Electrostatics_Formulae

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`* - Important Derivations`

Unit 1

Coulombs Law

$$F = \frac{1}{4\pi\varepsilon} \cdot \frac{Q_1 Q_2}{r^2}$$

Electric Fields

Point charge

$$D = \frac{Q}{4\pi r^2} a_r$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2} a_r$$

Line charge

$$E = \frac{-\rho_L}{4\pi\varepsilon_o \rho} \int_{\alpha_1}^{\alpha_2} [cos(\alpha) a_p + sin(\alpha) a_z] d\alpha$$

- Where α_1 and α_2 are angles that the line subtends to the point
- Infinite line charge -

$$E = \frac{\rho_L}{2\pi\varepsilon_o \rho} a_p$$

Surface charge

- Electric field due to surface charge is

$$E = \int_S \frac{\rho_s dS}{4\pi\varepsilon_0 R^2} a_R$$

- Substituting $dS \rightarrow \rho \, d\phi \, d\rho$ and solving for **Infinite Sheet Charge**

$$E = \frac{\rho_s}{2\varepsilon_0} a_n$$

- Where a_n is unit vector perpendicular to sheet

Volume charge

- Electric field due to volume charge is

$$E = \int_S \frac{\rho_s dV}{4\pi\varepsilon_0 R^2} a_R$$

- With $a_R = cos(\alpha)a_Z - sin(\alpha)a_p$

$$E = \frac{Q}{4\pi\varepsilon_o} z^2$$

$$Q = \rho_v \frac{4}{3}\pi a^3$$

Electric Flux Density

Also known as Electric Displacement

$$\overline{D} = \varepsilon_o \overline{E}$$

$$\Psi = \int_S \overline{D} \cdot d\overline{S}$$

Gradient

$$\nabla A = lim_{\Delta v \to 0} \frac{\int_S A \cdot dS}{\Delta V}$$

Cartesian

$$\nabla A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cylindrical

$$\nabla A = \frac{\partial A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical

$$\nabla A = \frac{\partial A_r}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{1}{rsin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Divergence

Note

General formula

$$\nabla \cdot A = \frac{1}{h_1 h_2 h_3}[\frac{\partial \, h_2 h_3}{\partial u_1} + \frac{\partial \, h_1 h_3}{\partial u_2} + \frac{\partial \, h_1 h_2}{\partial u_3}]A$$

Divergence Theorem

$$\int_S A \cdot dS = \int_V \nabla \cdot A \, dV$$

- Total outward flux of a vector field through closed surface S is same as volume integral of the divergence of A

Gauss law

$$\Psi = \int_S D \cdot dS = Q_{enclosed} = \int_V \rho_v \, dV$$

- Applying divergence theorem

$$\int_S D \cdot dS = \int_V \nabla \cdot D \, dV$$

- Comparing the equations :- We get Maxwell's 1st equation

$$\rho_V = \nabla \cdot D$$

Applying Gauss law

- Infinite line charge

$$\rho_L l = Q = \int_S D \cdot dS = D_\rho \int_S dS = D_\rho \, 2\pi \, \rho \, l$$

$$D_\rho = \frac{\rho_L l}{2\pi \, \rho \, l} \rightarrow D = \frac{\rho_L}{2\pi \, \rho} a_\rho$$

- Infinite sheet charge

$$\rho_S A = D_z(A + A) = 2D_z A$$

$$D = D_z a_z = \frac{\rho_s}{2} a_z \, (or) \, E = \frac{\rho_s}{2\varepsilon_o} a_z$$

- Uniformly charged sphere

$$D = \begin{cases} \frac{r}{3} \, \rho_o \, a_r & \text{if } 0 < r \leq a \\ \frac{a^3}{3\, r^2} \, \rho_o \, a_r & \text{if } r \geq a \end{cases}$$

Electric Potential

$$V_{BA} = \frac{W}{Q} = -\int_A^B E \cdot dl$$

- Absolute potential - work done per charge to bring it from infinity to r

$$V = -\int_{\infty}^{r} E \cdot dl$$

Relation between E and V

- For a closed loop:-

$$\int_L E \cdot dl = 0$$

> **Note**
>
> **Stoke's Theorem**
>
> $$\int\int_S (\overline{\nabla} \times \overline{B}).\, dS = \oint_L B.\, d\overline{l}$$
>
> **Curl**
>
> $$\nabla \times \mathbf{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

- Applying **stoke's theorem**

$$\int_L E \cdot dl = \int_S (\nabla \times E) \cdot dS = 0 \; (or) \; \nabla \times E = 0$$

- Electric field is the gradient of V :-

$$E = -\nabla V$$

Continuity Equation

$$I_{out} = \oint J.\, dS = -\frac{dQ_{in}}{dt}$$

- With divergence theorem :-

$$\oint J.\, dS = \int_v \nabla.\, J\, dV$$

- Also

$$-\frac{dQ_{in}}{dt} = -\int_v \frac{\partial \rho_v}{\partial t} dV$$

- Therefore **Continuity Equation**

$$\nabla . J = -\frac{\partial \rho_v}{\partial t}$$

> **Note**
>
> For DC current
>
> $\frac{\partial \rho_v}{\partial t} = 0$
>
> Therefore
>
> $\nabla . J = 0$

Relaxation time

$$\rho_v = \rho_{vo} \, e^{-\frac{t}{T_r}} \text{ where } T_r = \frac{\varepsilon}{\sigma}$$

Poisson's Equation

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon_o}$$

> **Note**
>
> Laplacian operator :-
> General Formula
>
> $$\nabla^2 A = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial a_1} \left(\frac{h_2 h_3}{h_1} \frac{\partial A}{\partial a_1} \right) + \frac{\partial}{\partial a_2} \left(\frac{h_1 h_3}{h_2} \frac{\partial A}{\partial a_2} \right) + \frac{\partial}{\partial a_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial A}{\partial a_3} \right) \right]$$
>
> - Cartesian (x, y, z) - $(h1, h2, h3) = (1, 1, 1)$
> - Cylindrical (ρ, ϕ, z) - $(h1, h2, h3) = (1, \rho, 1)$
> - Spherical (r, θ, ϕ) - $(h1, h2, h3) = (1, r, rsin(\theta))$

Laplace Equation

If there is no charge enclosed in the volume :-

$$\nabla^2 V = 0$$

Unit 2

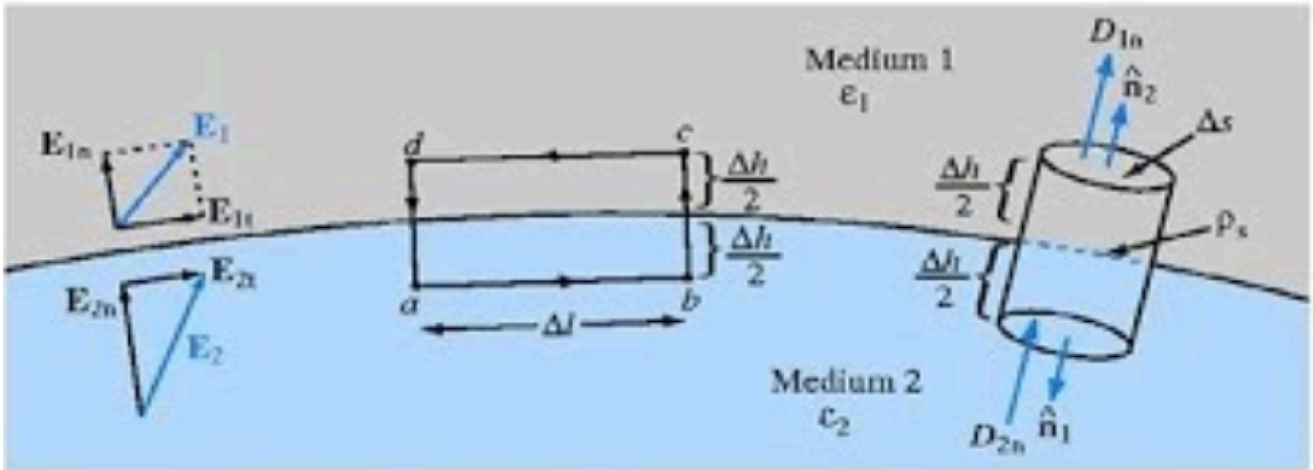
Boundary conditions

Maxwell's Equations :-

$$\oint_S \overline{D}.\, d\overline{S} = Q_{enc} \rightarrow 1^{st} \text{ Equation}$$

$$\oint_L \overline{E}.\, d\overline{l} = 0 \rightarrow 2^{nd} \text{ Equation}$$

Dielectric - Dielectric



$$\varepsilon_1 = \varepsilon_o\, \varepsilon_{r1} \text{ (and) } \varepsilon_2 = \varepsilon_o\, \varepsilon_{r2}$$

- For a closed path `abcda`. The tangential components of an electric field crossing the interface undergoes no change :-

$$E_{1t} = E_{2t}$$

- Or

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

- Similarly for cylindrical Gaussian surface

$$D_{1n} - D_{2n} = \rho_s$$

- Or when $\rho_s = 0$ where ρ_s is charge density in the interface surface

$$\varepsilon_1\, E_{1n} = \varepsilon_2\, E_{2n}$$

- Refraction

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

- θ_1, θ_2 -> angle of electric field with normal to surface in each medium

Conductor - Dielectric

$$E_t = 0$$

$$D_n = \rho_s$$

Conductor - Free space

$$D_t = \varepsilon_o E_t = 0$$

$$D_n = \varepsilon_o E_n = \rho_s$$

Drift velocity

$$v_e = -\mu_e \overline{E}$$

Ohms law

$$R = \frac{V}{I}$$

- Point form

$$\overline{J} = \sigma \overline{E}$$

Resistance

$$R = \frac{l}{\sigma A}$$

Conductivity

$$G = \frac{\sigma A}{l} = \frac{A}{\rho\, l}$$

Joules law

$$P(\text{power}) = \int_v \overline{J} \cdot \overline{E} dV$$

Polarisation

$$\overline{D} = \varepsilon_o \overline{E} + \overline{P}$$

- P -> electric polarisation field

$$\overline{P} = \varepsilon_o \chi_e \overline{E}$$

$$\overline{D} = \varepsilon_o \overline{E} + \overline{P} = \varepsilon \overline{E}$$

$$\varepsilon_r = \varepsilon_o(1 + \chi_e)$$

- χ_e -> Susceptibility
- ε_r -> relative permeability or dielectric strength

Capacitance

$$C = \frac{Q}{V}$$

$$RC = \frac{\varepsilon}{\sigma}$$

Parallel Plate

$$C = \frac{Q}{V} = \frac{\varepsilon S}{d}$$

$$W_E = \frac{1}{2}\int_v \varepsilon E^2 dv = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

Coaxial

$$E = \frac{Q}{2\pi\varepsilon L} ln\frac{b}{a}$$

$$C = \frac{2\pi\varepsilon L}{ln\frac{b}{a}}$$

Spherical

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

Configurations

Parallel

$$C = C_1 + C_2$$

Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$