

Graphs Implementation

Assignment Solutions



1. For a graph with n nodes, how many edges are there in a complete graph?

nC2: We can choose any two nodes out of n nodes and then connect them.

2. Minimum number of edges to make a graph on n nodes is?

0, since the nodes which are not at all connected still make a graph. It is known as a void graph.

3. Minimum number of edges to make a connected graph of n nodes?

n-1 nodes, which make up a tree as well.

4. Given the following adjacency list for an undirected graph:

```
[  
    [1, 2],  
    [0, 3],  
    [0, 3],  
    [1, 2]  
]
```

What are the adjacent vertices of vertex 1?

The adjacent vertices of a vertex are the vertices that are connected to it by an edge. In the given adjacency list, the adjacent vertices of vertex 1 are vertices 0 and 2.

5. Given the following adjacency matrix for a directed graph:

```
[[0, 1, 0, 0],  
 [0, 0, 1, 0],  
 [0, 0, 0, 1],  
 [0, 0, 0, 0]]
```

What is the indegree of vertex 3?

The indegree of a vertex in a directed graph is the number of edges that point to it. In the given adjacency matrix, there is one edge that points to vertex 3, namely edge (2, 3). Therefore, the indegree of vertex 3 is 1.

6. Consider a directed graph with 4 nodes (1, 2, 3, 4) and the following edges: (1->2), (2->3), (3->4), (4->1).**Represent this graph using an Edge List.**

The Edge List representation for the given directed graph would be:

```
(1, 2)  
(2, 3)  
(3, 4)  
(4, 1)
```

7. What is the maximum number of edges in a simple undirected graph with n nodes?

The maximum number of edges in a simple undirected graph with n nodes is $(n * (n-1)) / 2$. This is achieved in a complete graph.

8. What is the minimum number of edges in a simple undirected connected graph with n nodes?

The minimum number of edges in a simple undirected connected graph with n nodes is n-1. This forms a tree.

9. What is the minimum number of colors needed to color the nodes of a graph such that no two adjacent nodes have the same color?

The minimum number of colors needed to color the nodes of a graph such that no two adjacent nodes have the same color is called the chromatic number of the graph.

10. What is the minimum cut of a graph?

The minimum cut of a graph is the smallest number of edges that must be removed to disconnect the graph into two separate components.

11. What is the minimum number of nodes required to create a cycle in a directed graph?

At least 3 nodes are required to create a cycle in a directed graph.

12. What is the degree of a vertex in a graph?

The degree of a vertex is the number of edges incident to that vertex.

13. Consider a graph with 5 nodes, A, B, C, D, and E. The following edges are present in the graph:

A → B
B → C
C → D
D → E
E → A

What is the degree of vertex C?

The degree of a vertex is the number of edges connected to it. The degree of vertex C is 2, since it is connected to two other vertices, B and D.

14. What is the time complexity of finding the minimum spanning tree of a graph using Prim's algorithm?

The time complexity of finding the minimum spanning tree of a graph using Prim's algorithm is $O(|E| + |V| \log |V|)$, where $|E|$ is the number of edges in the graph and $|V|$ is the number of vertices in the graph.

15. In a complete graph, what is the sum of the degrees of all the vertices?

It is 2 times the number of vertices minus 2.

16. If a graph has 10 vertices and 25 edges, is it a tree?

No (A tree with n vertices has $n-1$ edges)

17. How many edges can be removed from a complete graph with 6 vertices to make it a tree?

9 (A complete graph with n vertices has $n(n-1)/2$ edges, and a tree with n vertices has $n-1$ edges)

18. How many vertices are there in a graph with 12 edges and a maximum degree of 4?

7 (You can use the Handshaking Lemma to find the number of vertices: $2 * \text{number of edges} = \text{sum of degrees}$)

19. What is the diameter of a complete graph with 10 vertices?

1 (In a complete graph, any two vertices are connected by an edge)

20. In a simple graph with 4 vertices and 3 edges, is there a cycle?

No (A graph with n vertices and $n-1$ edges is a tree, and trees have no cycles)