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* - Important Derivations

Unit 1

Coulombs Law

$$F = rac{1}{4\piarepsilon} \cdot rac{Q_1Q_2}{r^2}$$

Electric Fields

Point charge

$$D=rac{Q}{4\pi r^2}a_r$$

$$E = rac{Q}{4\piarepsilon r^2} a_r$$

Line charge

$$E=rac{-
ho_L}{4\piarepsilon_o
ho}\int_{lpha_1}^{lpha_2} [cos(lpha)a_p+sin(lpha)a_z]dlpha$$

- ullet Where $lpha_1$ and $lpha_2$ are angles that the line subtends to the point
- Infinite line charge -

$$E=rac{
ho_L}{2\piarepsilon_o
ho}a_p$$

Surface charge

• Electric field due to surface charge is

$$E=\int_{S}rac{
ho_{s}dS}{4\piarepsilon_{0}R^{2}}a_{R}$$

• Substituting $dS
ightarrow
ho \; d\phi \; d
ho$ and solving for **Infinite Sheet Charge**

$$E=rac{
ho_s}{2arepsilon_0}a_n$$

ullet Where a_n is unit vector perpendicular to sheet

Volume charge

• Electric field due to volume charge is

$$E=\int_{S}rac{
ho_{s}dV}{4\piarepsilon_{0}R^{2}}a_{R}$$

• With $a_R = cos(lpha) a_Z - sin(lpha) a_p$

$$E=rac{Q}{4\piarepsilon_o}z^2$$

$$Q=
ho_vrac{4}{3}\pi a^3$$

$$\overline{E}_z = rac{
ho_v}{4\piarepsilon_o z^2} rac{4\pi a^3}{3}$$

Electric Flux Density

Also known as Electric Displacement

$$\overline{D}=arepsilon_o\overline{E}$$

$$\Psi = \int_S \overline{D} \cdot d\overline{S}$$

Gradient

$$abla A = lim_{\Delta v
ightarrow 0} rac{\int_S A \cdot dS}{\Delta V}$$

Cartesian

$$abla A = rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$$

Cylindrical

$$abla A = rac{\partial A_
ho}{\partial
ho} + rac{1}{
ho} rac{\partial A_\phi}{\partial \phi} + rac{\partial A_z}{\partial z}$$

Spherical

$$abla A = rac{\partial A_r}{\partial r} + rac{1}{r}rac{\partial A_ heta}{\partial heta} + rac{1}{r sin heta}rac{\partial A_\phi}{\partial \phi}$$

Divergence



General formula

$$abla \cdot A = rac{1}{h_1 h_2 h_3} [rac{\partial \ h_2 h_3}{\partial u_1} + rac{\partial \ h_1 h_3}{\partial u_2} + rac{\partial \ h_1 h_2}{\partial u_3}] A$$

Divergence Theorem

$$\int_S A \cdot dS = \int_V
abla \cdot A \ dV$$

 Total outward flux of a vector field through closed surface S is same as volume integral of the divergence of A

Gauss law

$$\Psi = \int_S D \cdot dS = Q_{enclosed} = \int_V
ho_v \ dV$$

Applying divergence theorem

$$\int_S D \cdot dS = \int_V
abla \cdot D \; dV$$

Comparing the equations :- We get Maxwell's 1st equation

$$\rho_V = \nabla \cdot D$$

Applying Gauss law

Infinite line charge

$$ho_L l = Q = \int_S D \cdot dS = D_
ho \int_S dS = D_
ho \ 2\pi \
ho \ l \
onumber \ D_
ho = rac{
ho_L l}{2\pi \
ho \ l} \
ightarrow D = rac{
ho_L}{2\pi \
ho} a_
ho$$

Infinite sheet charge

Uniformly charged sphere

$$D = egin{cases} rac{r}{3}
ho_o \, a_r & ext{if } 0 < r \leq a \ rac{a^3}{3 \, r^2}
ho_o \, a_r & ext{if } r \geq a \end{cases}$$

Electric Potential

$$V_{BA} = rac{W}{Q} = -\int_A^B E \cdot dl$$

 Absolute potential - work done per charge to bring it from infinity to r

$$V = -\int_{-\infty}^{r} E \cdot dl$$

Relation between E and V

For a closed loop:-

$$\int_L E \cdot dl = 0$$

⊘ Note

Stoke's Theorem

$$\int\int_S (\overline{
abla} imes \overline{B}).\,dS = \oint_L B.\,dar{l}$$

Curl

$$abla imes \mathbf{F} = egin{array}{cccc} \hat{m{i}} & \hat{m{j}} & \hat{m{k}} \ rac{\partial}{\partial x} & rac{\partial}{\partial y} & rac{\partial}{\partial z} \ F_x & F_y & F_z \ \end{array}$$

Applying stoke's theorem

$$\int_L E \cdot dl = \int_S (
abla imes E) \cdot dS = 0 \; (or) \;
abla imes E = 0$$

• Electric field is the gradient of V :-

$$E = -\nabla V$$

Continuity Equation

$$I_{out} = \oint J.\,dS = -rac{dQ_{in}}{dt}$$

• With divergence theorem :-

$$\oint J.\,dS = \int_{v}
abla .\,J\,\,dV$$

Also

$$-rac{dQ_{in}}{dt}=-\int_{v}rac{\partial
ho_{v}}{\partial t}dV$$

• Therefore Continuity Equation

$$abla.J = -rac{\partial
ho_v}{\partial t}$$



For DC current

$$rac{\partial
ho_v}{\partial t} = 0$$

Therefore

$$\nabla . J = 0$$

Relaxation time

$$ho_v =
ho_{vo} \ e^{-rac{t}{T_r}} \ ext{where} \ T_r = rac{arepsilon}{\sigma}$$

Poisson's Equation

$$abla^2 V = -rac{
ho_v}{arepsilon_o}$$

Note

Laplacian operator :-

General Formula

$$\nabla^2 A = \frac{1}{h_1 h_2 h_3} \left[\, \frac{\partial}{\partial a_1} \, \left(\frac{h_2 h_3}{h_1} \, \frac{\partial A}{\partial a_1} \right) + \frac{\partial}{\partial a_2} \, \left(\frac{h_1 h_3}{h_2} \, \frac{\partial A}{\partial a_2} \right) + \frac{\partial}{\partial a_3} \, \left(\frac{h_1 h_2}{h_3} \, \frac{\partial A}{\partial a_3} \right) \, \right]$$

- Cartesian (x, y, z) (h1, h2, h3) = (1, 1, 1)
- Cylindrical $(
 ho,\phi,z)$ (h1,h2,h3)=(1,
 ho,1)
- Spherical $(r, heta,\phi)$ (h1,h2,h3)=(1,r,rsin(heta))

Laplace Equation

If there is no charge enclosed in the volume :-

$$\nabla^2 V = 0$$

Unit 2

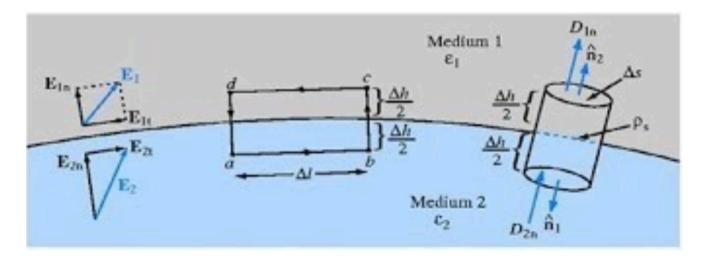
Boundary conditions

Maxwell's Equations :-

$$\oint_S \overline{D}.\,d\overline{S} = Q_{enc}
ightarrow 1^{st} ext{ Equation}$$

$$\oint_L \overline{E}.\, d\overline{l} = 0
ightarrow 2^{nd} ext{ Equation}$$

Dielectric - Dielectric



$$\varepsilon_1 = \varepsilon_o \, \varepsilon_{r1} \, (\mathrm{and}) \, \varepsilon_2 = \varepsilon_o \, \varepsilon_{r2}$$

• For a closed path abcda. The tangential components of an electric field crossing the interface undergoes no change :-

$$E_{1t} = E_{2t}$$

0r

$$rac{D_{1t}}{arepsilon_1} = rac{D_{2t}}{arepsilon_2}$$

• Similarly for cylindrical Gaussian surface

$$D_{1n} - D_{2n} = \rho_s$$

• Or when $ho_s=0$ where ho_s is charge density in the interface surface

$$arepsilon_1 E_{1n} = arepsilon_2 E_{2n}$$

Refraction

$$rac{ an heta_1}{ an heta_2} = rac{arepsilon_1}{arepsilon_2} = rac{arepsilon_{r1}}{arepsilon_{r2}}$$

• θ_1 , θ_2 -> angle of electric field with normal to surface in each medium

Conductor - Dielectric

$$E_t = 0$$

$$D_n = \rho_s$$

Conductor - Free space

$$D_t = \varepsilon_o E_t = 0$$

$$D_n = \varepsilon_o E_n = \rho_s$$

Drift velocity

$$v_e = -\mu_e \overline{E}$$

Ohms law

$$R = rac{V}{I}$$

Point form

$$\overline{J}=\sigma\overline{E}$$

Resistance

$$R = rac{l}{\sigma A}$$

Conductivity

$$G = rac{\sigma A}{l} = rac{A}{
ho \ l}$$

Joules law

$$P(ext{power}) = \int_v \overline{J} \cdot \overline{E} dV$$

Polarisation

$$\overline{D}=\varepsilon_o\overline{E}+\overline{P}$$

• P -> electric polarisation field

$$\overline{P} = arepsilon_o \chi_e \overline{E}$$
 $\overline{D} = arepsilon_o \overline{E} + \overline{P} = arepsilon \overline{E}$ $arepsilon_r = arepsilon_o (1 + \chi_e)$

- χ_e -> Susceptibility
- ullet $arepsilon_r$ -> relative permeability or dielectric strength

Capacitance

$$C = \frac{Q}{V}$$

$$RC = \frac{\varepsilon}{\sigma}$$

Parallel Plate

$$C=rac{Q}{V}=rac{arepsilon S}{d}$$
 $W_E=rac{1}{2}\int_v arepsilon E^2 dv=rac{1}{2}CV^2=rac{1}{2}QV=rac{Q^2}{2C}$

Coaxial

$$E=rac{Q}{2\piarepsilon L}lnrac{b}{a}$$

$$C=rac{2\piarepsilon L}{lnrac{b}{a}}$$

Spherical

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

Configurations

Parallel

$$C=C_1+C_2$$

Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$