# Lecture 2 - 08/03

## → 2.1 Quadratic Equations

- Quadratic eq. is in form $ax^2+bx+c=0$ for $a \neq 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2+4ac}}{2a} \qquad \Delta = b^2+4ac$$

**Theorem 1:** Let $A = ax^2+bx+c=0$, then

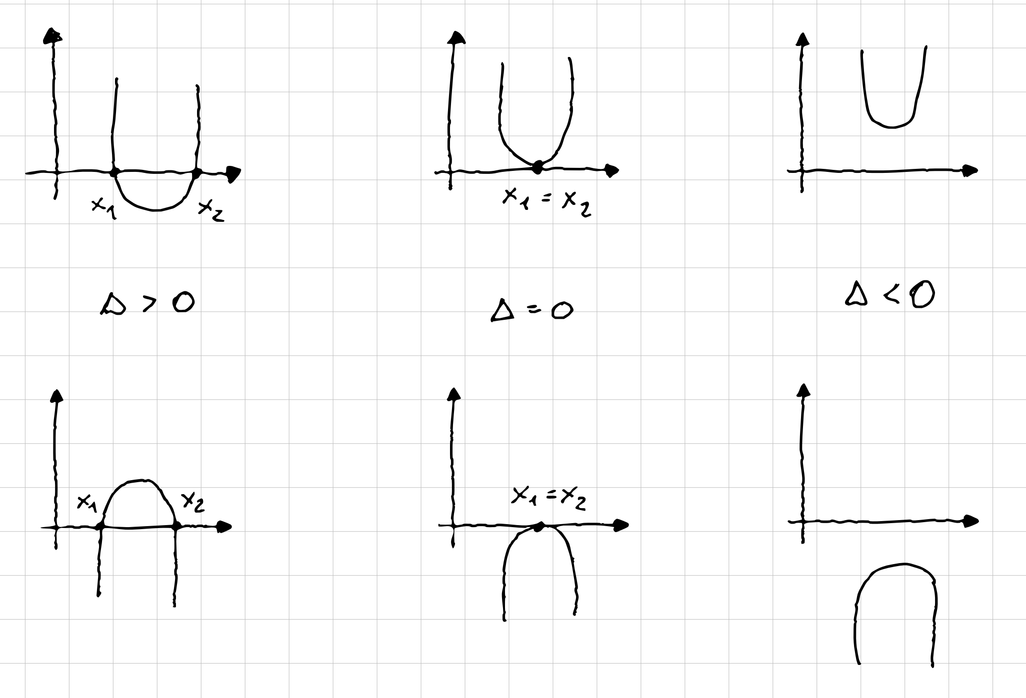
(i) $A \geq 0$ if $a > 0$ and $\Delta \leq 0$

(ii) $A$ has roots if $\Delta \geq 0$

(iii) $x_1$ and $x_2$ roots of $A$, then if $a > 0$ then $\{x \mid A \geq 0\}$ $y = [x_1, x_2]$

$a > 0$

$a < 0$




## → 2.2 Principles of Mathematical Induction

- Let $P(n)$ be a proposition concerning a natural number $n$.

ex. $P(n) =$ "$2^n = 2^{n-1} \cdot 2$" $P(n)$ is true for all $n$

$Q(n) =$ "2 devides $n$" can be true or false $Q(3)$ false, $Q(4)$ true

That is, $P(n)$ is either true or false for every $n \in \mathbb{N}$

**PiM** Principle of Mathematical Induction

Let $P(n)$ be a proposition for $n \in \mathbb{N}$, the following holds:

→ Base Case : $P(1)$ is true

→ Induction Step : For every $n \geq 1$ if

→ Inductive Step : $P(n)$ is true then $P(n+1)$ is true

then $P(n)$ is true for all $n \in \mathbb{N}$

ex. $P(n) =$ "$\sum_{k=1}^{n} k(k+1) = 1/3\, n(n+1)(n+2)$"

- $n=1$ $\quad P(1) = 1(1+1) = 1/3\; 1(1+1)(1+2)$
  $\qquad\qquad 2 = 2$

- Induction Step :

  assume $P(n)$ holds :

$$P(n+1) := \sum_{k}^{n+1} k(k+1) \stackrel{?}{=} 1/3\, n(n+1)(n+2)(n+3)$$

$$\downarrow$$

- Hypoth. $\sum_{k=1}^{n} k(k+1) + (n+1)(n+2) = 1/3\, n(n+1)(n+2) + (n+1)(n+2)$

$$= (n+1)(n+2)\left[1/3\, n + 1\right]$$

$$= 1/3\, (n+1)(n+2)(n+3)$$

so $P(n) \Rightarrow P(n+1)$

### Bernoulli's integrity

- For every $x \in \mathbb{R}$ such that $x > -1$ and for every integer $n \geq 0$

$$(1+x)^n \geq 1 + n \cdot x$$

- Base case $n=1$ ✓
- Inductive Step $(1+x)^{n+1} = (1+n)^n \cdot (1+n) \geq (1+x)(1+nx) \geq 1+(n+1)x$

$$\geq 1 + (n+1)x$$

## → 2.3 Binomial Formula

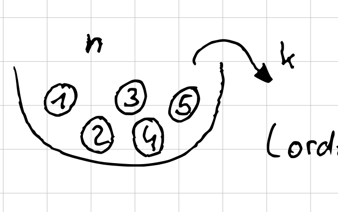
→ Factorial

$0! = 1$, $\quad 1! = 1 \qquad n! = n \cdot (n-1)(n-2) \ldots 1$

→ Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

- Observations: $\binom{n}{0} = 1 \quad \binom{n}{n} = 1 \quad \binom{n}{1} = n \quad \binom{n}{n-1} = n$




(order is not important)

→ Pascal's rule

- for $n \geq k \geq 1$

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

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        1            n = 0
      1   1          n = 1
    1   2   1        n = 2
  1   3   3   1      n = 3
1   4   6   4   1    n = 4
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→ Binomial Theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} \cdot b^r$$

$$= a^n + n \cdot a^{n-1} \cdot b + \binom{n}{r} a^{n-r} \cdot b^r \ldots + nab^{n-1} + b^n$$

Observation : all terms are in the form $a^\alpha b^\beta$, $\alpha + \beta = n$.

$\alpha$ from $n$ to $0$ while $\beta$ from $0$ to $n$