# Advanced techniques

## Lecture 5

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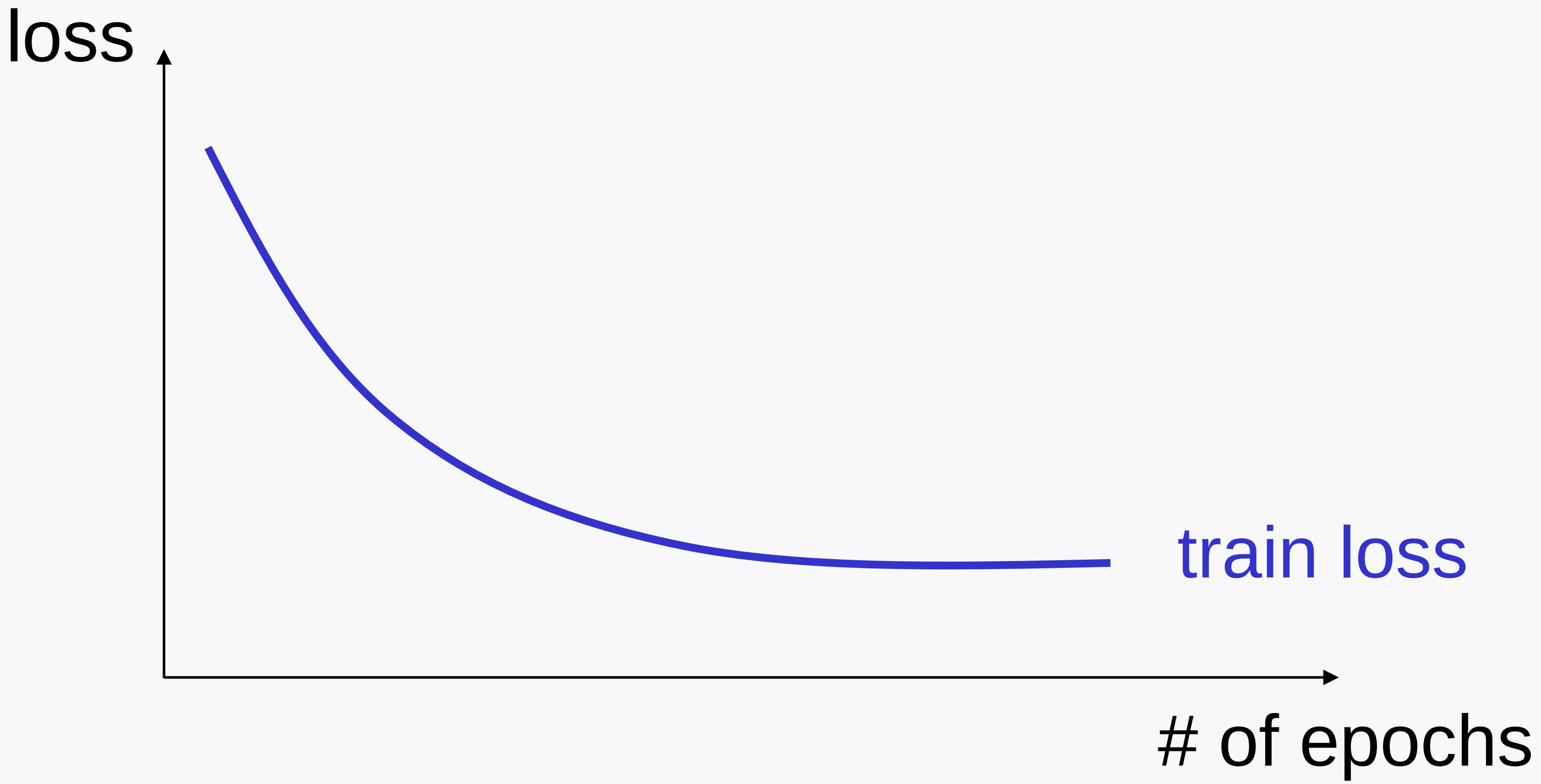
# Early stopping, dropout, Weight initialization & techniques for training stability

# Outline

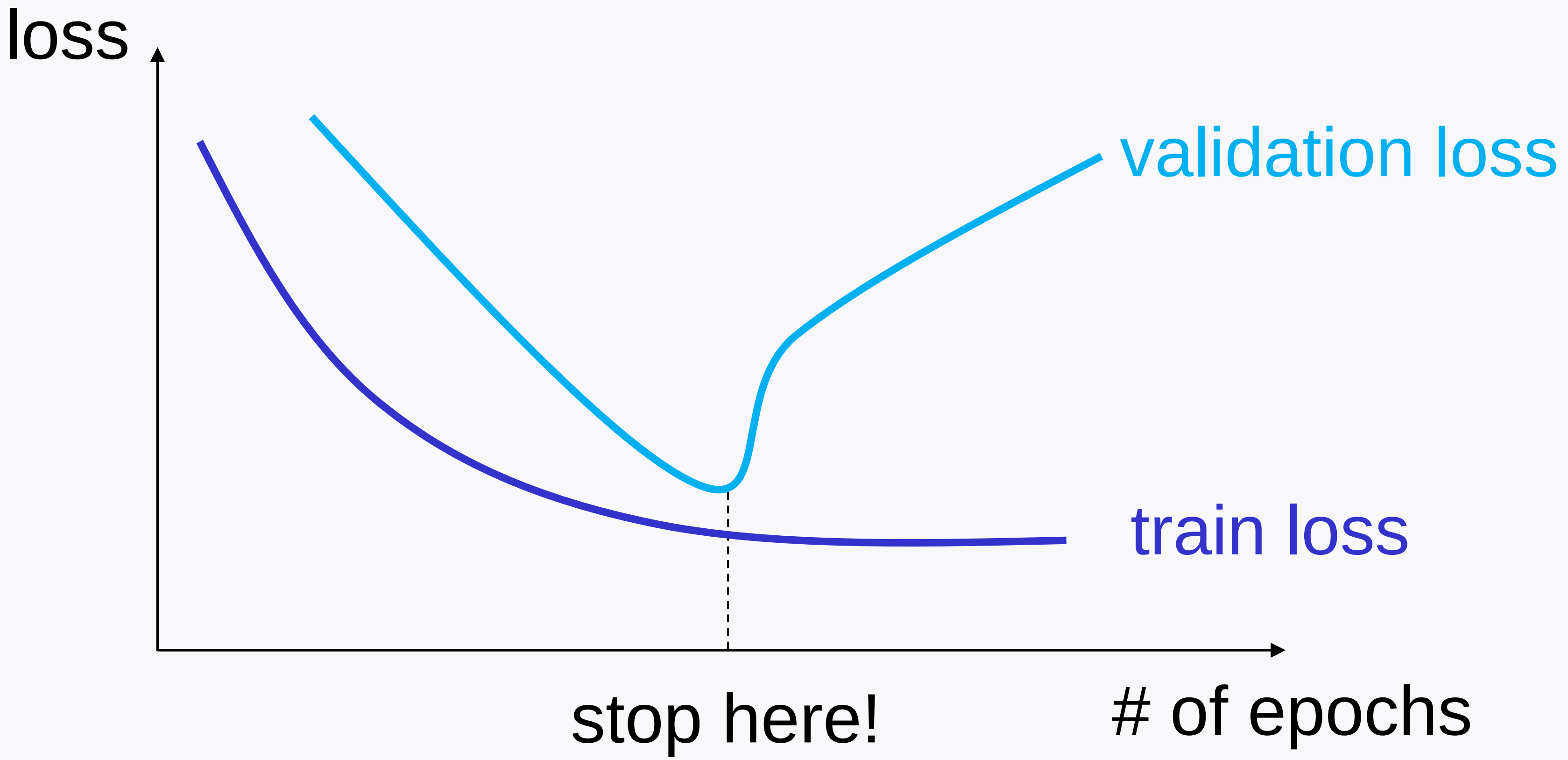
1. Generalization techniques
   - Early stopping
   - Dropout
2. Weight initialization
   - Xavier's initialization
   - He's initialization
3. Techniques for training stability
   - Adam optimizer
   - Learning rate decaying
   - Batch normalization

# Early stopping: Motivation




Large # of epochs: **Overfitting** to train data.

# Early stopping: Idea




To avoid overfitting: Rely on validation loss.

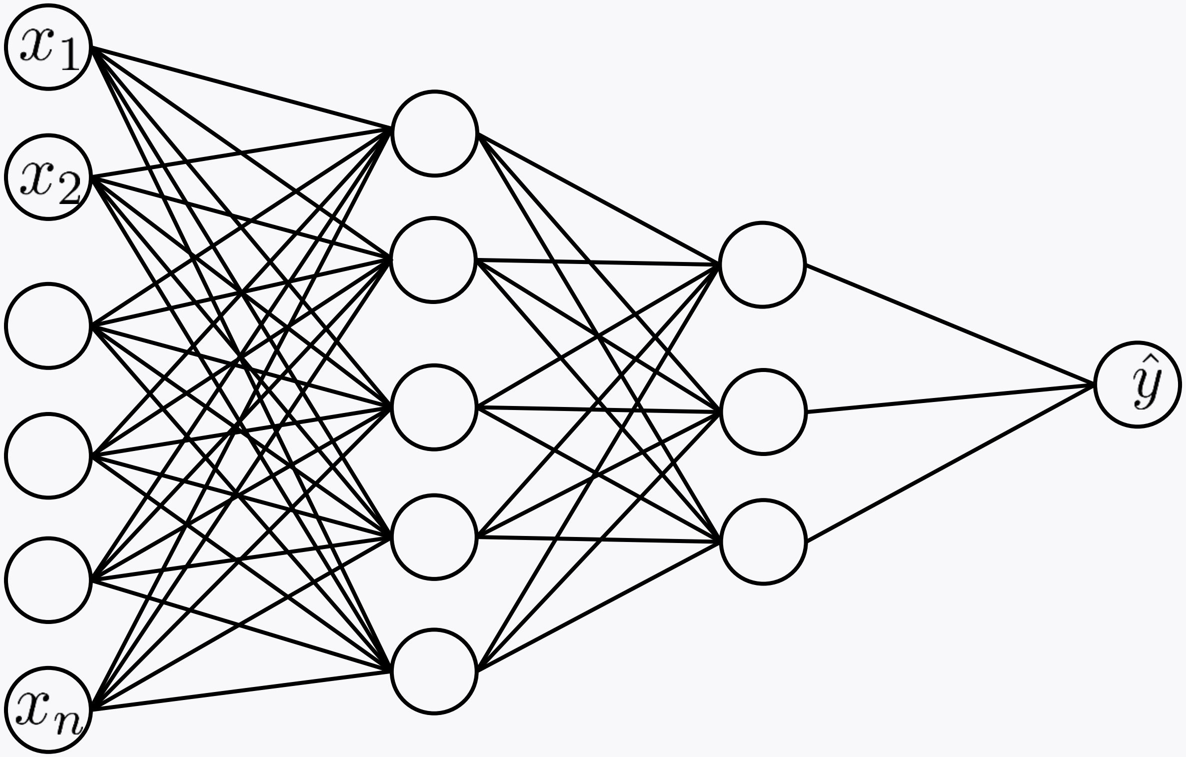
# Dropout

$$J(w) = \frac{1}{m_{\mathcal{B}}} \sum_{i=1}^{m_{\mathcal{B}}} \ell(y^{(i)}, \hat{y}^{(i)})$$

In computing a prediction $\hat{y}^{(i)}$ **per example**, construct a random neural network.

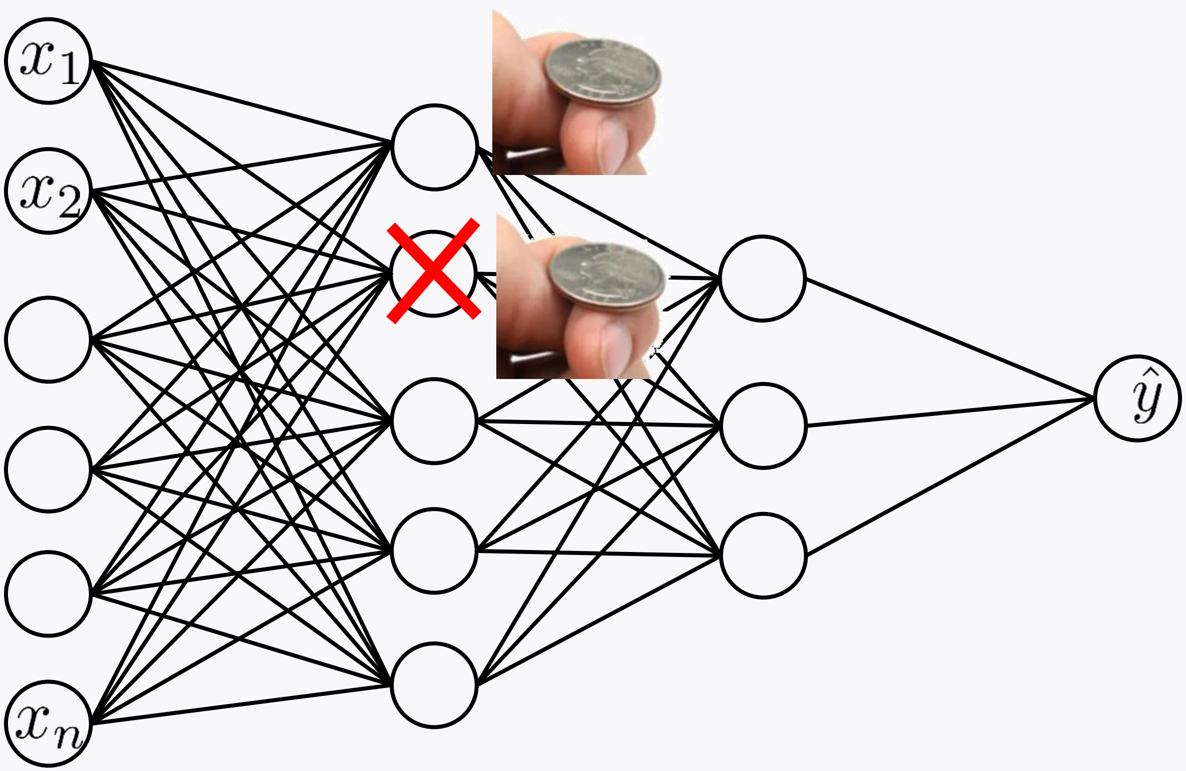
How to construct a random neural network?

# Construction of a random neural network



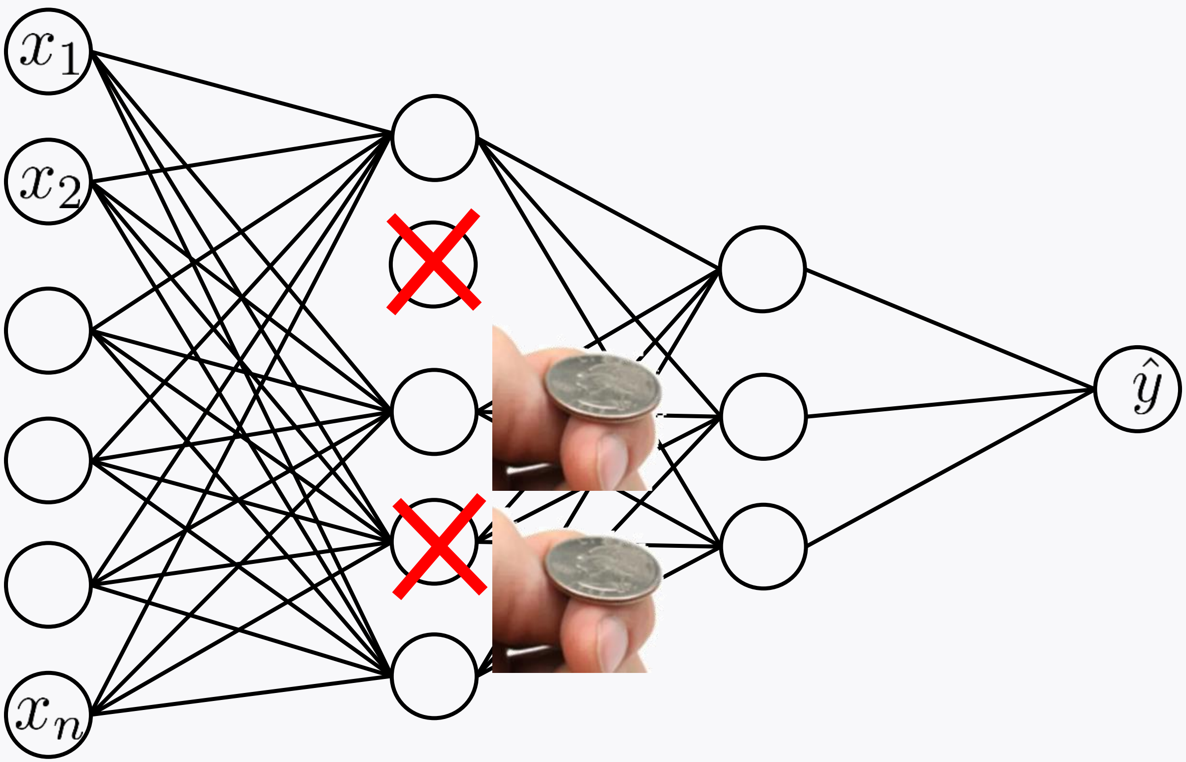
# Construction of a random neural network

Dropout rate: *p* (e.g., 0.5)



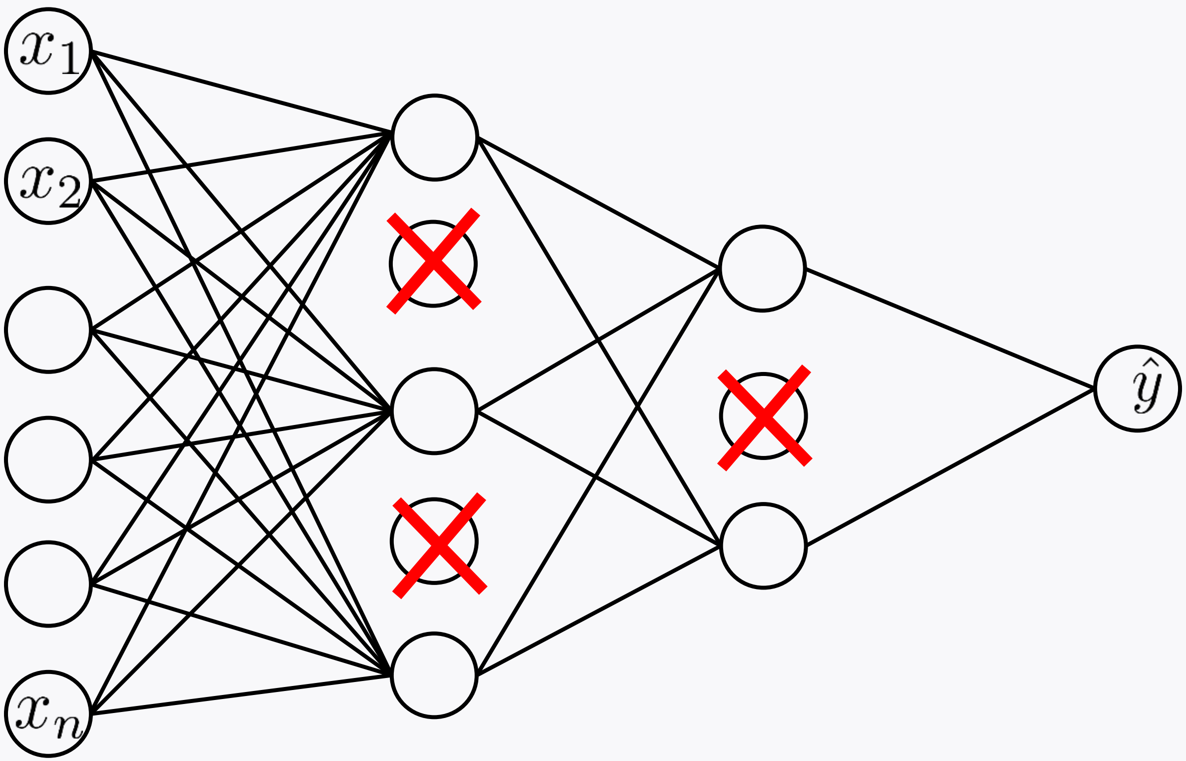
# Construction of a random neural network

Dropout rate: *p* (e.g., 0.5)



# Construction of a random neural network

Dropout rate: $p$ (e.g., 0.5)




Generate this partial NN per example.

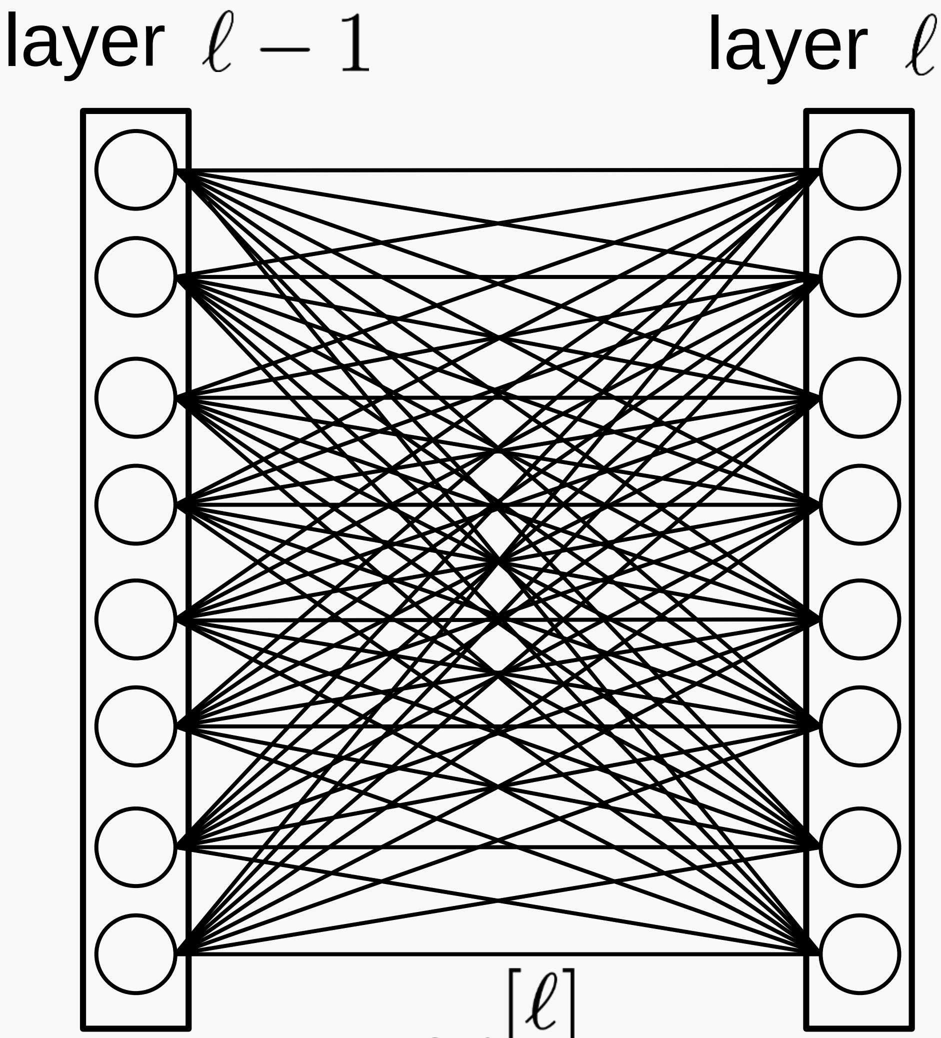
# Why dropout works?

Indirect effect to incorporate many smaller NNs.

Can interpret the resulting NN as an **averaging ensemble** of all these smaller NNs.

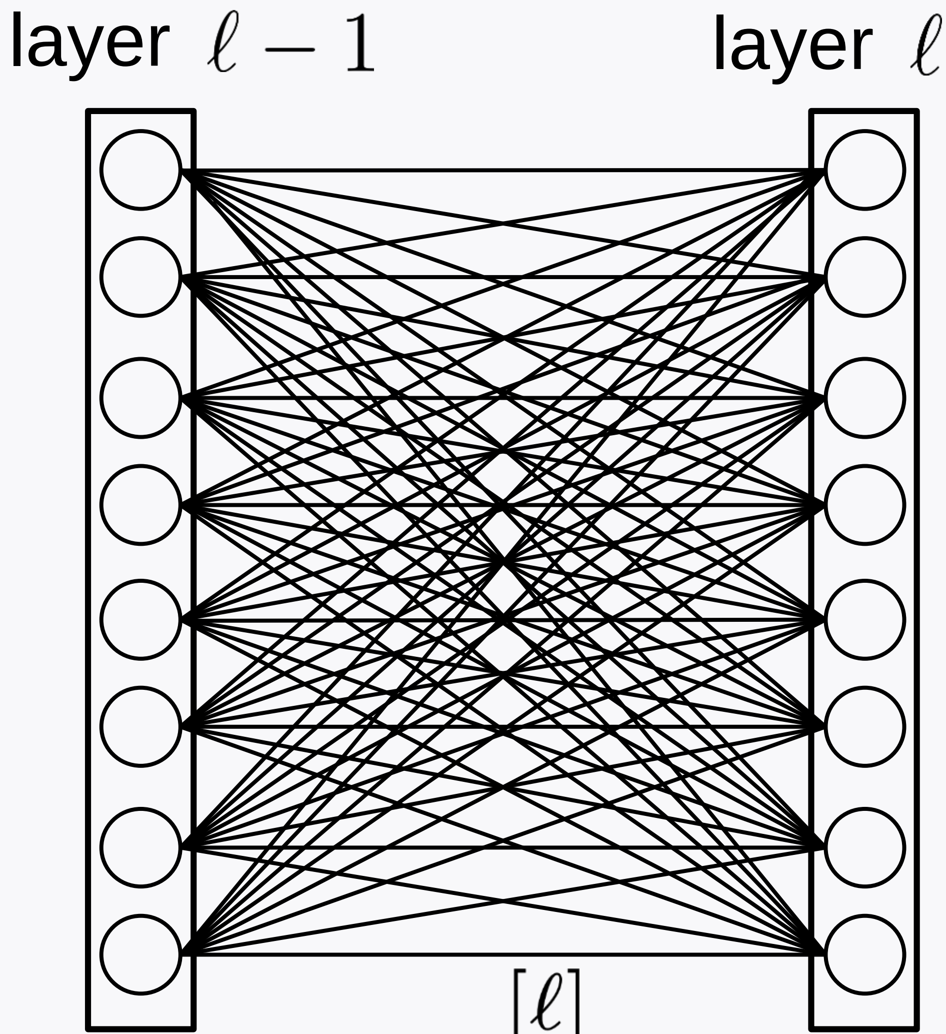
Not overfit to a particular NN; hence generalize better.

# Xavier's initialization: Motivation



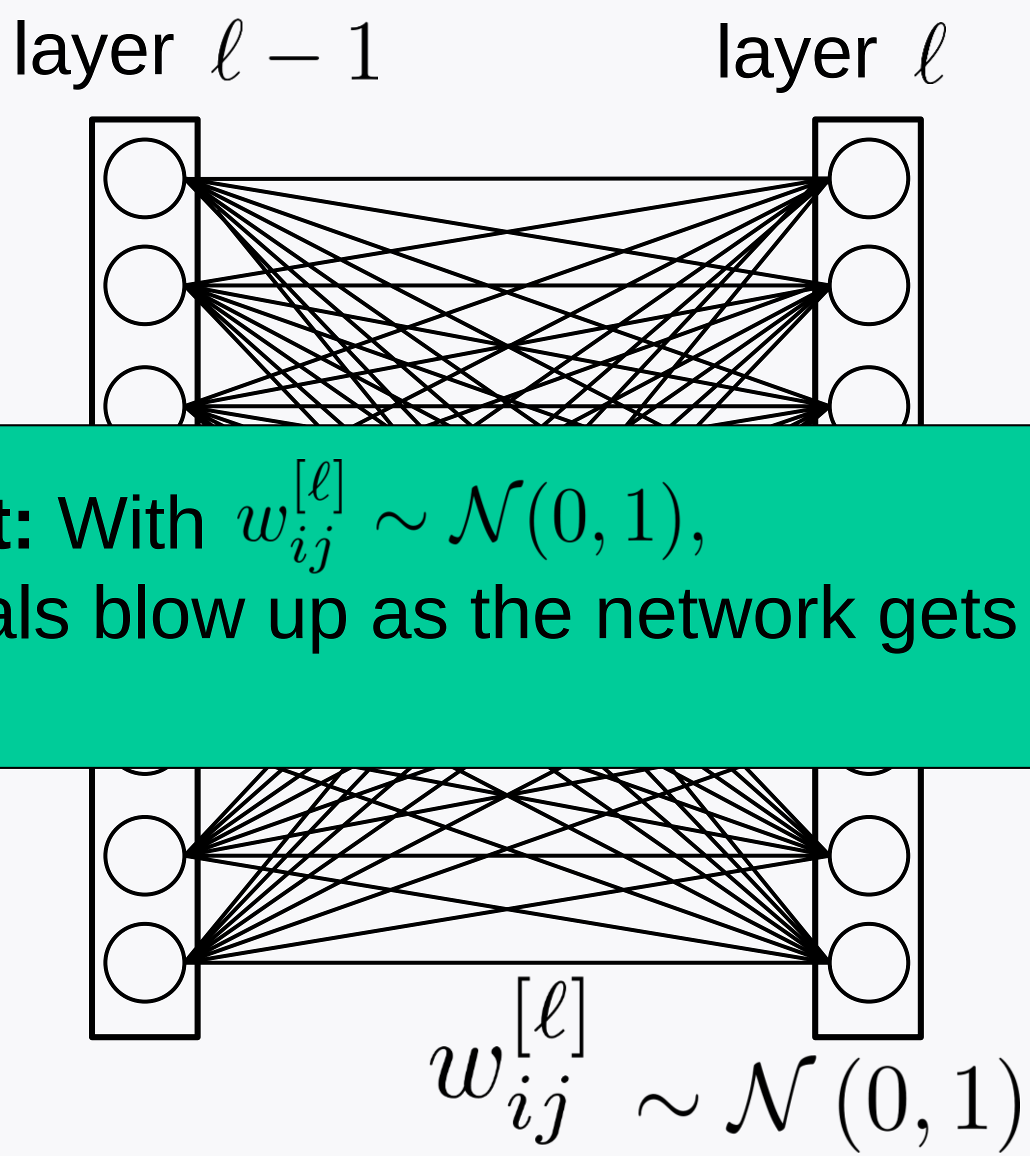

$w_{ij}^{[\ell]}$ random initialization?

# Xavier's initialization: Motivation




$$w_{ij}^{[\ell]} \sim \mathcal{N}(0,1)$$

# Xavier's initialization: Motivation




**Turns out:** With $w_{ij}^{[\ell]} \sim \mathcal{N}(0,1)$, signals blow up as the network gets deeper.

# Xavier's initialization: Motivation

To see this "exploding problem", consider:

$$z_1^{[\ell]} = \sum_{j=1}^{n^{[\ell-1]}} w_{1j}^{[\ell]} a_j^{[\ell-1]}$$

Signal dynamics can be quantified via:

$$\text{var}\left(z_1^{[\ell]}\right) = \text{var}\left(\sum_{j=1}^{n^{[\ell-1]}} w_{1j}^{[\ell]} a_j^{[\ell-1]}\right)$$

# Variance computation

$$\mathrm{var}\left(z_1^{[\ell]}\right) = \mathrm{var}\left(\sum_{j=1}^{n^{[\ell-1]}} w_{1j}^{[\ell]} a_j^{[\ell-1]}\right)$$

$$= \sum_{j=1}^{n^{[\ell-1]}} \mathrm{var}\left(w_{1j}^{[\ell]} a_j^{[\ell-1]}\right)$$

$$= \sum_{j=1}^{n^{[\ell-1]}} \mathbb{E}\left[(w_{1j}^{[\ell]})^2 (a_j^{[\ell-1]})^2\right] - \sum_{j=1}^{n^{[\ell-1]}} \left(\cancelto{0}{\mathbb{E}\left[w_{1j}^{[\ell]} a_j^{[\ell-1]}\right]}\right)^2$$

**Assumption:**

(i) weights independent

(ii) input independent

(iii) weights/input ind.

(iv) zero mean

# Variance computation

$$\text{var}\left(z_1^{[\ell]}\right) = \sum_{j=1}^{n^{[\ell-1]}} \mathbb{E}\left[(w_{1j}^{[\ell]})^2 (a_j^{[\ell-1]})^2\right]$$

$$= \sum_{j=1}^{n^{[\ell-1]}} \mathbb{E}\left[(w_{1j}^{[\ell]})^2\right] \mathbb{E}\left[(a_j^{[\ell-1]})^2\right]$$

$$= \sum_{j=1}^{n^{[\ell-1]}} \text{var}\left(w_{1j}^{[\ell]}\right) \text{var}\left(a_j^{[\ell-1]}\right)$$

**Assumption:**

(i) weights independent

(ii) input independent

(iii) weights/input ind.

(iv) zero mean

# Exploding problem

$$\text{var}\left(z_1^{[\ell]}\right) = \sum_{j=1}^{n^{[\ell-1]}} \text{var}\left(w_{1j}^{[\ell]}\right) \text{var}\left(a_j^{[\ell-1]}\right)$$

**Suppose:** $\text{var}\left(a_j^{[\ell-1]}\right) = 1, \ \text{var}\left(w_{1j}^{[\ell]}\right) = 1$

**Then:** $\text{var}\left(z_1^{[\ell]}\right) = n^{[\ell-1]}$

As the network gets deeper, explode!

# Xavier's initialization

$$\mathsf{var}\left(z_1^{[\ell]}\right) = \sum_{j=1}^{n^{[\ell-1]}} \mathsf{var}\left(w_{1j}^{[\ell]}\right) \mathsf{var}\left(a_j^{[\ell-1]}\right)$$

Suppose: $\mathsf{var}\left(a_j^{[\ell-1]}\right) = 1$

**Idea:** Set $\mathsf{var}\left(w_{1j}^{[\ell]}\right) = \frac{1}{n^{[\ell-1]}}$

$$\boxed{w_{ij}^{[\ell]} \text{ i.i.d. } \sim \mathcal{N}\left(0, \frac{1}{n^{[\ell-1]}}\right)}$$

# He's initialization: Motivation

$$a_1^{[\ell]} = \mathsf{ReLU}(z_1^{[\ell]})$$
$$= \max(0, z_1^{[\ell]})$$

$$\mathsf{var}\left(a_1^{[\ell]}\right) = \frac{1}{2}\mathsf{var}\left(z_1^{[\ell]}\right)$$

Xavier's initialization $w_{ij}^{[\ell]} \sim \mathcal{N}\left(0, \frac{1}{n^{[\ell-1]}}\right)$

$$\longrightarrow \mathsf{var}\left(a_1^{[\ell]}\right) = {\color{red}\frac{1}{2}}\mathsf{var}\left(a_j^{[\ell-1]}\right)$$

# He's initialization

$$w_{ij}^{[\ell]} \sim \mathcal{N}\left(0, \frac{2}{n^{[\ell-1]}}\right)$$

# Techniques for training stability:

Adam optimizer

Learning rate decaying

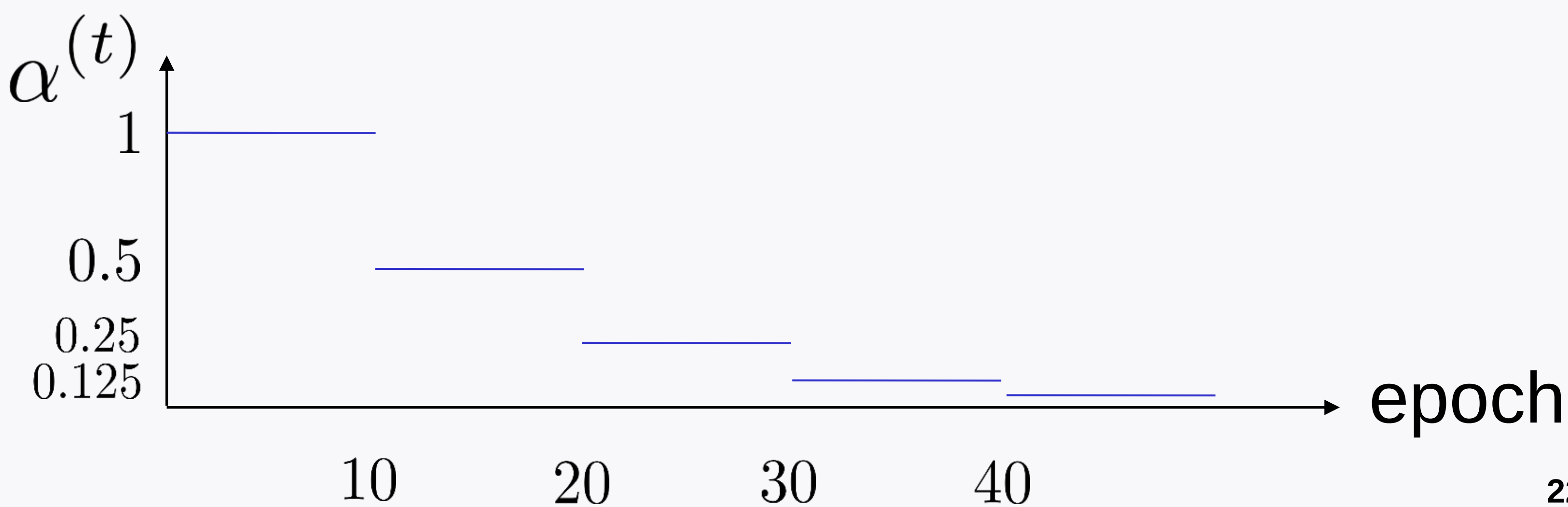
Batch normalization

# Learning rate decaying

Three popular choices:

1. $\alpha^{(t)} = \gamma^t \qquad 0 < \gamma < 1$

2. $\alpha^{(t)} = \dfrac{1}{\sqrt{t}}$

3. 

# Batch normalization: Motivation

**Turns out:** Different signal scalings across distinct layers incur training instability.

One prominent way to address this:

**Batch normalization**

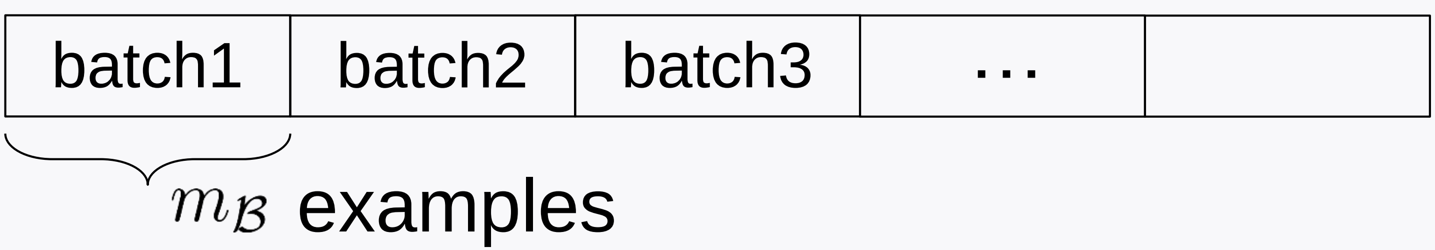
# Batch

**Recall the cost function** used for gradient descent:

$$J(w^{(t)}) := \frac{1}{m}\sum_{i=1}^{m} -y^{(i)}\log \hat{y}^{(i)} - (1-y^{(i)})\log(1-\hat{y}^{(i)})$$

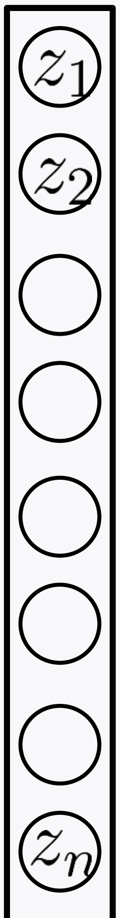
**Issue:** Computationally heavy for a large *m*.

**Hence:** In practice, use a chunk of examples, called a *batch*.



# Batch normalization

A hidden layer




**BN**

1. Normalization

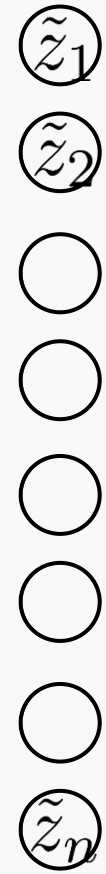
$$z_{\text{norm}} = \frac{z - \mu_{\mathcal{B}}}{\sigma_{\mathcal{B}}}$$

$$\mu_{\mathcal{B}} = \frac{1}{m_{\mathcal{B}}} \sum_{i \in \mathcal{B}} z^{(i)} \quad \sigma_{\mathcal{B}}^2 = \frac{1}{m_{\mathcal{B}}} \sum_{i \in \mathcal{B}} (z^{(i)} - \mu_{\mathcal{B}})^2$$

2. Customized scaling

$$\tilde{z} = \gamma z_{\text{norm}} + \beta$$

learnable parameters



# Look ahead

Will study:

hyperparameter search

cross validation