

## Case Study: Miles per Gallon Estimate

### Introduction

For this case study, we will be looking at the mileage data for 38 cars that were measured in 2005 to estimate miles per gallon. The variables for the dataset include Cylinders (number of cylinders), Size (engine displacement), HP (horsepower), and weight of the car (car weight). However, this dataset is missing values on numerous variables of the 38 records that it contains. In order to acquire a more complete dataset and therefore a more powerful analysis in theory, we will be using multiple imputations to explore this study.

### Literature review

From the initial information received on this dataset and the videos on 2ds, we know that this dataset is incomplete and will need figure out the best method to analyze the data with this in mind. This is a scenario that will likely come up on numerous occasions throughout the career of a Data Scientist and one must now how to combat this issue. Multiple imputations will likely be the technique used to properly analyze this information but we will need a comparison to single imputation to verify which method has more power.

### Method

First we will run a linear regression of the data in its current state using PROC REG in our SAS code; by default this uses list-wise deletion.

The REG Procedure					
Model: MODEL1					
Dependent Variable: MPG					
Number of Observations Read		38			
Number of Observations Used		20			
Number of Observations with Missing Values		18			

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	791.80042	158.36008	25.54	<.0001
Error	14	86.81158	6.20083		
Corrected Total	19	878.61200			

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	67.61816	7.12819	9.49	<.0001
CYLINDERS	1	-1.19508	1.13851	-1.05	0.3116
SIZE	1	0.05221	0.02938	1.78	0.0973
HP	1	-0.15009	0.07848	-1.91	0.0765
WEIGHT	1	-6.71776	3.98252	-1.69	0.1138
ACCEL	1	-0.68451	0.44024	-1.55	0.1423

From the image above, we can see that 38 observations were read but only 20 were use due to list-wise deletion and there only 19 degrees of freedom meaning that our analysis has lower power than expected. Next we will attempt imputation in order provide a more complete analysis of the dataset for cars.

The first step in looking at the data would be discover any missing value patterns using the SAS command PROC MI and then deciding which MI option to use.

Group	MPG	CYLINDERS	SIZE	HP	WEIGHT	ACCEL	Freq	Percent
1	X	X	X	X	X	X	20	52.63
2	X	X	X	X	X	.	2	5.26
3	X	X	X	X	.	X	3	7.69
4	X	X	X	X	.	.	1	2.63
5	X	X	X	.	X	X	5	13.16
6	X	X	.	X	X	X	2	5.26
7	X	X	.	X	.	X	1	2.63
8	X	.	X	X	X	X	2	5.26
9	X	.	X	X	X	.	1	2.63
10	X	.	X	X	.	X	1	2.63

From the image above, we can determine that pattern look likes it is non- monotone being that the values seem to missing randomly within the dataset provided. Now with this information we can use MCMC on the data to proceed with using multiple imputations due to its arbitrary nature.

Model Information	
Data Set	WORK.CARMPG
Method	MCMC
Multiple Imputation Chain	Single Chain
Initial Estimates for MCMC	EM Posterior Mode
Start	Starting Value
Prior	Jeffreys
Number of Imputations	25
Number of Burn-in Iterations	200
Number of Iterations	100
Seed for random number generator	3599

After running SAS code to create the imputation data, we can see that the MCMC method was used with a single imputation chain. Also, that the number of imputations is 25 meaning that there were 25 different datasets created from this imputation.

Now, that we have our imputed data we can run a regression analysis on each of the 25 full created datasets in order to estimate miles per gallon.

## Results

From imputation #1 below, we can see that all 38 cars are now included within the regression and 37 degrees of freedom are being used meaning that this observation has more power than our initial evaluation which only included only 19 degrees of freedom

<p>The REG Procedure Model: MODEL1 Dependent Variable: MPG</p>					
Number of Observations Read		38			
Number of Observations Used		20			
Number of Observations with Missing Values		18			

  

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	791.80042	158.36008	25.54	<.0001
Error	14	86.81158	6.20083		
Corrected Total	19	878.61200			

  

<p>Imputation Number=1</p>					
Number of Observations Read		38			
Number of Observations Used		38			

  

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	1413.23850	353.30962	67.45	<.0001
Error	33	172.85229	5.23795		
Corrected Total	37	1586.09079			

Next we will combine the results of all 25 imputations using PROC MIANALYZE for a single analysis

## The MIANALYZE Procedure

Model Information	
Data Set	WORK.OUTREG
Number of Imputations	25

Variance Information (25 Imputations)							
Parameter	Variance			DF	Relative Increase in Variance	Fraction Missing Information	Relative Efficiency
	Between	Within	Total				
CYLINDERS	0.061197	0.566872	0.630517	2355.5	0.112273	0.101703	0.995948
SIZE	0.000085537	0.000418	0.000507	779.12	0.212871	0.177619	0.992945
HP	0.000379	0.002012	0.002406	895.13	0.195804	0.165605	0.993419
WEIGHT	2.189402	8.202862	10.479840	508.4	0.277583	0.220333	0.991264
ACCEL	0.026085	0.108795	0.135923	602.51	0.249349	0.202227	0.991976
Intercept	3.400290	24.816680	28.352982	1542.8	0.142497	0.125857	0.994991

Parameter Estimates (25 Imputations)									
Parameter	Estimate	Std Error	95% Confidence Limits		DF	Minimum	Maximum	Theta0	t for H0: Parameter=Theta0 Pr >  t
CYLINDERS	-1.533464	0.794051	-3.0906	0.02365	2355.5	-1.904471	-0.671335	0	-1.93 0.0536
SIZE	0.055369	0.022514	0.0112	0.09956	779.12	0.033092	0.067195	0	2.46 0.0141
HP	-0.108087	0.049050	-0.2044	-0.01182	895.13	-0.145143	-0.074101	0	-2.20 0.0278
WEIGHT	-8.246574	3.237258	-14.6066	-1.88652	508.4	-10.190276	-4.864179	0	-2.55 0.0111
ACCEL	-0.684092	0.368678	-1.4081	0.03996	602.51	-1.063360	-0.355830	0	-1.86 0.0640
Intercept	68.052123	5.324752	57.6076	78.49664	1542.8	64.059281	71.562043	0	12.78 <.0001

While we do not expect the combined estimates to be similar to the original estimate, we do have confidence they are a better representation of the estimates for our parameters due reduced p values.

## Conclusion

In conclusion, using multiple imputations allowed us to provide analysis that represents the uncertainty of the missing value within the cars data. In theory, using this method as opposed to single imputation yields valid stats based inferences that reflect uncertainty of absent data.