AQP Formula Sheet

In SI units, the expressions for μ have no c.

$$\mu = IA = \tfrac{q}{2mc} L$$
$$\vec{\mu} = \tfrac{gq}{2mc} \vec{S}$$
$$F_z = \vec{\mu} \cdot \frac{\partial \vec{B}}{\partial z} \approx \mu_z \frac{\partial B_z}{\partial z}$$
$$\sigma_A = \Delta A$$
$$\Delta f^2 = \sum \left(\frac{\partial f}{\partial x}\right)^2 \Delta x^2$$

$$\langle A \rangle = \sum_i a_i \left|c_i\right|^2 = \langle \psi | \hat{A} | \psi \rangle$$
$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$$
$$\hat{R}(\mathrm{d}\phi\, \hat{k}) = 1 - \tfrac{i}{\hbar} \hat{J}_z \,\mathrm{d}\phi$$
$$\hat{R}(\phi \hat{k}) = \exp[-i \hat{J}_z \phi / \hbar]$$
$$\hat{R}(\phi \hat{k}) |\pm z\rangle = e^{\mp i\phi/2} |\pm z\rangle$$

$$\hat{P}^2 = \hat{P}$$
$$\sum_i \hat{P}_i = \mathbb{1}$$
$$\mathbb{1} = \sum_i |i\rangle\langle i|$$
$$A_{ij} |i\rangle\langle j| = |i\rangle\langle i|\hat{A}|j\rangle\langle j|$$
$$[\hat{J}_x, \hat{J}_y] = i\hbar \hat{J}_z$$

Useful Representations

$$|\pm x\rangle = \tfrac{1}{\sqrt{2}}|+z\rangle \pm \tfrac{1}{\sqrt{2}}|-z\rangle$$
$$|\pm y\rangle = \tfrac{1}{\sqrt{2}}|+z\rangle \pm \tfrac{i}{\sqrt{2}}|-z\rangle$$
$$|+n\rangle = \cos\tfrac{\theta}{2}|+z\rangle + e^{i\phi}\sin\tfrac{\theta}{2}|-z\rangle$$
$$|-n\rangle = \sin\tfrac{\theta}{2}|+z\rangle - e^{i\phi}\cos\tfrac{\theta}{2}|-z\rangle$$
$$|R\rangle = \tfrac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right)$$
$$|L\rangle = \tfrac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right)$$

Angular Momentum

$$\hat{R}(\Delta\phi \hat{k}) = \exp\left[-i\Delta\phi \hat{J}_z/\hbar\right]$$
$$\hat{J}^2|j,m\rangle = j(j+1)\hbar^2|j,m\rangle \implies \|J\| = \hbar\sqrt{j(j+1)}$$
$$\hat{J}_z|j,m\rangle = m\hbar|j,m\rangle$$

$$\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$$
$$\hat{J}_x = \tfrac{1}{2}\left(\hat{J}_+ + \hat{J}_-\right)$$
$$\hat{J}_y = \tfrac{1}{2i}\left(\hat{J}_+ - \hat{J}_-\right)$$

$$\hat{J}_+|j,m\rangle = \hbar\sqrt{j(j+1) - m(m+1)}|j,m+1\rangle$$
$$\hat{J}_-|j,m\rangle = \hbar\sqrt{j(j+1) - m(m-1)}|j,m-1\rangle$$
$$\Delta\hat{J}_x \Delta\hat{J}_y \geq \tfrac{1}{2}\left|\langle [\hat{J}_x, \hat{J}_y] \rangle\right| = \tfrac{\hbar}{2}\left|\langle \hat{J}_z \rangle\right|$$

$$\sigma_x = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S}_+ = \hbar\sqrt{2}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \hat{S}_- = \hbar\sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \hat{S}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ 1 & 0 & -i \\ 0 & 1 & 0 \end{pmatrix} \quad \hat{S}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Time Dependence

$$U(t) = \exp\left[i\hat{H}t/\hbar\right]$$
$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\hat{U}(t)|\psi(0)\rangle = \hat{H}\hat{U}(t)|\psi(0)\rangle$$
$$\frac{\mathrm{d}\langle A \rangle}{\mathrm{d}t} = \frac{i}{\hbar}\langle\psi(t)|\left[\hat{H}, \hat{A}\right]|\psi(t)\rangle + \langle\psi(t)|\frac{\partial\hat{A}}{\partial t}|\psi(t)\rangle$$

$$\hat{H}|E\rangle = E|E\rangle$$
$$U(t)|E\rangle = \exp[-iEt/\hbar]|E\rangle$$
$$\Delta E \Delta t \gtrsim \tfrac{\hbar}{2}$$

$$\omega_i \equiv -\frac{gqB_{i+1}}{2mc}$$
$$\hat{H}_{\vec{\mu}} = -\hat{\vec{\mu}} \cdot \vec{B} = \omega_0 S_z$$

$$\hat{H}_{mr} = \hat{H}_{\vec{\mu}} - \omega_1\cos(\omega t)\hat{S}_x$$
$$\hat{H}_{\mathrm{NH}_3} = E_0\sigma_z - A\sigma_x$$

$$\left|\langle -z|\psi_{mr}(t)\rangle\right|^2 = \frac{\omega_1^2/4}{(\omega_0 - \omega)^2 + \omega_1^2/4}\sin^2\frac{\sqrt{(\omega_0-\omega)^2 + \omega_1^2/4}}{2}t$$
$$\left|\langle I|\psi_{\mathrm{NH}_3'}(t)\rangle\right|^2 = \frac{(\mu_e E_1)^2}{(2A - \hbar\omega)^2 + (\mu_e E_1)^2}\sin^2\frac{\sqrt{(2A-\hbar\omega)^2 + (\mu_e E_1)^2}}{2\hbar}t$$