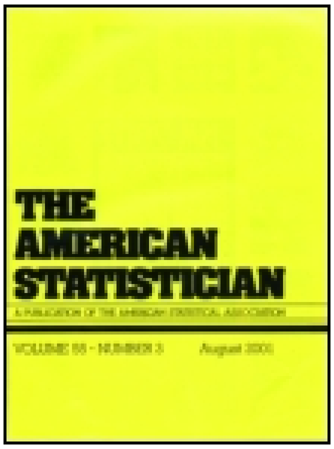

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Quantitative Graphics in Statistics: A Brief History

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Quantitative Graphics in Statistics: A Brief History

JAMES R. BENIGER AND DOROTHY L. ROBYN*

Quantitative graphics have been central to the development of science, and statistical graphics date from the earliest attempts to analyze data. Many familiar forms, including bivariate plots, statistical maps, bar charts, and coordinate paper, were used in the 18th century. Statistical graphics developed through attention to four problems: spatial organization (17th and 18th centuries), discrete comparison (18th and early 19th centuries), continuous distribution (19th century), and multivariate distribution and correlation (late 19th and early 20th centuries). Today, statistical graphics appear to be reemerging as an important analytic tool, with recent innovations exploiting computer graphics and related technologies.

KEY WORDS: History of statistics; Statistical graphics; Graphical data analysis; Computer graphics; History of science; Cartography in statistics.

From Sir Edmund Halley's graphical analysis of barometric pressure as a function of altitude, published in 1686, to the latest advertisements for computer graphic technology, the pages of scientific journals have recorded the importance of quantitative graphics to the scientific enterprise. Throughout the history of science, quantifiable imagery and numbers have served, side by side, in basic graphic forms like the table, coordinate system, and map, and in derivative forms like the line graph, histogram, and scatterplot.

Quantitative graphics did not originate with science; they can be traced back to prehistory (see Appendix). The earliest known map, extant on a clay tablet dated at 3800 B.C., depicts all of Northern Mesopotamia with conventions and symbols still familiar today. From about 3200 B.C., Egyptian surveyors abstracted their lands in terms of coordinates not unlike the Cartesian system still in use. By the tenth century A.D., medieval astronomers depicted planetary movements as cyclic lines on spatial-temporal grids, diagrams strikingly similar to modern line graphs (Figure A). Musical notation, standardized by the Vedic hymnists of the seventh century B.C., had become a true time series—following the Franconian reforms—by the 13th century A.D.

Statistical graphics, beginning with simple tables and plots, date from the earliest attempts to analyze empirical data; many of the most familiar forms and techniques were well-established at least 200 years ago. At the turn of the 19th century, to use a convenient benchmark, a statistical analyst might have resorted to the following graphical tools: bivariate plots of data points (used since the mid-17th century), line graphs of time series data (since 1724), curve-fitting and interpolation (1760), the notion of measurement error as deviation from a regular graphed line (1765), graphical analysis of periodic variation (1779), statistical mapping (1782), bar charts (1786), and printed coordinate paper (1794).

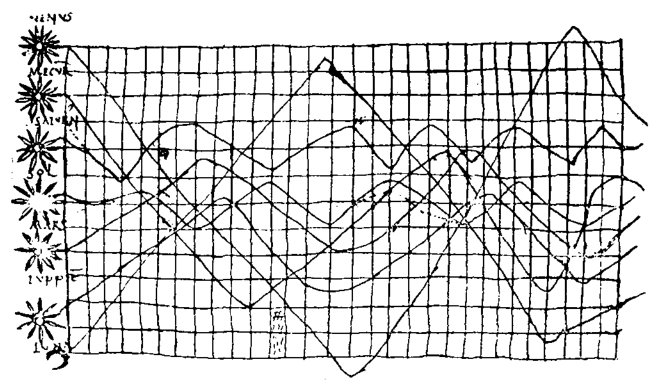

Figure A. Planetary Movements, Depicted as Cyclic Lines on a Spatial-Temporal Grid, by an Unknown Astronomer in a Transcription of Commentary of Macrobius on Cicero's *In Somnium Scipionis*, 10th or 11th Century A.D. Reprinted in [95].

Today, quantitative graphics are reemerging as an important statistical tool, as evidenced by developments as diverse as the growing use by statisticians of computer graphics, the proliferation of descriptive graphics in statistical publications of the Federal government [76, 85, 84], the progressive elaboration of graphics for exploratory data analysis [81, 82, 83], and the recent formation of an Ad Hoc Committee on Statistical Graphics in the American Statistical Association.

Despite the venerable tradition of quantitative graphics for data analysis and the recent revival of interest in statistical graphics, there remains only a single monographic history of the subject [96], now over 40 years old. This borrows considerably from a French text [101] published a century ago. With the exception of a modest revival of interest in quantitative graphics among historians of early statistics [104, 105], visual forms have passed virtually unnoticed by historians and sociologists of knowledge and science [90].

This article, limited to a brief overview of the history of quantitative graphics in statistics, is part of a larger effort by the senior author to trace similar developments in other disciplines [89, 90, 91]. The history of statistical graphics will be discussed in four general stages, corresponding approximately to successive historical periods, and characterized by a major graphical problem which preoccupied scientists and data analysts of that period. These include the

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problem of spatial organization for data analysis, in the 17th and 18th centuries; the problem of discrete quantitative comparison, in the 18th and early 19th centuries; the problem of continuous distribution, throughout the 19th century; and the problem of multivariate distribution and correlation, in the late 19th and early 20th centuries.

Spatial Organization for Data Analysis

The early problem of spatial organization grew with the amount of data to be analyzed. Multiple measurements proliferated with the Industrial Revolution in Europe, which brought a spate of new measuring devices: the air and water thermometer (c. 1590), micrometer (1636), barometer (1643), pendulum clock (1656), weather-clock (c. 1660), mercury thermometer (1714), etc. Spatial organization of multiple measurements was achieved in two competing forms, coordinate systems and tables, which dominated quantitative graphics in the 17th and 18th centuries (see Appendix).

The coordinate approach grew out of the analytic geometry developed by Descartes, Fermat, and other French mathematicians in the first half of the 17th century. Descartes himself was convinced that "imagination or visualization, and in particular the use of diagrams, has a crucial part to play in scientific investigation" [103, p. 28]. The first major success using coordinates belonged to Halley, who plotted barometric readings against elevation above sea level, then fitted a hyperbolic curve to the scatterplot. "The expansion of the air being reciprocal as the height of the mercury," Halley concluded, "it is evident, that by means of the curve of the hyperbola, the said expansion may be expanded to any given height of mercury" [8, p. 105]. Halley published this finding in 1686, only 43 years after the barometer's invention, and 39 years after Descartes' first Appendix on analytic geometry [7].

Despite Halley's impressive demonstration of the graphical method, Cartesian coordinate plots did not become an important tool of data analysis in the 18th century. After an exhaustive search of the 18th century journals and texts, Tilling concludes that "the use of experimental graphs, either with or without subsequent analysis, never became commonplace in that age" [105, p. 194]. From the beginning, coordinate plotting and graphical analysis were overshadowed by statistical tables and tabular inference, due largely to a vociferous group of social scientists organized under the catchphrase *Die Tabellen-Statistik* in Germany in the early 17th century and known as "Political Arithmetic" in Britain after 1687. These movements took impetus from the mounting collection and publication of state statistics or *Staatenkunde*—on population, land, and agricultural production—for the purpose of taxation in the new nation states of Europe.

The obsession of early applied scientists with tabular data, and their disregard for plotting and graphical analysis, is reflected in the early history of automatic recording devices. Between 1660 and 1800, scores of different mechanical recorders were invented that produced moving line graphs of various natural time series: temperature, barometric readings, wind speed and direction, tidal movements, etc. Such automatic graphs were considered useless for analysis and were routinely translated into tabular logs. [The literature is reviewed in 98, 99, 102.]

For example, when Christopher Wren's weather-clock, which produced an automatic and continuous line graph of temperature [20, 102, p. 245], broke down in 1684, the Royal Society moved "to reduce into writing some of the first papers marked by the weather-clock, that thereby the Society might have a specimen of the weather-clock's performances before they proceed to the repairing it" [97, p. 656; quoted in 99, p. 303]. Tilling finds, in examining the 18th-century descriptions of automatic barometers and tide recorders, "no emphasis placed on the fact that they produce graphical rather than tabular records" [105, p. 196].

Change in this attitude did not come until the 19th century, when Quetelet and his students demonstrated the usefulness of graphical analysis in their development of modern social statistics. By the 1830's, scientific journals began to record the painstaking graphing and extrapolation of data—much of which had originally come from machine-produced line graphs only decades earlier.

The simultaneous development of statistical cartography, one application for which tabular analysis could not suffice, also encouraged the reconciliation of coordinate plotting and tabular description [94]. Halley again showed the way, in 1701, when he published his celebrated map showing lines of magnetic declination for all navigated waters of the world [10; Figure B]. In 1778, Charpentier published the

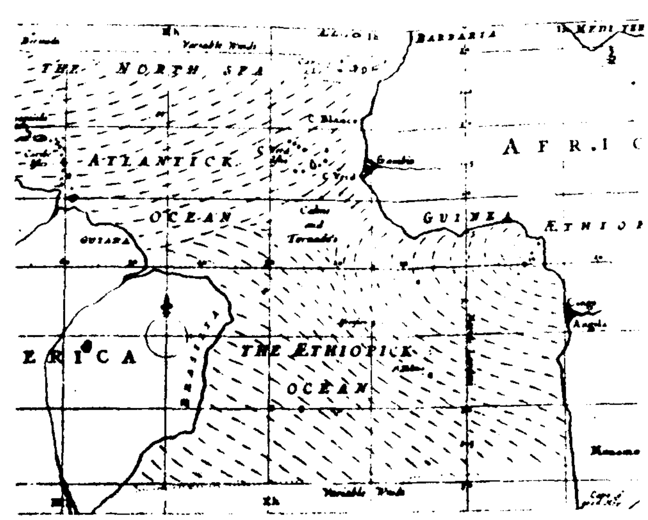

Figure B. Fragment of Map of World Showing Lines of Magnetic Declination, One of the First Published Plots of Empirical Data Points, by Sir Edmund Halley [10], 1701.

first geological map, on which he plotted the distribution of various soils and minerals, linking homogeneous areas by means of eight colored washes [3]. Four years later, using similar techniques, Crome published the first of his several geographical plots of demographic, political, and economic statistics [4]. Minard was first to plot statistical symbols—circles, squares, and small bar graphs—on a map published in 1861 [36].

The rise of coordinate plotting is also documented in the commercial development of graph paper. Rectangular grid paper was first offered for sale by a Dr. Buxton in London in 1794. Buxton's product first appeared in published research six years later, in an article on barometric variations which included a footnote advertising the product [27, p. 357]. Herschel made ingenious use of plotted data to calculate the elements of the elliptical orbits of double stars, and his 1832 paper on the subject included a ringing endorsement of graph paper: "Such charts may be obtained, neatly engraved; and are so very useful for a great variety of purposes, that every person engaged in astronomical computations, or indeed, in physico-mathematical inquiries of any description, will find his account in keeping a stock of them always at hand" [26, pp. 171–2].

Lalanne introduced both a logarithmic grid [34] and polar coordinates [33] to the French Academy in 1843. Twenty years later, Jevons developed semi-logarithmic paper in England [30], and in 1879 he included the first published instructions on the uses of graph paper in the third edition of his popular textbook, *Principles of Science* [31, pp. 492–5]. Four years later, the British government issued the first patent on logarithmic paper.

Discrete Quantitative Comparison

A second graphical problem, that of discrete quantitative comparison, arose in the state statistics or *Staatenkunde* of the early 18th century. Both tables of statistics in general, and comparative political data in particular, suggests the need for graphical comparison, especially in the growing volume of atlases and chartbooks intended for popular consumption.

Joseph Priestley made an early breakthrough in 1765, when he published the first of his several time-line charts, which used individual bars to compare the life-spans of some 2,000 celebrated persons who had lived from 1200 B.C. to 1750 A.D. [18]. Priestley's time-lines proved a commercial success and popular sensation, and went through dozens of editions, the last in 1820. These time-lines directly inspired William Playfair's invention of the bar chart [96, p. 280], a form which first appeared in his *Commercial and Political Atlas* [16], published in 1786.

Ironically, Playfair was driven to his invention by lack of data. With the exception of the single bar chart, all of the other 39 plates in his *Atlas* were line

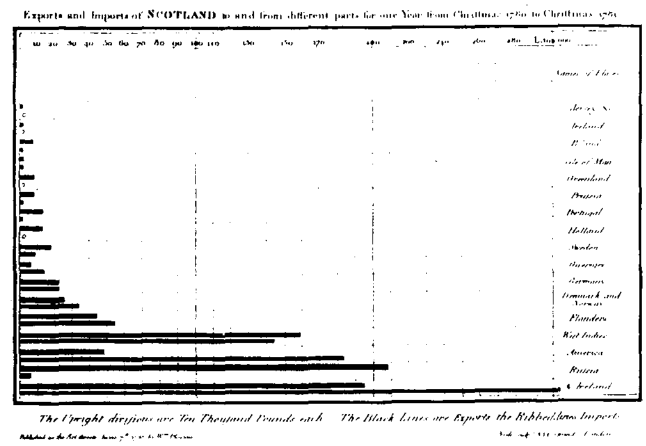

Figure C. Representation of Scotland's Imports and Exports for 17 Countries in 1781, the First Published Appearance of the Bar Chart, Invented by William Playfair and Included in his *Commercial and Political Atlas*, 1786.

graphs or surface charts (line graphs shaded or tinted between abscissa and function), the only published appearance of these forms in the 18th and early 19th centuries [105, p. 199]. Because Playfair lacked time series data for Scotland, however, he graphed its trade data for a single year as a series of 34 bars, one for each of the imports and exports of Scotland's 17 trading partners (Figure C).

Not only did Playfair fail to realize the importance of his invention, he felt compelled to apologize to his readers for what he saw as the limitations of the bar chart form [16, p. 101]:

> This Chart is different from the others in principle, as it does not comprehend any portion of time, and it is much inferior in utility to those that do; for though it gives the extent of the different branches of trade, it does not compare the same branch of commerce with itself at different periods; nor does it imprint upon the mind that distinct idea, in doing which, the chief advantage of Charts consists: for as it wants the dimension that is formed by duration, there is no shape given to the quantities.

Despite Playfair's own misgivings, his bar chart was the first quantitative graphic form that did not locate data either in space (as did coordinates and tables) or time (as did Priestley's time-lines). It constitutes a pure solution to the problem of discrete quantitative comparison.

During the next 15 years, Playfair came to recognize the importance of his contribution. In 1801, he published a third edition of the *Atlas* with a new preface. In contrast to his original misgivings concerning the bar chart, he now cited this form as the basis of his graphical method, which he called "lineal arithmetic" [16, 3rd ed., p. xii]:

> Suppose the money received by a man in trade were all in guineas, and that every evening he made a single pile of all the guineas received during that day, each pile would represent a day, and its height would be proportional to the receipts of that day; so that by this plain operation, time, proportion, and amount, would all be physically combined. Lineal arithmetic, then, it may be averred, is nothing more than those piles of guineas represented on paper, and on a

small scale, in which an inch (suppose) represents the thickness of five millions of guineas.

In the same year, Playfair increased the inventory of discrete comparative forms with two new inventions: the pie chart and the circle graph (both introduced in [41]).

Alexander von Humboldt, acknowledging Playfair's influence, combined the bar and pie chart ideas in the subdivided bar graph. Humboldt also introduced superimposed squares for the comparison of areas, a variation on Playfair's circle graphs. Both inventions were first published in 1811 in Humboldt's *Essai Politique sur le Royaume de la Nouvelle-Espagne*. "Without attaching much importance to these sketches," he wrote of Playfair's graphics and his own, "I cannot regard them as mere trifles foreign to science" [28, p. 186; trans. 96, p. 295].

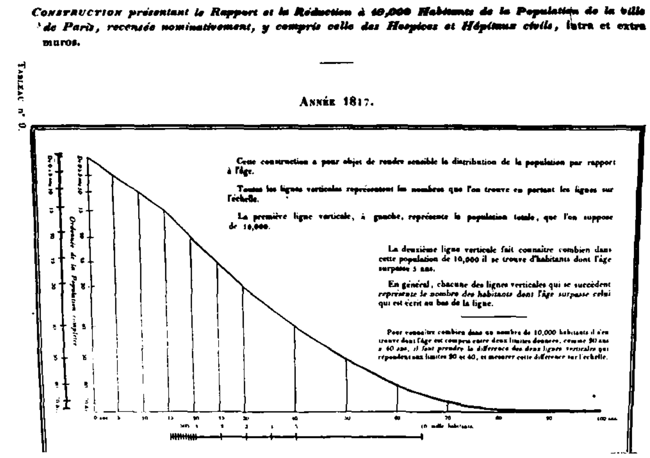

Figure D. Line Graph Representing the 1817 Population of Paris by Age Groupings, the First Appearance of the Cumulative Frequency Distribution Devised by J. B. J. Fourier, Published in [22], 1821.

Continuous Distribution

By the 1820's, an increasing number of the scientific journals in Europe began to publish graphs and charts that described and compared measurements of a wide range of natural and social phenomena. Graphical analysis of data finally emerged in the period 1830–1835 as a regular feature of scientific publication, particularly in England. At the same time, a relatively new field, vital statistics, generated a third graphical problem, that of representing continuous distributions. Two solutions, the ogive and the histogram, proved essential to the further development of vital statistics in the 19th century (see Appendix).

Although Halley had published the first scientifically constructed mortality tables in 1693 [9], he apparently never attempted a graphical representation of his figures. French mathematicians Loys de Cheseaux and d'Alembert graphed hypothetical curves of mortality, in the mid-18th century, but did not base their curves on actual data—although accurate mortality data had then existed for some 60 years. Eventual solutions for the representation of empirical distributions utilized the bar form, which was not widely known until the turn of the 19th century, following the work of Priestley and Playfair.

The first solution came in 1821 from J. B. J. Fourier [22], who began with a bar chart representing the population of Paris by age groupings, then placed the bars one atop the other to form the ordinate of a first quadrant graph. His abscissa contained the same age demarcations but at equal intervals. By connecting like age groupings with lines perpendicular to the axes, Fourier completed the first graph of a cumulative frequency distribution (Figure D), given the name "ogive" by Galton in 1875.

The second solution came when A. M. Guerry applied the bar chart form to crime data, which he had arranged in ordered categories for continuous variables like age and month, to produce histograms suggestive of various theoretical distributions. These he published in his 1833 *Essai sur la Statistique Morale de la France* [25], which won the medal of the French Academy. The word "histogram" was first used by Karl Pearson in his 1895 lectures on statistical graphics [39, p. 399].

Systematic development of the graphics of continuous distribution was due largely to the work of Adolphe Quetelet. Quetelet first used graphical analysis in his second statistical publication [42], and "published very few papers in which the graphical method does not play some part" [96, p. 299]. In 1828, he completed the work of Halley and the French mathematicians, publishing the first mortality curves from actual data [43]. In 1846, he published the results of an urn schemata as a symmetrical histogram, with the normal curve superimposed as the limiting "curve of possibility" [44].

Francis A. Walker, Superintendent of the U.S. Census, completed the 19th century inventory of forms for representing continuous distribution. Walker developed both the age pyramid and the bilateral frequency polygon, forms which he introduced in 1874 in the *Statistical Atlas of the United States*, based on data from the Ninth U.S. Census [48].

Multivariate Distribution and Correlation

By mid-19th century, quantitative graphics had become an accepted part of statistics. The Third International Statistical Congress, meeting in Vienna in 1857, organized an exhibition display of graphs and cartograms and debated the merits of various graphical methods [29]. In 1872, the U.S. Congress appropriated the first money for a graphical treatment of statistical data, the cartograms of Ninth Census results which appear in *Statistics of Wealth and Public Indebtedness* [46]. During the same period, vital statistics increasingly involved interrelationships among at least three variables—population, age, and time—which led to the graphical problem of representing multivariate distributions and correlations. Two general solutions, contour plots and stereograms (i.e., orthographic and axonometric projection) occupied statisticians into the 20th century.

The first contour map of physical geography had been published by Buache in 1752 [2], and such maps became commonplace by the early 19th century. In 1843, Lalanne used the contour form to reconstruct a three-way table, which gave mean temperatures by hour of the day and day of the year. Lalanne plotted months and hours on the abscissa and ordinate, then connected points of similar temperature to form contour lines [33; Figure E]. This is equivalent to an orthographic projection, in which locations in the z dimension are projected on the xy plane, a technique which was not formalized for another 30 years. Lalanne's inspiration came from nomographic tables, then essentially multiplication tables in contour form, first published by Pouchet [17] in 1795 and adopted by the French Artillery in the 19th century.

Lalanne speculated that his contours of tables might be applied to geographical distributions; 30 years later, in 1874, Vauthier published a map of Paris with contour lines showing densities of population [47; Figure F]. Galton cited the work of both

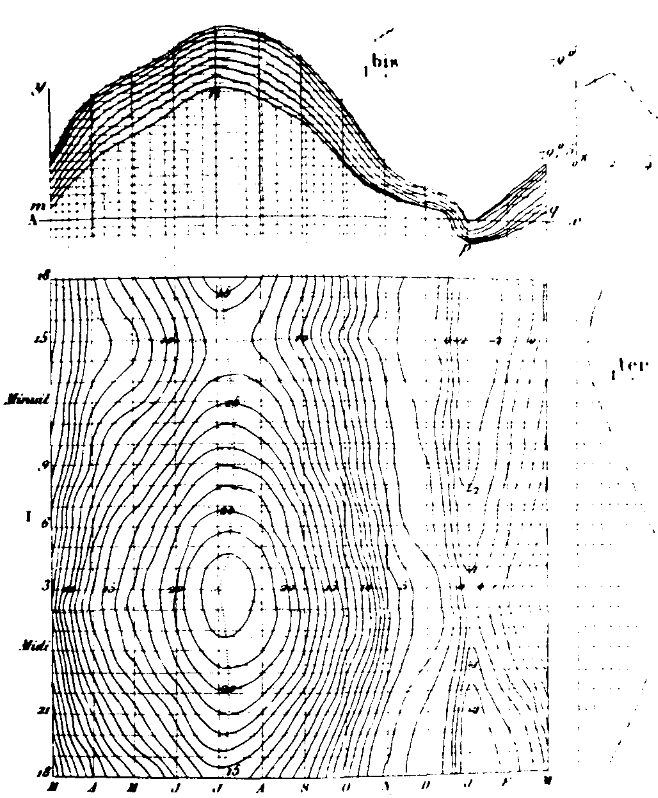

Figure E. Three-Way Table (Temperature × Hour × Month) Reconstructed as a Contour Map, the First Such Application of This Form, Invented by Léon Lalanne in 1843. Published in [33], 1845.

Figure F. Map of Paris with Population Density Depicted as Contour Lines, the First Such Application of This Form, Invented by L. L. Vauthier, 1874. Cited by Francis Galton as an Inspiration for His Normal Correlation Surface [23].

Lalanne and Vauthier as inspirations for his normal correlation surface, published in his 1885 paper, "Regression towards Mediocrity in Hereditary Stature" [23]. Two years later, Galton determined a coefficient of correlation by graphical means [24].

The problem of representing multivariate distributions suggested an alternative solution, that of the stereogram, which traces its origins from 1869, when Zeuner published a system that depicted demographic trends as surfaces in three coordinates [49]. Using axonometric projection, any "slice" of Zeuner's surface, including 45-degree slices representing the history of various cohorts, could be shown in two dimensions. Ten years later, Perozzo produced a colored relief drawing of Zeuner's theoretical surface, which was based on actual data from the Swedish censuses of 1750–1875 [40; Figure G]. Perozzo's drawing, given the name "stereogram" by an Italian colleague, was reproduced in statistical journals throughout Europe during the following decade. In various constructions of pasteboard and plaster of Paris, the three-dimensional surfaces of Galton and Perozzo became favorites of instructors of probability

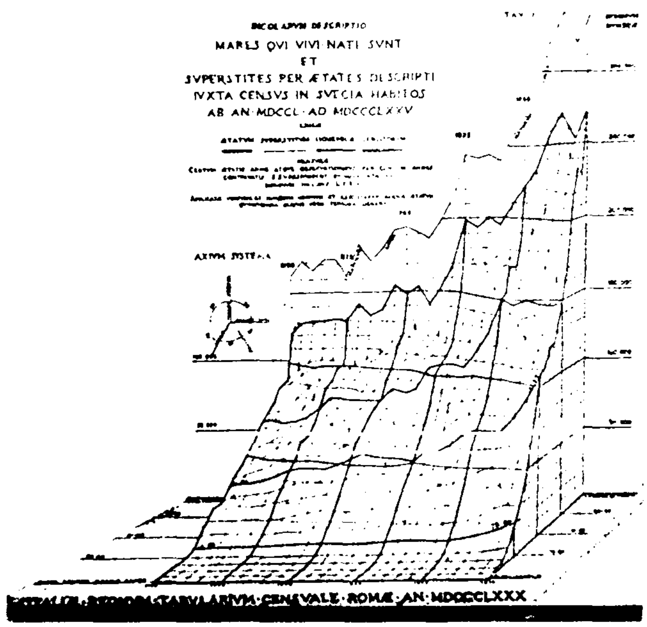

Figure G. Relief Drawing (Form Became Known as the "Stereogram") Representing the Population of Sweden, 1750–1875, by Age Groupings, Constructed by Luigi Perozzo, Italian Department of Statistics, in 1879. Published in Color in [40], 1880. Based on the Axonometric Projection System of Gustav Zeuner [49].

theory and demography, and they remained standard equipment in many statistical laboratories until the advent of computer graphics.

The Twentieth Century

At the turn of this century, statistical graphics had begun to diffuse—through textbooks, college courses, and the mass media—into the popular domain (see Appendix). A major vehicle for this diffusion was the pictogram, a comparative form based on similar drawings of different sizes developed by M. G. Mulhall for his popular *Dictionary of Statistics* [37] published in 1884. W. C. Brinton corrected the pictogram's most serious flaw—the ambiguity of whether comparisons are to be made in one, two, or three dimensions—with his suggestion to string out unit drawings, thus making the form analogous to the bar chart [51, pp. 39–41]. Otto Neurath was probably most influential in promoting pictorial statistics, first in the "Vienna Method" of the Social and Economic Museum, which popularized statistics of the city during the period 1924–1934, and later in Neurath's Isotype System, which became a profitable commercial enterprise [63].

Statistical diagrams were first used in school textbooks in France, beginning with Levasseur's *La France, Avec Ses Colonies* [35], published in 1868. Graphs of mathematical functions began to appear in U.S. textbooks after 1902, and statistical graphs of temperature and population trends appeared about 1910; bar and pie charts were added in the period 1915–1918 [96, pp. 357–8]. The first textbook in English devoted exclusively to graphics was J. B. Peddle's mathematical treatment, *The Construction of Graphical Charts* [64] published in 1910. The same decade brought more elementary texts by Brinton [51], S. Gilman [56] and A. C. Haskell [57]. During the period 1920–1926, textbooks on statistical graphics appeared at a rate of about two new titles each year.

The first American college course on graphical methods, offered by the Department of Agricultural Engineering at Iowa State in the academic year 1913–1914, was taught by M. F. P. Costelloe. In 1916, F. J. Warne offered a correspondence course, twenty weekly lessons for fifty dollars, out of the Southern Building in Washington, D.C. [65]. E. P. Cubberly taught a Stanford course, "Graphical Methods for Presenting Facts," annually during the period 1918–1933. In 1937, Funkhouser reported that "most of the present-day courses in statistics devote a portion of the time to graphical instruction" [96, p. 258].

As a result of invitations from the American Society of Mechanical Engineers, a joint committee of American scientific societies compiled standards of graphical presentation, published in 1914 [60]. This led members of the American Statistical Association to organize a standing committee on graphics. A 1919 government chartbook, *The War With Germany,* [50], published under the directorship of L. P. Ayres, did much to popularize the graphical method in the United States. In 1926, W. C. Eells published results of the first experiment with graphical forms, comparative tests of pie charts and subdivided bar charts, in the *Journal of the American Statistical Association* [55]. There ensued, in the next five years, a flurry of experimentation with graphical forms, including contributions by R. von Huhn [59], F. E. Croxton and collaborators [52, 54, 53], and J. N. Washburne [66].

With the advent of World War II, interest in graphical methods appeared to wane among academic statisticians, as attention turned to more mathematical concerns. This trend did not begin to reverse again until the mid-1960's (see Appendix), when developments in computer technology made possible the manipulation and analysis of large multivariate data sets. As W. H. Kruskall [100, p. 31] noted in 1975:

> The role of statistical graphics within statistics generally . . . has had tremendous ups and downs: at one time, graphical methods were near the core of statistics—Karl Pearson devoted considerable attention to graphics and he was following the emphasis of his hero, Francis Galton. Later on, statistical graphics became neglected and even scorned in comparison with the blossoming of the mathematical side of statistics. In recent years, however, there has been a renaissance of concern with graphics and some of our best statistical minds have suggested new graphical approaches of great interest.

New graphical approaches published during the past 20 years include, in 1957, Edgar Anderson's circular glyphs to represent multivariate data [67]; in 1965, the first of J. W. Tukey's innovations for exploratory data analysis [81]; in 1968, the "graphic rational patterns" of R. Bachi [69]; in 1972, D. F. Andrews' Fourier series to generate multivariate plots [68]; in 1973, H. Chernoff's cartoons of a human face to represent multiple variables [72]; in 1974, a color-coded matrix to represent two variables in a single map, developed by the U.S. Bureau of the Census [85]; in 1975, S. E. Fienberg's "Floating Four-Fold Circular Display" (FCD) to represent a 2×2 table [77].

All of these innovations exploit modern computer technology, and most have little practical value except when executed by computer. In the future, innovations in statistical graphics are likely to follow developments in computer graphics hardware and software, and might be expected to include solutions to problems generated or made tractable by the computer and associated technologies. These problems include:

(1) Embellishment of older graphical forms, as in Tukey's hanging histogram and rootogram [81, 82]; Fienberg's FCD [77], which combines the 2×2 table and circle and pie chart forms; the scatterplot Cleveland and Kleiner enhance with plots of moving statistics [74]; and the "Cartesian rectangle" of Wainer and Reiser [87], which combines the 2×2 table and subdivided bar chart.

(2) Representation of multivariate data in two dimensions, as in Anderson's glyphs [67]; the polygons of Siegal et al. [80]; Andrews' Fourier series [68]; and Chernoff's "faces" [72].

(3) Representation of variables simultaneously with geographical and population distributions (what Tukey has called the "patch map" problem), as in applications of Bachi's "graphical rational patterns" [69, ch. 12].

(4) Inclusion of two or more variables in a single map, as in Bachi's applications [69, ch. 13], or the color-coded cross-classification maps of the U.S. Census Bureau's *Urban Atlas* series [85].

(5) Representation of sampling and measurement error in traditional forms like the bar chart [e.g., 78, p. 10], line graph and statistical map [for a general discussion, see 93].

(6) Comparative testing of new forms, as in the test of the histogram vs hanging histogram and rootogram by Wainer [86]; of random permutations of features of Chernoff's faces by Chernoff and Rizvi [73]; of tables vs various graphical forms by A. S. C. Ehrenberg [75]; and of the Cartesian rectangle vs the table, bar chart, and FCD by Wainer and Reiser [87].

(7) General development of graphics for use in computer-assisted analysis of large, multivariate data sets, as in various adaptations of Tukey's exploratory data analysis [83, pp. 663–4].

Solutions might be expected to include innovative uses of both graphical and nongraphical dimensions only recently rendered technologically accessible, including color computer graphics, person-machine interaction, computer animation, three-dimensional computer graphics and holography, and mechanically controlled sound. A convenient bibliography of developments in these and related areas has recently been published [92].

Appendix: Developments in Statistical Graphics

Early (Pre-1600) Milestones in Quantitative Graphics

c. 3800 B.C.	Oldest known map (of Northern Mesopotamia) extant on clay tablet
c. 3200 B.C.	Coordinate systems to locate points in real space—Egyptian surveyors
c. 1500 B.C.	Systematic guide to practical problems of geometry, including graphical representations of areas (rectangular, trapezoidal, triangular and circular)—Rhind Papyrus, Scribe Ahmose, Egyptian
c. 700 B.C.	Musical notation (300 symbols, based on contemporary alphabet, each representing one set pattern of notes)—Vedic hymnists, Southern India
4th cent. B.C.	Terrestrial and celestial globes—Eudoxus, Greek astronomer
c. 300 B.C.	Formal statement of geometric principles—*Elements*, Euclid, Greek
10–11th cent. A.D.	Curves (planetary orbits) on time grid (Figure A)—unknown transcriber of commentary of Macrobius on Cicero's *In Somnium Scipionis*
12–13th cent.	Musical notation as true time series, with neumes (corresponding to notes) of fixed duration, introduction of bar to mark equal numbers of beats—Franconian reform, following Franco of Cologne's *Ars Cantus Mensurabilis*
c. 1350	Proto-bar graph (of theoretical function)—Nicole Oresme, French mathematician [15]
c. 1500	Rectangular coordinates to analyze velocity of falling objects—Leonardo da Vinci, Florentine [19]

Seventeenth and Eighteenth Century Developments in Statistical Graphics

early 17th cent.	Tables of empirical data, published tables of numbers—*Die Tabellen-Statistik*, Germany
1637	Coordinate system reintroduced in mathematics, analytic geometry, relationship established between graphed line and equation—René Descartes, French [7]
c. 1660	Automatic recording device (weather-clock) producing moving graph of temperature—Christopher Wren, English [20]
1686	Bivariate plot of observations (barometric reading vs altitude), graphical analysis of empirical data—Edmund Halley, English [8]
1693	Mortality tables—Halley [9]
1701	Commonly used isobar map, lines of equal magnetic declination for the world (Figure B)—Halley [10]
1712	Literal line graph, inspired by nature of observation (section of hyperbola, formed by capillary action of colored water between two glass plates)—Francis Hauksbee, English [11]
1724	Abstract line graph (of barometric observations), not analyzed—Nicolaus Cruquius [6]
1752	Contour map—Phillippe Buache, French [2]
1760	Curve-fitting and interpolation from empirical data points—J. H. Lambert, German [12]
1763	Graph of beta density—Thomas Bayes, English clergyman [1]
1765	Theory of measurement error as deviations from regular graphed line—Lambert [13]

	Historical time line (life spans of 2,000 famous people, 1200 B.C. to 1750 A.D.), quantitative comparison by means of bars—Joseph Priestley, English chemist [18]
1767–1796	Repeated systematic application of graphical analysis (line graphs applied to empirical measurements)—Lambert
1778	Geological map (distribution of soils, minerals)—J. F. W. T. von Charpentier, German [3]
1779	Graphical analysis of periodic variation (in soil temperature)—Lambert [14]
1782	Statistical map— A. W. F. Crome, German [4]
1785	Superimposed squares to compare areas (of European states)—Crome [5]
1786	Bar chart (Figure C)—William Playfair, English [16]
1794	Printed coordinate paper—a Dr. Buxton, English
1795	Multinumber graphical calculation (contours applied to multiplication table)—Louis Ezechiel Pouchet, French manufacturer [17]
1796	Automatic recording of bivariate data (pressure vs volume in steam engine)—"Watt Indicator," John Southern and James Watt, English (invention kept secret until 1822)

Nineteenth Century Developments in Statistical Graphics

1800	Use of coordinate paper in published research (graph of barometric variations)—Luke Howard, English [27]
	Idea for continuous log of automatically recorded time series graphs (of temperature and barometric pressure)—Alexander Keith, English [32]
1801	Pie chart, circle graph—William Playfair, English [41]
1811	Subdivided bar graph, superimposed squares—Alexander von Humboldt, German [28]
1819	Cartogram, map with shadings from black to white (distribution and intensity of illiteracy in France)—Charles Dupin, French geometer and statistician [21]
1820's	An increasing number of scientific publications begin to contain graphs which describe—but do not analyze—natural phenomena (magnetic variation, weather, tides, etc.)
1821	Ogive or cumulative frequency curve, inhabitants of Paris by age groupings (Figure D)—J. B. J. Fourier, French [22]
1828	Mortality curves drawn from empirical data (for Belgium and France)—Adolphe Quetelet, Belgium [43]
1830–1835	Graphical analysis of natural phenomena begins to appear on a regular basis in scientific publications, particularly in England
1832	Curve-fitting to a scatterplot, advocacy of graph paper as standard tool of science—J. F. W. Herschel, English [26]
1833	Histogram (crime by age groupings, months)—A. M. Guerry, French statistician [25]
	Rank lists, with lines showing shifts in rank order between categories (rank of types of crime from one age group to the next)—Guerry [25]
1843	Contour map of table, temperature × hour × month (Figure E)—Léon Lalanne, French engineer [33], published in 1845
	Polar coordinates (frequency of wind directions)—Lalanne [33]
	Logarithmic grid—Lalanne [34]
1846	Results of urn schemata as symmetrical histograms, with limiting "curve of possibility" (later called normal curve)—Quetelet [44]
1851	Map incorporating statistical diagrams (circles proportional to coal production)—Charles Joseph Minard, French engineer [36], published in 1861
1852	Statistical graphics used in lawsuit—Germany
1857	Discussion of graphical methods before Third International Statistical Congress, Vienna [29]
	Exhibition display of graphs and cartograms—Third International Statistical Congress, Vienna
	Polar area charts, known as "coxcombs"—Florence Nightingale, English, in anonymous publication for campaign to improve sanitary conditions of army [38]
1863	Semilogarithmic grid (showing percentage changes in commodities)—Stanley Jevons, English [30]
1868	Statistical diagrams in a school textbook—Émile Levasseur, French [35]
1869	Three-dimensional population surface or "stereogram," with axonometric projection to show curves of various "slices"—Gustav Zeuner, German [49]
1872	U.S. Congressional appropriation for graphical treatment of statistics
	Use of statistical graphics by U.S. Government in census reports (cartograms of data from Ninth Census)—U.S. Bureau of the Census [46]
	Classification of statistical graphical treatments by form—H. Schwabe, German [45]
1874	Age pyramid (bilateral histogram), bilateral frequency polygon—Francis A. Walker, Superintendent of U.S. Census [48]
	Population contour map (orthographic projection; Figure F)—L. L. Vauthier, French [47]
1879	Stereogram (three-dimensional population pyramid) modeled on actual data (Swedish census, 1750–1875; Figure G)—Luigi Perozzo, Italian [40]
	Published instructions on how to use graph paper—Jevons [31]
1883	Patent issued on logarithmic paper—England
1884	Pictogram—Michael George Mulhall, English [37]
1885	Normal correlation surface—Galton [23]
1892	Social data, diagrams, including regional survey, incorporated in museum—Outlook Tower, Edinburgh (Patrick Geddes, founder)
1899	Idea for "log-square" paper, ruled so that normal probability curve appears as a straight line—Galton [24]

Twentieth Century Developments in Statistical Graphics

1905	Lorenz curve (cumulative distribution by rank order, to facilitate study of concentrations)—M. O. Lorenz, U.S. [61]
c. 1910	Statistical diagrams in U.S. textbooks (graphs of temperature, population in texts of arithmetic, algebra)
1910	Textbook in English devoted exclusively to statistical graphics—John B. Peddle, U.S. [64]
1913	Arithmetic probability paper, ruled so that ogive appears as straight line—Allen Hazen, U.S. [58]
	Parade of statistical graphics—Employees of the City of New York
1913–1914	College course in statistical graphical methods—M. F. P. Costelloe, Department of Agricultural Engineering, Iowa State College
1914	Published standards of graphical presentation for U.S.—Joint Committee, convened by American Society of Mechanical Engineers [60]
	Pictogram of uniform size (combining the concepts of bar graph and pictogram of varying size)—Willard C. Brinton [51]

1915 Standing committee on graphics—American Statistical Association

1916 Correspondence course in graphical methods (20 lessons, 50 dollars)—Frank J. Warne, U.S. [65]

1918–1933 Annual college course in statistical graphical methods—E. P. Cubberly, Stanford University

1919 Social statistical chartbook, U.S. Government—L. P. Ayres [50]

1920–1926 Spate of U.S. textbooks on graphics, published at a rate of about two each year

1924 Museum of Social Statistical Graphics—Social and Economic Museum, Vienna (Otto Neurath, Director)

1926 Experimental test of statistical graphical forms (pie vs subdivided bar charts)—Walter C. Eells, U.S. [55]

1927–1932 Spate of articles on experimental tests of statistical graphical forms—R. von Huhn [59], F. E. Croxton, et al. [52, 54, 53], J. N. Washburne [66], U.S.

1931 "Log Square" paper—F. C. Martin and D. H. Leavens, U.S. [62]

1933 Standard statistical symbols (Neurath's Isotype method) established by government decree—Soviet Union (for schools, public posters, etc.)

Lapse of Interest in Statistical Graphics Innovation

1957 Circular glyphs, with rays to represent multivariate data—Edgar Anderson, U.S. [67]

1965 Improvements on histogram in analysis of counts, tail values—John W. Tukey, U.S. [81]

1966 Triangles to represent simultaneously four variables, using sides and orientation—R. Pickett and B. W. White, U.S. [79]

1968 Systematic "graphical rational patterns" for statistical presentation—Roberto Bachi, Israel [69]

1969 Graphical innovations for exploratory data analysis (stem-and-leaf, graphical lists, box-and-whisker plots, two-way and extended-fit plots, hanging and suspended rootograms)—Tukey [82]

Suggestion for displaying five variables by means of movements on a CRT—George Barnard, England [70]

1971 Irregular polygon to represent multivariate data (with vertices at equally spaced intervals, distance from center proportional to the value of a variable)—J. H. Siegel, R. M. Goldwyn, and H. P. Friedman, U.S. [80]

Proposal to use statistical graphics in social indicator reporting, particularly on television—Albert D. Biderman, U.S. [71]

1972 Form of Fourier series to generate plots of multivariate data—David F. Andrews, Canada [68]

1973 Cartoons of human face to represent multivariate data—Herman Chernoff, U.S. [72]

U.S. Government chartbook devoted exclusively to reporting social indicator statistics—Office of Management and Budget [76]

1973–1976 Revival of statistical graphics innovation, use by U.S. Bureau of the Census—Vincent P. Barabba, Director

1974 Color-coded bivariate matrix to represent two intervally measured variables in a single map—U.S. Bureau of the Census and Manpower Administration, *Urban Atlas* series [85]

Comparative experimental test of histogram, hanging histogram and hanging rootogram—Howard Wainer, U.S. [86]

1975 Weekly chartbook (eventually computer-generated) to brief U.S. President, Vice President on economic and social matters—Bureau of the Census and Office of Management and Budget, at request of Vice President Nelson Rockefeller

"Floating Four-Fold Circular Display" (FCD) to represent 2×2 table—Stephen E. Fienberg, U.S. [77]

Enhancement of scatterplot with plots of three moving statistics (midmean and lower and upper semimidmean)—W. S. Cleveland and Beat Kleiner, U.S. [74]

Experiment showing random permutations of features used in Chernoff's faces affect error rate of classification by about 25 percent—Chernoff and M. H. Rizvi, U.S. [73]

Experimental tests of statistical graphics vs tables, findings favoring latter—A. S. C. Ehrenberg, England [75]

1976 Monthly U.S. Government chartbook of economic and social trends—*StatUS*, U.S. Bureau of the Census [84]

1977 "Cartesian rectangle" to represent 2×2 table, experimentally tested against other forms—Wainer and Mark Reiser, U.S. [87]

Ad Hoc Committee on Statistical Graphics, American Statistical Association

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References

I. Early Quantitative Graphics (Pre-1800)

[1] Bayes, T. (1763), "Essay Towards Solving a Problem in the Doctrine of Chances," *Philosophical Transactions*, 370–418.

[2] Buache, P. (1752), "Essai de Géographie Physique," *Mémoires de l'Académie Royale des Sciences*, 399–416.

[3] Charpentier, J. F. W. T. von, (1778), *Mineralogische Geographie der Chursachsischen Lande*, Leipzig: Crusius.

[4] Crome, A. W. F. (1782), *Producten-Karte von Europa*, Dessau.

[5] ——— (1785), *Über die Grösse und Bevölkerung der Sämtlichen Europäischen Staaten*, Leipzig.

[6] Cruquius (1724), "Observationes Accuratae Captae . . . Circa Mediam Barometri Altitudinem, Mediam Thermometri Elevationem, Tum et Hydrometri Varietatem Mediam, etc.," *Philosophical Transactions*, 4–7.

[7] Descartes, R. (1637), "La Géométrie," Appendix to *Discours de la Méthode*, Paris.

[8] Halley, E. (1686), "On the Height of the Mercury in the Barometer at Different Elevations Above the Surface of the Earth, and On the Rising and Falling of the Mercury on the Change of Weather," *Philosophical Transactions*, 104–115.

[9] ——— (1693), "An Estimate of the Degrees of Mortality of Mankind, Drawn from Curious Tables of the Births and Funerals at the City of Breslaw, with an Attempt to Ascertain the Price of Annuities on Lives," *Philosophical Transactions*, 596–610.

[10] ——— (1701), *The Description and Uses of a New and Correct Sea-Chart of the Whole World, Shewing Variations of the Compass*, London.

[11] Hauksbee, F. (1712), "Account of the Experiment on the Ascent of Water between Two Glass Planes in an Hyperbolick Figure," *Philosophical Transactions*, 539–540.

[12] Lambert, J. H. (1760), *Photometria*, Augustae Vindelicorum, 1760.

[13] ——— (1765), "Theorie der Zuverlässigkeit," in *Beyträge zum Gebrauche der Mathematik und Deren Anwendung*, Vol. 1, Berlin, 424–488.

[14] ——— (1779), *Pyrometrie*, Berlin.

[15] Oresme, N. (1482), *Tractatus de Latitudinibus Formarum*, Padova.

[16] Playfair, W. (1786), *Commercial and Political Atlas*, London: Corry; 3rd edition, London: Stockdale, 1801.

[17] Pouchet, L. E. (1795), "Arithmétique Linéaire," Appendix to *Echelle Graphique des Nouveaux Poids, Mesures et Monnaies de la République Française*, Rouen.

[18] Priestley, J. (1765), *A Chart of Biography*, London.

[19] Vinci L. da (c. 1500), *Notebooks*, Vol. M, Verso 40, Paris: Manuscripts of the Institute of France.

[20] Wren, C. (1750), *Parentalia: or, Memoirs of the Family of the Wrens*, London: Osborn.

II. Nineteenth Century Statistical Graphics

[21] Dupin, C. (1819), *Carte de la France Éclairée et de la France Obscure*, Paris.

[22] Fourier, J. B. J. (1821), "Notions Generales, sur la Population," *Recherches Statistiques sur la Ville de Paris et le Departement de la Seine*, 1, 1–70.

[23] Galton, F. (1885–6), "Regression towards Mediocrity in Hereditary Stature," *Journal of the Anthropological Institute*, 15, 246–263.

[24] ——— (1899), "A Geometric Determination of the Median Value of a System of Normal Variants from Two of Its Centiles," *Nature*, 16, 102–104.

[25] Guerry, A. M. (1883), *Essai sur la Statistique Morale de la France*, Paris.

[26] Herschel, J. F. W. (1833), "On the Investigation of the Orbits of Revolving Double Stars," *Memoirs of the Royal Astronomical Society*, 5, 171–172; read January 13, 1832.

[27] Howard, L. (1800), "On a Periodical Variation of the Barometer, Apparently due to the Influence of the Sun and Moon on the Atmosphere," *Philosophical Magazine*, 7, 355–363.

[28] Humboldt, F. W. K. H. A. von (1811), *Essai Politique sur le Royaume de la Nouvelle-Espagne, avec un Atlas Physique et Géographique*, Paris.

[29] International Statistical Congress (1858), "Emploi de la Cartographie et de la Méthode Graphique en Général pour les Besoins Spéciaux de la Statistique," *Proceedings*, 3rd Session, August 31–September 5, 1857, Vienna.

[30] Jevons, W. S. (1863), *A Serious Fall in the Value of Gold Ascertained, and Its Social Effects Set Fourth*, London.

[31] ——— (1879), "Graphical Method," in *Principles of Science*, 3rd edition, London, 492–496.

[32] Keith, A. (1800), "Description of a Thermometer, Which Marks the Greatest Degree of Heat and Cold from One Time of Observation to Another, and May Also Register Its Own Height at Every Instant," *Journal of Natural Philosophy, Chemistry and the Arts*, 3, 266–268.

[33] Lalanne, L. (1845), "Sur la Représentation Graphique des Tableaux Météorologiques et des Lois Naturelles en Général," Appendix to *Cours Complet de Météorologie de L. F. Kaemtz*, translated and annotated by C. Martins, Paris.

[34] ——— (1846), *Mémoire sur les Tables Graphiques et sur la Géométrie Anamorphique Appliquées à Diverses Questions qui se Rattachent a l'Art de l'Ingénieur*, Paris; read 1843.

[35] Levasseur, E. (1868), *La France, Avec Ses Colonies*, Paris.

[36] Minard, C. J. (1861), *Des Tableaux Graphiques et des Cartes Figuratives*, Paris.

[37] Mulhall, M. G. (1884), *Dictionary of Statistics*, London.

[38] Nightingale, F. (1857), *Mortality of the British Army*, London.

[39] Pearson, K. (1895), "Contributions to the Mathematical Theory of Evolution—II, Skew Variation in Homogeneous Material," *Philosophical Transactions*, 186, Pt. 1, 343–414.

[40] Perozzo, L. (1880), "Della Rappresentazione Graphica di una Collettività di Individui nella Successione del Tempo," *Annali di Statistica*, 12, 1–16.

[41] Playfair, W. (1801), *Statistical Breviary; Shewing, on a Principle Entirely New, the Resources of Every State and Kingdom in Europe*, London.

[42] Quetelet, A. (1827), "Recherches sur la Population, les Naissances, etc., dans le Royaume des Pays-Bas," *Nouveaux Mémoires de l'Académie Royale des Sciences et Belles-Lettres de Bruxelles*, 4, 117–174.

[43] ——— (1828), *Instructions Populaires sur le Calcul des Probabilités*, Brussels.

[44] ——— (1846), *Lettres sur la Théorie des Probabilités, Appliquée aux Sciences Morales et Politiques*, Brussels.

[45] Schwabe, H. (1872), "Theorie der Graphischen Darstellungen," *Proceedings of the International Statistical Congress*, 8th Session, Pt. 1, St. Petersburg.

[46] U.S. Bureau of the Census (1872), *Statistics of Wealth and Public Indebtedness*, Washington, D.C.: U.S. Government Printing Office.

[47] Vauthier, L. L. (1874), "Note sur une Carte Statistique Figurant la Répartition de la Population de Paris," *Comptes Rendus des Séances de l'Académie des Sciences*, 78, 264–267.

[48] Walker, F. A. (1874), *Statistical Atlas of the United States, Based on the Results of Ninth Census, 1870*, Washington, D.C.: U.S. Bureau of the Census.

[49] Zeuner, G. (1869), *Abhandlungen aus der Mathematischen Statistik*, Leipzig.

III. Early Twentieth Century Statistical Graphics

[50] Ayres, L. P. (1919), *The War with Germany, A Statistical Summry*, Washington, D.C.: U.S. Government Printing Office.

[51] Brinton, W. C. (1914), *Graphic Methods for Presenting Facts*. New York: Engineering Magazine.

[52] Croxton, F. E. (1927), "Further Studies in the Graphic Use of Circles and Bars," *Journal of the American Statistical Association*, 22, 36–39.

[53] ———, and Stein, H. (1932), "Graphic Comparisons by Bars, Squares, Circles and Cubes," *Journal of the American Statistical Association*, 27, 54–60.

[54] ———, and Stryker, R. E. (1927), "Bar Charts versus Circle Diagrams," *Journal of the American Statistical Association*, 22, 473–482.

[55] Eells, W. C. (1926), "The Relative Merits of Circles and Bars for Representing Component Parts," *Journal of the American Statistical Association*, 21, 119–132.

[56] Gilman, S. (1917), *Graphic Charts for the Business Man*, Chicago.

[57] Haskell, A. C. (1919), *How to Make and Use Graphic Charts*, New York: Codex.

[58] Hazen, A. (1914), "Storage to be Provided in Impounding Reservoirs for Municipal Water Supply," *Transactions of the American Society of Civil Engineers*, 77, 1529–1669; read December 17, 1913.

[59] Huhn, R. von (1927), "A Discussion of the Eells' Experiment," *Journal of the American Statistical Association*, 22, 31–36.

[60] Joint Committee on Standards for Graphic Presentation (1914–15), "Preliminary Report Published for the Purpose of Inviting Suggestions for the Benefit of the Committee," *Publications of the American Statistical Association*, 14, 790–797.

[61] Lorenz, M. O. (1905), "Methods of Measuring the Concentration of Wealth," *Publications of the American Statistical Association*, 9, 209–219.

[62] Martin, F. C., and Leavens, D. H. (1931), "A New Grid for Fitting a Normal Probability Curve to a Given Frequency Distribution," *Journal of the American Statistical Association*, 26, 178–183.

[63] Neurath, O. (1973), "From Vienna Method to Isotype" (papers written 1925–45), in *Empiricism and Sociology*, eds. M. Neurath and R. S. Cohen, Dordrecht, Neth.: Reidel, 214–248.

[64] Peddle, J. B. (1910), *The Construction of Graphical Charts*, New York: McGraw-Hill.

[65] Warne, F. J. (1916), *Warne's Book of Charts, A Special Feature of Warne's Elementary Course in Chartography*, Washington, D.C.: F. J. Warne.

[66] Washburne, J. N. (1927), "An Experimental Study of Var-

ious Graphic, Tabular and Textual Methods of Presenting Quantitative Material," *Journal of Educational Psychology*, 18, 361–376, 465–476.

IV. Post-World War II Statistical Graphics

[67] Anderson, E. (1957), "A Semigraphical Method for the Analysis of Complex Problems," *Proceedings of the National Academy of Sciences*, 13, 923–927; reprinted in *Technometrics*, 2, No. 3 (August 1960), 387–391.

[68] Andrews, D. F. (1972), "Plots of High Dimensional Data," *Biometrics*, 28, 125–136.

[69] Bachi, R. (1968), *Graphical Rational Patterns, A New Approach to Graphical Presentation of Statistics*, Jerusalem: Israel Universities Press.

[70] Barnard, G. (1969), "Summary Remarks," in *New Developments in Survey Sampling*, eds. N. L. Johnson and H. Smith, Jr., New York: Wiley-Interscience, 696–711.

[71] Biderman, A. D. (1971), "Kinostatistics for Social Indicators," *Educational Broadcasting Review*, 5, 13–19.

[72] Chernoff, H. (1973), "The Use of Faces to Represent Points in k-Dimensional Space Graphically," *Journal of the American Statistical Association*, 68, 361–368.

[73] ———, and Rizvi, M. H. (1975), "Effect on Classification Error of Random Permutations of Features in Representing Multivariate Data by Faces," *Journal of the American Statistical Association*, 70, 548–554.

[74] Cleveland, W. S., and Kleiner, B. (1975), "A Graphical Technique for Enhancing Scatterplots with Moving Statistics," Paper presented at the Annual Meeting of the American Statistical Association, Atlanta, Georgia.

[75] Ehrenberg, A. S. C. (1975), "Numerical Information Processing Project, London Graduate School of Business Studies," unpublished personal communication.

[76] Executive Office of the President, Office of Management and Budget (1973), *Social Indicators 1973*, Washington, D.C.: U.S. Government Printing Office.

[77] Fienberg, S. E. (1975), "*Perspective Canada* as a Social Report," unpublished paper, Department of Applied Statistics, School of Statistics, University of Minnesota.

[78] Gonzalez, M. E., Ogus, J. L., Shapiro, G., and Tepping, B. J. (1975), "Standards for Discussion and Presentation of Errors in Survey and Census Data," *Journal of the American Statistical Association*, 70, No. 351, Pt. 2, 5–23.

[79] Pickett, R. and White, B. W. (1966), "Constructing Data Pictures," *Proceedings of the 7th National Symposium of the Society for Information Display*, 75–81.

[80] Siegel, J. H., Goldwyn, R. M., and Friedman, H. P. (1971), "Pattern and Process of the Evolution of Human Septic Shock," *Surgery*, 70, 232–245.

[81] Tukey, J. W. (1965), "The Future of Processes of Data Analysis," *Proceedings of the Tenth Conference on the Design of Experiment in Army Research Development and Testing*, ARO-D Report 65-3, Durham, N.C.: U.S. Army Research Office, 691–729.

[82] ——— (1972), "Some Graphic and Semigraphic Displays," in *Statistical Papers in Honor of George W. Snedecor*, ed. T. A. Bancroft, Ames, Iowa: Iowa State University Press, 293–316; presented at the Annual Meeting of the American Statistical Association, August 1969.

[83] ——— (1977), *Exploratory Data Analysis*, Reading, Mass.: Addison-Wesley.

[84] U.S. Bureau of the Census (1976), *StatUS*, Washington, D.C.: U.S. Government Printing Office.

[85] ———, and Manpower Administration (1974), *Urban Atlas*, Series GE-80, Washington, D.C.: U.S. Government Printing Office.

[86] Wainer, H. (1974), "The Suspended Rootogram and Other Visual Displays: An Empirical Validation," *American Statistician*, 28, 143–145.

[87] ———, and Reiser, M. (1976), "Assessing the Efficacy of Visual Displays," in American Statistical Association, *Proceedings of the Social Statistics Section*, Part 1, 19, 89–92.

V. Secondary Sources

[88] Barabba, V. (1975), "The Graphic Presentation of Statistics," in U.S. Bureau of the Census, *The Census Bureau, A Numerator and Denominator for Measuring Change*, Technical Paper 37, Washington, D.C.: U.S. Government Printing Office, 89–103.

[89] Beniger, J. R. (1975), "Annotated Bibliography: History of Quantitative and Statistical Graphics," Paper accompanying slide presentation at the Annual Meeting of the American Statistical Association, Atlanta, Georgia.

[90] ——— (1976), "Science's 'Unwritten' History: The Development of Quantitative and Statistical Graphics," Paper presented to the Annual Meeting of the American Sociological Association, New York.

[91] ——— (1976), "From Stylus to Light Pen: Technology and Innovation in the Development of Quantitative Graphics," *Computer Graphics and Art*, 1, 18–27.

[92] Feinberg, B. M., and Franklin, C. A., eds. (1975), *Social Graphics Bibliography*, Washington, D.C.: Bureau of Social Science Research.

[93] Fienberg, S. E., and Goodman, L. A. (1974), "Social Indicators 1973: Statistical Considerations," in *Social Indicators 1973: A Review Symposium*, ed. R. A. Van Dusen, Washington, D.C.: Social Science Research Council, Center for the Coordination of Research on Social Indicators, 63–85.

[94] Friis, H. R. (1974), "Statistical Cartography in the United States Prior to 1870 and the Role of Joseph C. G. Kennedy and the U.S. Census Office," *American Cartographer*, 1, 131–157.

[95] Funkhouser, H. G. (1936), "A Note on a Tenth Century Graph," *Osiris*, 1, 260–262.

[96] ——— (1937), "Historical Development of the Graphical Representation of Statistical Data," *Osiris*, Pt. 1 (November 1937), 269–405; reprinted Brugge, Belgium: St. Catherine Press, 1937.

[97] Gunther, R. T. (1930), "The Life and Work of Robert Hooke," *Early Science in Oxford*, Vol. 6, Oxford: Clarendon Press.

[98] Hoff, H. E., and Geddes, L. A. (1959), "Graphic Recording Before Carl Ludwig: An Historical Summary," *Archives Internationales d'Histoire des Sciences*, 12, 3–25.

[99] ———, and Geddes, L. A. (1962), "The Beginnings of Graphic Recording," *Isis*, 53, Pt. 3, 287–324.

[100] Kruskal, W. (1977), "Visions of Maps and Graphs," *Proceedings of the International Symposium on Computer-Assisted Cartography, Auto-Carto II*, 1975, 27–36.

[101] Marey, E. J. (1878), *La Méthode Graphique dans les Sciences Expérimentales et Principalement en Physiologie et en Médecine*, Paris: G. Masson.

[102] Middleton, W. E. K. (1964), *History of the Barometer*, Baltimore, Md.: Johns Hopkins University Press.

[103] Rée, J. (1974), *Descartes*, London: Allen Lane (Penguin).

[104] Royston, E. (1970), "Studies in the History of Probability and Statistics, III. A Note on the History of the Graphical Presentation of Data," *Biometrika*, 43, Pts. 3 and 4 (December 1956), 241–247; reprinted in *Studies in the History of Statistics and Probability Theory*, eds. E. S. Pearson and M. G. Kendall, London: Griffin.

[105] Tilling, L. (1975), "Early Experimental Graphs," *British Journal for the History of Science*, 8, 193–213.