Magnetism_Formulae

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`* - Important Derivations`

Part 1

Bio Savarts law

Differential

$$d\overline{H} = \frac{I}{4\pi}\frac{d\overline{L} \times \overline{r}}{|r|^3}$$

Note

Remember

- Cartesian

$$d\overline{L} = dxa_x + dya_y + dza_z$$

- Cylindrical

$$d\overline{L} = d\rho a_x + \rho d\phi a_\phi + dza_z$$

- Spherical

$$d\overline{L} = dra_r + rd\theta a_\theta + rsin(\theta)d\phi a_\phi$$

Magnetic Field Intensities

Line Segment

$$\overline{H} = \frac{I}{4\pi\rho}(cos(\alpha_1) + cos(\alpha_2))a_\phi$$

$$\overline{H} = \frac{I}{4\pi\rho}\frac{2L}{\sqrt{l^2 + 4\rho^2}}a_\phi$$

Circular coil

$$\overline{H} = \frac{I\rho^2}{2(\rho^2 + h^2)^{\frac{3}{2}}} a_z$$

Solenoid

$$\overline{H} = nIa_y$$

Amperes Law

$$\oint \overline{H} \cdot d\overline{l} = I_{enclosed} = \int_S \overline{J} \cdot d\overline{S}$$

$$\nabla \times \overline{H} = \overline{J}$$

Applications

Infinite Line Current

$$\overline{H} = \frac{I}{2\pi\rho} a_\phi$$

Infinite Sheet of current

$$\overline{H} = \pm\frac{k}{2} \times a_n$$

Infinite Length Coaxial Transmission Line

Hollow Cylinder

$$\overline{H} = 0\,;\ \rho < a$$

$$\overline{H} = \frac{I}{2\pi\rho} a_\phi\,;\ \rho > a$$

Solid Cylinder

$$\overline{H} = \frac{I\rho}{2\pi a^2} a_\phi\,;\ \rho < a$$

$$\overline{H} = \frac{I}{2\pi\rho} a_\phi\,;\ \rho > a$$

Coaxial Cable

$$\overline{H} = \frac{I\rho}{2\pi a^2} a_\phi\,;\ \rho < a$$

$$\overline{H} = \frac{I}{2\pi\rho} a_\phi\,;\ a \le \rho \le b$$

$$\overline{H} = \frac{I}{2\pi\rho}\,[1 - \frac{\rho^2 - b^2}{r_1^2 - b^2}]\,a_\phi\,;\,b \le \rho \le r_1$$

$$\overline{H} = 0\,;\,\rho > r_1$$

Toroid

$$\overline{H} = 0\,;\,\rho < \rho_o - a$$

$$\overline{H} = \frac{NI}{2\pi\rho}\,;\,\rho_o - a < \rho < \rho_o + a$$

$$\overline{H} = 0\,;\,\rho > \rho_0 + a$$

Stoke Law

$$\oint \overline{H} \cdot d\overline{l} = \int_S (\nabla \times \overline{H}) \cdot d\overline{S}$$

Magnetic Field Density

$$\overline{B} = \mu\overline{H}$$

Magnetic Flux

$$\Psi = \int_S \overline{B} \cdot d\overline{S}$$

Displacment Current

$$\overline{J_d} = \varepsilon\frac{d\overline{E}}{dt}$$

Maxwells Equations

Name	Integral Form	Point Form
Ampere's Law	$\oint \overline{H} \cdot d\overline{l} = \int_S \overline{J} \cdot d\overline{S}$	$\nabla \times \overline{H} = \overline{J} + \varepsilon\frac{d\overline{E}}{dt}$
Gauss' Law	$\oint \overline{D} \cdot d\overline{S} = \int_V \rho_v dV$	$\nabla \cdot \overline{E} = \frac{\rho_v}{\varepsilon}$
None Existence of magnetic dipoles	$\oint_S \overline{B} \cdot d\overline{S} = 0$	$\nabla \cdot \overline{B} = 0$

Name	Integral Form	Point Form
Faraday's Law	$\int \overline{E} \cdot d\overline{l} = 0$	$\nabla \times \overline{E} = -\frac{d\overline{B}}{dt}$

Magnetic Boundary Conditions

$$\mu_1 \overline{H}_{1n} = \mu_2 \overline{H}_{2n} \ \leftarrow or \rightarrow \ \overline{B}_{1n} = \overline{B}_{2n}$$

$$H_{1t} - H_{2t} = k$$

Refraction

$$\frac{tan(\theta_1)}{tan(\theta_2)} = \frac{\mu_1}{\mu_2}$$

> **Note**
>
> θ wrt to normal to plane

Force due to Magnetic Fields

$$F = I\overline{L} \times \overline{B}$$

$$\overline{F} = Q\overline{u} \times \overline{B}$$

Force on Current Element

$$d\overline{F} = Id\overline{l} \times \overline{B}$$

$$d\overline{F} = \overline{K} \, d\overline{S} \times \overline{B}$$

$$d\overline{F} = \overline{J} \, d\overline{V} \times \overline{B}$$

Force on charged particle

- Lorentz Force Equation

$$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B})$$

Force between Current Elements

$$\overline{F}_1 = -\overline{F}_2 = \frac{\mu_o I_1 I_2}{4\pi} \int_{l1} \int_{l2} \frac{dl_1 \times (dl_2 \times a_{R_{21}})}{|\overline{R_{21}}|^2}$$

Faradays Law

$$V_{emf} = \frac{-Nd\Psi}{dt} = \oint \overline{E}d\overline{l}$$

Stationary loop Varying magnetic Field

$$V_{emf} = \oint \overline{E} \cdot d\overline{l} = -\int_S \frac{d\overline{B}}{dt} \cdot d\overline{S}$$

- $V_{emf} \rightarrow$ Transformer EMF

Moving loop Static magnetic Field

$$V_{emf} = \oint \overline{E} \cdot d\overline{l} = \oint_L (\overline{u} \times \overline{B}) \cdot d\overline{l}$$

- Using Stokes Theorem

$$\nabla \times \overline{E} = \nabla \times (\overline{u} \times \overline{B})$$

Moving loop varying Magnetic Field

$$V_{emf} = \oint \overline{E} \cdot d\overline{l} = -\int_S \frac{d\overline{B}}{dt} \cdot d\overline{S} \; + \; \oint_L (\overline{u} \times \overline{B}) \cdot d\overline{l}$$

Part 2

Phasors

$$z = a + ib \rightarrow r\angle\theta = re^{j\theta}$$

$$X = cos(\omega t - \beta x) = Re\{\overline{X}_o e^{J\omega t}\}$$

- We can define the phasor X_s such that

$$\overline{X}(x, y, z, t) = Re\{\overline{X}_s(x, y, z)\; e^{J\omega t}\}$$

Maxwell equations

Name	Phasor Form
Ampere's Law	$\nabla \times \overline{H}_s = \overline{J}_s + j\omega \overline{D}_s$
Gauss' Law	$\nabla \cdot \overline{E}_s = \frac{\rho_v}{\varepsilon}$

Name	Phasor Form
None Existence of magnetic dipoles	$\nabla \cdot \overline{B}_s = 0$
Faraday's Law	$\nabla \times \overline{E}_s = -J\omega \overline{B}_s$

Wave Propagation #todo

- General Wave Equation:

$$\frac{d^2X}{dt^2} - u^2\nabla^2 X = 0$$

- Or

$$\frac{d^2E}{dt^2} - \frac{1}{\mu\varepsilon}\nabla^2 E = 0$$

- Solution to Wave Equation:

$$X\,(x, y, z, t) = A_o e^{-\alpha x} cos(\omega t - \beta x) = \ Re\{\overline{X}_s(x, y, z)\, e^{J\omega t}\}$$

$$\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda}$$

Lossy Dielectrics

- Wave Equation:

$$\nabla^2\, \overline{E}_s\ -\ \gamma^2\, \overline{E}_s = 0$$

γ - Propagation constant

$$\gamma = \sqrt{J\omega\mu(\sigma + j\omega\varepsilon)}$$

- $\gamma = \alpha + j\beta$

α - Attenuation constant

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + (\frac{\sigma}{\omega\epsilon})^2} - 1\right]}$$

β - Phase shift constant

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}+1\right]}$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha}$$

η - Intrinsic Impedance

$$\eta = \sqrt{\frac{J\omega\mu}{\sigma + J\omega\varepsilon}}$$

$$\eta = |\eta|e^{J\theta_\eta}$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left[1+(\frac{\sigma}{\omega\varepsilon})^2\right]^{\frac{1}{4}}}$$

θ_η - Phase

$$\theta_\eta = \frac{1}{2}tan^{-1}(\frac{\sigma}{\omega\varepsilon})$$

> **Note**
>
> - $\overline{E}$ leads $\overline{H}$ by angle θ_η
> - $\overline{H}$ lags $\overline{E}$ by angle θ_η

Plane Waves in Free Space

- We know that $\gamma^2 = J\omega\mu(\sigma + J\omega\varepsilon)$
- For free space $\sigma = 0$, therefore:

$$\gamma = J\omega\sqrt{\mu_o\varepsilon_o}$$

- $\alpha = 0$, therefore **no attenuation**

$$\beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c}$$

- $\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi$
- $\overline{E}$ and $\overline{H}$ are in phase

Lossless Dielectrics

- For free space $\sigma = 0$, therefore:

$$\gamma = J\omega\sqrt{\mu\varepsilon}$$

- And $\eta = \sqrt{\frac{\mu}{\varepsilon}}$
- And $\beta = \omega\sqrt{\mu\varepsilon}$, therefore:

$$c = \frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}}$$

Good Conductors

Plane Waves

Skin Effect

- Skin depth / penetration is the distance travelled by EM wave getting attenuated by a factor e

$$\delta = \frac{1}{\alpha}$$

- For good conductors,

$$\delta = \frac{1}{\sqrt{\pi\rho\mu\sigma}}$$

- δ is very small for good conductors $\therefore$ waves dies down at high rate
- Very high frequency (RF range) comes from good conductors, δ is small at high frequency $\therefore$ **Skin effect**

Wave Polarization

Linear

$$E_x = E_{ox}cos(\omega t - \beta z)$$

$$E_y = E_{oy}cos(\omega t - \beta z)$$

$$z = 0$$

$$\Delta\phi = \phi_y - \phi_x = 0$$

$$E_{ox} = E_{oy} \rightarrow \theta = 45^o$$

$$E_{ox} \neq E_{oy} \rightarrow \theta = tan^{-1}\left(\frac{E_{oy}}{E_{ox}}\right)$$

Circular

$$E_x = E_{ox}cos(\omega t - \beta z)$$

$$E_y = E_{oy} cos(\omega t - \beta z - \frac{\pi}{2})$$

$$E_{ox} = E_{oy}$$

$$\Delta\phi = \phi_y - \phi_x = \frac{\pi}{2}$$

Elliptical

$$E_x = E_{ox} cos(\omega t - \beta z)$$

$$E_y = E_{oy} sin(\omega t - \beta z)$$

$$E_{ox} \neq E_{oy}$$

$$\Delta\phi = \phi_y - \phi_x = \frac{\pi}{2}$$

$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 = 1$$

Poynting Theorem

$$\overline{P} = \overline{E} \times \overline{H}$$

- For perfect dielectric:

$$E_x = E_{ox} cos(\omega t - \beta z)$$

$$H_y = \frac{E_{ox}}{\eta} cos(\omega t - \beta z)$$

- Power density:

$$P_z = \frac{E_{ox}^2}{\eta} cos^2(\omega t - \beta z)$$

- For lossy dielectric, power density in phasor form

$$< P_z >= \frac{1}{2} \frac{E_{ox}^2}{|\eta|} e^{-2az} cos\theta_\eta$$