Lecture 1 - 06/03

→ 1.1 Set:
Collection of Objects called elements of the Set.

- $a \in A$ if a is element of A, otherwise $a \notin A$ & $A = \{1,2,3\}$
- infinite and finite sets

  ex. $\mathbb{N}$ set of natural numbers $\mathbb{N} = \{1,2,3,4 \ldots\}$
- Defining a set using a property:

  ex. let P be a Property → if it's either true or false for Set S

  $M = \{x \in S \mid P(x) \text{ is true}\}$

  $M = \{x \in \mathbb{R} \mid x^2 \leq 2\}$

- Set operations and relations:

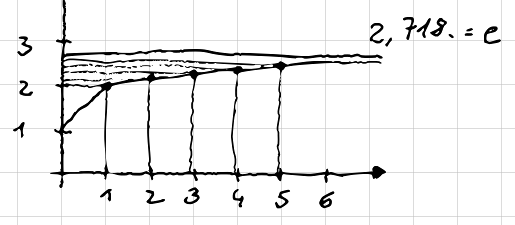
- Relations:
  - Subsets: $A \subseteq B$ if $x \in A \rightarrow x \in B$ (A could = B)
  - strict Subset: $A \subset B$ and $A \neq B$
  - empty Set: $\emptyset$ or $\{\}$
- Operations:
  - Union $\cup$ → $A \cup B = \{x \mid x \in A \overset{\vee}{\text{or}} x \in B\}$
  - Complement $\setminus$ → $A \setminus B = \{x \mid x \in A \overset{\wedge}{\text{and}} x \notin B\}$
  - Intersection $\cap$ → $A \cap B = \{x \mid x \in A \overset{\wedge}{\text{and}} x \in B\}$
  - Cartesian Product X → $A \times B = \{a, b \mid a \in A \overset{\wedge}{\text{and}} b \in B\}$

→ 1.2 Numbers:
- Natural Numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$

  Integers $\mathbb{Z} = \{\ldots -3, -2, 1, \ldots\}$
- Rational Numbers $\mathbb{Q} = \{\frac{1}{2}, \frac{2}{3} \ldots\}$
- Irrational Numbers $\mathbb{I} = \{\sqrt{3}, \sqrt{2}, \pi, e \ldots\}$
- Real Numbers $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$
- Complex Numbers $\mathbb{C}$ = not used in this course $\{\ldots \sqrt{-1}\}$

→ Rational Numbers
- definition $\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\}$
- density property: $p < q \rightarrow p < r < q$

ex. $x_n = (1 + \frac{1}{n})^n$



→ Axiom of completness

> For every non-empty set A of elements $\mathbb{R}$, that is bounded from above, exist $a \in \mathbb{R}$ s.t. for each $x \in A$; if holds, there exists a least upper bound for A in R

ex. $A = \{q \in \mathbb{Q} \mid q^2 < 2\}$

$A \subset \mathbb{Q}$, A is bounded from above by 2, but no least upper bound

→ Intervals:
- $[a, b]$: $a \leq x \leq b$
- (a, b) : $a < x < b$
- $[a, b)$: $a \leq x < b$
- $(a, b]$: $a < x \leq b$
- $[a, +\infty)$: $a \leq x$

→ 1.3 Absolute Magnitude and Inequalities
→ definition: for a number $x \in \mathbb{R}$ we define absolute magnitude:

$$|x| \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

abs (3) = 3
abs (0) = 0
abs (-3) = 3

→ Inequalities in $\mathbb{R}$

- an algebraic expresion expresses inequality with $\geq, \leq, >, <, \neq$

→ Property

$\boxed{x < y}$

↳ $x \cdot z \leq y \cdot z$ $\quad z \geq 0$

↳ $w \cdot x > y \cdot w$ $\quad w < 0$

↳ $-|-x| = |x|$

↳ $|xy| = |x|\,|y|$

↳ $|x + y| \leq |x| + |y|$

ex.

$|x - 1| < 3$

$$|x-1| \begin{cases} x-1 & x \geq 1 \\ -(x-1) & x < 1 \end{cases} \Leftrightarrow \begin{matrix} x < 4 & \text{when } x \geq 1 \\ x > 2 & \text{when } x < 1 \end{matrix} \Leftrightarrow \begin{matrix} x \in (1,4) \\ x \in (-2, 1) \end{matrix}$$

$\Rightarrow x \in (-2, 4)$

or $|x - 1| < 3 \rightarrow -3 < x - 1 < 3$