Magnetism_Formulae

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Part 1

Bio Savarts law

Differential

$$d\overline{H}=rac{I}{4\pi}rac{d\overline{L} imes\overline{r}}{|r|^3}$$

⊘ Note

Remember

• Cartesian

$$d\overline{L} = dxa_x + dya_y + dza_z$$

Cylindrical

$$d\overline{L} = d
ho a_x +
ho d\phi a_\phi + dz a_z$$

Spherical

$$d\overline{L} = dra_r + rd heta a_ heta + rsin(heta)d\phi a_\phi$$

Magnetic Field Intensities

Line Segment

$$\overline{H} = rac{I}{4\pi
ho}(cos(lpha_1) + cos(lpha_2))a_{\phi}$$

$$\overline{H} = rac{I}{4\pi
ho}rac{2L}{\sqrt{l^2+4
ho^2}}a_{\phi}$$

Circular coil

$$\overline{H}=rac{I
ho^2}{2(
ho^2+h^2)^{rac{3}{2}}}a_z$$

Solenoid

$$\overline{H}=nIa_y$$

Amperes Law

$$egin{aligned} \oint \overline{H} \cdot d\overline{l} &= I_{enclosed} = \int_S \overline{J} \cdot d\overline{S} \ \
abla imes \overline{H} &= \overline{J} \end{aligned}$$

Applications

Infinite Line Current

$$\overline{H}=rac{I}{2\pi
ho}a_{\phi}$$

Infinite Sheet of current

$$\overline{H}=\pmrac{k}{2} imes a_n$$

Infinite Length Coaxial Transmission Line

Hollow Cylinder

Solid Cylinder

$$egin{aligned} \overline{H} &= rac{I
ho}{2\pi a^2} a_\phi \ ; \
ho < a \ \ \overline{H} &= rac{I}{2\pi
ho} a_\phi \ ; \
ho > a \end{aligned}$$

Coaxial Cable

$$egin{aligned} \overline{H} &= rac{I
ho}{2\pi a^2} a_\phi \ ; \
ho < a \ \ \overline{H} &= rac{I}{2\pi
ho} a_\phi \ ; \ a \leq
ho \leq b \end{aligned}$$

$$\overline{H} = rac{I}{2\pi
ho} \ [1 - rac{
ho^2 - b^2}{r_1^2 - b^2}] \ a_\phi \ ; \ b \leq
ho \leq r_1 \ \ \overline{H} = 0 \ ; \
ho > r_1$$

Toroid

$$egin{split} \overline{H} &= 0 \ ; \
ho <
ho_o - a \ \ \overline{H} &= rac{NI}{2\pi
ho} \ ; \
ho_o - a <
ho <
ho_o + a \ \ \overline{H} &= 0 \ ; \
ho >
ho_0 + a \end{split}$$

Stoke Law

$$\oint \overline{H} \cdot d\overline{l} = \int_S (
abla imes \overline{H}) \cdot d\overline{S}$$

Magnetic Field Density

$$\overline{B} = \mu \overline{H}$$

Magnetic Flux

$$\Psi = \int_S \overline{B} \cdot d\overline{S}$$

Displacment Current

$$\overline{J_d} = arepsilon rac{d\overline{E}}{dt}$$

Maxwells Equations

Name	Integral Form	Point Form
Ampere's Law	$\oint \overline{H} \cdot d\overline{l} = \int_S \overline{J} \cdot d\overline{S}$	$ abla imes \overline{H} = \overline{J} + arepsilon rac{d\overline{E}}{dt}$
Gauss' Law	$\oint \overline{D} \cdot d\overline{S} = \int_V ho_v dV$	$ abla \cdot \overline{E} = rac{ ho_v}{arepsilon}$
None Existence of magnetic dipoles	$\oint_S \overline{B} \cdot d\overline{S} = 0$	$\nabla \cdot \overline{B} = 0$

Name	Integral Form	Point Form
Faraday's Law	$\int \overline{E} \cdot d\overline{l} = 0$	$ abla imes \overline{E} = -rac{d\overline{B}}{dt}$

Magnetic Boundary Conditions

$$\mu_1\overline{H}_{1n} = \mu_2\overline{H}_{2n} \ \leftarrow or
ightarrow \ \overline{B}_{1n} = \overline{B}_{2n}$$
 $H_{1t} - H_{2t} = k$

Refraction

$$rac{tan(heta_1)}{tan(heta_2)} = rac{\mu_1}{\mu_2}$$



 θ wrt to normal to plane

Force due to Magnetic Fields

$$F=I\overline{L} imes\overline{B}$$

$$\overline{F}=Q\overline{u} imes\overline{B}$$

Force on Current Element

$$d\overline{F} = Id\overline{l} imes \overline{B}$$

$$d\overline{F} = \overline{K} \ d\overline{S} \times \overline{B}$$

$$d\overline{F} = \overline{J} \; d\overline{V} \times \overline{B}$$

Force on charged particle

• Lorentz Force Equation

$$\overline{F} = Q(\overline{E} + \overline{u} imes \overline{B})$$

Force between Current Elements

$$\overline{F}_1=-\overline{F}_2=rac{\mu_o I_1 I_2}{4\pi}\int_{l1}\int_{l2}rac{dl_1 imes (dl_2 imes a_{R_{21}})}{\left|\overline{R_{21}}
ight|^2}$$

Faradays Law

$$V_{emf}=rac{-Nd\Psi}{dt}=\oint \overline{E}d\overline{l}$$

Stationary loop Varying magnetic Field

$$V_{emf} = \oint \overline{E} \cdot d\overline{l} = - \int_S rac{d\overline{B}}{dt} \cdot d\overline{S}$$

ullet $V_{emf}
ightarrow$ Transformer EMF

Moving loop Static magnetic Field

$$V_{emf} = \oint \overline{E} \cdot d\overline{l} = \oint_L (\overline{u} imes \overline{B}) \cdot d\overline{l}$$

• Using Stokes Theorem

$$abla imes \overline{E} =
abla imes (\overline{u} imes \overline{B})$$

Moving loop varying Magnetic Field

$$V_{emf} = \oint \overline{E} \cdot d\overline{l} = - \int_S rac{d\overline{B}}{dt} \cdot d\overline{S} \; + \; \oint_L (\overline{u} imes \overline{B}) \cdot d\overline{l}$$

Part 2

Phasors

$$z=a+ib
ightarrow roldsymbol{\angle} heta=re^{j heta} \ X=cos(\omega t-eta x)=Re\{\overline{X}_oe^{J\omega t}\}$$

ullet We can define the phasor X_s such that

$$\overline{X}(x,y,z,t) = Re\{\overline{X}_s(x,y,z) \ e^{J\omega t}\}$$

Maxwell equations

Name	Phasor Form
Ampere's Law	$ abla imes \overline{H}_s = \overline{J}_s + j\omega \overline{D}_s$
Gauss' Law	$ abla \cdot \overline{E}_s = rac{ ho_v}{arepsilon}$

Name	Phasor Form
None Existence of magnetic dipoles	$ abla \cdot \overline{B}_s = 0$
Faraday's Law	$ abla imes \overline{E}_s = -J\omega \overline{B}_s$

Wave Propagation #todo

• General Wave Equation:

$$rac{d^2X}{dt^2}-u^2
abla^2X=0$$

• 0r

$$rac{d^2E}{dt^2} - rac{1}{\muarepsilon}
abla^2E = 0$$

• Solution to Wave Equation:

$$X\left(x,y,z,t
ight) = A_{o}e^{-lpha x}cos(\omega t - eta x) = Re\{\overline{X}_{s}(x,y,z)\ e^{J\omega t}\}$$

$$\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda}$$

Lossy Dielectrics

Wave Equation:

$$abla^2 \, \overline{E}_s \, - \, \gamma^2 \, \overline{E}_s = 0$$

 γ - Propagation constant

$$\gamma = \sqrt{J\omega\mu(\sigma+j\omegaarepsilon)}$$

•
$$\gamma = \alpha + j\beta$$

lpha - Attenuation constant

$$lpha = \omega \sqrt{rac{\mu arepsilon}{2} igg[\sqrt{1 + (rac{\sigma}{\omega \epsilon})^2} - 1 igg]}$$

 β - Phase shift constant

$$eta = \omega \sqrt{rac{\mu arepsilon}{2} igg[\sqrt{1 + (rac{\sigma}{\omega \epsilon})^2} + 1 igg]} \ eta = rac{\omega \mu \sigma}{2 lpha}$$

η - Intrinsic Impedance

$$egin{aligned} \eta &= \sqrt{rac{J\omega\mu}{\sigma + J\omegaarepsilon}} \ \eta &= |\eta|e^{J heta_\eta} \ |\eta| &= rac{\sqrt{rac{\mu}{arepsilon}}}{\left[1 + (rac{\sigma}{\omegaarepsilon})^2
ight]^{rac{1}{4}}} \end{aligned}$$

$heta_\eta$ - Phase

$$heta_{\eta} = rac{1}{2} tan^{-1} ig(rac{\sigma}{\omega arepsilon}ig)$$

⊘ Note

- ullet \overline{E} leads \overline{H} by angle $heta_\eta$
- ullet \overline{H} lags \overline{E} by angle $heta_\eta$

Plane Waves in Free Space

- We know that $\gamma^2 = J\omega\mu(\sigma + J\omegaarepsilon)$
- For free space $\sigma=0$, therefore:

$$\gamma = J\omega\sqrt{\mu_oarepsilon_o}$$

• $\alpha=0$, therefore no attenuation

$$eta = \omega \sqrt{\mu_o arepsilon_o} = rac{\omega}{c}$$

- $ullet \ \eta_o = \sqrt{rac{\mu_o}{arepsilon_o}} = 120\pi$
- ullet \overline{E} and \overline{H} are in phase

Lossless Dielectrics

• For free space $\sigma=0$, therefore:

$$\gamma = J\omega\sqrt{\muarepsilon}$$

- And $\eta = \sqrt{\frac{\mu}{\varepsilon}}$
- And $\beta = \omega \sqrt{\mu arepsilon}$, therefore:

$$c=rac{1}{\sqrt{\mu_o\mu_rarepsilon_oarepsilon_r}}$$

Good Conductors

Plane Waves

Skin Effect

 Skin depth / penetration is the distance travelled by EM wave getting attenuated by a factor e

$$\delta = \frac{1}{\alpha}$$

For good conductors,

$$\delta = rac{1}{\sqrt{\pi
ho\mu\sigma}}$$

- δ is very small for good conductors $\dot{}$ waves dies down at high rate
- Very high frequency (RF range) comes from good conductors, δ is small at high frequency \therefore Skin effect

Wave Polarization

Linear

$$egin{aligned} E_x &= E_{ox} cos(\omega t - eta z) \ E_y &= E_{oy} cos(\omega t - eta z) \ z &= 0 \ \Delta \phi &= \phi_y - \phi_x = 0 \ E_{ox} &= E_{oy}
ightarrow heta = 45^o \ E_{ox}
eq E_{oy}
ightarrow heta = tan^{-1} \Big(rac{E_{oy}}{E_{ox}}\Big) \end{aligned}$$

Circular

$$E_x = E_{ox} cos(\omega t - \beta z)$$

$$E_y = E_{oy} cos(\omega t - eta z - rac{\pi}{2})$$
 $E_{ox} = E_{oy}$ $\Delta \phi = \phi_y - \phi_x = rac{\pi}{2}$

Elliptical

$$egin{aligned} E_x &= E_{ox} cos(\omega t - eta z) \ E_y &= E_{oy} sin(\omega t - eta z) \ E_{ox} &
eq E_{oy} \end{aligned} \ \Delta \phi = \phi_y - \phi_x = rac{\pi}{2} \ \left(rac{E_x}{E_{ox}}
ight)^2 + \left(rac{E_y}{E_{ox}}
ight)^2 = 1 \end{aligned}$$

Poynting Theorem

$$\overline{P}=\overline{E} imes\overline{H}$$

• For perfect dielectric:

$$E_x = E_{ox} cos(\omega t - eta z)$$

$$H_y = rac{E_{ox}}{\eta} cos(\omega t - eta z)$$

• Power density:

$$P_z = rac{E_{ox}^2}{n} cos^2 (\omega t - eta z)$$

For lossy dielectric, power density in phasor form

$$|< P_z> = rac{1}{2} rac{E_{ox}^2}{|\eta|} \, e^{-2az} cos heta_\eta \, .$$