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## Unit 1

#### Coulombs Law

$$F = rac{1}{4\piarepsilon} \cdot rac{Q_1Q_2}{r^2}$$

## **Electric Fields**

## Point charge

$$D=rac{Q}{4\pi r^2}a_r$$

$$E=rac{Q}{4\piarepsilon_o r^2}a_r$$

#### Line charge

$$E=rac{-
ho_L}{4\piarepsilon_o
ho}\int_{lpha_1}^{lpha_2}\left[cos(lpha)a_p+sin(lpha)a_z
ight]\!dlpha$$

- ullet Where  $lpha_1$  and  $lpha_2$  are angles that the line subtends to the point
- Infinite line charge -

$$E=rac{
ho_L}{2\piarepsilon_o
ho}a_p$$

#### Surface charge

• Electric field due to surface charge is

$$E=\int_{S}rac{
ho_{s}dS}{4\piarepsilon_{0}R^{2}}a_{R}$$

• Substituting  $dS 
ightarrow 
ho \; d\phi \; d
ho$  and solving for **Infinite Sheet Charge** 

$$E=rac{
ho_s}{2arepsilon_0}a_n$$

ullet Where  $a_n$  is unit vector perpendicular to sheet

#### Volume charge

• Electric field due to volume charge is

$$E=\int_{S}rac{
ho_{s}dV}{4\piarepsilon_{0}R^{2}}a_{R}$$

• With  $a_R = cos(lpha) a_Z - sin(lpha) a_p$ 

$$E=rac{Q}{4\piarepsilon_o}z^2$$

$$Q=
ho_vrac{4}{3}\pi a^3$$

# **Electric Flux Density**

Also known as Electric Displacement

$$\overline{D} = \varepsilon_o \overline{E}$$

$$\Psi = \int_S \overline{D} \cdot d\overline{S}$$

## **Gradient**

$$abla A = lim_{\Delta v 
ightarrow 0} rac{\int_S A \cdot dS}{\Delta V}$$

#### Cartesian

$$abla A = rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$$

## Cylindrical

$$abla A = rac{\partial A_
ho}{\partial 
ho} + rac{1}{
ho} rac{\partial A_\phi}{\partial \phi} + rac{\partial A_z}{\partial z}$$

# **Spherical**

$$abla A = rac{\partial A_r}{\partial r} + rac{1}{r}rac{\partial A_ heta}{\partial heta} + rac{1}{r sin heta}rac{\partial A_\phi}{\partial \phi}$$

# **Divergence**

**⊘** Note

General formula

$$abla \cdot A = rac{1}{h_1 h_2 h_3} [rac{\partial \ h_2 h_3}{\partial u_1} + rac{\partial \ h_1 h_3}{\partial u_2} + rac{\partial \ h_1 h_2}{\partial u_3}] A$$

#### **Divergence Theorem**

$$\int_S A \cdot dS = \int_V 
abla \cdot A \; dV$$

 Total outward flux of a vector field through closed surface S is same as volume integral of the divergence of A

#### Gauss law

$$\Psi = \int_S D \cdot dS = Q_{enclosed} = \int_V 
ho_v \ dV$$

Applying divergence theorem

$$\int_S D \cdot dS = \int_V 
abla \cdot D \ dV$$

Comparing the equations :- We get Maxwell's 1st equation

$$\rho_V = \nabla \cdot D$$

## **Applying Gauss law**

• Infinite line charge

$$ho_L l = Q = \int_S D \cdot dS = D_
ho \int_S dS = D_
ho \ 2\pi \ 
ho \ l \ 
onumber \ D_
ho = rac{
ho_L l}{2\pi \ 
ho \ l} \ 
ightarrow D = rac{
ho_L}{2\pi \ 
ho} a_
ho$$

Infinite sheet charge

$$ho_S A = D_z (A+A) = 2 D_z A \ 
onumber \ D = D_z a_z = rac{
ho_s}{2} a_z \ (or) \ E = rac{
ho_s}{2 arepsilon_o} a_z 
onumber$$

Uniformly charged sphere

$$D = egin{cases} rac{r}{3} 
ho_o \ a_r & ext{if} \ 0 < r \leq a \ rac{a^3}{3 \ r^2} 
ho_o \ a_r & ext{if} \ r \geq a \end{cases}$$

## **Electric Potential**

$$V_{BA} = rac{W}{Q} = -\int_{A}^{B} E \cdot dl$$

 Absolute potential - work done per charge to bring it from infinity to r

$$V = -\int_{\infty}^{r} E \cdot dl$$

## Relation between E and V

For a closed loop:-

$$\int_L E \cdot dl = 0$$

**⊘** Note

Stoke's Theorem

$$\int\int_S (\overline{
abla} imes \overline{B}).\,dS = \oint_L B.\,d\overline{l}$$

Curl

$$abla imes \mathbf{F} = egin{array}{cccc} \hat{m{i}} & \hat{m{j}} & \hat{m{k}} \ rac{\partial}{\partial x} & rac{\partial}{\partial y} & rac{\partial}{\partial z} \ F_x & F_y & F_z \ \end{array} egin{array}{ccccc}$$

• Applying stoke's theorem

$$\int_L E \cdot dl = \int_S (
abla imes E) \cdot dS = 0 \ (or) \ 
abla imes E = 0$$

• Electric field is the gradient of V :-

$$E = -\nabla V$$

# **Continuity Equation**

$$I_{out} = \oint J.\,dS = -rac{dQ_{in}}{dt}$$

• With divergence theorem :-

$$\oint J.\,dS = \int_{v} 
abla .\,J\,\,dV$$

Also

$$-rac{dQ_{in}}{dt}=-\int_{v}rac{\partial
ho_{v}}{\partial t}dV$$

• Therefore Continuity Equation

$$abla.J = -rac{\partial 
ho_v}{\partial t}$$

**⊘** Note

For DC current

$$\frac{\partial 
ho_v}{\partial t} = 0$$

Therefore

$$\nabla . J = 0$$

# Relaxation time

$$ho_v = 
ho_{vo} \ e^{-rac{t}{T_r}} ext{ where } T_r = rac{arepsilon}{\sigma}$$

# Poisson's Equation

$$abla^2 V = -rac{
ho_v}{arepsilon_o}$$

**⊘** Note

Laplacian operator :-

General Formula

$$\nabla^2 A = \frac{1}{h_1 h_2 h_3} \left[ \, \frac{\partial}{\partial a_1} \, \left( \frac{h_2 h_3}{h_1} \, \frac{\partial A}{\partial a_1} \right) + \frac{\partial}{\partial a_2} \, \left( \frac{h_1 h_3}{h_2} \, \frac{\partial A}{\partial a_2} \right) + \frac{\partial}{\partial a_3} \, \left( \frac{h_1 h_2}{h_3} \, \frac{\partial A}{\partial a_3} \right) \, \right]$$

- Cartesian (x,y,z) (h1,h2,h3)=(1,1,1)
- Cylindrical  $(
  ho,\phi,z)$  (h1,h2,h3)=(1,
  ho,1)
- Spherical  $(r, heta, \phi)$  (h1, h2, h3) = (1, r, rsin( heta))

# Laplace Equation

If there is no charge enclosed in the volume :-

$$abla^2 V = 0$$

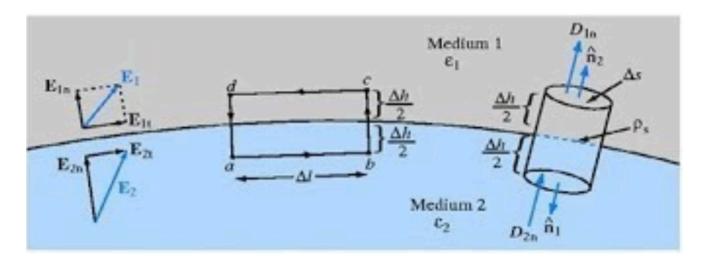
# **Boundary conditions**

Maxwell's Equations :-

$$\oint_S \overline{D}.\,d\overline{S} = Q_{enc} o 1^{st} ext{ Equation}$$

$$\oint_L \overline{E}.\, d\overline{l} = 0 
ightarrow 2^{nd} ext{ Equation}$$

# Dielectric - Dielectric



$$\varepsilon_1 = \varepsilon_o \ \varepsilon_{r1} \ (\mathrm{and}) \ \varepsilon_2 = \varepsilon_o \ \varepsilon_{r2}$$

• For a closed path <a href="mailto:abcda">abcda</a>. The tangential components of an electric field crossing the interface undergoes no change :-

$$E_{1t} = E_{2t}$$

• 0r

$$\frac{D_{1t}}{arepsilon_1} = \frac{D_{2t}}{arepsilon_2}$$

• Similarly for cylindrical Gaussian surface

$$D_{1n} - D_{2n} = \rho_s$$

• Or when  $ho_s=0$  where  $ho_s$  is charge density in the interface surface

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

Refraction

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

•  $\theta_1$ ,  $\theta_2$  -> angle of electric field with normal to surface in each medium

## Conductor - Dielectric

$$E_t = 0$$

$$D_n = \rho_s$$

# **Conductor - Free space**

$$D_t = \varepsilon_o E_t = 0$$

$$D_n = arepsilon_o E_n = 
ho_s$$

# **Drift velocity**

$$v_e = -\mu_e \overline{E}$$

#### Ohms law

$$R = rac{V}{I}$$

Point form

$$\overline{J}=\sigma\overline{E}$$

# Resistance

$$R = \frac{l}{\sigma A}$$

# Conductivity

$$G = rac{\sigma A}{l} = rac{A}{
ho \ l}$$

## Joules law

$$P( ext{power}) = \int_v \overline{J} \cdot \overline{E} dV$$

## **Polarisation**

$$\overline{D} = \varepsilon_o \overline{E} + \overline{P}$$

• P -> electric polarisation field

$$\overline{P} = arepsilon_o \chi_e \overline{E}$$

$$\overline{D} = \varepsilon_o \overline{E} + \overline{P} = \varepsilon \overline{E}$$

$$arepsilon_r = arepsilon_o (1 + \chi_e)$$

- $\chi_e$  -> Susceptibility
- $arepsilon_r$  -> relative permeability or dielectric strength

# Capacitance

$$C=rac{Q}{V}$$

$$RC = \frac{\varepsilon}{\sigma}$$

#### Parallel Plate

$$C = rac{Q}{V} = rac{arepsilon S}{d}$$

$$W_E=rac{1}{2}\int_v arepsilon E^2 dv =rac{1}{2}CV^2 =rac{1}{2}QV =rac{Q^2}{2C}$$

## Coaxial

$$E = \frac{Q}{2\pi\varepsilon L} ln \frac{b}{a}$$

$$C=rac{2\piarepsilon L}{lnrac{b}{a}}$$

# **Spherical**

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

# **Configurations**

#### **Parallel**

$$C = C_1 + C_2$$

#### **Series**

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$