Exercise 2.3.3.

Show that a factor of a topologically mixing system is also topologically mixing. (1)

Proof

Let $f : X \to X$ and $g : Y \to Y$ be topological dynamical systems, and π a topological semiconjugacy from f to g. (2)

Let U and V be nonempty open sets in Y. (3)

Since π is surjective and continuous, $\pi^{-1}(U)$ and $\pi^{-1}(V)$ are nonempty and open. (4)

By (2) and (4), there exists an $N \in \mathbf{N}$ such that for all $n \geq N$, $f^n(\pi^{-1}(U)) \cap \pi^{-1}(V) \neq \varnothing$. (5)

By (2),

$$\begin{aligned} \pi\left(f^n(\pi^{-1}(U)) \cap \pi^{-1}(V)\right) &\subseteq \pi(f^n(\pi^{-1}(U))) \cap \pi(\pi^{-1}(V)) \\ &= g^n(\pi(\pi^{-1}(U))) \cap \pi(\pi^{-1}(V)) \\ &= g^n(U) \cap V. \end{aligned} \tag{6}$$

By (6) and (5), $g^n(U) \cap V \neq \varnothing$, so g is topologically mixing, hence a factor of a topologically mixing system is topologically mixing. (7)