Formulae

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Unit 1

Point charge

$$D=rac{Q}{4\pi r^2}a_r$$

Line charge

$$E=rac{-
ho_L}{4\piarepsilon_o
ho}\int_{lpha_1}^{lpha_2} \left[cos(lpha)a_p+sin(lpha)a_z
ight]\!dlpha$$

- Where α_1 and α_2 are angles that the line subtends to the point
- Infinite line charge -

$$E=rac{
ho_L}{2\piarepsilon_o
ho}a_p$$

Surface charge

• Electric field due to surface charge is

$$E=\int_{S}rac{
ho_{s}dS}{4\piarepsilon_{0}R^{2}}a_{R}$$

- Substituting $dS o
ho \; d\phi \; d
ho$ and solving for Infinite Sheet Charge

$$E=rac{
ho_s}{2arepsilon_0}a_n$$

ullet Where a_n is unit vector perpendicular to sheet

Volume charge

Electric field due to volume charge is

$$E=\int_{S}rac{
ho_{s}dV}{4\piarepsilon_{0}R^{2}}a_{R}$$

ullet With $a_R=cos(lpha)a_Z-sin(lpha)a_p$

$$E=rac{Q}{4\piarepsilon_o}z^2$$

$$Q=
ho_vrac{4}{3}\pi a^3$$

Electric Flux Density

Also known as Electric Displacement

$$D = \varepsilon_o E$$

$$\psi = \int_S D \cdot dS$$

Divergence

$$abla \cdot A = lim_{\Delta v
ightarrow 0} rac{\int_S A \cdot dS}{\Delta V}$$

Cartesian

$$abla \cdot A = rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$$

Cylindrical

$$abla \cdot A = rac{\partial A_
ho}{\partial
ho} + rac{1}{
ho} rac{\partial A_\phi}{\partial \phi} + rac{\partial A_z}{\partial z}$$

Spherical

$$abla \cdot A = rac{\partial A_r}{\partial r} + rac{1}{r}rac{\partial A_ heta}{\partial heta} + rac{1}{r sin heta}rac{\partial A_\phi}{\partial \phi}$$

Divergence Theorem

$$\int_S A \cdot dS = \int_V
abla \cdot A \ dV$$

Total outward flux of a vector field through closed surface S is same as volume integral
of the divergence of A

Gauss law

$$\Psi = \int_S D \cdot dS = Q_{enclosed} = \int_V
ho_v \ dV$$

Applying divergence theorem

$$\int_S D \cdot dS = \int_V
abla \cdot D \ dV$$

Comparing the equations :- We get Maxwell's 1st equation

$$\rho_v = \nabla \cdot D$$

Applying Gauss law

Infinite line charge

$$ho_L l = Q = \int_S D \cdot dS = D_
ho \int_S dS = D_
ho \ 2\pi \
ho \ l \
onumber \ D_
ho = rac{
ho_L l}{2\pi \
ho \ l} \
ightarrow D = rac{
ho_L}{2\pi \
ho} a_
ho$$

Infinite sheet charge

$$ho_S A = D_z (A+A) = 2 D_z A \
onumber \ D = D_z a_z = rac{
ho_s}{2} a_z \ (or) \ E = rac{
ho_s}{2 arepsilon_o} a_z
onumber$$

Uniformly charged sphere

$$D = egin{cases} rac{r}{3}
ho_o \, a_r & ext{if } 0 < r \leq a \ rac{a^3}{3 \, r^2}
ho_o \, a_r & ext{if } r \geq a \end{cases}$$

Electric Potential

$$V_{BA} = rac{W}{Q} = -\int_A^B E \cdot dl$$

Absolute potential - work done per charge to bring it from infinity to r

$$V = -\int_{\infty}^{r} E \cdot dl$$

Relation b/w E and V

For a closed loop:-

$$\int_L E \cdot dl = 0$$



Stoke's Theorem

$$\int\int_S (\overline{
abla} imes \overline{B}).\,dS = \oint_S B.\,d\overline{l}$$

Curl

$$abla extbf{x} extbf{F} = egin{array}{ccccc} oldsymbol{\hat{i}} & oldsymbol{\hat{j}} & oldsymbol{\hat{k}} \ rac{\partial}{\partial x} & rac{\partial}{\partial y} & rac{\partial}{\partial z} \ F_x & F_y & F_z \ \end{array}$$

Applying stoke's theorem

$$\int_L E \cdot dl = \int_S (
abla imes E) \cdot dS = 0 \; (or) \;
abla imes E = 0 \; .$$

Electric field is the gradient of V :-

$$E = -\nabla V$$

Continuity Equation

$$I_{out} = \oint J.\,dS = -rac{dQ_{in}}{dt}$$

· With divergence theorem :-

$$\oint J.\,dS = \int_v
abla .\,J\,\,dV$$

Also

$$-rac{dQ_{in}}{dt}=-\int_{v}rac{\partial
ho_{v}}{\partial t}dV$$

• Therefore Continuity Equation

$$abla.J = -rac{\partial
ho_v}{\partial t}$$



For DC current

$$rac{\partial
ho_v}{\partial t} = 0$$

Therefore

$$\nabla . J = 0$$

Relaxation time

$$ho_v =
ho_{vo} \ e^{-rac{t}{T_r}} ext{ where } T_r = rac{arepsilon}{\sigma}$$

Poisson's Equation

$$abla^2 V = -rac{
ho_v}{arepsilon_o}$$



Laplacian operator :-

General Formula

$$\nabla^2 A = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial a_1} \left(\frac{h_2 h_3}{h_1} \frac{\partial A}{\partial a_1} \right) + \frac{\partial}{\partial a_2} \left(\frac{h_1 h_3}{h_2} \frac{\partial A}{\partial a_2} \right) + \frac{\partial}{\partial a_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial A}{\partial a_2} \right) \right]$$

Laplace Equation

If there is no charge enclosed in the volume :-

$$\nabla^2 V = 0$$

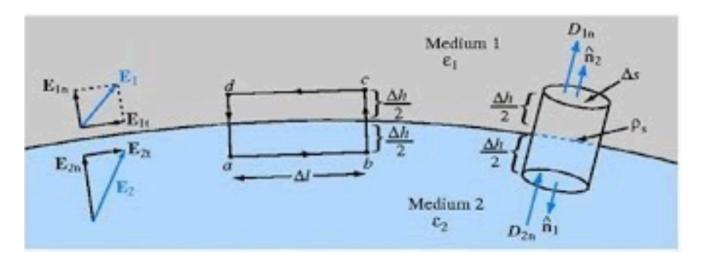
Boundary conditions

Maxwell's Equations :-

$$\oint_S \overline{D}.\,d\overline{S} = Q_{enc}
ightarrow 1^{st} ext{ Equation}$$

$$\oint_L \overline{E}.\, d\overline{l} = 0
ightarrow 2^{nd} ext{ Equation}$$

Dielectric - Dielectric



$$\varepsilon_1 = \varepsilon_o \ \varepsilon_{r1} \ (\mathrm{and}) \ \varepsilon_2 = \varepsilon_o \ \varepsilon_{r2}$$

• For a closed path abcda. The tangential components of an electric field crossing the interface undergoes no change :-

$$E_{1t}=E_{2t}$$

Or

$$rac{D_{1t}}{arepsilon_1} = rac{D_{2t}}{arepsilon_2}$$

Similarly for cylindrical Gaussian surface

$$D_{1n} - D_{2n} = \rho_s$$

- Or when $\rho_s=0$ where ρ_s is charge density in the interface surface

$$arepsilon_1 \, E_{1n} = arepsilon_2 \, E_{2n}$$

Refraction

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

• θ_1 , θ_2 -> angle of electric field with normal to surface in each medium

Conductor - Dielectric

$$E_t = 0$$

$$D_n = \rho_s$$

Conductor - Free space

$$D_t = \varepsilon_o E_t = 0$$

$$D_n = \varepsilon_o E_n = \rho_s$$

Unit 2

Drift velocity

$$v_e = -\mu_e \overline{E}$$

Ohm's law

$$R = rac{V}{I}$$

Point form

$$\overline{J} = \sigma \overline{E}$$

Resistance

$$R=rac{l}{\sigma A}$$

Conductivity

$$G = rac{\sigma A}{l} = rac{A}{
ho \ l}$$

Joule's law

$$P(ext{power}) = \int_v \overline{J} \cdot \overline{E} dV$$

Polarisation

$$\overline{D} = arepsilon_o \overline{E} + \overline{P}$$

• P -> electric polarisation field

$$\overline{P}=arepsilon_o\chi_e\overline{E}$$
 $\overline{D}=arepsilon_o\overline{E}+\overline{P}=arepsilon\overline{E}$

$$arepsilon_r = arepsilon_o (1 + \chi_e)$$

- χ_e -> Susceptibility
- ε_r -> relative permeability or dielectric strength

Capacitance

$$C=rac{Q}{V}$$

$$RC = rac{arepsilon}{\sigma}$$

Parallel Plate

$$C = rac{Q}{V} = rac{arepsilon S}{d}$$

$$W_E=rac{1}{2}\int_v arepsilon E^2 dv =rac{1}{2}CV^2 =rac{1}{2}QV =rac{Q^2}{2C}$$

Coaxial

$$E = rac{Q}{2\piarepsilon L} lnrac{b}{a}$$

$$C=rac{2\piarepsilon L}{lnrac{b}{a}}$$

Spherical

$$C = rac{4\piarepsilon}{rac{1}{a} - rac{1}{b}}$$

Configurations

Parrallel

$$C = C_1 + C_2$$

Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$