Lecture 18 - 01/06

→ 18.1 Method of Integration: Trigonometric Functions

$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

ex. $\int \sin^4(x)\, dx$

$\sin^4 x = (\sin^2 x)^2 = \left[\frac{1}{2}(1-\cos 2x)\right]^2 =$

$= \frac{1}{4}(1 - 2\cos(2x) + \cos^2(2x)] = \frac{1}{4}(1 - 2\cos(2x) + \frac{1}{2}(1 + \cos 4x))$

$\int \sin^4(x)\, dx = \int \left[\left(\frac{1}{4} + \frac{1}{8}\right) - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos 4x\right] dx$

$= \int \frac{5}{8}\, dx - \frac{1}{2}\int \cos(2x)\, dx + \int \frac{1}{8}\cos(4x)\, dx =$

$= \frac{5}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$

If we have a Integral of the form

$\int \sin^m(x)\cos^n(x)\, dx, \quad m,n \in \mathbb{N}$

then, if $n = 2k+1$

$\cos^2 x = 1 - \sin^2(x)$

$\cos^{2k+1} x = \cos^{2k} x \cdot \cos x = (1 - \sin^2 x)^k \cdot \cos(x)$

the Integral becomes

$\int \sin^m x (1 - \sin^2 x)^2 \cos(x)\, dx$

$\left[\begin{array}{l} u(x) = \sin(x) \\ du = \cos(x)\, dx \end{array}\right]$

$\int u^m (1-u^2)^k\, du = \ldots$

ex. $\int \sin^3 x \cos^8 x\, dx$

$= \int (1 - \cos^2(x)) \cdot \cos^8 x \cdot \underbrace{\sin(x)\, dx}_{du}$

$\left[\begin{array}{l} u(x) = \cos(x) \\ du = -\sin(x)\, dx \end{array}\right.$

$= -\int (1-u^2) \cdot u^8\, du = -\int u^8 - u^{10}\, du$

$= -\left(\frac{1}{9}u^9 - \frac{1}{11}u^{11}\right) + c = -\frac{1}{9}\cos^9(x) + \frac{1}{11}\cos^{11}(x) + c$

→ 18.2 Method of Integration: Partial Fraction Expansion (PFE)

If we have a rational function, $R(x) = \frac{P(x)}{Q(x)}$

where P, Q are Polynomials of the form

$a_n x^n + a_{n-1}x^{n-1} \ldots a_0$, and $\deg(P) < \deg(Q)$.

$Q(x) = (x - a_1)(x - a_2)\ldots(x - a_n)$ a: roots of Q

Them we have that $\frac{P(x)}{Q(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} \ldots + \frac{A_n}{x-a_n}$

- ex.

$\frac{x+1}{(2x+1)(x+3)(x+2)} = \frac{A}{2x+1} + \frac{B}{x+3} + \frac{C}{x+2}$

→ Lecture 4.4

$\frac{(x+1)}{(2x+1)(x+3)(x+2)} = \frac{A(x+3)(x+2) + B(2x+1)(x+2) + C(2x+1)(x+3)}{(2x+1)(x+3)(x+2)}$

⇒ multiply A, B, C and then group by power

$(x+1) = x^2(\underbrace{\quad}_{=0}) + x(\underbrace{\quad}_{=1}) + 1(\underbrace{\quad}_{=1})$

$A = \frac{2}{5}, \quad B = -\frac{2}{5}, \quad C = \frac{1}{3}$

$\int \frac{(x+1)}{(2x+1)(x+3)(x+2)} = \int \left(\frac{2}{5}\frac{1}{2x+1} - \frac{2}{5}\frac{1}{x+3} + \frac{1}{3}\frac{1}{x+2}\right) dx$

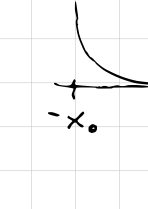
$= \frac{2}{5}\ln|2x+1| - \frac{2}{5}\ln|x+3| + \frac{1}{3}\ln|x+2| + C$

→ 18.3 Inverse Functions

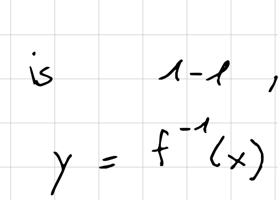
def. A function f is one-to-one ("1-1")

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for $x_1, x_2 \in \mathrm{dom}(f)$

ex $f(x) = x^3$ is 1-1 on $\mathbb{R}$



$f(x) = x^2$



$f(-x_0) = f(x_0)$

f is not 1-1 on $\mathbb{R}$

f is 1-1 on $[0, +\infty]$

def:

If f is 1-1, then it has an Inverse Function f^{-1}

$y = f^{-1}(x)$ IFF $x = f(y)$

ex $f(x) = x^3$

$x = y^3 \Rightarrow f^{-1}(x) = x^{1/3} = \sqrt[3]{x}$

<u>Observations</u>:

- $\mathrm{dom}(f) = \mathrm{range}(f^{-1})$
- $\mathrm{range}(f) = \mathrm{dom}(f^{-1})$

From the definition:

$y = f^{-1}(x)$ and $x = f(y)$

$f(f^{-1}(x)) = x = id(x)$ ⊗

$f^{-1}(f(x)) = y = id(y)$ ⊗⊗

<u>Graphically</u>:

f^{-1} is "mirrowing" f, with respect to $y = x$