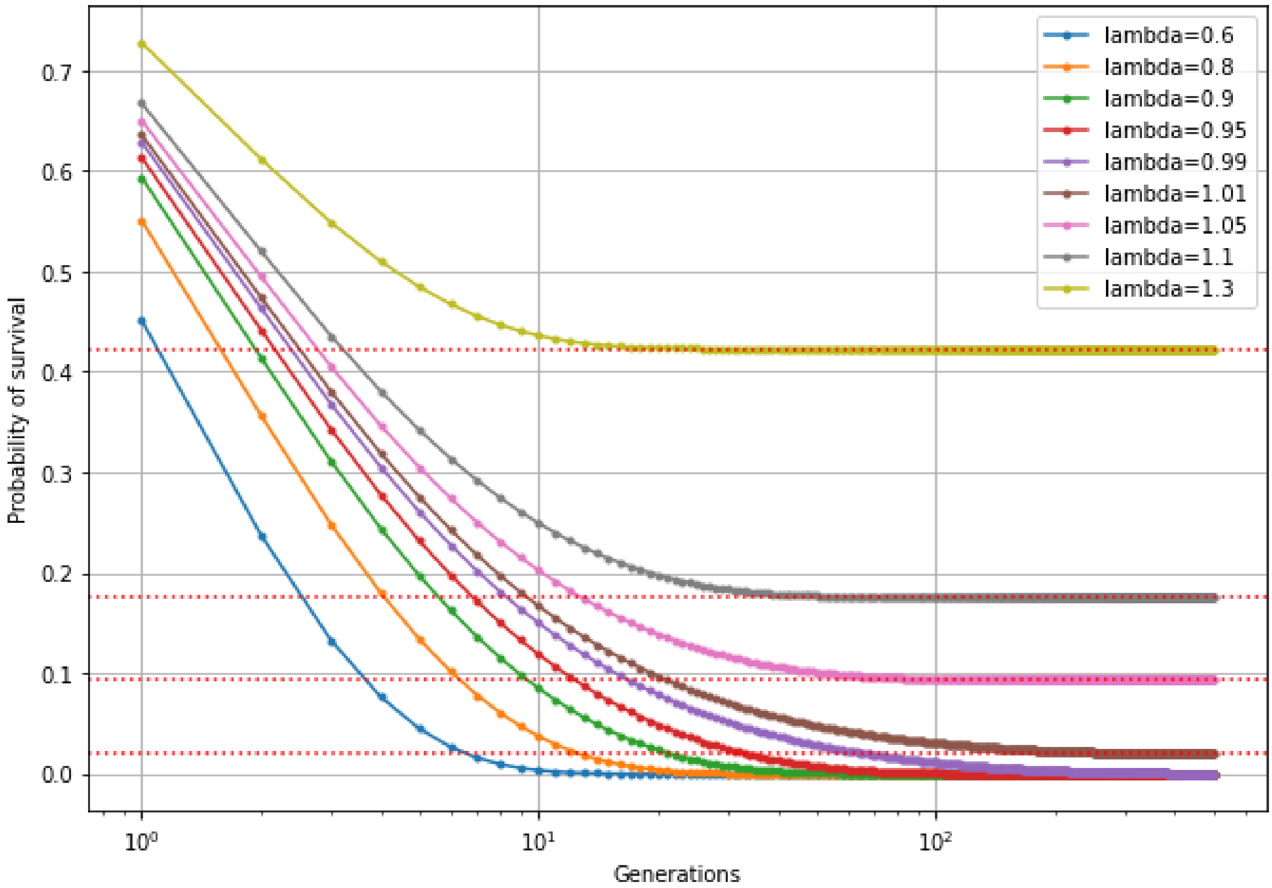
$$P(X = 0) \sim Poisson(0, \lambda) = \frac{\lambda^{\,0}}{0!} e^{-\lambda} = e^{-\lambda}$$

$$P(exitintion\ at\ gen. n, with\ k\ nodes) = P(X = 0)^k = e^{-k\lambda}$$

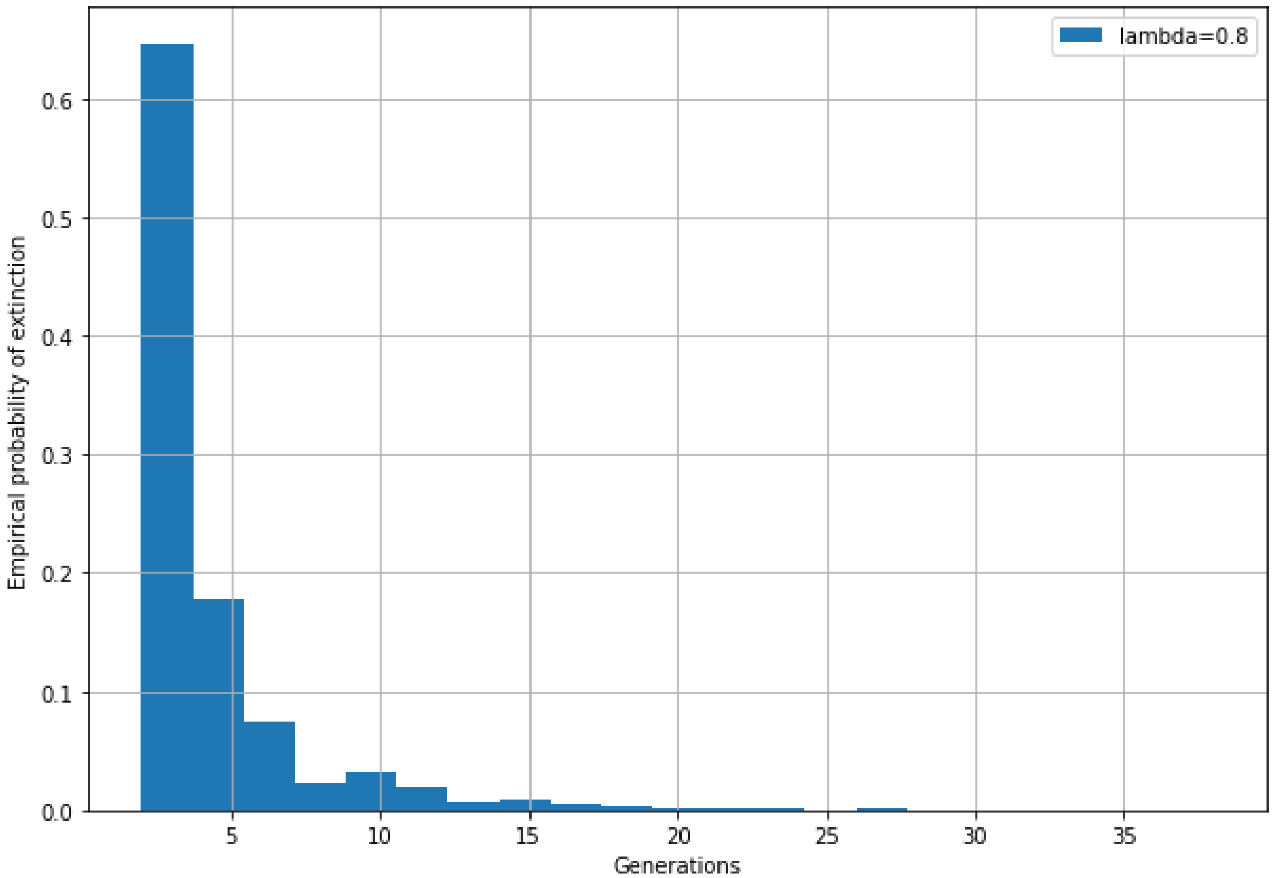
Results



Probability of survival
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^0
10^1
10^2
Generations
lambda=0.6
lambda=0.8
lambda=0.9
lambda=0.95
lambda=0.99
lambda=1.01
lambda=1.05
lambda=1.1
lambda=1.3



Probability of survival
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^0
10^1
10^2
Generations
lambda=0.6
lambda=0.8
lambda=0.9
lambda=0.95
lambda=0.99
lambda=1.01
lambda=1.05
lambda=1.1
lambda=1.3



Conclusions

From the results we can conclude that our simulation is working as expected and that the implemented stopping condition interrupts the simulation near the theoretical non-extinction condition.

The histogram of the probability distribution shows that with lambda = 0.8, the process ends with more than 80% probability in 5 generations.