Lecture 6 - 22/03

6.1 Functions

- def: Function f from A to B is $f \subseteq A \times B$, each element x from A can be paired with at most one y from B:

  $((x, y_1) \in f$ and $(x, y_2) \in f \Rightarrow y_1 = y_2)$

- We say A is the <u>DOMAIN</u> of f - $\mathrm{dom}(f)$, and B is the <u>CODOMAIN</u> of f

  ex. $f(x) = x^2$, $f: \mathbb{R} \to \mathbb{R}$ $\mathrm{dom}(f) = \mathbb{R}$

- To define a function we need to define the domain

  ex. $f(x) = \sqrt{x}$, where $x \geq 0$, $\mathrm{dom}(f) = [0, +\infty)$

  $f(x) = \sqrt{1-x^2}$, $\mathrm{dom}(f) = [-1, 1]$

  $f(x) = \frac{x}{x^2-4}$, $\mathrm{dom}(f) = \mathbb{R} \setminus \{-2, 2\}$

- Finally we define <u>IMAGE</u> of a function f as follows

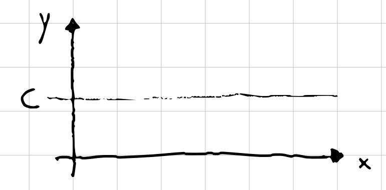
  $\mathrm{IM}(f) = \{ f(x) \mid x \in \mathrm{dom}(f) \}$

  ex $f(x) = x^2$, $\mathrm{IM}(f)$

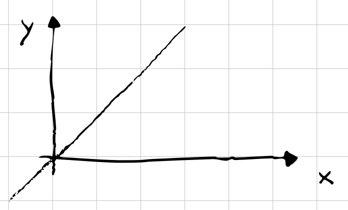
6.2 Graphs of functions

- We can only plot smooth functions.

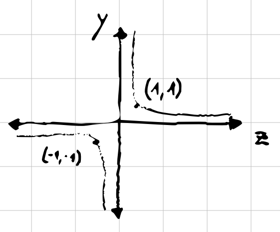
  a) $f(x) = c$



  b) $f(x) = x$



  c) $f(x) = 1/x$



6.3 Combining Functions

- Let f and g be functions, then we define

  $(f+g)(x) = f(x) + g(x)$, as a new function, we can also define

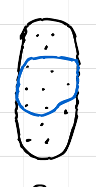
  $\mathrm{dom}(f+g) = \mathrm{dom}(f) \cap \mathrm{dom}(g)$

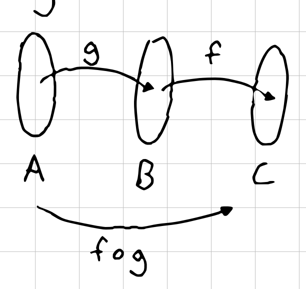
  $(f \cdot g)(x) \to \mathrm{dom}(f \cdot g) = \mathrm{dom}(f) \cap \mathrm{dom}(g)$

- Composition of functions

  $f: B \to C$ and $g: A \to B$, we define $f \circ g: A \to C$

  $(f \circ g)(x) = f(g(x))$





  $\mathrm{dom}(f \circ g) = \{ x \in \mathrm{dom}(g) \mid g(x) \in \mathrm{dom}(f) \}$

  $\mathrm{Img}(g) \cap \mathrm{dom}(f)$

ex. $f(x) = \sqrt{x}$, $g(x) = x + 1$

  $\mathrm{dom}(f) = [0, +\infty)$ $\mathrm{dom}(x) = \mathbb{R}$

  $(f \circ g)(x) = f(g(x)) = \sqrt{x+1}$, $\mathrm{dom}(f \circ g) = [-1, +\infty)$

  $(g \circ f)(x) = g(f(x)) = \sqrt{x} + 1$, $\mathrm{dom}(g \circ f) = [0, +\infty)$

ex. $G(x) = \frac{1-x}{1+x}$

  $(G \circ G)(x) = G\left(\frac{1-x}{1+x}\right) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = \frac{\frac{1+x-1+x}{1+x}}{\frac{1+x+1-x}{1+x}} = \frac{2x}{2} = x$

  $\mathrm{dom}(G \circ G) = \{ x \in \mathrm{dom}(G) \mid G(x) \in \mathrm{dom}(G) \}$

  $= \{ x \in \mathbb{R} \setminus \{-1\} \mid G(x) \in \mathbb{R} \setminus \{-1\} \}$

6.4 Polynomial and Rational Functions

- Polynomial: $P: \mathbb{R} \to \mathbb{R}$ of form $P(x): a_n x^n + a_n x^{n-1} \cdots a_0$

  n is the degree of the Polynomial

- Rational: $R: \mathbb{R} \to \mathbb{R}$ is a function of the form

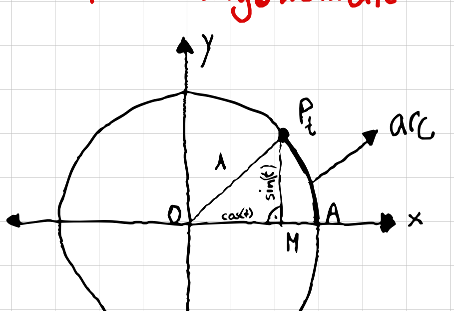
  $R(x): \frac{P(x)}{Q(x)}$, where P and Q are Polynomial functions with

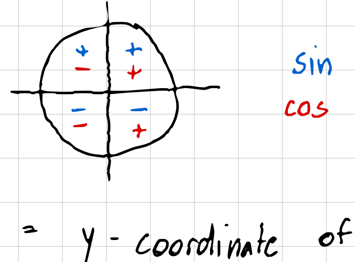
  $\mathrm{dom}(R) = \{ x \in \mathbb{R} \mid Q(x) \neq 0 \}$

- Roots of a Polynomial P are numbers r, s.t. $P(r) = 0$.

  P of $\deg(P) = n$ can have at most n roots

6.5 The trigonomeric functions





arc length: $\widehat{AP_t} = t$

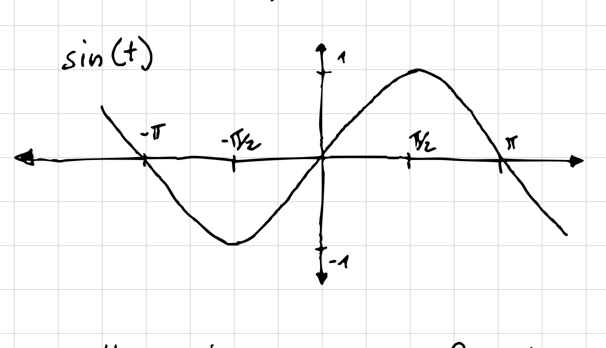
$\sin(t) = |P_t M| =$ y - coordinate of P_t

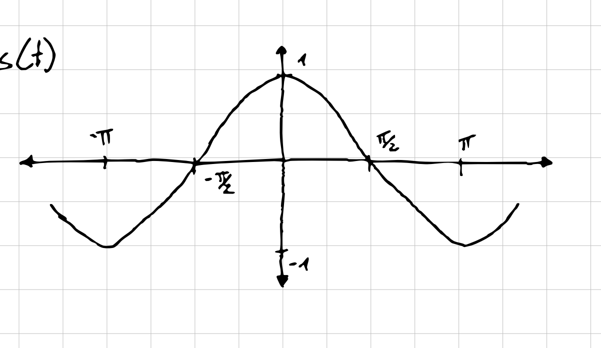
$\cos(t) = |OM| =$ x - coordinate of P_t

Observation: circumference $= 2\pi r$

- We measure t anti-clockwise, sin and cos repeat/reset after 2π

  - $\sin(t) = \sin(2\pi) = \sin(t + 2\pi)$ for $k \in \mathbb{N}$, same for cos
  - $\sin(0) = 0$, $\sin(\pi) = 0$, $\sin(\pi/2) = 1$, $\sin(-\pi/2) = -1$
  - $\cos(0) = 1$, $\cos(\pi) = -1$, $\cos(\pi/2) = 0$, $\cos(-\pi/2) = 0$





- Other trigonometric function:

  - $\tan(t) = \frac{\sin(t)}{\cos(t)}$
  - $\cot(t) = \frac{\cos(t)}{\sin(t)} = \frac{1}{\tan(t)}$
  - $\sec(t) = \frac{1}{\cos(x)}$
  - $\mathrm{cosec}(t) = \frac{1}{\sin(t)}$