

ELEC 221 Lecture 06

Introducing filters

Tuesday 27 September 2022

Announcements

- Assignment 2 due tomorrow
- Assignment 3 available tomorrow
- Quiz 3 today

Last time

We explored periodic DT complex exponential signals:

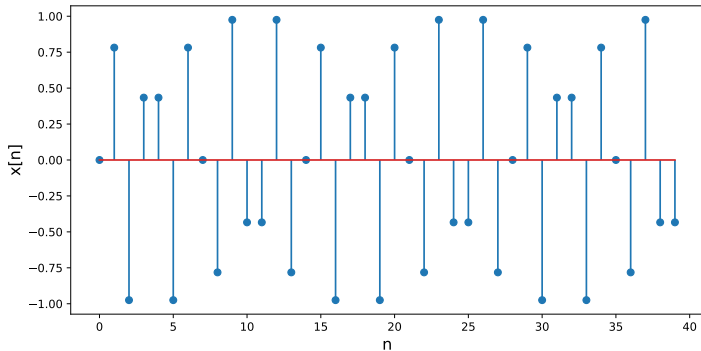
We found that these signals behave differently than CT signals...

Difference 1: we only need to consider ω in the range $[0, 2\pi)$.

Difference 2: there are additional criteria for periodicity.

Last time

Example: $x[n] = \sin(5\pi n/7)$ is periodic.

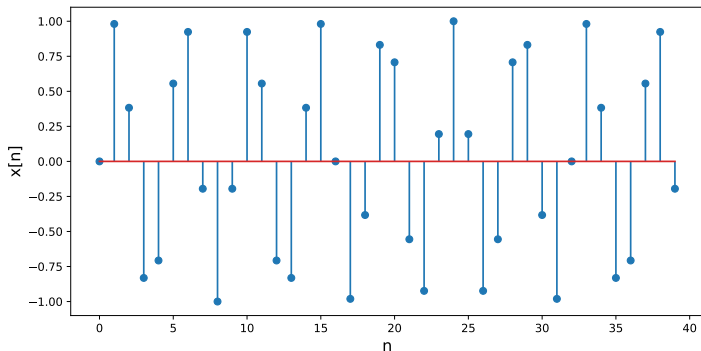


In CT, period of $x(t) = \sin(5\pi t/7)$ is $T = 14/5$.

In DT, period of $x[n] = \sin(5\pi n/7)$ is $N = 14$.

Last time

Example: $x[n] = \sin(5n/7)$ is NOT periodic.



In CT, period of $x(t) = \sin(5t/7)$ is $T = 14\pi/5$.

Difference 3: there are only finitely many harmonics.

Last time

We expressed DT complex exponential signals with fundamental period N as Fourier series.

DT synthesis equation:

DT analysis equation:

We introduced the idea of **system functions** and **frequency response** of LTI systems.

$H(j\omega)$ in CT, and $H(e^{j\omega})$ in DT, are called the **frequency response** of the system.

Today

This week, we are going to look at our first major application of signals and systems: filters.

Learning outcomes:

- Define filters and identify a basic set of ideal frequency-selective filters
- Plot the frequency response of a filter in CT and DT
- Define and distinguish between finite-impulse-response (FIR) and infinite-impulse-response (IIR) filters in DT

In-class activity on Thursday: apply filters in the context of audio processing.

Filters are LTI systems that can be used to separate out, combine, or modify the components of a signal at specific frequencies.

Two key types:

- **Frequency-shaping**: change the amplitudes of parts of a signal at specified frequencies
- **Frequency-selective**: eliminate or attenuate parts of a signal at specified frequencies

CT frequency-selective filters

A convenient way to understand (and to design) filters is by looking at their **frequency response**.

Recall that the **system function** of a CT system $H(j\omega)$, is a measure of how much a system “modifies” a particular frequency:

By selecting the frequency response at will, we can change the behaviour of a system.

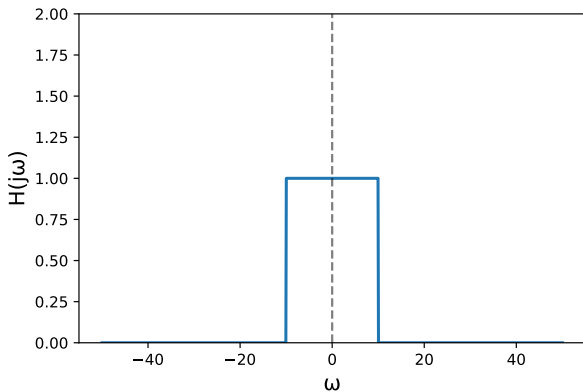
Example. Consider a system function defined as follows.

What does this look like, and what does it do?

CT frequency-selective filters

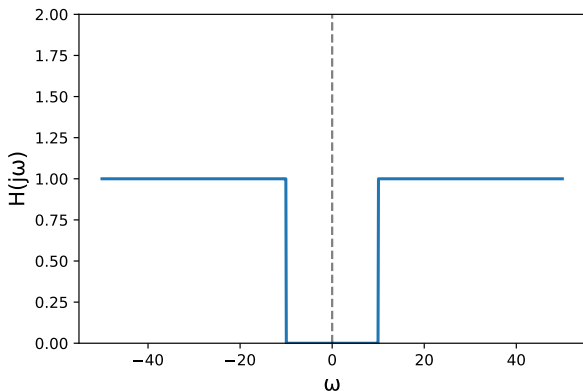
This is an **ideal lowpass filter**.

We often represent the frequency response graphically:



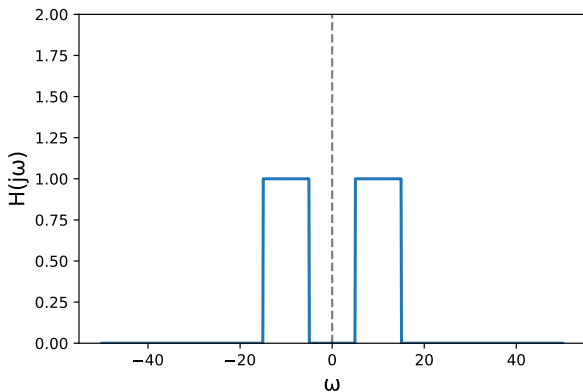
CT frequency-selective filters

We can also consider an ideal **highpass filter**:



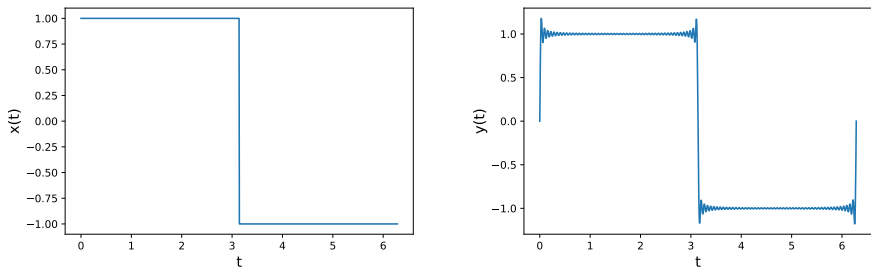
CT frequency-selective filters

Or an ideal **bandpass** filter:



Filters and ringing

We have inadvertently already encountered an example of what can happen when we apply a lowpass filter: the Gibbs phenomenon.

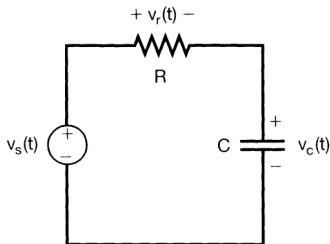


We can actually reduce the effect of the Gibbs phenomenon using a different kind of frequency response called σ approximation.

Lowpass filters in practice

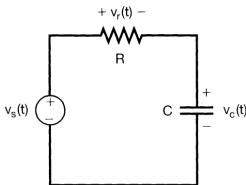
In reality some filters are made up of real, physical components, described by differential equations, and involve tradeoffs in their construction.

Example: We can construct a lowpass filter using a resistor and a capacitor.



Lowpass filters in practice

Suppose we input some complex exponential signal $v_s(t) = e^{j\omega t}$.



Can derive two expressions for the current, using the resistor and capacitor respectively:

Put these together to form a differential equation:

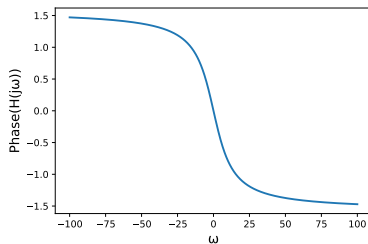
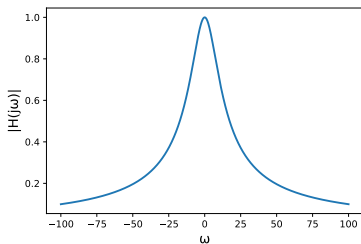
What is the voltage across the capacitor?

Can show that if $v_s(t) = e^{j\omega t}$, then $v_c(t) = H(j\omega)e^{j\omega t}$ for some scaling $H(j\omega)$.

Plug this into the ODE:

Lowpass filters in practice

Results in the following frequency response (setting $RC = 0.1$):



Adjusting the value of RC controls the frequency response. Increasing RC cuts off more frequencies.

Lowpass filters in practice

However, there are tradeoffs involved in the design of such filters.

Example: we want the system to response *quickly* when we start giving it input. How does it respond to the unit step?

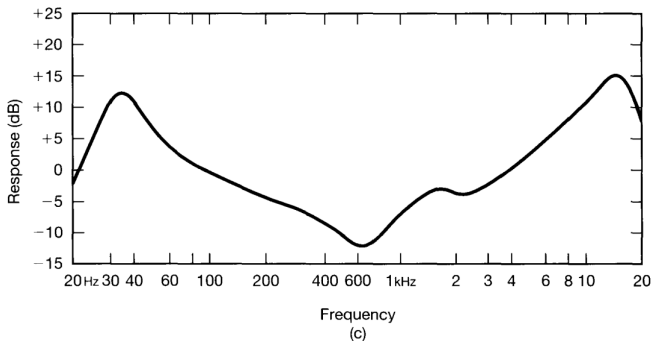
You can show that the unit impulse response of this filter is

and the unit step response is

High RC means tight frequency response, but slow step response.

Frequency shaping filters

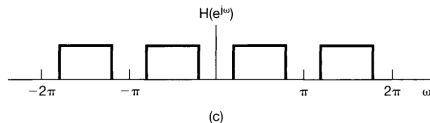
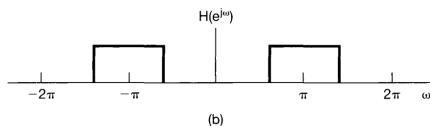
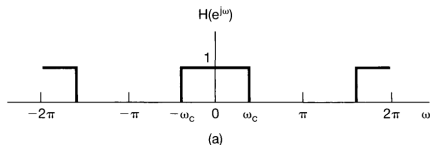
Especially common in audio process: not eliminating any frequencies, just changing their strengths.



Decibels are units of $20 \log_{10} |H(j\omega)|$; frequency in Hz is $\omega/2\pi$.
(We will play with these on Thursday!)

DT filters

Recall that in DT, the frequency increases up until $\omega = \pi$, then decreases as it approaches 2π .



There are two major categories of DT filters:

1. Infinite impulse response (IIR)
2. Finite impulse response (FIR)

Example: we can generate a low- or high-pass filter using a system described by a first-order difference equation.

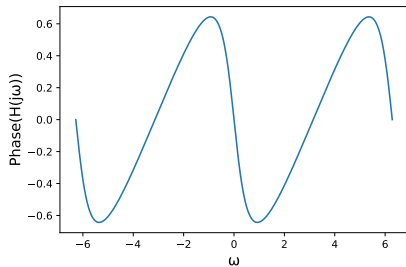
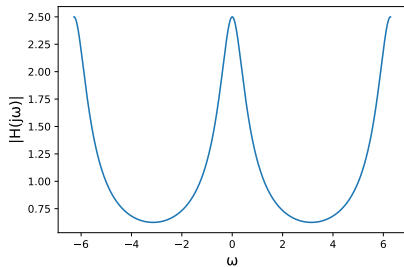
We can use the same method as in the CT case to compute the frequency response: we know that complex exponential signals are eigenfunctions, so $y[n] = H(e^{j\omega})e^{j\omega n}$.

Work through the math:

Depending on the value of a , this could be either a highpass or lowpass filter.

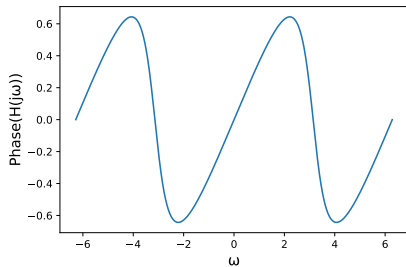
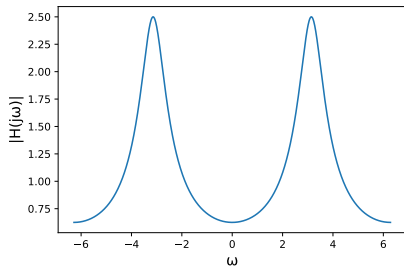
DT filters (IIR)

Case: $a > 0$



DT filters (IIR)

Case: $a < 0$



DT filters (IIR)

What happens to the impulse response of this filter? Need to solve the differential equation... consider a signal $x[n] = K\delta[n]$.

Assuming the system starts at rest ($y[-1] = 0$), we can solve recursively:

DT filters (IIR)

If we put in the unit impulse ($K = 1$), we get

so the impulse response is

This system has an *infinite impulse response*!

DT filters (FIR)

An example of a DT filter with a finite impulse response is the moving average.

Suppose the output of the system is moving after in a window of size M around each point:

What is its impulse response and frequency response?

Impulse response: set $x[n] = \delta[n]$

This is clearly finite (as a result: these filters are stable systems).

DT filters (FIR)

Frequency response: set $x[n] = e^{j\omega n}$,

So

What does this look like? Let's plot it...

Today's learning outcomes were:

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What topics did you find unclear today?

For next time

Content:

- Fun with filters! In-class activity with opportunity for some **extra credit**.
- To prepare for class:
 - Produce a 10-15s clip of your favourite song in .wav format
 - Set up a Python environment (Google Colab works fine)
 - Loosely organize yourselves into groups of 3-4 (it will help for at least one person to have musical training)

Action items:

1. Assignment 2 is due tomorrow at 23:59
2. Assignment 3 is available tomorrow

Recommended reading:

- From today's class: Oppenheim 3.9-3.12
- For next class: Oppenheim 5.0-5.3