Asymmetric Key Cryptography

Using a private key (not shared) and a public key (which is distributed) for encryption

Maths

Euler's Phi/Totient Function

- $\phi(1) = 0$
- $\phi(p) = p - 1$ for p -> Prime number
- $\phi(m\ x\ n) = \phi(m) \times \phi(n)$ for `m`, `n` are relative primes
- $\phi(p^e) = p^e - p^{e-1}$ for p-> prime number

Fermat's Little Theorem

> **Note**
>
> Only apply if p -> Prime number

1. $a^{p-1} = 1\ mod\ p$
2. $a^p = a\ mod\ p$
3. $a^{-1} mod\ p\ = a^{p-2}\ mod\ p$

Euler's Theorem

1. $a^{\phi(n)} = 1\ (mod\ n)$
2. $a^{k\ x\ \phi(n)+1} = a\ (mod\ n)$
3. $a^{-1} mod\ n\ = a^{\phi(n)-1}\ mod\ n$

Principles

Authentication

- Can be implemented by using the keys (public and private) from the same key source
- For example: in the communication between A and B if A's private key is used for encryption and A's public key is used for decryption by B, A is the aunthenticated sender

Confidentiality

- Using double encryption and decryption:
  - Encrypt using the private key of A and public key of B
  - Decrypt using the private key of B and public key of A

RSA

Key Generation

1. Select 2 **relatively prime** numbers (p,q)
2. $n = p * q$
3. $\phi(n) = (p-1)(q-1)$
4. Choose value of `e` such that $1 < e < \phi(n)$ and $gcd(e, \phi(n)) = 1$
5. $d = e^{-1} mod\, \phi(n)$

Private Key

$$\{e, n\}$$

Public Key

$$\{d, n\}$$

Encryption

$$c = m^e mod\, n$$

Decryption

$$m = c^d mod\, n$$

Potential Attacks

- Factorisation
- Chosen Cipher Text - Intercept and masking plaintext
- Encryption Exponent - When the encryption exponent is set too low (Recommended: Prime numbers around $2^{16} + 1$
  - Coppersmith
    - For $C = M^e mod\, N$
    - If $M < N^{\frac{1}{e}}$, M can be directly recovered by taking `e`th root of C
  - Broadcast
    - If Alice sends the same message with same exponent to 3 different recipients, attacker (Eve) can use chinese remainder theorem to decrypt the message
  - Related messages (Franklin Reiter)

        - If 2 messages (P1 and P2) were encrypted with same `e`, and if P1 is related to P2 in by a *linear function*, Eve can decrypt the corresponding C1 and C2 in feasible computation time
    - Short pad (Coppersmith)
        - Alice pads with r1, encrypts and sends message to Bob
        - Eve intercepts and drops message (C1)
        - Bob requests message again and Alice sends again with padding of r2
        - Eve intercepts again (C2)
        - Eve knows C1 and C2 has same plaintext
        - If r1 and r2 are short, Eve can recover original plaintext
- Decryption Exponent - Releaved and low exponent
- Plaintext
    - Short message
        - Short messages can be easily decrypted by brute force
        - Use padding (in the start or end of message) using `OAEP` to prevent this
    - Cyclic
    - Unconcealed
- Modulus - Common modulus
- Implementation
    - Timing and power (Paul Kocher)
        - Blinding and Random Delays can help with this

Diffie Hellman Key Exchange Algorithm

Procedure

1. Consider a prime number q
2. Select α such that $\alpha < q$ and α is *primitive root* of q
3. Assume X_A (Private key of A) and $X_A < q$. Calculate $Y_A = \alpha^{X_A} mod\, q$ (Public Key of A)
4. Repeat previous step for B
5. Calculate secret keys

- $K_A = (Y_B)^{X_A} mod\, q$
- $K_B = (Y_A)^{X_B} mod\, q$

6. If $K_A == K_B$, key exchange succesfull

Man in the middle attack

Insert a man in the middle of A and B (Eve) and carry out diffie hellman exchange with each A and B

> **Todo**
>
> Complete this section

El-Gamal

Key Generation

1. Select prime number P
2. Select private key d
3. Select 2nd part of encryption key e_1
4. Select 3rd part of encryption key e_2
    - $e_2 = e_1^d mod\ P$
5. Public key = (e_1, e_2, P), Private Key = d

Encryption

1. Select random integer r
2. Calculate $C_1 = e_1^r mod\ P$
3. Calculate $C_2 = (M * e_2^r) mod\ P$
4. Cipher Text = (C_1, C_2)

Decryption

1. $M = [C_2 * (C_1^D)^{-1}] mod\ P$

Knapsack Algorithm

> **Todo**
>
> Complete this section

Given:

- Sum (S)
- Weights (W) - Superincreasing tuple

Key Generation

Public Key (Hard Knapsack)

Private Key (Easy Knapsack)

Key Distribution

Methods

- Public Announcement
- Public Key Directory
- Public Key Authority
- Certificate Authority

Digital Signature

A cryptosystem uses the private and public keys of the receiver: a digital signature uses the private and public keys of the sender

Services

- Message Authentication
- Message Integrity
- Nonrepudiation

Attacks Types

- Key Only Attack
- Known Message Attack
- Chosen Message Attack

Forgery Types

- Existential Forgery
- Selective Forgery

Algorithms

RSA Digital Signature

- Signing
  - $S = M^d mod\, n$
- Verifying
  - $M' = S^e mod\, n$

- If $M' == M$, accept

With Message Digest

- Signing
  - $D = h(M)$ (Digest)
  - $S = D^d mod\, n$
- Verifying
  - $D' = S^e mod\, n$
  - If $D' == h(M)$, accept

ElGamal Digital Signature

- Public key - (e_1, e_2, p), Private key - d
- Signing
  - $S_1 = e_1^r mod\, p$
  - $S_2 = (M - dS_1)r^{-1} mod(p-1)$
- Verifying
  - $V_1 = e_1^M mod\, p$
  - $V_2 = e_2^{S_1} S_1^{S_2} mod\, p$
  - If $V_1 == V_2$, accept

Schnorr Digital Signature

- Public key - (e_1, e_2, p, q), Private key - d
- Signing
  - $S_1 = h(\ M \mid e_1^r mod\, p\ )$
  - $S_2 = r + dS_1\ mod\ q$
- Verifying
  - $V = h(\ M \mid e_1^{S_1} e_2^{-S_2} mod\, p\ )$
  - If $V == S_1$, accept (S_1 is congruent to $V\, mod\, p$)

Digital Signature Standard

- Public key - (e_1, e_2, p, q), Private key - d
- Signing
  - $S_1 = (e_1^r\ mod\, p)\ mod\, q$
  - $S_2 = (h(M) + dS_1)\ r^{-1}\ mod\, q$
- Verifying
  - $V = (e_1^{h(M)S_2^{-1}}\ e_2^{S_1 S_2^{-1}}\ mod\, p)\ mod\, q$
  - If $V == S_1$, accept (S_1 is congruent to $V\, mod\, p$)
- Properties:

    - Faster computation wrt RSA
    - Smaller signatures wrt ElGamal

Elliptic Curve Digital Signature

Application

- Time Stamped Signatures
    - Time stamped to prevent replay of documents
- Blind Signatures (David Chaum)
    - Signing without knowing content of message
    - Procedure
        - Blinding the message from Bob: $B = M \times b^e mod\, n$ (`e` -> Alice Public Key, `b` -> Blinding Factor)
        - Signing by Alice: $S_{blind} = B^d\, mod\, n$
        - Unblind by Bob: $S = S_b b^{-1}\, mod\, n$